Type-2 Fuzzy Logic Controller Using SRUKF-Based State Estimations for Biped Walking Robots

Liyang Wang, Zhi Liu, Yun Zhang, C. L. Philip Chen, and Xin Chen

Abstract

Walking motions of biped robots have features of humanoid and intelligent. Generally, when we human are walking, we may control the walking motions using qualitative instructions, such as linguistic “slow” or “fast,” “forward” or “backward,” “big step” or “small step”. This motivates us to design a fuzzy controller for biped robots. Furthermore, to improve the robustness of the biped walking system, a framework of biped gait control based on a predictable type-2 fuzzy logic controller (T2FLC) is proposed to ensure the dynamic balance of the biped under the condition of complex process noises and measurement noises. Different with existing controllers for biped robots, a state estimator based on square root unscented Kalman filter (SRUKF) is incorporated in the proposed control strategy. Using the estimated biped states as inputs, the proposed T2FLC can predictably adjust the posture of the trunk timely and properly to ensure the dynamic balance of the whole legged system. Simulation results are presented to verify the effectiveness of the proposed method.

Keywords: Biped robot, square root unscented Kalman filter, state estimation, type-2 fuzzy logic system.

1. Introduction

Biped robots always suffer from complex process noises and measurement noises. As a result, to ensure the dynamic balance of biped robots under the noises be-

Corresponding Author: Liyang Wang is with the Department of Automation, Guangdong University of Technology, Guangzhou Guangdong 510006, China. Liyang Wang is also with the Department of Electronic Engineering, Shunde Polytechnic, Foshan Guangdong 528300, China.
E-mail: ddd0wwl@sohu.com
Zhi Liu and Yun Zhang are with the Department of Automation, Guangdong University of Technology, Guangzhou Guangdong 510006, China.
C. L. Philip Chen is with the Faculty of Science and Technology, University of Macau, Macau, China.
Xin Chen is with the Department of Mechatronics Engineering, Guangdong University of Technology, Guangzhou Guangdong 510006, China.
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and timely.

Nonlinear filters, such as extended Kalman filter (EKF) [28], particle filter (PF) [29], unscented Kalman filter (UKF) [30] and SRUKF [31-32], are known as powerful tools for nonlinear state estimations. SRUKF is an improved version of the UKF. The most computationally expensive operation in the UKF is to calculate the new set of sigma points at each time update. This requires the computation of a matrix square-root of the state covariance matrix, and an integral part of the UKF requires computation of full covariance matrix be recursively updated. In the SRUKF implementation, square-root of the state covariance matrix will be propagated directly, avoiding the requirement of re-factorizing at each time step. Therefore, the SRUKF provides the same results as the UKF to within machine accuracy and it can be reposed with less complexity for nonlinear state estimations. From the above, the SRUKF is a kind of potential estimator for the biped states.

In this work, a SRUKF-based predictable T2FLC is proposed to ensure the dynamic balance of biped robots under complex process noises and measurement noises. First, a method of SRUKF-based biped state estimation is proposed, which provides an optimal estimation of the biped states. Secondly, a T2FLC is built, which realizes a dynamical compensation for the biped gait using trunk rotations. Two following modules are implemented to realize the proposed SRUKF-based predictable T2FLC. To move the trunk timely and properly, the state of the biped robot for the coming sampling time of the system is predicted by using the SRUKF-based biped state estimator with low computational and the high preciseness. Furthermore, the T2FLC is proposed to compensate the biped gait according to the dynamic kinematics relationships between the legs and the trunk. The essential of the T2FLC is moving the ZMP far away from the edge of the support area to ensure the dynamic balance of the biped walking robots.

The organization of this paper is as follows. In Section II, criteria for the stability of biped motions are presented, and the proposed SRUKF-based biped state estimation is presented in Section III. The T2FLC with estimated biped states is proposed in Section IV. In Section V, simulations are conducted and the conclusions are presented in Section VI.

2. Background of Biped Robots

A. Criteria for the stability of biped motions

The general definition of motion stability was first proposed by Lyapunov, which mainly describes the stability of equilibrium points [33-34]. However, biped robots have inherent characteristics of unilateral constraint, hybrid and variable topology of the drive mode. As a result, it is difficult to analyze the stability of biped robots using Lyapunov stability theory. Some researches [35-36] proposed that stable gaits of biped robots show themselves in the form of limit cycles in the phase space, and limit cycles are fixed points of Poincaré mappings. So the study on stable gaits of biped robots can be simplified as the study on the stability of the fixed points of Poincaré mappings. However, it is difficult to get the analytic solution of the Poincaré mappings because of the complexity of the biped dynamic. The fixed points of Poincaré mappings can only be found using numerical method, which determines that this kind of method can only be applied to passive robots, biped robots without feet and jumping robots based on simple models.

