Adaptive Fuzzy Decentralized Control for Nonlinear Time-Delay Large-Scale Systems Based on DSC Technique and High-Gain Filters

Yongming Li, Shaocheng Tong, and Tieshan Li

Abstract

This paper proposes a novel adaptive fuzzy decentralized output-feedback control approach for a class of uncertain nonlinear large-scale systems. The considered nonlinear systems possess the unknown nonlinear functions, unmeasured states and unknown time-varying delays. Fuzzy logic systems are employed to approximate the unknown nonlinear functions, and adaptive high-gain filters are presented to estimate the unmeasured states. Based on the designed high-gain observer and Lyapunov-Krasovskii functionals, and using the backstepping design and dynamics surface control (DSC) approaches, a novel adaptive fuzzy tracking controller is designed. It is proved that the proposed control approach can guarantee all signals of the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB), the observer and the tracking errors converge to a small neighborhood of the origin. Finally, the simulation results are provided to demonstrate the effectiveness of theoretical results.

Keywords: Nonlinear large-scale systems, fuzzy logic systems, time-delay, adaptive decentralized control, high-gain filters, dynamic surface control.

1. Introduction

A large-scale system is often considered as a set of interconnected subsystems, such as power systems, computer and telecommunications networks, economic systems and multi-agent systems, et al. Owing to the complexity of control synthesis and the physical restrictions on information exchange among subsystems, it is often required to design a decentralized controller for each subsystem depending only on the local measurements, even if to achieve an objective for the whole large-scale system.

Since fuzzy logic system (FLS) is a universal approximator, and it can approximate any nonlinear function with arbitrary precision [1-3]. During the past two decades, various adaptive fuzzy decentralized control approaches have been developed for uncertain large-scale nonlinear systems, see [4-6] for example. In [4], an adaptive fuzzy decentralized control scheme was developed for a class of uncertain nonlinear large-scale systems with measurable states. By designing the state observer in [5-6], two adaptive decentralized fuzzy output feedback control approaches were proposed for the uncertain nonlinear large-scale systems with immeasurable states, respectively. Generally, these adaptive fuzzy decentralized control approaches can deal with a larger class of nonlinear large-scale systems with unknown nonlinear functions and without a priori knowledge on the interconnections, and achieve satisfactory control performances. However, the main restriction in the above control methods is that the controlled uncertain nonlinear large-scale systems must satisfy the so-called matching condition [7-9]. Thus they can not be applied to those uncertain nonlinear large-scale systems without satisfying the matching condition.

To handle the uncertain nonlinear large-scale systems without satisfying the matching condition, several adaptive fuzzy and neural networks (NN) decentralized controllers were developed in [10-17] by using the backstepping design technique [18]. In [10-12], some adaptive fuzzy or NN decentralized backstepping state feedback control approaches were developed for the uncertain large-scale systems, respectively. To remove the restriction in [10-12] that states are available for controller design, [13-16] investigated the adaptive fuzzy or NN decentralized backstepping output feedback controllers for a class of uncertain large-scale systems. Then, [17] developed an adaptive fuzzy decentralized output feedback controller for a class of the uncertain large-scale systems with immeasurable states and unknown gain signs. As pointed out in [19-20], however, these control approaches in [11, 12, 14-16, 18] suffered from a drawback of the so-called problem of “explosion of complexity” caused by repeated differentiations of certain nonlinear functions such as virtual controllers,
which inevitably leads to a complicated algorithm with heavy computation burden. In addition, [13, 14, 16, 17] did not considered the problem of time delays. It is well known that time delays frequently occur in real engineering systems, and they may destroy the stability or degrade the performance of the controlled systems. Therefore, the controller synthesis and stability analysis for the nonlinear systems with time delays are important both in theory and applications [21-23]. On the other hand, based on Takagi-Sugeno (T-S) fuzzy model [24-25], some novel fuzzy decentralized control methods were proposed in [26-29]. [24] focused on the delay-dependent stability analysis and stabilization for uncertain T-S fuzzy control systems with state and input delays. [25] presented a new robust fuzzy controller design for a class of T-S fuzzy bilinear systems with time-delay. [26] for the first time studied the robust output-feedback control strategy via a decentralized fuzzy-observer-based fuzzy controller where the premise output-feedback control strategy via a decentralized time-delay. [27] for the first time studied the robust output-feedback control strategy via a decentralized fuzzy-observer-based fuzzy controller where the premise output-feedback control strategy via a decentralized time-delay. [26-29] for the first time studied the robust output-feedback control strategy via a decentralized fuzzy-observer-based fuzzy controller where the premise output-feedback control strategy via a decentralized time-delay. [27] for the first time studied the robust output-feedback control strategy via a decentralized fuzzy-observer-based fuzzy controller where the premise output-feedback control strategy via a decentralized time-delay.

The rest of the paper is organized as follows: Section 2 presents the problem formulations and some preliminaries. Section 3 and 4 investigate the fuzzy high-gain filters and controllers designs, respectively. Section 5 gives the stability analysis of the closed-loop systems.

