Adaptive Neuro-Fuzzy Formation Control for Leader-Follower Mobile Robots

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Abstract

This paper aims to investigate the formation control of multi-robot systems, where the kinematic model of a differentially driven wheeled mobile robot is considered. Based on the graph-theoretic concepts and locally distributed information, an adaptive neural fuzzy formation controller is designed with the capability of on-line learning. The learning rules of controller parameters can be derived from the analyzing of Lyapunov stability. In addition to simulations, the proposed techniques are applied to an experimental multi-robot platform for performance validations. From simulation and experimental results, the proposed adaptive neural fuzzy protocol can provide better formation responses compared to conventional consensus algorithms.

Keywords: Multi-robot systems, formation control, neural fuzzy control, Lyapunov stability.

1. Introduction

Over the last decade, robot control has attracted great attention for its applications in service robot, rescue robot, and unmanned autonomous vehicle, to name but a few. In designing robot system, the challenges how to derive the nonlinear model and how to deal with the uncertainty are repeatedly encountered. It is well known that classical control strategies are usually relied on the availability of precise models. To overcome this problem, fuzzy logic control is suitable for controlling a robot system due to the tolerance to noise and disturbance even under uncertainty. However, the design of fuzzy rules leaks systematic methodology and it is frequently reliant on heuristic experience. Therefore, in recent year, the neural fuzzy control that combining with the capability of fuzzy reasoning to handle uncertain nonlinear information and the capability of artificial neural networks to learn from processes has been popularly addressed in robot systems. A number of neuro-fuzzy logic approaches have been implemented for mobile robot systems. In [1], a neuro-fuzzy reasoning algorithm, having the advantage of greatly reducing the number of fuzzy rules, was proposed to fulfill the navigation task of mobile robots. In [2], a behavior-based neuro-fuzzy controller for mobile robot system was addressed for the navigation problem. In this work, a neuro-fuzzy method was applied to implement the behavioral function. A neuro-fuzzy algorithm was presented and implemented in an 8-bit microcontroller to control the mobile robot for multiple tasks [3]. The authors Alavander and Nigam proposed a new approach to deal with the problem of inverse kinematics in robot control by adaptive neuro-fuzzy method [4]. In the work of [5], a torque controller was presented to manipulate a PUMA-600 robotic arm using an adaptive neuro-fuzzy inference. In [6], a neural integrated fuzzy controller was successfully implemented for the navigation task of a multi-robot system under the information exchange in a complete topology. However, the stability of multi-robot system associated with the communication topology has not been well considered.

Distributed multi-agent coordination has attracted much attention in many fields, such as autonomous vehicles [7], wireless sensor networks [8], smart buildings [9], group decision making [10], multi-agent architecture with cooperative fuzzy control [11], and power systems [12], where only the information available locally is required for each agent. In general, information transmitted among agents is based on a communication topology that could be static or dynamic. Recently, graph theory has been used to describe network topologies in the study of consensus protocols [13]-[16]. A general consensus problem solving is to find a distributed control strategy such that the states of agents converge to a common value. In [13], average-consensus problem was investigated for distributed networks, in which the analysis of consensus protocols was based on the graph theory, matrix theory, and control theory. A distributed impulsive control
protocol was presented for multi-agent linear dynamic systems [17]. In [18], leader-following consensus problem was discussed for agents with a general $n$-th order linear model. Qin et. al. [19] discussed the condition of communication delays to achieve consensus for second-order agents under switching topology. In [20], consensus problems were addressed for a group of high-order dynamic agents with switching topology and time-varying communication delays. Moreover, linear consensus protocol and saturated consensus protocol were presented for a heterogeneous multi-agent system [21]. In most of the existing results, first- and second-order linear models are commonly considered to address the consensus stability of multi-agent systems. However, aforementioned the cooperative problem of multi-agent system associated with parameter uncertainty or external disturbance has not been well considered.

This paper aims to investigate the formation control of multi-robot systems with parameter uncertainty or external disturbance, where the kinematic model of a differential wheeled robot is considered. The graph theory is used to model the communication topology between robots. To improve the control performance, a novel formation algorithm, adaptive neural fuzzy formation control, is proposed for multi-robot systems in directed graphs. The proposed formation controller has the capability of on-line learning, and the adaptive learning rules can be derived using the Lyapunov stability analysis. An experimental platform is also applied to validate the formation performance.

This paper is organized as follows. In Sec. 2, some graph-theoretic concepts and a differential wheeled robot with kinematics are introduced. In Sec. 3, the framework of an adaptive neural fuzzy formation control is presented. In Sec. 4, the stability analysis of leader-follower mobile robots are investigated. In Sec. 5, simulation and experimental results are provided for performance validations. Some concluding remarks are given in Sec. 6.

2. Preliminaries

A. Graph theory

In this section, some fundamental concepts in algebraic graph theory used for multi-agent systems will be introduced. Considering a multi-agent system of $n$ agents, let $G=(V,E)$ be a directed graph (digraph), consisting of a vertex set $V=\{v_1,v_2,\ldots,v_n\}$ and an edge set $E \subseteq V \times V$. The vertexes $v_i$ and $v_j$ represent the $i$th and $j$th agents, respectively. In digraphs, an edge of $G$ is an ordered pair of distinct nodes $(v_i,v_j) \in E$, in which $v_i$ and $v_j$ are the head and tail of the edge, respectively [14]. The weighted adjacency matrix of a digraph $G$ is denoted as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \in \mathbb{R}^{n \times n},$$  \hspace{1cm} (1)

where $a_{ij}$ is the weight of link $(v_i,v_j)$,

$\begin{align*}
    a_{ij} = 1, & \quad (v_i,v_j) \in E \\
    a_{ij} = 0, & \quad (v_i,v_j) \notin E
\end{align*}$

The degree matrix of $G$ is a diagonal matrix, $D=\text{diag}\{d_1,d_2,\ldots,d_n\} \in \mathbb{R}^{n \times n}$, where the in-degree of node $v_j$ is defined as

$$d_j = \sum_{i=1}^{n} a_{ij}, i=1,2,\ldots,n.$$  \hspace{1cm} (2)

