Fuzzy Fault Tolerant Control via Takagi-Sugeno Fuzzy Models for Nonlinear Systems with Multiplicative Noises

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Abstract

This paper studies the model based fault tolerant tracking control problem for continuous-time nonlinear systems that are represented by the Takagi-Sugeno fuzzy models with multiplicative noises. The multiplicative noise terms are introduced in the consequent part of the Takagi-Sugeno fuzzy system to represent the stochastic behaviors. The faults considered in this paper are characterized as time varying functions that are modeled by exponential functions or first order polynomials. The purpose of this paper is to ensure system states can track the state references when faults occur. Based on Lyapunov theory and descriptor redundancy property, the sufficient conditions are derived in terms of linear matrix inequalities. Finally, some numerical examples are provided to illustrate the applicability and effectiveness of the proposed control methodology.

Keywords: Fault tolerant tracking control, nonlinear systems with multiplicative noise, Takagi-Sugeno fuzzy model, linear matrix inequality.

1. Introduction

Reliability is an important performance requirement for the control of complex systems. Conventional feedback control design for such complex systems may result in unacceptable degradation in performance or even instability in the event of an actuator, sensor or component failure. Hence, it is desirable to have a certain degree of fault tolerance with respect to various faults. Various approaches for Fault Tolerance Control (FTC) design have been proposed in the literature. Overviews on the development of FTC have been provided in survey books and papers as for example in [1] and [2]. FTC design approaches can be broadly classified into two types: the passive approach and the active approach. One can refer to robust fault accommodation approach [3] etc. for the passive approach. The active approach can be referred to such as model following [4]; pseudo-inverse [5]; eigenstructure assignment [6]; multiple model [7]; and adaptive approach [8] etc. In the passive approach, the same controller is used throughout normal and fault cases. On the other hand, a FTC system based on active approach can compensate for faults either by selecting a pre-computed control law or by synthesizing a new control strategy on-line.

The main idea of FTC is relied on the adaptation of the control law on the basis of the estimation of the faults affecting the systems. The FTC approach is usually developed for the linear control systems [3-10]. However, most of dynamic systems are nonlinear in practical engineering systems. Hence, the development of nonlinear FTC approaches is an important issue in the FTC design. Some researchers have studied the FTC approaches for the nonlinear control systems [11-13]. But these design methods are applied only when the operating points change or the fault occurs. Thus, the approaches taking into account the changes caused by both operating point variations and fault are not yet available. For more details, the readers can refer to [2]. Indeed, a way to overcome the existing drawbacks of previous FTC approaches is to consider the nonlinear Takagi-Sugeno (T-S) fuzzy models [14]. This class of models allows representing exactly the overall nonlinear systems on its operating region in the state space.

In the past few years, the control engineers have witnessed rapidly growing interest in fuzzy logic control of nonlinear systems. In the T-S fuzzy model, local dynamics in different state space regions are represented by linear subsystems. That is, each linear subsystem represents the behavior of nonlinear system in a specific region in state space. The overall system dynamics of the T-S fuzzy models is obtained by blending the linear subsystems with membership functions. According to the T-S fuzzy models, the fuzzy controller design technique is usually accomplished by taking the Parallel Distributed Compensation (PDC) scheme [15-22] into account. Besides, the adaptive fuzzy backstepping control
[23-24] is also an important issue for the fuzzy control systems. In [23-24], a fuzzy state observer was designed for estimating the unmeasured states. This approach is similar to the FTC approach. Nevertheless, the FTC design problem for the T-S fuzzy models has been few treated [25-29]. In [27], the proportional integral observer was employed to deal with the case of faults being constant signals or slowly time-varying ones. The adaptive fuzzy decentralized FTC problem was investigated in [29] for a class of nonlinear large-scale systems in strict-feedback form.

In this paper, the Fuzzy Fault Tolerance Control (FFTC) for the nonlinear systems is investigated via the T-S fuzzy models with multiplicative noises. Comparing with the nominal dynamic equation, the stochastic noise usually appears as a multiplicative noise term. The multiplicative noise term is structured as that states multiplied by the zero-mean white noise. According to the T-S fuzzy models with multiplicative noises, the stability and stabilization problems have been investigated in [30-33]. However, the FFTC problem was not considered in [30-33]. In addition, previous FTC approaches developed in [25-29] did not deal with the multiplicative noises for the T-S fuzzy systems. Referring to [29], it can be found that in practical control mechanisms, various components such as actuators, sensors, and processors may undergo abrupt failures individually or simultaneously during operation. It is thus important to develop a FTC scheme accommodating such failures and maintaining acceptable system performance. Thus, the main motivation of this paper is to develop a methodology to deal with the FFTC problem for the T-S fuzzy models with multiplicative noises. Two specific fault models considered in this paper include exponential function and first order polynomial. Therefore, two approaches for exponential function fault and first order polynomial fault are proposed to ensure the state tracking between the healthy systems and the faulty ones. The contribution of this paper is to develop a FTC law to ensure the tracking between the faulty system states and the reference model ones for the T-S fuzzy models with multiplicative noises. To the best of the authors’ knowledge, there has been less works on it, partly due to the difficulties in analyses and syntheses for the existence of multiplicative noises. Based on Lyapunov theory, some stability sufficient conditions are developed to find FTC law. These stability sufficient conditions can be solved by the Linear Matrix Inequality (LMI) technique [34].

The organization of this paper is structured as follows. The reference model, fault system and observer for the FTC design problem are introduced in Section 2. Considering exponential function fault and first order polynomial fault, the sufficient conditions for obtaining the FTC law are developed in Section 3. Applying the LMI technique to solve the sufficient conditions developed in Section 3, some numerical examples and simulation results are provided in Section 4 to demonstrate the applicability and effectiveness of proposed FTC design methodology. Finally, some concluding remarks are made in Section 5.