Notation: The following notations will be used throughout this paper. $R^n$ denotes the real $n$ -dimensional space. For a given vector or matrix $X$, $X^T$ denotes its transpose; $\|X\|$ denotes the 2-norm for vector or matrix $X$; For a given matrix $Y$, $Y^{-1}$ denotes its inverse, and $Y^2 = Y \times Y$, and $\lambda_{\min}(Y)$ denotes the smallest eigenvalue of matrix $Y$; $I_n$ denotes identity matrix; $\|X\|$ denotes For a given scalar $x$, $|x|$ denotes its absolute value.

2. Problem Formulations and Some Preliminaries

A. System descriptions

Consider an uncertain nonlinear time-delay large-scale system composed of $N$ subsystems interconnected by their outputs. The $i$ th subsystem $\Sigma_i (i=1,\cdots,N)$ is given as

$$
\dot{x}_{i1} = x_{i2} + f_{i1}(y_i) + h_{i1}(y_i(t-\tau_{i1}(t))) + g_{i1}(\vec{y}) \\
\dot{x}_{i2} = x_{i3} + f_{i2}(y_i) + h_{i2}(y_i(t-\tau_{i2}(t))) + g_{i2}(\vec{y}) \\
\vdots \\
\dot{x}_{i,i-1} = x_{i,i} + f_{i,i-1}(y_i) + h_{i,i-1}(y_i(t-\tau_{i,i-1}(t))) + g_{i,i-1}(\vec{y}) \\
\dot{x}_{i,i} = f_{i,i}(y_i) + h_{i,i}(y_i(t-\tau_{i,i}(t))) + g_{i,i}(\vec{y}) + u_i \\
y_i = x_{i1},
$$

where $x_i = [x_{i1},\cdots,x_{i,i}]^T \in R^n$, $u_i \in R$, $y_i \in R$ are the
state vector, the input and the output of subsystem \( \Sigma_i \), respectively. \( \bar{y} = (y_1, \ldots, y_n) \). \( f_{i,j}(y_i) \), \( 1 \leq i \leq N \), \( 1 \leq j \leq n_i \) is an unknown smooth nonlinear function. \( g_{i,j}(\bar{y}) \), \( 1 \leq i \leq N \), \( 1 \leq j \leq n_i \) is completely unknown smooth functions, representing the nonlinearities in the \( i \) th subsystem and the interconnection effects between the \( i \) th subsystem and other subsystems. \( h_{i,j}() \), \( 1 \leq i \leq N \), \( 1 \leq j \leq n_i \) are unknown smooth nonlinear functions, and \( \tau_{i,j}(t) \) is an unknown time delay such that \( |f_{i,j}(t)| \leq \delta^* \) and \( \tau_{i,j}(t) \leq \tau < 1 \), \( 1 \leq i \leq N \), \( 1 \leq j \leq n_i \), where \( \delta^* \) and \( \tau \) are known constants. In this paper, it is assumed that only \( y_i \) is available for measurement.

Remark 1: If no time-delay terms are included in (1), i.e., \( h_{i,j}() = 0 \), then (1) becomes the interconnected nonlinear strict-feedback systems studied widely, see [13] and [17].

The purpose of this paper is to design adaptive fuzzy output feedback decentralized control scheme such that all the signals in the closed-loop system are SGUUB, and the tracking errors \( z_{i,j} = y_i - y_{i,r} \) can be made as small as possible.

Throughout this paper, the following assumptions are made on the system (1).

Assumption 1: \[ |g_{i,j}(\bar{y})| \leq \sum_{k=1}^{n_i} q_{i,j}^k |y|_p^k, \] where \( q_{i,j}^k \) is an unknown constant and \( p = \max\{p_{i,j} | 1 \leq i \leq N, 1 \leq j \leq n_i \} \) is a known integer.

Assumption 2[10]: Nonlinear functions \( h_{i,j}(\cdot) \), \( i = 1,2,\ldots,N \), \( j = 1,2,\ldots,n_i \) satisfy the following inequalities for \( \bar{z}_{i,j}(t) = (z_{i,j}(t), H_{i,j}(z_{i,j}(t)) + \kappa_{i,j}, h_{i,j}(y_{i,r}(t)) \) where \( H_{i,j}(\cdot) \) is a known function, \( h_{i,j}(0) = 0 \), and \( \kappa_{i,j} \) is a positive scalar and \( z_{i,j} = y_{i,r}(t) - y_{i,r}(t) \).

Assumption 3: For the given reference signal \( y_{i,r}(t) \) is a sufficiently smooth function of \( t \), \( y_{i,r}, \hat{y}_{i,r}, \) and \( \hat{y}_{i,r} \) are bounded, that is, there exists a known positive constant \( B_{i,0} \) such that \[ \Pi_{i,0} = \{ (y_{i,r}, \hat{y}_{i,r}, \hat{y}_{i,r}) : y_{i,r}^2 + \hat{y}_{i,r}^2 + \hat{y}_{i,r}^2 \leq B_{i,0} \} \).