Then the Laplacian matrix associated with the digraph $G$ is defined as $L = D - A \in \mathbb{R}^{n \times n}$. In this paper, a leader-follower problem will be dealt with, where the multi-agent system consists of $n$ agents, one leader and $n-1$ followers. In notations, the agents indexed by $1,2,\ldots,n-1$ are followers and the item $n$ is the leader. Assume that the leader agent has only transmitting capability, i.e. the leader acquire no information from followers, $a_{ij}=0$, $j=1,\ldots,n$ . In this case, let the topology relationship of follower agents be denoted as $\overline{G}$, a subgraph of $G$. Then, the associated adjacency matrix of $\overline{G}$ is represented as

$$\overline{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(n-1)} \\ a_{21} & a_{22} & \cdots & a_{2(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{(n-1)1} & a_{(n-1)2} & \cdots & a_{(n-1)(n-1)} \end{bmatrix} \in \mathbb{R}^{(n-1) \times (n-1)},$$  \hspace{1cm} (3)

Similarly, let $\overline{D}=\text{diag}\{\overline{d}_1,\overline{d}_2,\ldots,\overline{d}_{n-1}\}$ be the degree matrix of digraph $\overline{G}$, where $\overline{d}_i=\sum_{j=1}^{n-1} a_{ij}$, $i=1,2,\ldots,n-1$. The Laplacian matrix of the digraph $\overline{G}$ can be analogously defined as $\overline{L} = \overline{D} - \overline{A}$. Then the following condition holds

$$\overline{L}1_{n-1} = 0,$$  \hspace{1cm} (4)

where $1_{n-1}=[1\cdots1]^T \in \mathbb{R}^{n-1}$. Consequently, the connection relationship between the leader and followers can be described as $\overline{B}=\text{diag}\{\overline{b}_1,\overline{b}_2,\ldots,\overline{b}_{n-1}\}$, where $\overline{b}_i=a_{i,n}$, $i=1,2,\ldots,n-1$.

B. Kinematic model of wheeled mobile robot

Considering a group of $n$ identical wheeled mobile robots, the robot structure is shown in Fig. 1. The
kinematic characteristics of the \( i \)th robot are described by
\[
\begin{align*}
\dot{x}_i(t) &= v_i(t) \cos(\theta_i(t)), \\
\dot{y}_i(t) &= v_i(t) \sin(\theta_i(t)), \\
\dot{\theta}_i(t) &= \omega_i(t),
\end{align*}
\]
(5)
in which \( l_i \) is the length between the center position and the head position. Taking the derivative of (6), the governing equations of the head position for the \( i \)th robot can be determined as follows
\[
\begin{align*}
\dot{x}_i(t) &= \cos(\theta_i(t)) l_i \sin(\theta_i(t)) v_i(t), \\
\dot{y}_i(t) &= \sin(\theta_i(t)) l_i \cos(\theta_i(t)) v_i(t),
\end{align*}
\]
(7)

According to the configuration in Fig. 1, the motion equations of \( v_i(t) \) and \( \omega_i(t) \) corresponding to the left- and right-wheel angular velocities, \( \omega_L(t) \) and \( \omega_R(t) \), can be obtained as
\[
\begin{align*}
\dot{v}_i(t) &= -0.5r \omega_L(t) + 0.5r \omega_R(t), \\
\dot{\omega}_i(t) &= \frac{1}{I_a} \left[ T_{1i}(t) - T_{2i}(t) \right],
\end{align*}
\]
(8)
where \( r \) is the radius of wheel, and \( l_a \) is the distance between the center of robot and the wheel. Substituting (8) into (7) and defining \( l_a = 0.5l_i/l_r \), it leads to the following equivalent model of a wheeled robot
\[
\begin{align*}
\dot{x}_i(t) &= -l_a (l_i \sin(\theta_i(t))) v_i(t), \\
\dot{y}_i(t) &= -l_a (l_i \cos(\theta_i(t))) v_i(t), \\
\dot{\theta}_i(t) &= \frac{1}{l_a} \left[ T_{1i}(t) - T_{2i}(t) \right],
\end{align*}
\]
(9)
where \( T_{1i}(t) = 0.5r \cos(\theta_i(t)) + rl_a \sin(\theta_i(t)) \), \( T_{2i}(t) = 0.5r \cos(\theta_i(t)) - rl_a \sin(\theta_i(t)) \).

(From 9), the perturbed model of a wheeled robot can be represented as [22]
\[
\begin{align*}
\dot{x}_i(t) &= (T_i(t) + \Delta T_i(t)) \omega_i(t), \\
\dot{y}_i(t) &= (T_i(t) + \Delta T_i(t)) \omega_i(t), \\
\dot{\theta}_i(t) &= \frac{1}{l_a} \left[ T_{1i}(t) - T_{2i}(t) \right] \omega_i(t),
\end{align*}
\]
(10)
in which \( \Delta T_i(t) \) is the perturbation term of augmented uncertainties.

Let
\[
\begin{align*}
u_i(t) &= T_i(t) \omega_i(t) \delta_i(t) + \Delta T_i(t) \omega_i(t),
\end{align*}
\]
where \( u_i(t) \) and \( v_i(t) \) are considered as control actions to the head position, and \( \delta_i(t) \) and \( \delta_i^\prime(t) \) are described as the uncertainty terms of the \( x \)-axis and \( y \)-axis, respectively. Therefore, the kinematic equations of a wheeled robot with uncertainties can be described in the following compact form,
\[
\begin{align*}
\dot{x}_i(t) &= u_i(t), \\
\dot{y}_i(t) &= v_i(t) + \delta_i(t), \\
\dot{\theta}_i(t) &= \frac{1}{l_a} \left[ T_{1i}(t) - T_{2i}(t) \right] \omega_i(t).
\end{align*}
\]
(11)

It is noted that the kinematic equations of a wheeled robot in (11) can be decoupled into two one-dimensional subsystems. To derive consensus controllers, it suffices to consider only the subsystem in the following.