On the other hand, the concept of ZMP has been applied to many famous biped robots successfully, such as ASIMO of Honda [37]. ZMP [38] is a point on the ground at which the net moment of the inertial forces and the gravity forces has no component along the horizontal axes. At a given time instant, dynamic balance of legged systems is ensured if the ZMP is inside the support area. ZMP is the most popular criterion for the stability of the biped locomotion up to now. Therefore, ZMP theory is used as the criterion of dynamic biped balance in this work.

B. Ensuring the dynamic balance of the biped using ZMP theory

According to the ZMP theory, the dynamic balance of the biped can be ensured if the ZMP is inside the support area, as shown in Figure 1. In addition, the minimum distance between the ZMP and the boundary of the support area is called ZMP stability margin. To avoid the falling down of the biped, the biped motions should guarantee the ZMP criterion, and the ZMP stability margin should be positive.

Furthermore, the ZMP can be computed using the following equations [39]:

\[ x_{\text{zmp}} = \sum_{j=1}^{n} m_j (\dddot{x}_j + g) x_j - \sum_{j=1}^{n} m_j \dddot{x}_j y_j + \sum_{j=1}^{n} f_j \Omega_y \]

\[ y_{\text{zmp}} = \sum_{j=1}^{n} m_j (\dddot{y}_j + g) y_j - \sum_{j=1}^{n} m_j \dddot{y}_j z_j + \sum_{j=1}^{n} f_j \Omega_z \]

where \((x_{\text{zmp}}, y_{\text{zmp}}, 0)\) is the coordinate of the ZMP, and \((x_j, y_j, z_j)\) is the coordinate of the mass center of link \(j\) on an absolute Cartesian coordinate system. \(m_j\) is the mass of link \(j\), \(I_x\) and \(I_y\) are the inertial components, \(\Omega_x\) and \(\Omega_z\) are the absolute angular acceleration components around \(x\) axis and \(y\) axis at the center of
gravity of link j, g is the gravitational acceleration. ZMP stability margins can be calculated as follows:
\[
\begin{align*}
\Delta_{\text{ZMP}_\text{left}} &= \min\left|Y_{\text{left}} - Y_{\text{ref}}\right| \cdot \left|X_{\text{left}} - X_{\text{ref}}\right| \\
\Delta_{\text{ZMP}_\text{right}} &= \min\left|Y_{\text{right}} - Y_{\text{ref}}\right| \cdot \left|X_{\text{right}} - X_{\text{ref}}\right|
\end{align*}
\]
where \(\Delta_{\text{ZMP}_\text{left}}\) and \(\Delta_{\text{ZMP}_\text{right}}\) are the ZMP stability margins. \(X_{\text{ref}}\) and \(Y_{\text{ref}}\) are the ZMP positions.

Let \(1 \in \mathbb{R}_{n}\), we have:
\[
\begin{align*}
\phi(\Theta) &= \begin{bmatrix} 0 \\
-\mathbf{M}^{-1} \mathbf{C} \end{bmatrix} \Theta + \begin{bmatrix} 0 \\
\mathbf{M}^{-1} [\mathbf{r} - \mathbf{G}] \end{bmatrix} \\
\mathbf{B}(\Theta) &= \begin{bmatrix} 0 \\
\mathbf{M}^{-1} \end{bmatrix}
\end{align*}
\]
A zero-holder between the controller and the plant is considered, and the process noise is considered. Referring to [40], the sampled-data representation of Eq. (9) can be expressed as:
\[
\Theta(k+1) = \overline{A}\Theta(k) + \overline{G}(\Theta(k)) \tau(k) + \overline{W}(\Theta(k), \tau(k)) + w(k)
\]
where \(\overline{A}, \overline{G}(\Theta(k))\) and \(\overline{W}(\Theta(k), \tau(k))\) can be referenced in [40] for detail. \(w(k)\) is the process noise. And Eq. (12) can be rewritten as
\[
\Theta(k+1) = f(\Theta(k), \tau(k)) + v(k)
\]
Eq. (13) is the needed discrete time state equation of biped robots.

3. Estimating Biped States Using a SRUKF

To estimate biped states using a SRUKF, a discrete-time model for biped state estimation (including the state equation and the measure equation) is developed first. After that, a SRUKF filter algorithm for the biped state estimation is deduced in detail.