2. System Descriptions and Problem Statements

In this section, a reference model is introduced as a T-S fuzzy model that is applied to describe a nonlinear system with multiplicative noises. The i-th fuzzy rule of the present reference T-S fuzzy model is given as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left( A_i x(t) + B_i u(t) + (\overline{A}_i x(t) + \overline{B}_i u(t)) \right) \beta(t)$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left( C_i x(t) + D_i u(t) \right)$$

(1a) (1b)

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^p$ and $u(t) \in \mathbb{R}^m$ respectively denote the state, the output and the nominal control vectors. The multiplicative noise $\beta(t)$ is a zero-mean white noise and $r$ is the number of fuzzy rules. Referring to [19], it is assumed that $E[\beta(t)] = 0$ and $E[\beta(t)^T \beta(t)] = 1$. The matrices $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, $C_i \in \mathbb{R}^{p \times n}$, $D_i \in \mathbb{R}^{p \times m}$, $\overline{A}_i \in \mathbb{R}^{n \times n}$, $\overline{B}_i \in \mathbb{R}^{n \times m}$ are constant and $\mu_i(\xi(t))$ are nonlinear functions depending on the variable $\xi(t)$ which can be unmeasurable $x(t)$ or measurable $u(t)$ or $y(t)$. These nonlinear functions satisfy the convex sum property, i.e., for all $t$, one has $0 \leq \mu_i(\xi(t)) \leq 1$ and $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$, $i = 1, \ldots, r$.

The faulty system considered in this paper has the following form:

$$\dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left( A_i x_f(t) + B_i u_f(t) + G_i f(t) \right) + (\overline{A}_i x_f(t) + \overline{B}_i u_f(t)) \beta(t)$$

$$y_f(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left( C_i x_f(t) + D_i u_f(t) + W_i f(t) \right)$$

(2a) (2b)

where $x_f(t) \in \mathbb{R}^n$, $y_f(t) \in \mathbb{R}^p$, $f(t) \in \mathbb{R}^r$, and $u_f(t) \in \mathbb{R}^m$ are respectively the fault state, the fault output, the fault affecting the system and the fault tolerant control vectors. Besides, $G_i \in \mathbb{R}^{n \times r}$ and $W_i \in \mathbb{R}^{p \times r}$ are the transfer matrices of the faults on the system. The fault tolerant fuzzy controller design scheme is shown in Figure 1.
In order to ensure the tracking between the healthy system states and the faulty system ones, the FFTC law employed in this paper is given as follows:

\[
u_f(t) = u(t) + \sum_{i=1}^{n} \mu_i(\xi_i(t)) \left[ K_i(x(t) - \hat{x}_i(t)) - K'_i \hat{f}(t) \right]
\]  

(3)

where \( K_i \in \mathbb{R}^{n_x \times n} \) and \( K'_i \in \mathbb{R}^{n_y \times q} \) are the state feedback gain matrices to be determined. In this study, an observer is needed to simultaneously estimate the faults and the faulty system states. The observer is also constructed by the T-S fuzzy model as follows:

\[
\dot{\hat{x}}_i(t) = \sum_{r=1}^{n} \mu_r(\xi_r(t)) \left[ A_r \hat{x}_i(t) + B_r u_r(t) + G_r \hat{f}(t) + \varphi_i(t) \right] + \left( \overline{A}_r \hat{x}_i(t) + \overline{B}_r u_r(t) \right) \beta(t)
\]

(4a)

\[
\dot{\hat{y}}_i(t) = \sum_{r=1}^{n} \mu_r(\xi_r(t)) \left[ C_r \hat{x}_i(t) + D_r u_r(t) + W_r \hat{f}(t) \right]
\]

(4b)

\[
\dot{\hat{f}}(t) = \sum_{r=1}^{n} \mu_r(\xi_r(t)) \left[ H_r y(t) - \hat{y}_r(t) \right] - H'_r \hat{f}(t)
\]

(4c)

with \( \varphi_i(t) = H'_i (y(t) - \hat{y}_i(t)) \). The observer’s gain matrices \( H'_i \in \mathbb{R}^{m_{out} \times q} \), \( H'_j \in \mathbb{R}^{m_{out} \times r} \) and \( H'_j \in \mathbb{R}^{m_{in} \times r} \) are to be designed. With this fault tolerant controller structure, one can remark that fault detection and isolation are performed since an estimate of the fault affecting the system is available.

In this paper, it is assumed that the faults and the system states are observable from the output as well as the considered fault functions are differentiable. Besides, the nonlinear functions depend only on the measurable variables \( y(t) \) or \( u(t) \). The purpose of this paper is to design a model based fuzzy fault tolerant controller (3) such that the state tracking error, the state and fault estimation errors converges to a specified bound. The following lemmas are useful for the derivations of the subsequent theorems that provide sufficient conditions to achieve the stability and fault tolerant tracking performance for the closed-loop system.

\[
X'Y + Y'X \leq \delta X'X + \delta' Y'Y
\]

(5)

where \( \delta' \) is a positive scalar.

\[\text{Lemma 1 [35]: For any two real matrices } X \text{ and } Y \text{ with appropriate dimensions, one has} \]

\[\text{Lemma 2 [34]: Consider the matrices } T_i = T_i^t, \quad i \in \{0, \ldots, k\}. \quad \text{The following expressions are equivalent:} \]

\[
\forall \eta, \quad \eta^T T_q \eta \geq 0 \quad \text{and} \quad \eta^T T_q \eta \geq 0, \quad \forall i \in \{1, \ldots, k\}
\]

(6)

\[
\exists \sigma_0 \geq \ldots, \sigma_k \geq 0 \text{ such that } T_q - \sum_{i=1}^{k} \sigma_i T_i \geq 0
\]

(7)

\#

3. Fuzzy Fault Tolerant Tracking Controller Design

In this section, the fuzzy fault tolerant tracking control for the T-S fuzzy models with multiplicative noises is studied. The faults considered in this section are characterized as time varying functions modeled by exponential functions and first order polynomials.