3. Adaptive Neural Fuzzy Formation Control

In this section, an adaptive neural fuzzy formation control (ANFFC) is proposed to deal with the leader-following formation problem, where the perturbed kinematic model of (11) is considered. First, let the \( x \)-axis error functions be defined as
\[
\begin{align*}
e_{x, j}(t) &= \sum_{j \neq i} a_i [(x_i(t) - p_{x,i}) - (x_j(t) - p_{x,j})], \\
&+ \bar{b}_i [(x_i(t) - p_{x,i}) - (x_m(t) - p_{x,m})],
\end{align*}
\]
(12)
where \( p_{x,i} \) indicates the coordinate position regarding to a desired formation pattern in \( x \)-axis, \( i = 1, 2, \ldots, n \). It is noticed that \( a_{y, i} = 1 \) means that the \( j \)th robot can send position information to the \( i \)th robot. In addition, \( \bar{b}_i = 1 \) implies that the leader robot can send position information to the \( i \)th robot. The controller input of the \( i \)th robot is designated as follows
\[
S_i = c_1 e_{x,i} + c_2 \int e_{x,i} dt,
\]
(13)
in which \( c_1 \) and \( c_2 \) are positive constants.

The network structure of neural fuzzy control (NFC) is shown in Fig. 2. The fuzzy rules are given in Table 1, where the input and output spaces are fuzzily partitioned into six fuzzy sets, Negative Big (NB), Negative
Medium (NM), Negative Small (NS), Positive Small (PS), Positive Medium (PM) and Positive Big (PB). The input and output membership functions are depicted in Fig. 3.

Table 1. Fuzzy rule base.

<table>
<thead>
<tr>
<th>$S_{ik}$ ($S_{yi}$)</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{nc,NFC}$ ($y_{yi,NFC}$)</td>
<td>PB</td>
<td>PM</td>
<td>PS</td>
<td>NS</td>
<td>NM</td>
<td>NB</td>
</tr>
</tbody>
</table>

Using the centroid defuzzification technique, the NFC output can be calculated as follows:

$$u_{nc,NFC}(S_{yi}, \xi_{yi}, \theta_{yi}) = \frac{\sum_{k=1}^{n} \xi_{yi,k} \theta_{yi,k}}{\sum_{k=1}^{n} \xi_{yi,k}} = \theta_{yi}^{*} \xi_{yi}, \quad (16)$$

where $\theta_{yi,k}$ are the values of the corresponding output fuzzy singletons, $\theta_{yi} = [\theta_{yi,1}, \theta_{yi,2}, \ldots, \theta_{yi,6}]^T$ and

$$\xi_{yi} = [\xi_{yi,1} / \xi_{yi,2} / \xi_{yi,3} / \xi_{yi,4} / \xi_{yi,5} / \xi_{yi,6}]^T, \quad \hat{\xi}_{yi} = \sum_{k=1}^{6} \xi_{yi,k}.$$

Since the robot kinematics is transformed to a collection of decoupled subsystems as (11), it suffices to design an ANFFC for one subsystem. Consider that

$$\hat{u}_{nc,NFC} = u_{nc,NFC}(S_{yi}, \xi_{yi}, \theta_{yi}) = \hat{u}_{nc,NFC}(S_{yi}, \hat{\xi}_{yi}, \hat{\theta}_{yi})$$

$$= \theta_{yi}^{*} \hat{\xi}_{yi} - \hat{\theta}_{yi} \hat{\xi}_{yi}, \quad (17)$$

where $u_{nc,NFC}$ and $\hat{u}_{nc,NFC}$ are the optimal and estimated NFC control actions, respectively,

$$\xi_{yi} = [\xi_{yi,1} / \xi_{yi,2} / \xi_{yi,3} / \xi_{yi,4} / \xi_{yi,5} / \xi_{yi,6}]^T,$$

$$\xi_{yi} = [\hat{\xi}_{yi,1} / \hat{\xi}_{yi,2} / \hat{\xi}_{yi,3} / \hat{\xi}_{yi,4} / \hat{\xi}_{yi,5} / \hat{\xi}_{yi,6}]^T,$$

$$\theta_{yi} = [\theta_{yi,1} / \theta_{yi,2} / \theta_{yi,3} / \theta_{yi,4} / \theta_{yi,5} / \theta_{yi,6}]^T,$$

$$\hat{\theta}_{yi} = [\hat{\theta}_{yi,1} / \hat{\theta}_{yi,2} / \hat{\theta}_{yi,3} / \hat{\theta}_{yi,4} / \hat{\theta}_{yi,5} / \hat{\theta}_{yi,6}]^T.$$

It is noticed that $\hat{\xi}_{yi}$, $\hat{\theta}_{yi}$, and $u_{nc,NFC}$ are unknown optimal parameters and control output. In addition, $\hat{\xi}_{yi}$ and $\hat{\theta}_{yi}$ are the estimated parameters corresponding to $\xi_{yi}$ and $\theta_{yi}$, respectively. Let the estimation errors of controller parameters be defined as $\hat{\xi}_{yi} = \xi_{yi} - \hat{\xi}_{yi}$ and $\hat{\theta}_{yi} = \theta_{yi} - \hat{\theta}_{yi}$.

The vector of Gaussian functions is linearized using Taylor expansion, $\hat{\xi}_{yi}$ can be written as [23]

$$\hat{\xi}_{yi} = G_{yi} \hat{\xi}_{yi} + H_{yi} \hat{\sigma}_{yi} + \text{h.o.t.}, \quad (18)$$

in which

$$G_{yi} = \begin{bmatrix} \frac{\partial \hat{\xi}_{yi}}{\partial C_{yi}} & \frac{\partial \hat{\xi}_{yi}}{\partial C_{yi}} & \ldots & \frac{\partial \hat{\xi}_{yi}}{\partial C_{yi}} \\ \frac{\partial \hat{\xi}_{yi}}{\partial \sigma_{yi}} & \frac{\partial \hat{\xi}_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \hat{\xi}_{yi}}{\partial \sigma_{yi}} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial \hat{\xi}_{yi}}{\partial \sigma_{yi}} & \frac{\partial \hat{\xi}_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \hat{\xi}_{yi}}{\partial \sigma_{yi}} \end{bmatrix}_{yi = \hat{\xi}_{yi}},$$

$$H_{yi} = \begin{bmatrix} \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} \\ \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} \end{bmatrix}_{yi = \hat{\xi}_{yi}},$$