B. Fuzzy logic systems

A fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for a FLS comprises a collection of fuzzy if-then rules of the following form:

\[ R^i : \text{If } y_1 \text{ is } F_1^i \text{ and } y_2 \text{ is } F_2^i \text{ and } \ldots \text{ and } y_n \text{ is } F_n^i, \]

\[ \text{then } s \text{ is } G^i, \ l = 1,2,\ldots,N \]

where \( y = [y_1,\ldots,y_n]^T \) and \( s \) are the fuzzy logic system input and output, respectively. Fuzzy sets \( F^i_1 \) and \( G^i \), are associated with the fuzzy functions \( \mu^i_1(y_1) \) and \( \mu^i_2(s) \), respectively. \( N \) is the rules number.

By use of singleton function, center average defuzzification and product inference [1], the FLS can be expressed as

\[ \hat{f}(y) = \frac{\sum_{i=1}^{N} \tilde{s}_i \prod_{i=1}^{n} \mu^i_1(y_i)}{\sum_{i=1}^{N} \prod_{i=1}^{n} \mu^i_1(y_i)} \] (3)

where \( \tilde{s}_i = \max_{s \in R} \mu^i_2(s) \). Define the fuzzy basis functions as

\[ \varphi_i = \frac{\prod_{i=1}^{n} \mu^i_1(y_i)}{\sum_{i=1}^{N} \prod_{i=1}^{n} \mu^i_1(y_i)} \] (4)

Denoting \( \theta = [\tilde{s}_1, \tilde{s}_2,\ldots,\tilde{s}_n]^T = [\theta_1, \theta_2,\ldots,\theta_n]^T \) and \( \varphi^T(y) = [\varphi_1(y),\ldots,\varphi_n(y)] \), then the FLS (3) can be rewritten as

\[ \hat{f}(y) = \theta^T \varphi(y) \] (5)

Lemma 1[1]: Let \( f(y) \) be a real continuous function defined on a compact set \( \Omega \). Then, for any constant \( \delta > 0 \), there exists an FLS (5) such that \( \sup_{y \in \Omega} |f(y) - \theta^T \varphi(y)| \leq \delta \).

3. Fuzzy Adaptive High-Gain Filters Design

According to Lemma 1, because \( f_{i,j}(y_i) \) is a smooth function in (1), \( f_{i,j}(y_i) \) can be approximated by the following FLS.

\[ f_{i,j}(y_i) = \theta_{i,j}^T \varphi_{i,j}(y_i) + \delta_{i,j}(y_i), j = 1,\ldots,n_i \]

\[ i = 1,\ldots,N \] (6)

where \( \varphi_{i,j}(y_i) \) is fuzzy basis function, \( \theta_{i,j} \) is the optimal parameter vector, \( \delta_{i,j}(y_i) \) is the approximation errors and satisfies \( |\delta_{i,j}(y_i)| \leq \delta_{i,j}^* \), where \( \delta_{i,j}^* \) is an unknown constant.
By substituting (6) into (1), the system (1) can be presented in the following form:

\[
\begin{align*}
\dot{x}_i &= A_i x_i + \Phi_i^T(y_i) \Theta_i + \delta_i + h_i + e_{i,n} u_i + g_i \\
y_i &= e_i^T x_i
\end{align*}
\]

where

\[
A_i = \begin{bmatrix}
0 & I_{n-1} & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & I_{n-1}
\end{bmatrix}, \quad \Phi_i^T(y_i) = \begin{bmatrix}
\phi_i^T(y_i) \\
\vdots \\
\phi_{i,n}^T(y_i)
\end{bmatrix}, \quad \Theta_i = \begin{bmatrix}
\theta_{i,1} \\
\vdots \\
\theta_{i,n}
\end{bmatrix}, \quad \delta_i = \begin{bmatrix}
\delta_{i,1} \\
\vdots \\
\delta_{i,n}
\end{bmatrix}, \quad g_i = \begin{bmatrix}
g_{i,1} \\
\vdots \\
g_{i,n}
\end{bmatrix}, \quad h_i = \begin{bmatrix}
h_{i,1}(y_i(t - \tau_{i1}(t))) \\
\vdots \\
h_{i,n}(y_i(t - \tau_{in}(t)))
\end{bmatrix}, \quad e_{i,n} = [0,0,\ldots,0]^T.
\]

Rewrite (7) as

\[
\begin{align*}
\dot{x}_i &= \tilde{A}_i x_i + K_i y_i + \Phi_i^T(y_i) \Theta_i + \delta_i + h_i + e_{i,n} u_i + g_i \\
y_i &= e_i^T x_i
\end{align*}
\]

where

\[
K_i = \left[\frac{k_{i,1}}{l_i}, \cdots, \frac{k_{i,n}}{l_i}\right]^T \quad \text{and} \quad \tilde{A}_i = A_i - K_i e_i^T.
\]

\(l_i(0 < l_i < 1)\) is a positive design constant. The vector \(k_i = [k_{i,1}, \ldots, k_{i,n}]^T\) is chosen such that \(\tilde{A}_i\) is a strict Hurwitz matrix. And hence there exists a symmetric and positive definite matrix \(P_i\) satisfying the Lyapunov equation:

\[
\tilde{A}_i^T P_i + P_i \tilde{A}_i = -4I_i
\]

where \(\tilde{A}_i = A_i - K_i e_i^T\).