Case I: FFTC for exponential faults

Assume that the faults affecting the system are modeled by exponential functions \( f_i(t) = e^{\omega_i t} \) with \( \alpha_i, \omega_i \in \mathbb{R} \), \( i = 1, \ldots, q \). In this case, it is assumed that \( \alpha_0 = \alpha_0 + \omega_0 \alpha \), which allows defining a set of exponential functions, \( \alpha_0 \) and \( \omega_0 \alpha \), representing respectively the nominal and the uncertain parts of \( \alpha_i \). Let us define

\[
\alpha = \text{diag} \left( \alpha_1, \ldots, \alpha_q \right) \quad \alpha_0 = \text{diag} \left( \alpha_0, \ldots, \alpha_0 \right) \quad \text{and} \quad \omega_0 \alpha = \text{diag} \left( \omega_0, \ldots, \omega_0 \right), \quad \text{where } \text{diag} \left( \Theta_1, \ldots, \Theta_\ell \right) \text{ is a block diagonal matrix which diagonal entries are defined by } \left( \Theta_1, \ldots, \Theta_\ell \right). \text{ The uncertain part can be bounded as:}
\]

\[
(\omega_0 \alpha)^T \omega_0 \alpha \leq \nu
\]

(8)

where \( \nu \in \mathbb{R}^{n_{q}} \) is a known diagonal positive definite matrix. Let us respectively define the state and fault estimation errors as: \( e_s(t) = x(t) - \hat{x}_i(t) \) and \( e_f(t) = f(t) - \hat{f}(t) \). Let us also define the state tracking error \( e_x(t) = x(t) - x_\beta(t) \), the output error \( e_y(t) = y(t) - \hat{y}_i(t) \) and the error between the nominal and FFTC signals \( e_u(t) = u(t) - u_f(t) \). According to the above notations, the dynamics of \( e_x(t) \) and \( e_y(t) \) can be obtained as:

\[
\dot{e}_x(t) = (A_x + \overline{A}_x \beta(t)) e_x(t) + (B_x + \overline{B}_x \beta(t)) e_f(t) - G_x f(t)
\]

(9)

\[
\dot{e}_y(t) = (A_y + \overline{A}_y \beta(t)) e_y(t) + G_y e_f(t) - H'_y e_f(t)
\]

(10)
where \( X_\mu = \sum_{i=1}^{\mu} \mu(\xi(t))X_i \).

Since \( \dot{f}(t) = \alpha f(t) \), the dynamics of the fault estimation error is given as follows:
\[
\dot{\varepsilon}_\mu(t) = \alpha f(t) - H_\mu^\top \varepsilon_\mu(t) - H_\mu^\top \dot{f}(t) \tag{11}
\]
By adding and subtracting \( H_\mu^\top f(t) \) in (11), one has
\[
\dot{\varepsilon}_\mu(t) = -H_\mu^\top \varepsilon_\mu(t) - H_\mu^\top \varepsilon_\mu(t) + (\alpha + H_\mu^\top) f(t) \tag{12}
\]
The substitution of the expression of \( \varepsilon_\mu(t) \) in (10) and (12) and the expression of \( \varepsilon_\mu(t) \) in (9) leads to introduce multiplications between the system matrices and the ones of the observer and the controller. This coupling leads to conservative results. A way to overcome this problem is to introduce a virtual dynamics in the \( \varepsilon_\mu(t) \) and \( \varepsilon_\mu(t) \) equations [35-36]. This craftiness allows decoupling the system, observer and FFTC controller matrices in the expressions of the error dynamics. Hence, \( \varepsilon_\mu(t) \) and \( \varepsilon_\mu(t) \) can be rewritten as
\[
\dot{\varepsilon}_\mu(t) = -\varepsilon_\mu(t) + C_\mu \varepsilon_\mu(t) + W_\mu \varepsilon_\mu(t) \tag{13}
\]
where \( \dot{0} \) is a zero matrix. Adding and subtracting \( K_\mu \xi(t) + K_\mu^\top f(t) \) in \( \varepsilon_\mu(t) \), one can obtain
\[
\dot{\varepsilon}_\mu(t) = -K_\mu \varepsilon_\mu(t) + K_\mu \varepsilon_\mu(t) + K_\mu \varepsilon_\mu(t) + \varepsilon_\mu(t) \tag{14}
\]
The concatenation of (9)-(10) and (12)-(14) leads to the following descriptor representation:
\[
K \dot{\varepsilon}(t) = \overset{\mu}{\ddot{A}} \ddot{\varepsilon}(t) + \overset{\mu}{\ddot{B}} \dot{f}(t) + \overset{\mu}{\ddot{C}} \ddot{\varepsilon}(t) \beta(t) \tag{15}
\]
where \( K = \text{diag}[I_\mu I_\mu 0_0 0_0] \),
\[
\ddot{\varepsilon}(t) = \begin{bmatrix} \varepsilon_\mu(t) \\ \varepsilon_\mu(t) \\ \varepsilon_\mu(t) \\ \varepsilon_\mu(t) \end{bmatrix},
\]
\[
\ddot{A}_\mu = \begin{bmatrix} A_\mu & 0 & 0 & B_\mu \\ 0 & A_\mu & G_\mu & -H_\mu^\top \\ 0 & 0 & -H_\mu^\top & -H_\mu^\top \\ 0 & C_\mu & W_\mu & -I \end{bmatrix},
\]
\[
\ddot{B}_\mu^\top = \begin{bmatrix} -G_\mu^\top \\ \alpha + H_\mu^\top \end{bmatrix} \text{and}
\]
\[
\ddot{C}_\mu = \begin{bmatrix} \ddot{A} & 0 & 0 & \ddot{B} \\ 0 & \ddot{A} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\]

From the structure of the matrices \( K \) and \( \ddot{A}_\mu \), one can find that the descriptor (15) is impulse-free. The main interest of the descriptor approach is to avoid products of system, observer and controller gains in the LMI. Thus, the more tractable and less restrictive conditions can be obtained. The following theorem provides the sufficient conditions for obtaining the FFTC law.