$G_{yi}$ and $H_{yi}$ are matrices of $R^{6 \times 6}$, $\hat{C}_{yi}$, $\hat{\sigma}_{yi}$, $\hat{C}_{yi}$, $\hat{\sigma}_{yi}$, $\hat{C}_{yi}$, $\hat{\sigma}_{yi}$, and

$$\frac{\partial \xi_{yi}}{\partial C_{yi}} = \begin{bmatrix} \frac{\partial \xi_{yi}}{\partial C_{yi}} & \frac{\partial \xi_{yi}}{\partial C_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial C_{yi}} \\ \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} \end{bmatrix},$$

$$\frac{\partial \xi_{yi}}{\partial \sigma_{yi}} = \begin{bmatrix} \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} \\ \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} & \ldots & \frac{\partial \xi_{yi}}{\partial \sigma_{yi}} \end{bmatrix}.$$

The corresponding if-then fuzzy rules for the $i$th robot are expressed as

$$R_{ik} : \text{IF} \ S_{yi} \text{ is } M_{yi} \ \text{ THEN } u_{nc,NFC} \text{ is } G_{yi} \quad (14)$$

where $M_{yi}$ and $G_{yi}$ are the fuzzy sets of antecedent and consequent parts, respectively, $i = 1, 2, \ldots, n-1$, $k = 1, 2, \ldots, 6$.

In Fig. 3, the $k$th nodes of the membership layer are Gaussian functions represented as

$$\hat{\xi}_{yi} = \hat{\xi}_{yi}(S_{yi}, \sigma_{yi}, C_{yi}) = \exp \left[ - \left( \frac{S_{yi} - C_{yi}}{\sigma_{yi}} \right)^2 \right], \quad (15)$$

where $\sigma_{yi} = [\sigma_{yi,1}, \sigma_{yi,2}, \ldots, \sigma_{yi,6}]^T$, $C_{yi} = [C_{yi,1}, C_{yi,2}, \ldots, C_{yi,6}]^T$, $i = 1, 2, \ldots, n-1$. In (15), $C_{yi}$ and $\sigma_{yi}$ are means and standard deviations, respectively, $k = 1, 2, \ldots, 6$.
In (18), the term h.o.t. stands for the high-order terms associated with the Taylor expansion. From (17) and (18), it can be obtained that
\[
\hat{u}_{a,NFC} = u_{a,NFC} - \hat{u}_{a,NFC} = (\hat{\theta}_a + \hat{\theta}_a \hat{\xi}_a + \hat{\theta}_a \hat{\xi}_a) - \hat{\theta}_a \hat{\xi}_a
\]
\[
= \hat{\theta}_a(G_{a} \hat{C}_a + H_a \hat{\sigma}_a + h.o.t.) + (\hat{\theta}_a + \hat{\theta}_a \hat{\xi}_a) \hat{\xi}_a.
\]
Let \( \epsilon_a = \hat{\theta}_a(h.o.t.) + \hat{\theta}_a \hat{\xi}_a \). The proposed ANFFC for the \( x \)-axis dynamic equation of (11) is designed as
\[
u_{a}(t) = \left[ c_1 \left( \bar{b}_j + \sum_{j=1}^{n} a_{ij} \right) \right]^{-1} \left[ \hat{\theta}_a \hat{\xi}_a \right] + \hat{\theta}_a(G_{a} \hat{C}_a + H_a \hat{\sigma}_a) + \epsilon_a + u_{a}(t) - c_1 \epsilon_a
\]
+ \( c_1 \sum_{j=1}^{n} a_{ij} \hat{u}_{a,ij} + c_1 \hat{\theta}_a u_{a} \)\].

From (11) and (21), the derivative of (13) can be rewritten as
\[
\dot{S}_a = \hat{\theta}_a(G_{a} \hat{C}_a + H_a \hat{\sigma}_a) + \hat{\theta}_a \hat{\xi}_a + \epsilon_a + u_{a}(t)
\]
+ \( c_1 \sum_{j=1}^{n} a_{ij} (\hat{\sigma}_a - \hat{\sigma}_a) + \bar{b}_j (\hat{\sigma}_a - \hat{\sigma}_a) \).

Then (22) can be equivalently written as
\[
\dot{S}_a = \hat{\theta}_a(G_{a} \hat{C}_a + H_a \hat{\sigma}_a) + \hat{\theta}_a \hat{\xi}_a + u_{a} + \eta_a,
\]
where
\[
\eta_a = \epsilon_a + c_1 \sum_{j=1}^{n} a_{ij} (\hat{\sigma}_a - \hat{\sigma}_a) + \bar{b}_j (\hat{\sigma}_a - \hat{\sigma}_a),
\]
\( \eta_a \) is an uncertain term. Suppose \( \eta_a \) is bounded such that \( |\eta_a| \leq E_a \) for a constant \( E_a \). However, \( E_a \) is unknown a priori, and then it is rather to be estimated. Denoted \( \hat{E}_a \) as the estimation of \( E_a \), the estimation error is defined as
\[
\check{E}_a = E_a - \hat{E}_a.
\]

The adaptive laws and compensation control \( u_{a}(t) \) for the dynamic equation of (11) are designed as follows
\[
\dot{\hat{u}}_a = r_1 |S_a| \hat{C}_a = r_1 S_a G^T \hat{\mu}_a, \quad \hat{\sigma}_a = r_1 S_a H^T \hat{\mu}_a,
\]
\[
\dot{\hat{\theta}}_a = r_2 \hat{S}_a \hat{\xi}_a, \quad \hat{u}_a = -\check{E}_a \text{sgn}(S_a).
\]
where \( r_1 > 0 \), \( i = 1, 2, 3, 4 \).