Let \(\tilde{I}_i = \text{diag} [I_{l_i}, I_{l_i}, \ldots, I_{l_i}] \) and \(\tilde{P}_i = \tilde{I}_i P_i \tilde{I}_i\). Then, it is easy to see that \(\tilde{P}_i\) is a symmetric and positive definite matrix. From this and the fact \(I_i \tilde{A}_i = \tilde{A}_i I_i\), it follows that \(\tilde{P}_i\) satisfies \(\tilde{A}_i^T \tilde{P}_i + \tilde{P}_i \tilde{A}_i = -4I_i\). This shows that for any \(l_i > 0\), the matrix \(\tilde{A}_i\) is Hurwitz as well.

Since the states of the controlled system (1) cannot be measured directly, the following fuzzy high-gain filters are constructed

\[
\begin{align*}
\hat{e}_i &= \tilde{A}_i \hat{e}_i + K_i y_i \\
\Omega_i &= \tilde{A}_i \Omega_i + \Phi_i^T(y_i) \\
\hat{\lambda}_i &= \tilde{A}_i \hat{\lambda}_i + e_{i,n} u_i
\end{align*}
\]

The designed state estimation is \(\hat{x}_i = \hat{e}_i + \Omega_i \hat{\lambda}_i + \lambda_i\).

Denote state estimation error vector

\[
e_i = [e_{i,1}, \ldots, e_{i,n}]^T = x_i - \hat{x}_i
\]

From (8), (10)-(12) and (13), it can be shown that the state estimation error vector satisfies

\[
\dot{e}_i = \tilde{A}_i e_i + \delta_i + h_i + g_i
\]

Consider the following Lyapunov-Krasovskii functionals for the error system (14)

\[
V_0 = \sum_{i=1}^{N} \left( \frac{1}{2} e_i^T P_i e_i + \frac{N}{i} \tilde{V}_{i,0} \right)
\]

where

\[
\tilde{V}_{i,0} = e_i^T (\tilde{I}_i P_i) e_i + e_i^T P_i \delta_i 
\]

By using Young’s inequality, we obtain

\[
e_i^T P_i \delta_i \leq \frac{1}{2} e_i^T e_i + \frac{1}{2} \|P_i\|^2 \sum_{i=1}^{N} \delta_{i,n}^2 (15)
\]

By using Assumption 1 and Young’s inequality, we have

\[
\sum_{i=1}^{N} e_i^T P_i g_i \leq \sum_{i=1}^{N} \|e_i\|^2 + \sum_{i=1}^{N} (\|P_i\|^2 \sum_{k=1}^{n} \Gamma_k^2)
\]

where \(\Gamma_k = \sum_{i=1}^{N} q_{k,i}^T |y_i|^k\).

Applying the Cauchy-Schwartz inequality \((\sum_{i=1}^{n} a_i b_i)^2 \leq (\sum_{i=1}^{n} a_i^2)(\sum_{i=1}^{n} b_i^2)\) with \(a_k = \Gamma_k\) and \(b_k = 1\) gives:

\[
\sum_{i=1}^{N} \sum_{k=1}^{n} \Gamma_k^2 \leq \sum_{k=1}^{n} \sum_{i=1}^{N} \Gamma_k^2
\]

So that:

\[
\sum_{i=1}^{N} \sum_{k=1}^{n} q_{k,i}^T |y_i|^k \leq \sum_{k=1}^{n} \sum_{i=1}^{N} |q_{k,i}^T |y_i|^k
\]

Similarly, applying Cauchy-Schwartz inequality \((\sum_{i=1}^{n} a_i b_i)^2 \leq (\sum_{i=1}^{n} a_i^2)(\sum_{i=1}^{n} b_i^2)\) with \(a_k = q_{k,i}^T |y_i|^k\) and \(b_k = 1\) to (19), we have

\[
\sum_{i=1}^{N} \sum_{k=1}^{n} q_{k,i}^T |y_i|^k \leq \sum_{k=1}^{n} \sum_{i=1}^{N} |q_{k,i}^T |y_i|^k
\]
Substituting (21) into (20) results in
\[
\begin{aligned}
\sum_{i=1}^{N} \| \Phi_i \| \sum_{j=1}^{N} \left( \sum_{k=1}^{n} q_{i,j,k} |y_i|^2 \right)^{\frac{1}{2}} & \\
& \leq pN \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \sum_{k=1}^{n} q_{i,j,k} |z_i|^2 + |y_i|^2 \right)^{\frac{1}{2}}
\end{aligned}
\] (22)

Also, applying Cauchy-Schwartz inequality
\[
\sum_{i=1}^{N} |e_i|^2 \leq 2^N \sum_{i=1}^{N} |e_i|^{2^N}
\]
(23)

\[d_{i,k} = \sum_{m=1}^{N} \| \Phi_{m,i} \| \sum_{j=1}^{N} q_{m,j,i} y^2 \]
where
\[
\sum_{j=1}^{N} \rho_{i,j}^{*} \text{ is a constant defined as}
\]
\[
\rho_{i,j}^{*} > \| \Phi_{i,j} \| (\hat{h}_i(t), (y_i(t), (t - \tau_{i,j}(t)))) + \kappa_{i,j},
\]
Noting that
\[