\textbf{Theorem 1 (FFTC with exponential faults):}

If there exists matrices \( P_1 = P_1^\top \geq 0 \), \( P_2 = P_2^\top \geq 0 \), \( P_3 = P_3^\top \geq 0 \), \( P_4 \), \( P_5 \), \( P_6 \), \( P_7 \), \( F \), \( R \), \( S \), \( Q \), \( M \), positive scalars \( \gamma \) and \( \tau \) such that the following LMI conditions are satisfied for \( i = 1, \ldots, r \), then the system (15) that describes the different estimation errors is stable and the \( L_2 \)-gain from the faults to the state tracking error, the state and fault estimation errors is bounded by \( \sqrt{\beta} \).
\[
\begin{bmatrix}
\gamma_1^{11} & * & * & * & * & 0 \\
C_1^\top P_6 & \gamma_2^{12} & * & * & 0 & 0 \\
W_1^\top P_6 & \gamma_3^{13} & * & * & * & 0 \\
-\gamma_2^{12} & \gamma_4^{14} & \gamma_5^{15} & 0 & 0 & 0 \\
-\gamma_3^{13} & -F_i & -Q & 0 & \gamma_5^{15} & * & 0 \\
-G_i^\top P_0 & 0 & \gamma_5^{15} & Q_i^\top & -\gamma f + \tau & 0 \\
0 & 0 & 0 & 0 & 0 & -t_i
\end{bmatrix} < 0 \tag{16}
\]

where the star "*" denotes the transposed element in the symmetric position of a block matrix,
\[
\begin{align*}
\gamma_1^{11} &= P_1 A_1 + A_1^\top P_1 + \tilde{A}_1^\top \tilde{P}_1 \tilde{A}_1 + I, \\
\gamma_2^{12} &= P_2 A_1 + A_1^\top P_2 + C_1^\top P_1 + \tilde{A}_1^\top \tilde{P}_1 \tilde{A}_1 + I, \\
\gamma_3^{13} &= P_3 C_1 + C_1^\top P_3 + W_1^\top P_3, \\
\gamma_4^{14} &= -P_1^\top C_1 - P_3^\top C_1 - S, \\
\gamma_5^{15} &= -P_2^\top P_3 - \tilde{P}_1^\top, \\
\gamma_5^{15} &= -P_6^\top P_6 - \tilde{P}_3^\top, \\
\gamma_5^{15} &= -P_5^\top P_5 - \tilde{P}_2^\top + \tilde{P}_1^\top \tilde{P}_1 \tilde{P}_1 \\
\gamma_5^{15} &= \alpha_0 P_0 + M_i.
\end{align*}
\]

The proof of Theorem 1 can be referred to Appendix A.

\textbf{Case II: FFTC for first order polynomial faults}

Let us now consider the FFTC problem for the fault modeled by the first order polynomial given as follows:
\[
f_i(t) = \lambda_i t + \delta_i \tag{17}
\]
where \( \lambda_i \in \mathbb{R} \) and \( \delta_i \in \mathbb{R} \), \( i = 1, \ldots, q \). As well as for exponential function, \( \lambda = \lambda_0 + \delta \lambda \), with \( \delta \lambda \) verifying:
\[
(\delta \lambda)^\top \delta \lambda \leq v \tag{18}
\]
where \( v \in \mathbb{R}^{+q} \) is a known diagonal positive definite matrix. According to (12), the fault estimation error dynamics is given by:
\[
\dot{\varepsilon}_\mu(t) = -H_\mu^\top \varepsilon_\mu(t) - H_\mu^\top \varepsilon_\mu(t) + \lambda f + \lambda f \tag{19}
\]
Combining (9)-(10), (13)-(14) and (19), the following descriptor representation can be obtained:
\[
E \dot{\varepsilon}(t) = \ddot{A}_\mu \ddot{\varepsilon}(t) + \ddot{B}_\mu f(t) + E \tag{20}
\]
where the matrices \( E \) and \( \ddot{A}_\mu \) are defined in equation (16) and \( E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \), \( \ddot{B}_\mu ^\top = \begin{bmatrix} -G_\mu ^\top & 0 & (H_\mu^\top)^\top & 0 \end{bmatrix} \).

As well as for the case of exponential faults, the obtained descriptor (20) is impulse-free. The main results of the FFTC with first order polynomial faults are summarized in the following theorem.
Theorem 2 (FFTC with first order polynomial faults):

If there exists matrices $P_i = P_i^T \geq 0$, $P_i = P_i^T \geq 0$, $R_i = R_i^T \geq 0$, $M_i$ and positive scalars $\gamma$, $\rho$ and $\sigma$ such that the following LMI conditions are satisfied for $i=1,...,r$, then the system (20) that describes the different estimation errors is stable and the $L_2$-gain from the faults to the state tracking error, the state and fault estimation errors is bounded by $\sqrt{\gamma}$.

$$
\begin{bmatrix}
\Phi_i^{1,1} & * & * & * & * & 0 & 0 \\
\Phi_i^{1,2} & \Phi_i^{2,2} & * & * & * & 0 & 0 \\
\Phi_i^{1,3} & \Phi_i^{2,3} & \Phi_i^{3,3} & * & * & * & * \\
-P_i & \Phi_i^{2,4} & \Phi_i^{3,4} & \Phi_i^{4,4} & 0 & 0 & 0 \\
\Phi_i^{5,1} & -R_i & -Q_i & 0 & \Phi_i^{5,5} & * & 0 \\
0 & 0 & M_i & 0 & Q_i & -\gamma I & 0 \\
0 & 0 & 0 & P_{i,3} & 0 & 0 & 0 & -\sigma I
\end{bmatrix} < 0 \quad (21)
$$

where $\Phi_i^{1,1} = \gamma_i^1 + \rho I$, $\Phi_i^{2,2} = C_i^T P_{i,16}$, $\Phi_i^{3,3} = \gamma_i^2 + \rho I$, $\Phi_i^{4,4} = W_i^T P_{i,16}$, $\Phi_i^{5,1} = \gamma_i^3 + \rho I$, $\Phi_i^{5,5} = \gamma_i^4 + \rho I$, $\Phi_i^{5,6} = -G_i^T P_i$, $\Phi_i^{6,3} = \lambda_i P_i$, $\Phi_i^{6,6} = -\rho I + \sigma V$.