**Theorem 1:** Consider the \( x \)-axis dynamic equation of (11). With the ANFFC and adaptive laws of neural fuzzy networks designed as (20) and (25), the \( x \)-axis formation stability of the \( i \)-th robot is guaranteed. **Proof:** A Lyapunov function candidate is chosen as
\[
V_a = \frac{1}{2} \hat{S}_a^T \hat{C}_a + \hat{\theta}_a^T \hat{\theta}_a + \hat{\sigma}_a^T \hat{\sigma}_a + \hat{\theta}_a^T \hat{\theta}_a,
\]
\( i = 1, 2, \ldots, n - 1 \). From (23), the derivative of (26) can be obtained as
\[
\dot{V}_a = \check{C}_a (S_a G_a \hat{\mu}_a - r_2 \hat{\sigma}_a) + \hat{\sigma}_a (S_a H_a \hat{\mu}_a - r_2 \hat{\theta}_a)
\]
\[
+ \hat{\theta}_a (S_a \hat{\xi}_a - r_2 \hat{\theta}_a) + S_a (u_{a} + \eta_a) - r_1 \hat{E}_a \hat{E}_a.
\]
Substituting (25) into (27), it gives that
\[
\dot{V}_a = S_a (u_{a} + \eta_a) - \frac{1}{r_1} \hat{E}_a \hat{E}_a.
\]
Furthermore, substituting (25) into (28), it leads to
\[
\dot{V}_a = S_a (u_{a} + \eta_a) - r_1 \hat{E}_a \hat{E}_a
\]
\[
= S_a (-\hat{E}_a \text{sgn}(\eta_a) + \eta_a - r_1 (S_a \hat{E}_a - \hat{E}_a)) \hat{E}_a
\]
\[
= -\dot{\hat{E}}_a \text{sgn}(\eta_a) + \eta_a - S_a \text{sgn}(\eta_a) + S_a \hat{E}_a
\]
\[
\leq S_a (|\eta_a| - |\eta_a|).
\]
From the assumption that \( |\eta_a| \leq E_a \), it can be concluded that \( \dot{V}_a \) is negative semi-definite. From the negative semi-definiteness of \( \dot{V}_a \), it implies that \( S_a \), \( \hat{E}_a \), \( \hat{C}_a \), \( \hat{\sigma}_a \), and \( \hat{\theta}_a \) are all bounded. Furthermore, it can be obtained that \( \dot{V}_a \) is also bounded. Then using the Barbalat's lemma [24], it can be concluded that \( \dot{V}_a \to 0 \) as \( t \to \infty \), i.e. \( S_a \to 0 \) and \( e_a \to 0 \) as \( t \to \infty \). Hence, it can be concluded that the \( x \)-axis formation stability of the \( i \)-th robot is guaranteed.

**Remark 1:** The proposed ANFFC for the \( y \)-axis dynamic equation of (11) is designed as
\[
u_{a,NFC}(t) = \left[ c_1 \left( \bar{b}_j + \sum_{j=1}^{n} a_{ij} \right) \right]^{-1} \left[ \hat{\theta}_a \hat{\xi}_a \right] + \check{E}_a \text{sgn}(S_a)
\]
+ \( c_1 \sum_{j=1}^{n} a_{ij} \hat{u}_{a,ij} + c_1 \check{E}_a u_{a} \)\].

The adaptive laws and compensation control \( u_{a}(t) \) for the \( y \)-axis dynamic equation of (11) are designed as follows
\[
\dot{\hat{u}}_a = r_1 |S_a| \hat{C}_a = r_2 S_a G^T \hat{\mu}_a, \quad \hat{\sigma}_a = r_2 S_a H^T \hat{\mu}_a,
\]
\[
\dot{\hat{\theta}}_a = r_2 \hat{S}_a \hat{\xi}_a, \quad \hat{u}_a = -\check{E}_a \text{sgn}(S_a),
\]
where \( r_2 > 0 \), \( i = 1, 2, 3, 4 \).

As a result, the guaranteed stability of the \( i \)-th robot implies that the required formation goal for the \( i \)-th robot is guaranteed.
robot can be asymptotically achieved by employing the proposed ANFFC approach.

4. Stability Analysis of Leader-Follower Mobile Robots

The proposed ANFFC protocol can be extended to multi-robot systems with one leader and \( n-1 \) followers. From (20) and (30), the output of ANFFC in a multi-robot system can be grouped together as

\[
U = [c_1((\bar{D} + \bar{B}) \otimes I_n)](-c_2((\bar{D} + \bar{B}) \otimes I_n)(\zeta - \Gamma)) + c_3([\bar{B} I_{n+1} \otimes I_n](Z_n - P_n) + c_4(\bar{A} \otimes I_n)U)
\]

(32)

where the symbol \( \otimes \) stands for the Kronecker product, \( I_n = \text{diag}[1,1,1, \ldots, 1] \), and

\[
\hat{U}_{\text{ANFFC}} = [\hat{U}_{\text{ANFFC}}^T \hat{U}_{\text{ANFFC}} \ldots \hat{U}_{\text{ANFFC}}^T] \in \mathbb{R}^{2(n-1)},
\]

\[
U_i = [U_{i1}^T U_{i2}^T \ldots U_{(n-1)}^T] \in \mathbb{R}^{2(n-1)},
\]

\[
\zeta = [Z_1^T Z_2^T \ldots Z_{n-1}^T] \in \mathbb{R}^{2(n-1)},
\]

\[
\Gamma = [P_1^T P_2^T \ldots P_{n-1}^T] \in \mathbb{R}^{2(n-1)},
\]

\[
U = [U_{11}^T U_{12}^T \ldots U_{(n-1)}^T] \in \mathbb{R}^{2(n-1)},
\]

and

\[
\hat{U}_{\text{ANFFC}} = [\hat{U}_{n,\text{ANFFC}} \hat{U}_{n,\text{ANFFC}} \ldots \hat{U}_{n,\text{ANFFC}}] \in \mathbb{R}^{2(n-1)},
\]

(33)

Further, the term on the left-hand side of (33) can be rewritten as

\[
[U_{2(n-1)} - [c_1((\bar{D} + \bar{B}) \otimes I_n)]([c_4(\bar{A} \otimes I_n)]U)
\]

\[
= [c_1((\bar{D} + \bar{B}) \otimes I_n)]([c_4(\bar{A} \otimes I_n)]U)
\]

\[
= [c_1((\bar{D} + \bar{B}) \otimes I_n)]([c_4(\bar{A} \otimes I_n)]U)
\]

(34)