\hat{\nu}_{i,0} = -\rho_{i,0} + \frac{\epsilon_{i}^{\tau} \epsilon_{i}}{(1-\tau) \| \Phi_{i} \| \| \sum_{j=1}^{N} z_{i,j}(H_{i,j}(z_{i,j}(t))) \|
\]
(24)

By using Assumption 2 and Young’s inequality, we have
\[
\hat{\nu}_{i,0} = -\rho_{i,0} + \frac{\epsilon_{i}^{\tau} \epsilon_{i}}{(1-\tau) \| \Phi_{i} \| \| \sum_{j=1}^{N} z_{i,j}(H_{i,j}(z_{i,j}(t))) \|
\]
(26)

Substituting (17), (24), (25) and (26) into (16) results in
\[
\begin{aligned}
\dot{V}_0 & \leq \sum_{i=1}^{N} \left\{ -\left( \frac{\lambda_{i}}{I_i} \| \Phi_{i} \| \sum_{j=1}^{N} \delta_{i,j}^{\alpha} \right) - \frac{7}{4} e_{i}^{\tau} e_{i} + \frac{1}{2} \| \Phi_{i} \| \| \sum_{j=1}^{N} \delta_{i,j}^{\alpha} \|
\right. \\
& \quad + \sum_{k=1}^{n} 2^{k} d_{i,k} |y_{i,k}|^{2k} + |\bar{e}_{i,k}|^{2k} + \sum_{j=1}^{N} \rho_{i,j}^{*} \\
& \quad - \rho_{i,0} + \frac{\epsilon_{i}^{\tau} \epsilon_{i}}{(1-\tau) \| \Phi_{i} \| \| \sum_{j=1}^{N} z_{i,j}(H_{i,j}(z_{i,j}(t))) \|
\right\}
\end{aligned}
\] (27)

It can be seen from (27) that the fuzzy adaptive high-gain filters (10)-(12) can not guarantee the convergence of the observer errors, thus it is necessary to design a controller to make the resulting closed-loop system stable in the next section.

4. Adaptive Fuzzy Decentralized Dynamic Surface Control Design

In this section, we will incorporate the DSC technique proposed in [19] into an adaptive fuzzy decentralized control design scheme for the $n_i$-order system described by (1). Similar to the traditional backstepping design method, the recursive design procedure contains $n_i$ steps.

From (6) and (11), we have
\[
\begin{aligned}
\dot{y}_i &= x_{i,2} + \Phi_{i}^{T} \hat{\alpha}_i \gamma_i \theta_i + \delta_i \\
& + h_i(y_i(t - \tau_{i,1}(t))) + g_i(\overline{\tilde{y}})
\end{aligned}
\] (28)

\[
\begin{aligned}
\hat{x}_i &= \frac{k_1}{I_1} \lambda_{i,1} + \lambda_{i,2} \\
& \vdots \\
\end{aligned}
\]
where $\Phi_{i(1)}$ is the first row of $\Phi_i$.

The $n_i$-step adaptive fuzzy decentralized output feedback backstepping design is based on the change of coordinates:
\[
\begin{aligned}
z_{i,1} &= y_i - y_{i,v} \\
z_{i,j} &= \hat{x}_{i,j} - \alpha_{i,j-1}, \quad j = 2, \ldots, n_i
\end{aligned}
\] (29)

where $z_{i,j}$ is called the error surface, $\alpha_{i,j-1}$ is a state variable, which is obtained through a first-order filter on the intermediate function $\alpha_{i,j-1}$, and $\lambda_{i,j}$ is called the output error of the first-order filter.

4.1 The time derivative of $z_{i,1}$

The time derivative of $z_{i,1}$ along (12), (28) and (29) is
\[
\begin{aligned}
\dot{z}_{i,1} &= \dot{x}_{i,1} - \dot{y}_{i,v} = \lambda_{i,2} + e_{i,2} + \epsilon_{i,2} + (\Omega_{i,1}) \\
& + \Phi_{i}^{T} \hat{\alpha}_i \gamma_i \theta_i + \delta_i \\
& + h_i(y_i(t - \tau_{i,1}(t))) + g_i(\overline{\tilde{y}}) - \dot{y}_{i,v}
\end{aligned}
\] (30)

where $\Omega_{i,1}$ is the second row of $\Omega_i$. Consider the Lyapunov function candidate $V_1$ as
\[
V_1 = V_0 + \sum_{i=1}^{n_i} \left[ \frac{1}{2} \hat{\alpha}_i^2 + \frac{1}{2} \hat{\beta}_i^2 + \frac{1}{2} \lambda_i^2 \right] + \left[ \frac{1}{2} \lambda_i^2 + \lambda_i^2 \right]
\] (31)

where $\gamma_i$ and $\lambda_i$ are positive design constants.