The proof of Theorem 2 is described in Appendix B. The sufficient LMI conditions for obtaining the FFTC law are provided in Theorem 1 and Theorem 2 for exponential faults and first order polynomial faults, respectively. Employing the convex optimal programming algorithm [34] to solve these sufficient LMI conditions, the FFTC law can be obtained to achieve the stability and fault tolerant tracking performance for the T-S fuzzy models with multiplicative noises. In the following section, two numerical examples are provided to show the applicability and effectiveness of the proposed FFTC methodology.

### 4. Numerical Examples

In order to illustrate the effectiveness and applicability of the proposed design approach, two examples are provided in this section. An academic example is given in Example 1 to compare the proposed design approach with previous FFTC method. In order to illustrate the applicability of proposed FFTC approach, the fault tolerant control problem of a nonlinear pendulum system is considered in Example 2.

#### Example 1

Let us consider the following T-S fuzzy model:

$$
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(u(t)) \begin{bmatrix}
A_i x_i(t) + B_i u_i(t) + G_i f_i(t)
\end{bmatrix} + \begin{bmatrix}
\bar{A}_i x_i(t) + \bar{B}_i u_i(t)\end{bmatrix} \beta_i(t)
$$

where $r$ is the number of rules, $A_i$, $B_i$, $G_i$, $\bar{A}_i$, $\bar{B}_i$ and $\beta_i$ are the system matrices.

#### Case 1: Case of exponential fault

Let us consider the following exponential fault affecting the system behavior occurs at $8s \leq t \leq 13s$:

$$
f(t) = e^{0.8t-3}
$$

The controller and the observer are synthesized for $\alpha_0 = 1$ and $\alpha = 0.2$. Using the LMI-Toolbox of MATLAB to solve the conditions of Theorem 1, one can obtain controller gains and observer gains as follows:

$$
K_i = [0.5771 \ 0.1690 \ 0.1868],
K_i' = [0.6126 \ 0.2322 \ 0.4875],
K_i'' = -0.1731,
\]$$

$$
H_i = \begin{bmatrix}
7.8940 & -4.3408 & 1.8940 \\
8.6243 & -1.4707 & 2.7081
\end{bmatrix},
\]$$

In order to show the advantages of the proposed FFTC method, some comparisons between proposed FFTC approach and the method developed in [28] are made in the simulations. For the space limit, only the responses of state $x_i(t)$ are shown. The responses of state $x_i(t)$ controlled by the proposed FFTC method are shown in Figure 2. Using the fault tolerant control method of [28],
the responses of state $x_1(t)$ are shown in Figure 3. From the simulation results, one can find that tracking responses of Figure 2 are better than Figure 3 in the case of considering the exponential faults.

$K_2 = \begin{bmatrix} 0.7880 & 0.4097 & 0.5765 \end{bmatrix}, \quad K'_2 = \begin{bmatrix} 7.2572 & 0.9826 \end{bmatrix},$

$H_2^1 = \begin{bmatrix} 6.6357 & 8.2301 \end{bmatrix}, \quad H_2^1 = \begin{bmatrix} 7.4678 & 6.0658 \end{bmatrix}, \quad H_2^1 = \begin{bmatrix} -0.5923 & 2.3715 \end{bmatrix}$

Applying the above controller gains and observer gains, the simulation results of state $x_1(t)$ are illustrated in Figure 4. Similarly, the simulation results of state $x_1(t)$ controlled by the FTC method of [28] are shown in Figure 5. For considering the first order polynomial fault, one can find that the tracking performance of Figure 4 is better than that of Figure 5. From the results of Figures 2-5, it can be found that the proposed FFTC method provides a better tracking performance than [28] when the systems having multiplicative noises.

$K_3 = \begin{bmatrix} 1.0556 & 2.6003 \end{bmatrix}, \quad H_3^1 = \begin{bmatrix} 6.6357 & 8.2301 \end{bmatrix}, \quad H_3^1 = \begin{bmatrix} 7.4678 & 6.0658 \end{bmatrix}, \quad H_3^1 = \begin{bmatrix} -0.5923 & 2.3715 \end{bmatrix}$

Case II: Case of first order polynomial fault

It is assumed that the fault affecting the system occurs at $10s \leq t \leq 15s$ and is given by following first order polynomial:

$f(t) = 0.3t - 3$ \hspace{1cm} (24)

Note that the FFTC gains and observer gains are computed for $\varepsilon = 10^{-3}$, $\lambda_0 = 0.11$ and $\Delta \lambda = 0.25$. Similarly, the FFTC law and observer gains can be obtained by solving the conditions of Theorem 2 via the LMI-Toolbox of MATLAB. By solving the conditions of Theorem 2, one can obtain the following controller gains and observer gains:

$K_1 = \begin{bmatrix} 0.7307 & 0.3398 & 0.2374 \end{bmatrix},$
Example 2:
In order to illustrate the effectiveness and applicability of the proposed FFTC approach, a nonlinear pendulum system is considered in this example. The nonlinear pendulum system considered in this paper is given as follows [37]:
\[
\dot{\theta}(t) = -a \sin(\theta(t)) - b \dot{\theta}(t) - c T(t)
\]  
(25)
where \( a > 0 \), \( b > 0 \) and \( c > 0 \) are constant, \( \theta(t) \) is the angle by the rod and the vertical axis, \( T(t) \) is the control input torque. Let \( \dot{\theta}(t) = x_1(t) \), \( \theta(t) = x_2(t) \) and \( T(t) = u(t) \) for tie in with the forging variables and define the constant values \( a = c = 10 \) and \( b = 1 \). Besides, the multiplicative noise \( v(t) \) is considered in the nonlinear pendulum system. Then, the nonlinear pendulum system (25) can be represented as state space type such as
\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) + 0.2 x_1(t) \cos x_1(t) \nu(t) + 0.1 x_1(t) v(t) \\
\dot{x}_2(t) &= -10 \sin x_1(t) - x_2(t) + 10 u(t) + 0.1 x_1(t) \cos x_1(t) \nu(t) + \\
&\quad 0.2 x_2(t) \sin x_1(t) v(t)
\end{align*}
\]  
(26a, 26b)
In order to obtain the T-S fuzzy model for the pendulum system (26), it is assumed that the torque \( T(t) \) works in the range of \( x_1(t) \in [-90, 90] \). Here, we choose two operation points such as
\[
\begin{align*}
(x^0(t),u^0(t))_{opt_1} &= (\pm 90^0,0|0) \\
(x^0(t),u^0(t))_{opt_2} &= (0^0,0|0)
\end{align*}
\]  
(27a, 27b)
Note that the point \( (x^0(t),u^0(t))_{opt_2} = (0^0,0|0) \) is the equilibrium point for the pendulum system. Through the above two operation points, the corresponding linear subsystems for the T-S fuzzy model can be obtained via linearizing technique [15]. The membership function for the fuzzy blending is presented in Figure 6. The following corresponding fuzzy model is thus represented for the nonlinear pendulum system (26).
\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{2} \mu_i(x_i(t)) \left( A_i x_i(t) + B_i u_i(t) + G_i f(t) \right) \\
&\quad + (\overline{A}_i x_i(t) + \overline{B}_i u_i(t)) \beta(t) \\
y(t) &= \sum_{i=1}^{2} \mu_i(x_i(t)) \left( C_i x_i(t) + D_i u_i(t) + W_i f(t) \right)
\end{align*}
\]  
(28a, 28b)
with \( A_i = \begin{bmatrix} 0 & 1 \\ -6.3662 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -10 & -1 \end{bmatrix} \),
\[
\overline{A}_i = \begin{bmatrix} 0 & 0.01 \\ -0.5 & -0.01 \end{bmatrix}, \quad \overline{A}_2 = \begin{bmatrix} 0 & 0.01 \\ -0.01 & -0.01 \end{bmatrix}, \quad B_i = B_2 = \begin{bmatrix} 0 \\ 10 \end{bmatrix},
\]
\[
\overline{B}_i = \overline{B}_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \quad G_i = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix},
\]
\[
W_i = -0.5, \quad W_2 = -0.3, \quad C_i = [0.5 \ 0.5], \quad C_2 = [1 \ 0.5] \text{ and } D_i = D_2 = -0.8.
\]
The nominal input signal is \( u(t) = \sin(\cos(0.5t)) \).

Figure 6. Membership function of \( x_1(t) \) for the pendulum system (26).

Case I: Case of exponential fault
Let us consider the following exponential fault affecting the system behavior occurs at \( 8s \leq t \leq 13s \):
\[
f(t) = e^{0.2t-3}
\]  
(29)
The controller and the observer are synthesized for \( \alpha_0 = 1 \) and \( \Delta \alpha = 0.2 \). Using the LMI-Toolbox of MATLAB to solve the conditions of Theorem 1, one can obtain the following matrices:
\[
P_1 = 10^4 \begin{bmatrix} 4.3016 & 0.2982 \\ 0.2982 & 0.5309 \end{bmatrix},
\]
\[
P_7 = 10^4 \begin{bmatrix} 1.5498 & 0.4496 \\ 0.4496 & 0.215 \end{bmatrix}, \quad P_{13} = 8.9421 \times 10^4,
\]
\[
P_{16} = [809.5621 \ 247.7521],
\]
\[
P_{17} = 10^4 \times [0.8833 -2.5047], \quad P_{18} = 6.9564 \times 10^4,
\]
\[
P_{19} = 3.0131 \times 10^4, \quad P_{20} = 391.8225,
\]
\[
F_1 = [2.7721 \ 5.2125] \times 10^4, \quad F_2 = [3.6865 \ 4.9686] \times 10^4,
\]
\[
R_1 = 4.8525 \times 10^4, \quad R_2 = 5.9634 \times 10^4,
\]
\[
S_1 = 10^4 \times [0.5361 \ 3.8954], \quad S_2 = 10^4 \times [2.1033 \ 3.9968],
\]
\[
Q_1 = 1.267 \times 10^5, \quad Q_2 = 1.546 \times 10^5, \quad M_1 = 8.3367 \times 10^4, \quad M_2 = 5.4495 \times 10^4,
\]
\[
\gamma = 3.3134 \times 10^5, \quad \tau = 6.092 \times 10^4.
\]
In addition, the controller gains and observer gains can be obtained as follows:
\[
K_1 = [0.0915 \ 0.172], \quad K_2 = [0.1216 \ 0.1639],
\]
\[
K_1' = 0.0418, \quad K_2' = 0.051, \quad H_1 = 5.4266,
\]
\[
H_1 = 6.6685, \quad H_2 = -12.473, \quad H_1' = -10.2509,
\]
\[
H_2' = 44.1918, \quad H_1' = 40.0176.
\]
Applying the above controller gains and observer gains, the simulation results are illustrated in the Figures 7-8. From these responses, one can find that the synthesized
observer and FFTC law showed their effectiveness. The fault and the system states are estimated and the tracking between the faulty system states and the reference model ones is ensured.