Substituting (34) into (33), the control actions to a leader-following multi-robot system can be obtained as

\[
U = [c_1((\bar{D} + \bar{B}) \otimes I_n)]([c_4(\bar{A} \otimes I_n)]U)
\]

\[
+ c_2([\bar{B} I_{n+1} \otimes I_n](Z_n - P_n) + c_4(\bar{A} \otimes I_n)U).
\]

(35)

It is known that if graph \( G \) has a spanning tree, then a right eigenvector of \( L \) with the zero eigenvalue is \( 1_n \). In other words, \( \text{Rank}(L) = n-1 \) if and only if \( L \) has a simple zero eigenvalue \([14]\). In a leader-follower system, \( L \) can be rewritten as

\[
L = \begin{bmatrix}
    d_1 & -a_{12} & \cdots & -a_{1(n-1)} & -a_{1n} \\
    -a_{21} & d_2 & \cdots & -a_{2(n-1)} & -a_{2n} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    -a_{(n-1)1} & -a_{(n-1)2} & \cdots & d_{(n-1)} & -a_{(n-1)n} \\
    0 & 0 & \cdots & 0 & 0
\end{bmatrix}
\]

(36)

It means that \( \text{Rank}(L) = \text{Rank}(\bar{L} + \bar{B}) = n-1 \) and \( (\bar{L} + \bar{B}) \) is invertible. Thus, the ANFFC of (35) is computable. The adaptive laws in a multi-robot system can be grouped together as

\[
\hat{E} = r \hat{r} | S |
\]

\[
= r [S_1 | S_2 | \ldots | S_{(n-1)} | S_{(n-1)}] \hat{r}^T,
\]

\[
\dot{\hat{C}} = r \hat{C} \hat{F} \hat{\xi} \hat{r} \hat{F} \hat{\xi} \hat{r},
\]

(37)

where \( \hat{E} = [\hat{E}_1^T \cdots \hat{E}_{n-1}^T] \) and \( S = [S_1^T \cdots S_{n-1}^T] \) are vectors of \( \mathbb{R}^{2(n-1)} \); \( \hat{C} = [\hat{C}_1^T \cdots \hat{C}_{n-1}^T] \) \( \hat{r} = [\hat{r}_1^T \cdots \hat{r}_{n-1}^T] \), \( \hat{\xi} = [\hat{\xi}_1^T \cdots \hat{\xi}_{n-1}^T] \) are vectors of \( \mathbb{R}^{n-1} \); \( \hat{\xi}_i, \hat{\xi}_i, \hat{\xi}_i, \hat{\xi}_i, \) and \( \hat{\xi}_i, \hat{\xi}_i, \hat{\xi}_i, \hat{\xi}_i \) are vectors of \( \mathbb{R}^{n-1} \). Moreover,

\[
F = \begin{bmatrix}
    F_1 & 0 & \cdots & 0 \\
    0 & F_2 & \cdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & \cdots & F_{n-1} & 0
\end{bmatrix},
\]

\[
G = \begin{bmatrix}
    G_1 & 0 & \cdots & 0 \\
    0 & G_2 & \cdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & \cdots & 0 & G_{n-1}
\end{bmatrix},
\]

\[
H = \begin{bmatrix}
    H_1 & 0 & \cdots & 0 \\
    0 & H_2 & \cdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & \cdots & 0 & H_{n-1}
\end{bmatrix}
\]

and

\[
F_i = \text{diag}(S_{i1}, S_{i2}, S_{i3}, S_{i4}, \ldots, S_{in}, S_{in}),
\]

\[
G_i = \text{diag}(\hat{C}_{i1}, \hat{C}_{i2}, \hat{C}_{i3}, \hat{C}_{i4}, \ldots, \hat{C}_{in}, \hat{C}_{in})
\]
It is noticed that $F_{i}$, $G_{i}$, and $H_{i}$ are matrices of $\mathcal{R}^{12\times12}$, $i = 1,2, \ldots, n-1$.

The compensation control $U_{c}(i)$ in a multi-robot system can be grouped together as

$$
U_{c} = \begin{bmatrix}
\begin{array}{c}
u_{c1} \\
\vdots \\
\nu_{c(n-1)}
\end{array}
\end{bmatrix} = \begin{bmatrix}
\begin{array}{c}
-\tilde{E}_i \text{sgn}(S_{i1}) \\
-\tilde{E}_i \text{sgn}(S_{i2}) \\
\vdots \\
-\tilde{E}_{i(n-1)} \text{sgn}(S_{i(n-1)})
\end{array}
\end{bmatrix}.
$$

(38)

In matrix forms, $\tilde{E}_{i}$, $\tilde{C}_{i}$, $\tilde{\sigma}$, and $\tilde{\theta}$ in multi-robot systems can be represented as

$$
\tilde{E} = E - \tilde{E}, \quad \tilde{C} = C - \tilde{C}, \quad \tilde{\sigma} = \sigma - \tilde{\sigma}, \quad \tilde{\theta} = \theta - \tilde{\theta},
$$

where $E = [E_{1}^{T} \cdots E_{n}^{T}]^{T} \in \mathcal{R}^{2(n+1)}$; $\theta^{*} = [\theta_{1}^{*T} \cdots \theta_{n}^{*T}]^{T}$; $C^{*} = [C_{u}^{*T} \cdots C_{u}^{*T}]^{T}$, and $\sigma^{*} = [\sigma_{u}^{*T} \cdots \sigma_{u}^{*T}]^{T}$ are vectors of $\mathcal{R}^{12(n+1)}$; $E_{i} = [E_{i} \ E_{i}]^{T}$ is a vector of $\mathcal{R}^{2}$; $C_{i}^{*} = [C_{u}^{*T} C_{y}^{*T}]^{T}$, $\sigma^{*} = [\sigma_{u}^{*T} \sigma_{y}^{*T}]^{T}$, and $\theta^{*} = [\theta_{u}^{*T} \theta_{y}^{*T}]^{T}$, $i = 1,2, \ldots, n-1$, are vectors of $\mathcal{R}^{12}$.

**Theorem 2:** Considering the dynamic equation of (11), it is assumed that the communication graph of a multi-robot system has a directed spanning tree. With the ANFFC and adaptive law designed as (35) and (37)-(38), the stability of leader-follower formation control is guaranteed.