A. System modeling for biped state estimation

To design a SRUKF for biped robots, the dynamic of the robot should be described using state equation firstly. A discrete-time model for predicting joint states of biped robots is built based on Lagrange dynamics. The dynamic of biped robots in single support phase can be written as:
\[
\begin{align*}
M(\dot{q}) \ddot{\dot{q}} + C(q, \dot{q}) \dot{\dot{q}} + G(q) &= \tau \\
\dot{q} &= \dot{x} \quad \text{and} \quad \dot{x} = \dot{\dot{x}} \dot{x}
\end{align*}
\]
where \(q\) is the joint variable vector, \(M(q) \in \mathbb{R}^{n \times n}\) is the inertia matrix, and \(C(q, \dot{q}) \in \mathbb{R}^{n \times n}\) and \(G(q) \in \mathbb{R}^{n}\) are Coriolis and centrifugal forces, \(G(q) \in \mathbb{R}^{n}\) is the gravity term. \(\tau\) is the control input torque.

Let \(x_1 = \dot{q}\) and \(x_2 = \ddot{q}\), the dynamic of biped robots can be written as follows:
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= M^{-1}(x_1) \tau - M^{-1}(x_1) [C(x_1, x_2) x_2 + G(x_1)]
\end{align*}
\]
i.e.,
\[
\begin{align*}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\
-\mathbf{M}^{-1}(x_1) \mathbf{C}(x_1, x_2) & \mathbf{M}^{-1}(x_1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\
\mathbf{M}^{-1}(x_1) [\mathbf{r} - \mathbf{G}(x_1)] \end{bmatrix}
\end{align*}
\]
Submitting \(\dot{\Theta} = [x_1, x_2]^T\) and \(\Theta = [x_1, x_2]^T\) to Eq. (7), we get
\[
\dot{\Theta} = \begin{bmatrix} 0 \\
\mathbf{M}^{-1} \end{bmatrix} \Theta + \begin{bmatrix} 0 \\
\mathbf{M}^{-1} \mathbf{G} \end{bmatrix} \\
\mathbf{B}(\Theta) &= \begin{bmatrix} 0 \\
\mathbf{M}^{-1} \end{bmatrix}
\]

Figure1. Support areas for biped robots.
of the joint angle and the velocity angle. Without loss of generality, \( w(k) \) and \( v(k) \) are both white, zero-mean, uncorrelated, and have known covariance matrices \( Q(k) \) and \( R(k) \), respectively.

B. SRUKF algorithm for biped state estimation

Here we denote the joint angle estimate of \( \Theta(k+1) \) as \( \hat{\Theta}(k+1) \) when all of the measurements before (but not including) time \( k+1 \) are available for our joint angle estimate. The “-” superscript denotes that the estimate is a priori.

Step 1: Initialization

The estimation of the initial state \( \Theta(0) \) is denoted as \( \hat{\Theta}(0) \), \( E(\cdot) \) is the expectation. Then we have

\[
\hat{\Theta}(0) = E[\Theta(0)]
\]

\[
S^0(0) = \sqrt{E[(\Theta(0) - \hat{\Theta}(0))^2]} \tag{17}
\]

Step 2: Calculate Sigma Points

The scaled \( 2L+1 \) sigma points set \{\( \Theta(i); i = 0, \ldots, 2L \), where the 0 inside the parentheses denotes time step \( k = 0 \), is generated by

\[
\Theta(i) = \hat{\Theta}(0) + \sqrt{\lambda} S(i,0), \quad i = 0, \ldots, L
\]

\[
\Theta(i) = \hat{\Theta}(0) - \sqrt{\lambda} S(i,0), \quad i = L + 1, \ldots, 2L
\]

Let \( \alpha^\sigma \) and \( \alpha^c \) denote the weights associated with the \( i \) th sigma point. These weights can be calculated by

\[
\alpha^\sigma_i = \lambda (L + \lambda)
\]

\[
\alpha^c_i = \lambda (L + \lambda + 1 - \alpha^2 + \beta)
\]

\[
\alpha^o_i = \alpha^c_i / (2L + 2\lambda), \quad i = 0, \ldots, 2L
\]

where \( \lambda = \alpha^2 (L + \gamma - L) \), \( \alpha \) controls dispersing extent of sampling points in a small positive value range \( 10^{-3} \sim 1 \). \( \beta \) is a secondary scaling factor, \( \beta = 2 \) is optimal under the condition of the Gaussian noise. \( \gamma \) is a tertiary scaling factor and is usually set equal to 0. The superscripts \( m \) and \( c \) denote mean and covariance, respectively.

Step 3: Time update

\[
\hat{\Theta}^m(k+1) = \sum_{i=0}^{2L} \alpha^m_i \Theta(i + \|k\)
\]

\[
S^m(k+1) = q_r(\sqrt{\alpha^m_i} (\Theta(i + \|k) - \hat{\Theta}^m(k+1)))
\]

\[
S^m(k+1) = cholupdate\{S^m(k+1), \Theta(i + \|k), \alpha^o_i \}
\]

\[
\hat{y}^c(k+1) = \sum_{i=0}^{2L} \alpha^c_i y_i(k + \|k)
\]

Step 4: Measurement update

\[
S^c(k+1) = q_r(\sqrt{\alpha^c_i} (y_i(k + \|k) - \hat{y}^c(k+1)))
\]

\[
S^c(k+1) = cholupdate\{S^c(k+1), y_i(k + \|k), \alpha^o_i \}
\]

4. T2FLC Using Estimated Biped States

Trunk rotation is a usual way for effective dynamic human walking [41-43]. When the bipeds try to maintain the dynamic balance by trunk rotations, the controller should make the decision accurately and timely, even there exists unavoidable noise from the biped system and the sensors.