$\hat{\gamma}_i = \gamma_i - \hat{\gamma}_i, \quad \hat{\beta}_i = \beta_i - \hat{\beta}_i, \quad \hat{\lambda}_i$ and $\hat{\beta}_i$ are the estimates of $\gamma_i$ and $\beta_i$.

\[
\begin{aligned}
d_{i,1,k} &= \sum_{i=1}^{n_i} \left( q_{i,j,k}^2 \right) \dot{N}_p, \quad \text{respectively;}
\end{aligned}
\]

\[
\begin{aligned}
\dot{V}_1 &= \epsilon_{i}^{\tau} \epsilon_{i} / (2(1-\tau)) e_{i}^{\tau} \int_{t_{i,1}(t)}^{t_{i,1}(s)} e_{i}^{\tau} z_{i,1}(s)(H_{i,1}(z_{i,1}(s)))ds.
\end{aligned}
\]
The time derivative of $V_1$ along (32) is
\[
V'_1 \leq \dot{V}_0 + \sum_{i=1}^{N} \left[ z_{i,1}(\lambda_{i,2} + \xi_{i,2} - \dot{y}_{i,r}) + z_{i,1}(\Omega_{i,12}) \\
+ \Phi^T_{i,12}(y) \right] \theta_i + \frac{1}{\gamma_i} \theta_i \dot{\gamma}_i + \frac{1}{\gamma_i} \beta_i \ddot{\gamma}_i + \ddot{V}_i 
\]
By using Young's inequality, we have
\[
z_{i,1}(\Omega_{i,12}) + \Phi^T_{i,12}(\theta_i) \leq z_{i,1}^2 \left[ \left\| \Omega_{i,12} + \Phi^T_{i,12} \right\|^2 + \frac{1}{4} \right] \]
Using Cauchy-Schwarz inequality, Young's inequality and Assumptions 1 and 2, the following inequalities are obtained
\[
\sum_{i=1}^{N} \left| g_{i,1} z_{i,1} \right| \leq \sum_{i=1}^{N} z_{i,1}^2 \sum_{i=1}^{N} \sum_{k=1}^{p} 2^{k+1} d_{i,k} \left( |y_{i,r}| \right)^{2k} \\
+ \left[ z_{i,1}^{2k} \right] \\
\sum_{i=1}^{N} \left| z_{i,1} h_{i,1} \right| \leq \sum_{i=1}^{N} \frac{1}{2} z_{i,1}^2 + \frac{1}{2} z_{i,1} (t - \tau_{i,1}(t)) \\
\times H_{i,1}(z_{i,1}(t - \tau_{i,1}(t)) + d_{i,1}) 
\]
where $\ddot{d}_{i,1} = 1/(2 \|P\|) \rho_{i,1}$. Note that
\[
\sum_{i=1}^{N} \tilde{P}_{i,1} = \sum_{i=1}^{N} \left\{ -r F_{i,0} + \frac{e^{c_{i,1}}}{2(1-\tau)} z_{i,1} (t) H_{i,1} \right\} \]
Substituting (27), (35)-(40) into (34) results in
\[
V'_1 \leq \sum_{i=1}^{N} \left[ -\left( \frac{2\lambda_{\min}(I_{i,1}^2)}{l_i} - \frac{9}{4} \right) e_{i,1} \right] e_{i,1} - r F_{i,0} - r F_{i,1} \\
+ \beta_i \sum_{k=1}^{p} 2^{k+1} \left( |y_{i,r}|^{2k} \right) + z_{i,1} \left( \frac{5}{2} \right) z_{i,1} \\
+ \lambda_{i,2} + \xi_{i,2} - \dot{y}_{i,r} + z_{i,1} \left( \Omega_{i,12} + \Phi^T_{i,12} \right) \left( 2 \right) \\
+ \frac{e^{c_{i,1}}}{2(1-\tau)} \left\| P_{i,1} \right\| \sum_{i=1}^{N} H_{i,1} (z_{i,1}(t)) + \frac{e^{c_{i,1}}}{2(1-\tau)} \right\| P_{i,1} \right\| \\
\times H_{i,1}(z_{i,1}(t)) + \frac{1}{\gamma_i} \beta_i \ddot{\gamma}_i + \frac{1}{\gamma_i} \beta_i \ddot{\gamma}_i + \ddot{V}_i 
\]
where $\ddot{d}_{i,1} = \ddot{d}_{i,1} + \frac{1}{4} \left( P_{i,1} \right) \sum_{i=1}^{N} \delta_{i,1}^{2k+1} + \frac{1}{2} \delta_{i,1}^{2k+1}$. From (28) and (29), we obtain
\[
\lambda_{i,2} = z_{i,1} + \chi_{i,2} + \alpha_{i,1} 
\]
Substituting (42) into (41) results in
\[
V'_1 \leq \sum_{i=1}^{N} \left[ -\left( \frac{2\lambda_{\min}(I_{i,1}^2)}{l_i} - \frac{9}{4} \right) e_{i,1} \right] e_{i,1} - r F_{i,0} - r F_{i,1} \\
+ z_{i,1} \left( \frac{5}{2} \right) z_{i,1} + z_{i,2} + \chi_{i,2} + \alpha_{i,1} + \xi_{i,2} - \dot{y}_{i,r} \\
+ z_{i,1} \left( \Omega_{i,12} + \Phi^T_{i,12} \right) \left( 2 \right) \\
+ \frac{e^{c_{i,1}}}{2(1-\tau)} \left\| P_{i,1} \right\| \sum_{i=1}^{N} H_{i,1} (z_{i,1}(t)) + \frac{e^{c_{i,1}}}{2(1-\tau)} \right\| P_{i,1} \right\| \\
\times H_{i,1}(z_{i,1}(t)) + \frac{1}{\gamma_i} \beta_i \ddot{\gamma}_i + \frac{1}{\gamma_i} \beta_i \ddot{\gamma}_i + \ddot{V}_i 
\]
Design the intermediate control function $\alpha_{i,1}$, parameter adaptive laws $\beta_i$ and $\dot{\theta}_i$ as
\[
\begin{align*}
\alpha_{i,1} &= -c_{i,1} z_{i,1} - \frac{5}{2} z_{i,1} - \xi_{i,2} + \dot{y}_{i,r} - z_{i,1} \dot{\theta}_i \\
&\times H_{i,1}(z_{i,1}(t)) \left( 2 \right) \\
- \frac{e^{c_{i,1}}}{2(1-\tau)} H_{i,1} \\
\beta_i &= \gamma_i \sum_{k=1}^{p} 2^{k+1} \left( \frac{2}{2} \right) \mu_i \dot{\theta}_i \\
\dot{\theta}_i &= \gamma_i \left( \frac{2}{2} \right) \Omega_{i,12} + \Phi^T_{i,12} \left( 2 \right) - \mu_i \dot{\theta}_i \\
\end{align*}
\]
where $c_{i,1}$ and $\mu_i$ are positive design constants.
Substituting (44), (45) and (46) into (43) results in
\[
V'_1 \leq \sum_{i=1}^{N} \left[ -\left( \frac{2\lambda_{\min}(I_{i,1}^2)}{l_i} - \frac{9}{4} \right) e_{i,1} \right] e_{i,1} - r F_{i,0} - r F_{i,1} \\
+ \left( \frac{5}{2} \right) z_{i,1} + z_{i,2} + \chi_{i,2} \left( H_{i,1} \right) \left( 2 \right) \\
+ \frac{e^{c_{i,1}}}{2(1-\tau)} \left\| P_{i,1} \right\| \sum_{i=1}^{N} H_{i,1} (z_{i,1}(t)) + \frac{e^{c_{i,1}}}{2(1-\tau)} \right\| P_{i,1} \right\| \\
\times H_{i,1}(z_{i,1}(t)) + \frac{1}{\gamma_i} \beta_i \ddot{\gamma}_i + \frac{1}{\gamma_i} \beta_i \ddot{\gamma}_i + \ddot{V}_i 
\]
According to Refs [19]-[20], introduce a new state variable $\pi_{i,2}$ and let $\alpha_{i,1}$ pass through a first-order filter with the constant $\bar{r}_{i,2}$ to obtain $\pi_{i,2}$
\[
\pi_{i,2} = \pi_{i,2} + \pi_{i,2} = \alpha_{i,1}, \pi_{i,2}(0) = \alpha_{i,1}(0) 
\]
By defining the output error of this filter $\chi_{i,2} = \pi_{i,2} - \alpha_{i,1}$,
\[ \dot{x}_{i,2} = \hat{\alpha}_{i,2} - \dot{\alpha}_{i,2} = \frac{\dot{x}_{i,2}}{r_{i,2} + B_{i,2}(\cdot)} \]