**Proof:** To verify the overall stability of an ANFFC-controlled multi-robot system, a Lyapunov function candidate is chosen as

$$
V = \frac{S^{T} S}{2} + \frac{\tilde{E}^{T} \tilde{E}}{2r_{C}} + \frac{\tilde{C}^{T} \tilde{C}}{2r_{C}} + \frac{\tilde{\sigma}^{T} \tilde{\sigma}}{2r_{C}} + \frac{\tilde{\theta}^{T} \tilde{\theta}}{2r_{C}}.
$$

(40)

From (35), the derivative of (40) can be obtained as

$$
\dot{V} = \tilde{C}^{T} (F \tilde{C}^{T} \tilde{C} \dot{\theta} - E \tilde{E}^{T} \tilde{C}) + \tilde{\sigma}^{T} (F \tilde{H}^{T} \tilde{H} \dot{\sigma} - E \tilde{E}^{T} \tilde{\sigma}) + \tilde{\theta}^{T} (F \tilde{E} \dot{\theta} - E \dot{E}^{T} \tilde{E}) + \dot{\eta}^{T} (S^{T} + \eta) - r_{C}^{-1} \tilde{E}^{T} \tilde{E},
$$

(41)

where $\eta = [\eta_{u}^{T} \ \eta_{y}^{T} \ \eta_{u1}^{T} \ \eta_{u2}^{T} \ \cdots \ \eta_{u(n+1)}^{T} \ \eta_{y(n+1)}^{T}]^{T}$.

Furthermore, substituting (37)-(38) into (41), it leads to

$$
\dot{V} \leq S^{T} \ |(\eta \ | -E) \leq 0.
$$

(42)

From (42), implies that $S$, $\tilde{E}$, $\tilde{C}$, $\tilde{\sigma}$, and $\tilde{\theta}$ are all bounded, $i = 1,2, \ldots, n-1$. Furthermore, it can be obtained that $\dot{V}$ is also bounded. Then using the Barbata's lemma [24], it can be concluded that $V \rightarrow 0$ as $t \rightarrow \infty$, i.e. $S_{u} \rightarrow 0$, $S_{y} \rightarrow 0$, $\dot{\eta}_{u} \rightarrow 0$, and $\dot{\eta}_{y} \rightarrow 0$, as $t \rightarrow \infty$, $i = 1,2, \ldots, n-1$. From (12), the error function in a multi-robot system can be grouped together as

$$
\Xi = [(\overline{L} + \overline{B}) \otimes I_{j}] (\zeta - \Gamma) - (\overline{B} I_{ii} \otimes I_{j}) (Z_{e} - P_{s}).
$$

(43)

where $0_{n \times (n-1)} = [0 \cdots 0 \ 0]^{T} \in \mathcal{R}^{2(n+1)}$, $\Xi = [\Xi_{u} \ \Xi_{y}]^{T}$, $\Xi_{u} = [e_{u} \ e_{y}]^{T}$, and $i = 1,2, \ldots, n-1$. From (4), it can be obtained that $[(\overline{L} + \overline{B}) \otimes I_{j}] (\zeta - \Gamma) = [(\overline{L} + \overline{B}) \otimes I_{j}] (I_{n} \otimes I_{j}) (Z_{e} - P_{s})$. (44)

It is noticed that $(\overline{L} + \overline{B})$ is invertible. From (44), it can be equivalently obtained that

$$
\zeta = (I_{n} \otimes I_{j}) (Z_{e} = [\Gamma (I_{n} \otimes I_{j})] (P_{s})
$$

(45)

It concludes that formation control in the multi-robot systems is guaranteed. Hence the stability of the multi-robot systems is guaranteed, and the leader-following formation control can be asymptotically achieved using the proposed ANFFC protocol.

### 5. Simulation and Experimental Results

#### A. Simulation results

The case of four robots, one leader and three followers, is considered, where $r = 0.029$, $l_{u} = 0.034$, and $l_{y} = 0.04$ (m). The communication topology is shown in Fig. 4, where the circles labeled 1 to 3 denote the follower robots and the circle 4 represents the leader robot.

![Communication topology of multi-robot systems](image)

Figure 4. Communication topology of multi-robot systems.

It is clear that the information exchange graph forms a spanning tree, where the adjacency matrix and Laplacian matrix can be determined as follows

$$
A = \begin{bmatrix}
0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad L = \begin{bmatrix}
2 & -1 & 0 & -1 \\
-1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0
\end{bmatrix}.
$$

Consequently, it can be obtained that

$$
\delta = \begin{bmatrix}
0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad \delta = \begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad \delta = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}.
$$

The follower robots are initially placed on $(0,0.1)$, $(0,0.8)$, and $(0,0.3)(m)$, respectively, and the initial
position of leader is \((0, 0.6) (m)\). The formation pattern are designated as
\((p_{s1}, p_{s2}) = (-0.2, 0.2), (p_{s3}, p_{s4}) = (-0.4, 0), (p_{s5}, p_{s6}) = (-0.2, -0.2), \text{ and } (p_{s7}, p_{s8}) = (0, 0)(m)\).

The parameters for the ANFFC are initially chosen as
c\_1 = 3, c\_2 = 2, \tilde{E}\_u(0) = \tilde{E}\_y(0) = 0,
r\_1 = 0.03, r\_2 = 0.19, r\_3 = 0.19, r\_i = 0.19,
\tilde{C}\_u(0) = \tilde{C}\_y(0) = [-1 - 0.67 - 0.33 0.33 0.67 1]^T
\tilde{\theta}\_u(0) = \tilde{\theta}\_y(0) = [-1 - 0.67 - 0.33 0.33 0.67 1]^T

and
\(\tilde{\sigma}\_u(0) = \tilde{\sigma}\_y(0) = [0.5 0.5 0.5 0.5 0.5 0.5]^T, i = 1, 2, 3\).