The T2FLC with estimated biped states is shown in Figure 2. To make the decision accurately and timely, the inputs of the proposed T2FLC are preprocessed by a SRUKF. Using the estimated ankle trajectory and hip trajectory as inputs, and the corresponding trunk trajectory as outputs, the essential of the T2FLC is moving the ZMP far away from the boundary of the support area, for ensuring the dynamic balance of the biped walking robots.

This section designs trunk trajectory corresponding to the estimated biped states using a T2FLC. The antecedent part of the T2FLC uses interval type-2 fuzzy sets, and the consequent part is of the Mamdani-type. The \( i \) th rule in the system has the following form:
Rule $i$: IF $\hat{y}_{iy}$ is $\tilde{\lambda}_{i1}$ AND $\hat{y}_{iabd}$ is $\tilde{\lambda}_{i2}$ THEN $y_{\text{root}}$ is $\bar{G}_i$, $i = 1, \ldots, M$ (19)

where $\hat{y}_{iy}$ and $\hat{y}_{iabd}$ denote the estimated centroid positions of the supporting hip and the swing ankle respectively, $y_{\text{root}}$ is the centroid positions of the trunk. $\tilde{\lambda}_{i1}$ and $\tilde{\lambda}_{i2}$ are interval type-2 fuzzy sets, $\bar{G}_i$ is the output interval type-2 fuzzy set of the $i$th rule, and $M$ is the number of rules. Based on the rule base in (19), the computation of the T2FLC involves the fuzzifier, inference engine, type reducer, and defuzzifier, which will be described in detail next.

A. Fuzzification

The fuzzifier maps crisp input values to fuzzy sets. For the $i$th fuzzy sets $\tilde{\lambda}_{i1}$ and $\tilde{\lambda}_{i2}$ in the input variables, a Gaussian primary membership function is used, which has a fixed standard deviation and an uncertain mean that takes on values in $[\bar{m}_i, \bar{m}_i^{\prime}]$.

For example, the membership degree of the centroid positions of the supporting hip is

$$\mu_{\lambda_{i1}}(\hat{y}_{iy}) = \exp \left\{ \frac{1}{2} \left( \frac{\hat{y}_{iy} - \bar{m}_i}{\sigma} \right)^2 \right\}$$

(20)

Here, the membership degree $\mu_{\lambda_{i1}}(\hat{y}_{iy})$ is an interval set and is denoted by $\mu_{\lambda_{i1}}(\hat{y}_{iy}) = [\mu_{\lambda_{i1}}(\hat{y}_{iy}), \bar{\mu}_{\lambda_{i1}}(\hat{y}_{iy})]$. The mathematical functions of the lower and upper MFs, $\bar{\mu}_{\lambda_{i1}}(\hat{y}_{iy})$ and $\bar{\mu}_{\lambda_{i1}}(\hat{y}_{iy})$, are described as follows:

$$\bar{\mu}_{\lambda_{i1}}(\hat{y}_{iy}) = \begin{cases} N(m_i, \sigma_i; \hat{y}_{iy}), & \hat{y}_{iy} < \bar{m}_i \\ 1, & \bar{m}_i \leq \hat{y}_{iy} \leq \bar{m}_i^{\prime} \\ N(m_i, \sigma_i; \hat{y}_{iy}), & \hat{y}_{iy} > \bar{m}_i^{\prime} \end{cases}$$

(21)

Similarly, the membership degree of the centroid positions of the supporting ankle can be expressed as

$$\mu_{\lambda_{i2}}(\hat{y}_{iabd}) = \begin{cases} N(m_i^{\prime}, \sigma_i^{\prime}; \hat{y}_{iabd}), & \hat{y}_{iabd} \leq \bar{m}_i^{\prime} \\ \frac{1}{2}, & \bar{m}_i^{\prime} \leq \hat{y}_{iabd} \leq \bar{m}_i^{\prime} \\ N(m_i^{\prime}, \sigma_i^{\prime}; \hat{y}_{iabd}), & \hat{y}_{iabd} > \bar{m}_i^{\prime} \end{cases}$$

(22)

And the membership degree of the centroid positions of the trunk can be expressed as

$$\mu_{\lambda_{i2}}(y_{\text{root}}) = N(m_i^{\prime}, \sigma_i^{\prime}; y_{\text{root}}), m_i^{\prime} \in [m_i^{\prime}, \bar{m}_i]$$