![Figure 7. Responses of $x_1(t)$ and $x_{1f}(t)$ with exponential faults.](image)

![Figure 8. Responses of $x_2(t)$ and $x_{2f}(t)$ with exponential faults.](image)

**Case II: Case of first order polynomial fault**

It is assumed that the fault affecting the system occurs at $10s \leq t \leq 15s$ and is given by following first order polynomial:

$$f(t) = 0.3t - 2.5$$

(30)

Note that the FFTC controller and observer gains are computed for $\varepsilon = 10^{-9}$, $\lambda_0 = 0.11$ and $\delta \lambda = 0.25$. Similarly, the FFTC law and observer gains can be obtained by solving the conditions of Theorem 2 via the LMI-Toolbox of MATLAB. The corresponding parameters can be obtained as follows:

$$P_1 = 10^6 \times \begin{bmatrix} 9.5241 & 0.6156 \\ 0.6156 & 1.1689 \end{bmatrix}, P_2 = 10^6 \times \begin{bmatrix} 2.7781 & 0.7967 \\ 0.7967 & 0.3814 \end{bmatrix},$$

$$P_3 = 1.2819 \times 10^4, P_4 = 10^4 \times \begin{bmatrix} -1.3044 & 0.4028 \end{bmatrix},$$

$$P_5 = 5.2798 \times 10^7, P_6 = 5.3758 \times 10^7,$$

$$F_1 = 10^3 \times \begin{bmatrix} 0.5391 & 1.1427 \end{bmatrix}, F_2 = 10^7 \times \begin{bmatrix} 0.8067 & 1.0824 \end{bmatrix},$$

$$R_1 = 8.7337 \times 10^6,$$

$$R_2 = 1.0832 \times 10^6, S_1 = 10^3 \times \begin{bmatrix} 0.8132 & 6.8729 \end{bmatrix},$$

$$S_2 = 10^3 \times \begin{bmatrix} 3.6796 & 7.1652 \end{bmatrix}, Q_1 = 3.2087 \times 10^6,$$

$$Q_2 = 3.4761 \times 10^6, M_1 = 1.4941 \times 10^8,$$

$$M_2 = 9.8993 \times 10^7, \tilde{\gamma} = 7.5931 \times 10^8, \rho = 2.1262 \times 10^5,$$

$$\sigma = 1.6086 \times 10^4.$$}

Then, one can obtain the following controller gains and observer gains:

$$K_1 = \begin{bmatrix} 0.1003 & 0.2126 \end{bmatrix}, K_2 = \begin{bmatrix} 0.1501 & 0.2013 \end{bmatrix},$$

$$K_1^f = 0.0597, K_2^f = 0.0647, H_1^f = \begin{bmatrix} -12.1608 \\ 43.4229 \end{bmatrix},$$

$$H_2^f = \begin{bmatrix} 12.1608 \\ -43.4229 \end{bmatrix}.$$
Considering the first order polynomial faults, the responses of states of nonlinear pendulum system (26) are shown Figures 9-10. Similar to the results of exponential fault case, the nonlinear pendulum system (26) can be successfully controlled by proposed FFTC approach with first order polynomial faults. From Figures 7-10, it can be concluded that the proposed FFTC approach is a useful fault compensating method that ensures the tracking between the reference model and faulty system states for the nonlinear pendulum systems.

5. Conclusions

The fuzzy fault tolerant tracking controller design approach for T-S fuzzy models with multiplicative noises has been proposed in this paper. It concerned the case when the fault affecting the system behavior are time varying. The faults considered in this paper were modeled by exponential function and first order polynomial, respectively. The conditions derived in the theorems of this paper were easily formulated in term of LMI that can be solved by convex optimal programming algorithm. The effectiveness and applicability of the proposed design approach have been illustrated by numerical examples. In the future, the proposed control approach can be extended to combine adaptive control or sliding mode control to deal with more complex performances for the nonlinear stochastic systems.

Appendix A (Proof of Theorem 1)

To study the stability of (15), the quadratic Lyapunov function is chosen as follows:

\[ V(e_r(t),e_s(t),e_f(t)) = \varepsilon^T(t)KP\varepsilon(t) \]  

(A1)

with

\[ KP = P^TK \geq 0 \]  

(A2)

The choice of the Lyapunov function (A1) will ensure the stability of the tracking error and of the state and fault estimations. To attenuate the fault effect on the error dynamics, the \( L_1 \) constraint [34] is employed as follows:

\[ \int_0^{t_f} \varepsilon^T(t)K\varepsilon(t) \leq \gamma^2 \int_0^{t_f} f^T(t)f(t) \]  

(A3)

where \( t_f \) and \( \gamma \) represent respectively the final time and the attenuation level. The tracking error \( e_r(t) \), state \( e_s(t) \) and fault \( e_f(t) \) estimation errors must therefore satisfy the following inequality:

\[ \varepsilon^T(t)KP\varepsilon(t) + \varepsilon^T(t)K\varepsilon(t) + \varepsilon^T(t)K\varepsilon(t) - \gamma^2 f^T(t)f(t) < 0 \]  

Considering (A2) and substituting (15) into (A4), one can obtain

\[ \begin{bmatrix} \dot{\varepsilon}(t) \\ f^T(t) \end{bmatrix} \Lambda \begin{bmatrix} \dot{\varepsilon}(t) \\ f(t) \end{bmatrix} < 0 \]  

(A5)

where \( \Lambda = A_1^TP + P^TA_2 + C_1^TPK\tilde{C} + K^TP^TB_1 \). The inequality (A5) is fulfilled if the following condition is satisfied:

\[ \Lambda < 0 \]  

(A6)

Let us choose the structure of the Lyapunov matrix \( P \) as follows:

\[ P = \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 \\ 0 & P_7 & 0 & 0 & 0 \\ 0 & 0 & P_3 & 0 & 0 \\ P_{16} & P_{17} & P_{18} & P_{19} & 0 \\ 0 & 0 & 0 & 0 & P_{25} \end{bmatrix} \]  

(A7)

Obviously, the above structure is not the general one but it limits the coupling between the unknown submatrices to be determined and allows the convergence conditions to be expressed using LMI. According to (A2), this latter imposes that \( P_1 = P_1^T \geq 0 \), \( P_7 = P_7^T \geq 0 \), \( P_3 = P_3^T \geq 0 \), and \( P_{16}, P_{17}, P_{18}, P_{19}, P_{25} \) are free slack matrices with appropriate dimensions. Considering (A7) and the matrices defined in (15), the mathematical development of (A6) leads to

\[ \Xi_\mu + \Gamma^T \Omega + \Omega^T \Gamma < 0 \]  