where \( B_{i,2}(\cdot) \) is a continuous function with the following expression
\[
B_{i,2}(\cdot) = c_{i,2,1} \dot{z}_{i,1} + 2z_{i,1} \dot{z}_{i,2} + \dot{\hat{\alpha}}_{i,2} - \ddot{\hat{\alpha}}_{i,2} + \dot{\hat{\alpha}}_{i,2} \dot{\hat{\alpha}}_{i,2} + \frac{d}{dt} \left[ \Omega_{i,2} + \Phi_{i,2} \right] + \frac{d}{dt} \left[ \Omega_{i,2} + \Phi_{i,2} \right] + \beta \sum_{k=1}^{p} 2k z_{i,1}^{2k-1} \\
+ \beta (2k-1) \sum_{j=1}^{n} 2^{2k} z_{i,1}^{2k-2} z_{i,1} + \frac{e^{\varepsilon t}}{2(1-\tau)} d(H_{i,1}(z_{i,1})) + \frac{e^{\varepsilon t}}{2(1-\tau)} d(H_{i,1}(z_{i,1})) 
\]

**4.2 The time derivative of \( z_{i,2} \)**

From (28), (30) and (31), differentiating the second error variable \( z_{i,2} \), we have
\[
\dot{z}_{i,2} = -k_{i,2,1} \dot{\dot{\hat{\alpha}}}_{i,2} + z_{i,1} + \chi_{i,2} + \alpha_{i,2} - \hat{\alpha}_{i,2} 
\]

Choose intermediate control function \( \alpha_{i,2} \) as
\[
\alpha_{i,2} = -c_{i,2,1} z_{i,2} + k_{i,2,1} \dot{\dot{\hat{\alpha}}}_{i,2} + \hat{\alpha}_{i,2} \]  
(52)

where \( c_{i,2,1} > 0 \) is a design constant. Substituting (52) into (51) results in
\[
\dot{z}_{i,2} = \dot{\hat{\alpha}}_{i,2} - \dot{\alpha}_{i,2} = \frac{\dot{x}_{i,2}}{r_{i,2} + B_{i,2}(\cdot)} 
\]