(23)

B. Inference

The inference engine operation performs the fuzzy meet operation by using an algebraic product operation. The result of the input and antecedent operations $F_i$ is an interval type-1 set, i.e., $F_i = [\bar{f}_i, \bar{f}_i^{\prime}]$, where

$$\bar{f}_i = \bar{p}_{\lambda_{i1}} \cdot \bar{p}_{\lambda_{i2}}, \bar{f}_i = \mu_{\lambda_{i1}} \cdot \mu_{\lambda_{i2}}$$

(25)

The $i$th rule fired output consequent set $\mu_{\bar{G}_i}(y_{\text{root}})$ is

$$\mu_{\bar{G}_i}(y_{\text{root}}) = \int_{y_{\text{root}} \in \{y_{\text{root}} | y_{\text{root}} \leq \bar{f}_i \}} \mu_{\bar{G}_i}(y_{\text{root}}) 1/y_{\text{root}}$$

(26)

where $\bar{p}_{\lambda_{i1}}(y_{\text{root}})$ and $\bar{p}_{\lambda_{i2}}(y_{\text{root}})$ are the lower and upper membership grades of $\mu_{\lambda_{i1}}(y_{\text{root}})$ and $\mu_{\lambda_{i2}}(y_{\text{root}})$. The output fuzzy set $\mu_{\bar{G}_i}(y_{\text{root}})$ is

$$\mu_{\bar{G}_i}(y_{\text{root}}) = \int_{y_{\text{root}} \in \{y_{\text{root}} | y_{\text{root}} \leq \bar{f}_i \}} \mu_{\bar{G}_i}(y_{\text{root}}) 1/y_{\text{root}}$$

(27)

C. Type reduction

For T2FLCs, the final crisp output is the center of the type-reduced set. There exist many kinds of type-reduction, such as centroid, center-of-sets, height, and modified height [44]. In this paper, center-of-sets type-reduction is used as follows:
where $L$ and $R$ denote the left and right crossover positions, respectively. $y_{\text{cent}}$ is the centroid of the type-2 interval consequent set $\tilde{G}_i$.

The outputs $y'_{\text{cent}}$ and $y''_{\text{cent}}$ can be computed as follows:

$$y'_{\text{cent}} = \frac{\sum_{i=1}^{r} (Q_i^f) y_{\text{cent}} + \sum_{i=r+1}^{M} (Q_i^f) y_{\text{cent}}}{\sum_{i=1}^{r} + \sum_{i=r+1}^{M} (Q_i^f)}$$

$$y''_{\text{cent}} = \frac{\sum_{i=1}^{r} (Q_i^f) y_{\text{cent}} + \sum_{i=r+1}^{M} (Q_i^f) y_{\text{cent}}}{\sum_{i=1}^{r} + \sum_{i=r+1}^{M} (Q_i^f)}$$

where $L$ and $R$ denote the left and right crossover points, respectively. $y_{\text{cent}} = (y_{\text{cent}}^{\text{left}}, \ldots, y_{\text{cent}}^{\text{right}})$ denotes the original rule-ordered consequent values and $y_{\text{cent}}^{\text{left}} \leq y_{\text{cent}} \leq \ldots \leq y_{\text{cent}}^{\text{right}}$. $Q$ is an $M \times M$ permutation matrix. These two points vary with different inputs. Karnik-Mendel iterative procedure can be used here to find these two points [45].

D. Defuzzification

The defuzzifier computes the system output variable $y_{\text{out}}$ by performing the defuzzification operation on the interval set $[y'_{\text{cent}}, y''_{\text{cent}}]$ from the type reduction. Based on the centroid defuzzification operation, the centroid of the interval set $[y'_{\text{cent}}, y''_{\text{cent}}]$ is the average of $y'_{\text{cent}}$ and $y''_{\text{cent}}$. Hence, the defuzzified output is

$$y_{\text{out}} = \frac{y'_{\text{cent}} + y''_{\text{cent}}}{2}$$

Up to now, with type-2 fuzzy inputs of the centroid positions for the supporting hip and the swing ankle, the corresponding position for the trunk is deduced using a T2FLC. The trunk trajectory provided by the T2FLC is then used as the reference trajectory for control, which ensures the dynamic balance of biped walking robots.

5. Simulations Research

A. Reference trajectories for the biped joints

A typical gait is planned for the biped robot to walk on the horizontal ground. The whole walking period of the biped robot is considered to be composed of a single support phase and an instantaneous double support phase. The supporting foot is assumed to remain in full contact with the ground during the single support phase, and both the feet revolve around the point contacting with the ground during the double support phase. The walk cycle is $T_w$, and let $T_s = 1\text{s}$.