(A8)

where \( \Gamma = [0 \ 0 \ 0 \ 0 \ \alpha \alpha \alpha] \), \( \Omega = [0 \ P_1 \ 0 \ 0 \ 0] \),

\[ \Xi_\mu = \begin{bmatrix} \psi^{1,1}_{\mu} & * & * & * & * \\ C^\mu \ P^T_{16} & \psi^{2,2}_{\mu} & * & * & 0 \\ W^\mu \ P^T_{16} & \psi^{3,2}_{\mu} & \psi^{3,3}_{\mu} & * & * \\ -P^\mu & \psi^{4,2}_{\mu} & \psi^{4,4}_{\mu} & 0 & 0 \\ \psi^{5,1}_{\mu} & -P^\mu_{25} K^\mu & -P^\mu_{25} K^\mu & 0 & \psi^{5,5}_{\mu} \\ -G^\mu_{1,1} P^T & 0 & \psi^{6,3}_{\mu} & 0 & \psi^{6,5}_{\mu} \end{bmatrix} \gamma^2 I \]  

with

\[ \psi^{1,1}_{\mu} = P_1 A_1 + A_1^T P_1 + \tilde{A}_1^T \tilde{P}_1 \tilde{A}_1 + I \]  

\[ \psi^{2,2}_{\mu} = P_7 A_1 + A_1^T P_7 + P_7^T C_1 + C_1^T P_7 + \tilde{A}_1^T \tilde{P}_1 \tilde{A}_1 + I \]  

\[ \psi^{3,3}_{\mu} = P_{16} C_1 + C_1^T P_{16} + W_{16} P_{16} + I \]  

\[ \psi^{4,4}_{\mu} = P_{19} C_1 + P_{19}^T C_1 - (H_\mu)^T P_{19} + P_{19}^T W_{19} P_{19} + I \]  

\[ \psi^{5,5}_{\mu} = -P_{25}^T K^\mu_\mu - P_{25}^T K^\mu_\mu 0 0 \psi^{5,5}_{\mu} \\ -G_{1,1}^\mu P_1^T 0 \psi^{6,3}_{\mu} 0 \psi^{6,5}_{\mu} \]  

Considering (8) and applying Lemma 1 on (A8), then
(A8) can be bounded as
\[ \Xi_\mu + \tau \Lambda + \tau^{-1} \Omega \Omega < 0 \quad (A9) \]
with \( \Lambda = [0 \ 0 \ 0 \ 0 \ \nu] \). To provide LMI conditions, let us define \( (H_\mu^T)^T P_{13} = M_\mu \), \( P_{20}^T K_s = F_\mu \), \( P_{13}^T H_\mu = R_\mu \), \( (H_\mu^T)^T P_1 = S_\mu \), \( P_{20}^T K_s' = Q_\mu \), \( \hat{\gamma} = \gamma^2 \). Applying Schur complement [34] on the term \( \tau^{-1} \Omega \Omega \), thus the sufficient LMI conditions proposed in Theorem 1 hold.

# Appendix B (Proof of Theorem 2)

Consider (A1), (A2) and (20). Following the same procedure of the proof of Theorem 1, one can obtain
\[ \begin{bmatrix} \hat{e}^T & f^T & I \end{bmatrix} \Omega \begin{bmatrix} \hat{e} \\ f \\ I \end{bmatrix} < 0 \quad (B1) \]
where \( \Omega = \begin{bmatrix} \tilde{A}_\mu^T P + P \tilde{A}_\mu + \tilde{C}_\mu^T K P \tilde{C}_\mu + K & * & * \\ \hat{B}_\mu^T P & -\gamma^2 I & 0 \\ N^T P & 0 & 0 \end{bmatrix} \).

To ensure the asymptotic convergence to a ball of radius \( \varepsilon \) of the error dynamics, the following constraint is considered.
\[ \| \hat{e} \| \geq \varepsilon I \quad (B2) \]
with \( \varepsilon \) is a known small positive scalar. It is easy to find that the inequality (B2) is equivalent to the following inequality.
\[ \begin{bmatrix} \hat{e}^T & f^T & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & -\varepsilon I \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \hat{e} \\ f \\ I \end{bmatrix} > 0 \quad (B3) \]
Applying Lemma 2 on (B2) and (B3), one has
\[ \begin{bmatrix} \hat{e}^T & f^T & I \end{bmatrix} \begin{bmatrix} \Psi_\mu & * & * \\ \hat{B}_\mu^T P & -\gamma^2 I & 0 \\ N^T P & 0 & -\delta \varepsilon I \end{bmatrix} \begin{bmatrix} \hat{e} \\ f \\ I \end{bmatrix} < 0 \quad (B4) \]
where \( \Psi_\mu = \tilde{A}_\mu^T P + P \tilde{A}_\mu + \tilde{C}_\mu^T K P \tilde{C}_\mu + K + \rho I \). Considering the matrix \( N \) defined in (20) and \( P \), (B4) is fulfilled if the following inequality is satisfied.
\[ \chi_\mu + \Sigma^T \phi + \phi^T \Sigma < 0 \quad (B5) \]
where \( \Sigma = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \Delta \lambda \end{bmatrix}, \phi = [0 \ 0 \ P_{13} \ 0 \ 0 \ 0] \)
and \( \chi_\mu = \begin{bmatrix} \Psi_\mu & * & * \\ \hat{B}_\mu^T P & -\gamma^2 I & 0 \\ 0 & 0 & -\lambda_\mu^2 P_{13} & 0 & 0 & -\delta \varepsilon I \end{bmatrix} \).

Applying Lemma 1 on the inequality (B5), then one can obtain
\[ \chi_\mu + \sigma \Sigma^T \Sigma + \sigma^{-1} \phi^T \phi < 0 \quad (B6) \]
Applying the Schur complement [34] on the term \( \sigma^{-1} \phi^T \phi \) and following the same steps of the proof of Theorem 1 from (A6) to the end, the sufficient LMI conditions for the FFTC law described in Theorem 2 can be obtained.

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# References


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