In the following, three cases are respectively investigated:
C1. A constant velocity command is assigned to the leader, \((u\_u, u\_y) = (0.1, 0.04) (m/s)\). In addition, \(\delta\_u(t)\) and \(\delta\_y(t)\) in (11) are given as 0.03*rand(\cdot), \text{ where } rand(\cdot)\in[-1,1] \text{ is a uniformly distributed random number.}

C2. The uncertainties addressed in the case C1 are taken. Moreover, a time-varying leader velocity, \((u\_u, u\_y) = (0.06, 0.08 \cos(0.2\pi t)) (m/s)\), is considered.

Accordingly, the simulation results are shown in Figs. 5-6 corresponding to the cases C1 and C2, respectively. In each subplot, both the robot trajectories and formation errors are presented, where unique distributed strategies, CA [25], NFC, and ANFFC, are considered. The performance comparisons for the \(x\)- and \(y\)-axis error functions are summarized in Table 2, where ITSE is the integral time square error, ITAE is the integral time absolute error,
\[
\text{ITSE} = \int_0^T t \left( \sum_{i=1}^n (e_{u_i} + e_{y_i}) \right)^2 dt,
\]
\[
\text{ITAE} = \int_0^T t \left( \sum_{i=1}^n (e_{u_i} + e_{y_i}) \right) dt.
\]

#### Table 2. Performance comparison (simulation).

<table>
<thead>
<tr>
<th>Performance index</th>
<th>CA C1</th>
<th>NFC C1</th>
<th>ANFFC C1 C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITSE</td>
<td>47.4</td>
<td>50.9</td>
<td>10.4 11.4</td>
</tr>
<tr>
<td>ITAE</td>
<td>256.7</td>
<td>418.0</td>
<td>113.6 188.6</td>
</tr>
</tbody>
</table>

In Fig. 5, it is illustrated that the desired formation can not be achieved using conventional consensus algorithm [25]. The error divergence is improved with the NFC control, however, there exists some steady-state formation errors in the \(x\)-axis. Moreover, the formation errors are significantly reduced using the proposed ANFFC subject to uncertainties. Formation results associated with time-varying leader velocity are shown in Fig. 6. It is noticed that formation outcomes of CA and NFC become much worse. However, the formation errors of robots asymptotically converge and the desired formation pattern can be achieved using the proposed ANFFC. In addition, from Table 2, it can be observed that the proposed ANFFC has the advantages of formation responses in all performance indexes.

#### B. Experimental results

In this paper, the e-puck education robot is used as the test platform for the formation control of multi-robot systems, in which there are three followers and one leader. It is noticed that the e-puck robot is a modular, robust, and desktop-sized robot that is designed by Francesco Mondada and Michael Bonani from Ecole...
Polytechnique Federale de Lausanne (EPFL) for research and educational purposes [22], [26]. The e-puck robot is powered by a dsPIC processor and is equipped with sensors, and its mobility is ensured by a differential driven configuration. In addition, the e-puck hardware and software are fully open source, providing low-level access to every electronic device and offering extension possibilities. The e-puck robot can communicate with a computer or with other devices by Bluetooth communication. The control scheme of the multi-robot system is shown in Fig. 7. The control scheme of the multi-robot system consists of one leader, three followers, Bluetooth communication module, a computer, and a webcam. The webcam is utilized to obtain the position of each robot by the techniques of image processing. One robot can receive the position information from its neighbors, and the control algorithm is implemented in the dsPIC processor to achieve the purpose of formation control. Two main tasks are executed in the PC, namely, transmitting the position information to each robot through the Bluetooth and recording the responses of all robots.

The initial positions and desired formation pattern of robots are the same as simulation settings. The sampling time of experiments is selected to be 100 ms. The communication topology is the same digraph shown in Fig. 4. In experimental validations, two scenarios are considered, constant and time-varying leader velocities. The assigned velocities are the ones given in simulation cases. Since the position of each robot is determined through certain image processing, the associated position deviations can be viewed as uncertainties.

Experimental results are shown in Figs. 8-9, where three formation strategies, CA, NFC, and ANFFC, are considered. In Fig. 8, there exhibit significant formation errors in both axes using the conventional CA, i.e. the designated formation pattern of this four-robot system cannot be achieved. Alternatively, either NFC or ANFFC can effectively improve the formation responses. Furthermore, the associated formation results are getting worse using CA and NFC under the circumstance of time-varying leader velocity, shown in Fig. 9. In particular, comparing with the formation results of NFC and ANFFC, it can be seen that the steady-state responses of ANFFC is much better than that of NFC. Ultimately, the proposed parameter tuning rules can further improve the formation responses as expected.

Experimental results are shown in Figs. 8-9, where three formation strategies, CA, NFC, and ANFFC, are considered. In Fig. 8, there exhibit significant formation errors in both axes using the conventional CA, i.e. the designated formation pattern of this four-robot system cannot be achieved. Alternatively, either NFC or ANFFC can effectively improve the formation responses. Furthermore, the associated formation results are getting worse using CA and NFC under the circumstance of time-varying leader velocity, shown in Fig. 9. In particular, comparing with the formation results of NFC and ANFFC, it can be seen that the steady-state responses of ANFFC is much better than that of NFC. Ultimately, the proposed parameter tuning rules can further improve the formation responses as expected. The comparison of experiment results are shown in Table 3, it can be seen that the ANFFC has better performance than other methods.

### Table 3. Performance comparison (experimentation).

<table>
<thead>
<tr>
<th>Performance index</th>
<th>CA</th>
<th>NFC</th>
<th>ANFFC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITSE</td>
<td>408.9</td>
<td>50.1</td>
<td>231.6</td>
</tr>
<tr>
<td>ITAE</td>
<td>326.4</td>
<td>182.3</td>
<td>246.6</td>
</tr>
</tbody>
</table>
6. Conclusion

This paper presents an adaptive neural fuzzy formation control for multi-robot systems. A graph theoretic digraph is used to describe the communication topology between agents. The kinematic model of wheeled robots is addressed with the consideration of uncertainties. With the proposed ANFFC, the desired leader-following formation of multiple robots can be asymptotically achieved. The stability of the distributed control system is guaranteed by the Lyapunov theorem. Moreover, updating rules for ANFFC parameters are derived. In addition to simulations, the effectiveness of the designed ANFFC for multi-robot systems is validated by experimental results. Compared to the conventional consensus algorithm, the proposed ANFFC is superior in performance and robustness.

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