B. Implementation of SRUKF-based biped state estimation

1) Simulation conditions

Firstly, the biped system is as follows.

The details of the humanoid can be referenced in Table 1. Simulation model of the biped system is as follows:

$$\begin{align*}
\Theta(k+1) &= f(\Theta(k), \tau(k)) + w(k) \\
\gamma(k) &= \Theta(k) + \nu(k)
\end{align*}$$

where $\Theta(k) = [\theta_1(k), \ldots, \theta_4(k), \dot{\theta}_1(k), \ldots, \dot{\theta}_4(k)]^T \in \mathbb{R}^4$ is the state vector, and the joint angle $\theta_n(k)$ and the angle velocity $\dot{\theta}_n(k)$ are physical quantities at time $k$. The process noise $w(k) \sim (0,10^{-3})$, and the measurement noise $\nu(k) \sim (0,10^{-3})$. $f(\cdot)$ is defined in the Eq. (14). $\gamma(k)$ is the measure matrix of the joint angle and the angle velocity. Let $k = 1, \ldots, 41$, and the sampling interval $T = 0.025s$. The control input torque $\tau(k) = [\tau(1), \tau(2), \ldots, \tau(41)]$ is designed using the method of computer torque:

$$\tau(k) = [\hat{M} \dot{\theta}(k) + K_e \dot{\theta}(k) + K_p \theta(k)] + [\hat{C} \dot{\theta}(k) + \hat{G} \theta(k)], \quad k = 1, \ldots, 41$$

where $\hat{M}$ denotes the estimation parameter. $e(k) = \theta(k) - \hat{\theta}(k)$, $\dot{e}(k) = \dot{\theta}(k) - \ddot{\theta}(k)$, $\theta^p(k)$, and $\ddot{\theta}(k)$ are the desired reference trajectories for the robot. $K_p$ and $K_e$ are the proportional and differential gains. It is noted that discrete-time models are discussed in this work. The variables such as $\dot{e}(k)$, $\dot{\theta}(k)$, and $\ddot{\theta}(k)$ are all physical quantities. They are not continuous-time variables.

Secondly, initial state values of the biped system are as follows.

$$\Theta(0) = [0.0873, 0.1183, 0.7210, 0, 0.3550, 0.5989, 0, 0]$$

Thirdly, the three parameters of the SRUKF estimator are set to be $\alpha = 1$, $\beta = 2$, and $\gamma = 0$ because the additive noise is Gaussian [46].

2) Simulation results of the biped state estimations

Although the reference trajectories are planned as previously mentioned, the biped could deviate from the planned gait because of the process noise and measure-

Table 1. Parameters of the biped system.

<table>
<thead>
<tr>
<th>Link</th>
<th>Length (m)</th>
<th>Mass (Kg)</th>
<th>Inertia moment (Kg m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trunk</td>
<td>0.175</td>
<td>1.060</td>
<td>0.01623</td>
</tr>
<tr>
<td>Thigh</td>
<td>0.130</td>
<td>0.075(*2)</td>
<td>0.00063(*2)</td>
</tr>
<tr>
<td>Shank</td>
<td>0.130</td>
<td>0.075(*2)</td>
<td>0.00037(*2)</td>
</tr>
<tr>
<td>Foot</td>
<td>0.020</td>
<td>0.060(*2)</td>
<td>0.00001(*2)</td>
</tr>
</tbody>
</table>
ment noise. To reduce the influence of the noises, SRUKF-based biped state estimation is implemented to provide optimal estimated inputs to the T2FLC.

In this work, Matlab 7.0 is used to model the biped system and the estimators. The estimated trajectories of the joint angles are shown in Figure 3. As we can see, the SRUKF-based estimated results are close to the true trajectories of the joint angles. One other thing should be noted is that the true trajectories here in the simulations are determined in advance while true trajectories in the practical engineering are unknown because of various noises. That is why biped state estimations are needed.

3 ) Comparison of different filters

In order to evaluate the performance of the estimator, the mean of the squared error (MSE) of the SRUKF estimator is calculated using the following formula, and the result is compared with those of the EKF, the PF, and the UKF:

$$MSE = \frac{1}{N_r} \sum_{r=1}^{N_r} \sum_{k=1}^{N} [\Theta(k) - \hat{\Theta}_i(k)]^2$$

where \( k = 1, \ldots, N \) (\( N = 41 \)) is the index of the samples, \( r = 1, \ldots, N_r \) (\( N_r = 100 \)) is the index of different runs. \( \hat{\Theta}_i(k) \) is computed using the \( r \)th realization of the recursive filters. In Figure 4, the SRUKF and the PF (the number of the particles is 200 here) have been shown to present better performance than the others in term of MSE. However, the execution time for the PF is much longer than that of the others, which is disadvantageous for the real-time control of the robot. On the other hand, the execution time of the PF can be reduced if the number of the particles is decreased, while this will leads to significant decreasing of the MSE. As a result, the performance of the SRUKF is the best among the four typical nonlinear filters if both the MSE and the execution time are considered.

![Figure 3](image_url)  
*Figure 3. The true trajectories and the SRUKF-estimated joint angles: true (dotted black line with circles), SRUKF-estimated (blue solid line) (a)supporting ankle (b)supporting knee (c) supporting hip (d)swing hip (e) swing knee (f) swing ankle.*

![Figure 4](image_url)  
*Figure 4. Comparison of EKF/PF/UKF/SRUKF (mean of 100 runs).*

C. Implementation of the proposed T2FLC

The proposed T2FLC has the estimated positions of the supporting hip and the swing ankle as inputs, and the centroid positions of the trunk as outputs. The \( i \)th rule in the system has the following form:

$$\text{Rule } i: \text{ IF } \hat{y}_\text{hyp} \text{ AND } \hat{y}_\text{ankle} \text{ THEN } y_{\text{next}} \text{, } i = 1, \ldots, 35$$

where \( \hat{y}_\text{hyp} \) and \( \hat{y}_\text{ankle} \) denote the estimated centroid positions of the supporting hip and the swing ankle respectively, \( y_{\text{next}} \) is the centroid position of the trunk. \( \hat{A}_j \), \( j = 1, 2 \) is an interval type-2 fuzzy set, \( \hat{G}_i \) is the output interval type-2 fuzzy set of the \( i \)th rule, and the number of rules is 35.

As shown in Figure 2, in this work, ZMP stability of the biped is ensured by trunk rotations [41, 43] based on fuzzy rules. About the ZMP of biped robot, there are some facts derived as follows [43]: The desired ZMP is greatly influenced by the position of the supporting hip, and in a less degree by the position of the foot of the swing leg. According to these facts, to ensure ZMP stability of the biped, the rule base of the proposed T2FLC is designed as shown in Table 2. Here, two steps are involved in the design procedure of the fuzzy rules: First, the interval type-2 fuzzy sets of the variables and the rule base of the T2FLC are designed manually. Secondly, the final interval type-2 fuzzy sets are obtained using the method of trial-and-error.
Table 2. Rule base of the proposed T2FLC.

<table>
<thead>
<tr>
<th>( \hat{\mathbf{y}}_{\text{ankle}} )</th>
<th>( \hat{\mathbf{y}}_{\text{hip}} )</th>
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<tbody>
<tr>
<td>UB</td>
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<td>B</td>
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</table>

The details about Table 2 will be described next.

1) To express the linguistic and numerical uncertainty, interval type-2 fuzzy membership functions of \( \hat{\mathbf{y}}_{\text{hip}} \) are designed as Figure 5. Gaussian primary membership functions are used, which have a fixed standard deviation \( \sigma = 0.01 \) and uncertain means that takes on values in the following intervals:

\[
[m^0_i, \bar{m}^0_i] = [0.027, 0.029], [m^1_i, \bar{m}^1_i] = [0.044, 0.046],
\]

\[
[m^2_i, \bar{m}^2_i] = [0.078, 0.080], [m^3_i, \bar{m}^3_i] = [0.095, 0.097],
\]

\[
[m^4_i, \bar{m}^4_i] = [0.112, 0.114], [m^5_i, \bar{m}^5_i] = [0.128, 0.130]
\]

2) Considering the linguistic and numerical uncertainty of the system, interval type-2 fuzzy membership functions of \( \hat{\mathbf{y}}_{\text{ankle}} \) are designed as Figure 6. Also, Gaussian primary membership functions are used here, which have a fixed standard deviation \( \sigma = 0.02 \) and uncertain means that takes on values in the following intervals:

\[
[m^0_i, \bar{m}^0_i] = [0, 0.002], [m^1_i, \bar{m}^1_i] = [0.048, 0.052],
\]

\[
[m^2_i, \bar{m}^2_i] = [0.098, 0.102], [m^3_i, \bar{m}^3_i] = [0.148, 0.152],
\]

\[
[m^4_i, \bar{m}^4_i] = [0.198, 0.200]
\]

3) Interval type-2 fuzzy membership functions for the positions of the trunk are designed as Figure 7. Still, Gaussian primary membership functions are used, which have a fixed standard deviation \( \sigma = 0.01 \) and uncertain means that takes on values in the following intervals:

\[
[m^0_i, \bar{m}^0_i] = [-0.0728, -0.0708], [m^1_i, \bar{m}^1_i] = [-0.0593, -0.0573],
\]

\[
[m^2_i, \bar{m}^2_i] = [-0.0468, -0.0448], [m^3_i, \bar{m}^3_i] = [-0.0343, -0.0323],
\]

\[
[m^4_i, \bar{m}^4_i] = [-0.0093, -0.0073], [m^5_i, \bar{m}^5_i] = [0.0032, 0.0052],
\]

\[
[m^6_i, \bar{m}^6_i] = [0.0157, 0.0177], [m^7_i, \bar{m}^7_i] = [0.0282, 0.0302]
\]

D. Analysis and comparisons of the dynamic biped balance

To analyze the performance index of the dynamic biped balance, mean of the ZMP stability margin (MZSM) is calculated during one whole walking step using the next formula:

\[
MZSM = \frac{1}{N} \sum_{k=1}^{N} \min \left( |x_{\text{toe}}(k)-x_{\text{heel}}(k)|, |x_{\text{heel}}(k)-x_{\text{toe}}(k)| \right)
\]

where \( x_{\text{toe}}(k) \) is the position of the ZMP on the \( k \)th sampling point in a walk cycle, which can be obtained using Eq.(1), and \( k = 1, 2, ..., N (N = 41) \). \( x_{\text{toe}} \) and \( x_{\text{heel}} \) are the positions of the toe and the heel of the biped robot.
Comparisons for MZSM using different methods are shown in Table 3. When the simulation is implemented without adding the process noises and the measurement noises, the T2FLCs exhibits better performance slightly than the other two methods, and the proposed method provides similar result as that of the T2FLC without SRUKF. However, when process noise $w(k) \sim (0, 10^{-4})$ and measurement noise $v(k) \sim (0, 10^{-4})$ are added in the simulated model, an interesting phenomenon occurs. Influenced by the added noises, the PID controller can not keep the MZSM to be positive, which means that the ZMP stability criterion can not be guaranteed any more. Intelligent controller like T1FLC and T2FLC without SRUKF lead to a positive MZSM with small value, which is dangerous for the biped because the ZMP is very close to the edge of the foot. On the other hand, the proposed method improves the MZSM remarkably. Compared with the traditional T2FLC, the proposed SRUKF-based T2FLC obtains better performance using “estimation and compensation” strategy.

6. Conclusions

An effective method for biped robot based on a predictable T2FLC is proposed to ensure the dynamic balance of the biped walking. Simulation results demonstrate the superiority of the proposed methods, and the proposed method shows superior performance when compared to the PID controller, the T1FLC and the T2FLC without SRUKF. We believe that the proposed framework will be very promising for dynamic balance of biped robot systems suffering from various uncertainties. Future works include automatic optimization on the predictable T2FLC.

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References


Liyang Wang received the B.S. degree in communication engineering from Tongji University, Shanghai, China, in 2002, the M.S. degree in electrical and communication engineering from Guangdong University of Technology, Guangzhou, China, in 2007. During 2010-2011, she was a Research Associate in Intelligent Systems Research Institute, Sungkyunkwan University, Korea. She is currently pursuing a Ph.D. degree in control theory and control engineering from Guangdong University of Technology, Guangzhou, China. Her research interests include robotics and learning control.

Zhi Liu received the B.S. degree from Huazhong University of Science and Technology, Wuhan, China, in 1997, the M.S. degree from Hunan University, Changsha, China, in 2000, and the Ph. D degree from Tsinghua University, Beijing, China, in 2004, all in electrical engineering.

He is currently a Professor in the Department of Automation, Guangdong University of Technology, Guangzhou, China. His research interests include fuzzy logic systems, neural networks, robotics, and robust control.

C. L. Philip Chen received the M.S. degree from the University of Michigan, Ann Arbor, in 1985, and the Ph.D. degree from Purdue University, West Lafayette, IN, in 1988. He is currently a Dean and Chair Professor with the Faculty of Science and Technology, University of Macau, Macau, China. He is also a Professor and the Chair of the Department of Electrical and Computer Engineering, Associate Dean for Research and Graduate Studies of the College of Engineering, University of Texas at San...
Antonio, San Antonio.

**Yun Zhang** received the B.S. and M.S. degrees in automatic engineering from Hunan University, Changsha, China, in 1982 and 1986, respectively, and the Ph.D. degree in automatic engineering from the South China University of Science and Technology, Guangzhou, China, in 1998.

He is currently a Professor with the Department of Automation, Guangdong University of Technology, Guangzhou, China. His research interests include intelligent control systems, network systems, and signal processing.

**Xin Chen** received the B.S. degree from Changsha Railway Institute, Hunan, China, in 1982, the M.S. degree from Harbin Institute of Technology, Heilongjiang, China, in 1988, the Ph.D. degree from Huazhong University of Science and Technology, Wuhan, China, in 1995, all in mechanical engineering.

He is currently a Professor with the Department of Mechatronics Engineering, Guangdong University of Technology, Guangzhou, China. His research interests include Mechatronics and robotics.