4.3 The time derivative of \( z_{i,j} \)

A similar procedure in step \( i,2 \) is employed recursively for step \( j = 3, \cdots, n_i - 1 \). From (28), (30) and (31), differentiating the error variable \( z_{i,j} \), we have
\[
\dot{z}_{i,j} = -k_{i,j} \dot{\dot{\hat{\alpha}}}_{i,j} + z_{i,j+1} + \chi_{i,j+1} + \alpha_{i,j} - \hat{\alpha}_{i,j} 
\]  
(57)

Choose intermediate control function \( \alpha_{i,j} \) as
\[
\alpha_{i,j} = -c_{i,j} z_{i,j} + k_{i,j} \dot{\dot{\hat{\alpha}}}_{i,j} + \hat{\alpha}_{i,j} \]  
(58)

where \( c_{i,j} > 0 \) is a design constant. Substituting (58) into (57) results in
\[
\dot{z}_{i,j} = z_{i,j+1} + \chi_{i,j+1} - c_{i,j} z_{i,j} 
\]

(59)

Introduce a new state variable \( \pi_{i,j+1} \) and let \( \alpha_{i,j} \) pass through a first-order filter with the constant \( r_{i,j+1} \) to obtain \( \pi_{i,j+1} \)
\[
\pi_{i,j+1} = \pi_{i,j+1} - c_{i,j} \pi_{i,j+1} \]  
(60)

By defining the output error of this filter
\[
\chi_{i,j+1} = \pi_{i,j+1} - \alpha_{i,j}, \text{ it yields } \pi_{i,j+1} = \frac{\dot{\pi}_{i,j+1}}{r_{i,j+1}} 
\]

(61)

where \( B_{i,j+1}(\cdot) \) is a continuous function with the following expression
\[
B_{i,j+1}(\cdot) = c_{i,j} \dot{\alpha}_{i,j} - k_{i,j} \dot{\dot{\hat{\alpha}}}_{i,j} - \frac{\dot{x}_{i,2}}{r_{i,2}} 
\]

(62)

4.4 The time derivative of \( z_{i,n_i} \)

In the final step, differentiating the error variable \( z_{i,n_i} \), we have
\[
\dot{z}_{i,n_i} = \dot{\hat{\alpha}}_{i,n_i} - \dot{\hat{\alpha}}_{i,n_i} = -k_{i,n_i} \dot{\dot{\hat{\alpha}}}_{i,n_i} + u_i - \hat{\alpha}_{i,n_i} 
\]

(63)

Choose the actual control function \( u_i \) as
\[
u_i = -c_{i,n_i} z_{i,n_i} + k_{i,n_i} \dot{\dot{\hat{\alpha}}}_{i,n_i} + \hat{\alpha}_{i,n_i} \]  
(64)

where \( c_{i,n_i} > 0 \) is a design constant. Substituting (64) into (63) results in
\[
\dot{z}_{i,n_i} = -c_{i,n_i} \dot{z}_{i,n_i} 
\]  
(65)

5. The Stability Analysis of the Closed-Loop Systems

The goal of this section is to establish that the resulting closed-loop system possesses the semi-globally uniformly ultimately bounded property.

**Assumption 4:** For a given \( p > 0 \), for all initial conditions satisfying
\[
\epsilon_{p}, P_{i}, \gamma, \bar{\gamma}, \bar{\beta}, \sum_{k=1}^{n_i} \chi_{i,k}^{2}, \sum_{k=1}^{n_i} \dot{z}_{i,k}^{2} + P_{i}^{T} P_{i} + \bar{P}_{i}^{T} \bar{P}_{i} \leq 2p \]

**Theorem 1:** Consider the closed-loop system (1). Under Assumptions 1-4, the fuzzy adaptive controller (64) with fuzzy high-gain filters (10), (11) and (12), the intermediate control (44), (52) and (58) and parameter adaptive laws (45) and (46) guarantees that all the signals in the resulting closed-loop system is SGUUB.
Moreover, the tracking errors can be made arbitrarily small by choosing appropriate design parameters.

Proof: Consider the following Lyapunov-Krasovskii functionals

\[ V = V_i + \frac{1}{2} \sum_{k=2}^{N} \left[ \sum_{i=1}^{n} X_{i,k}^2 + \sum_{i=1}^{n} z_{i,k}^2 \right] \tag{66} \]

The time derivative of the Lyapunov function \( V \) is

\[ \dot{V} = \dot{V}_i + \sum_{i=1}^{n} \left[ \sum_{k=2}^{N} X_{i,k} \dot{X}_{i,k} + \sum_{k=2}^{N} z_{i,k} \dot{z}_{i,k} \right] \tag{67} \]

Substituting (47), (59) and (61) into (67) yields

\[ \dot{V} \leq \sum_{i=1}^{n} \left[ -\frac{2 \lambda_{\max} (I_i^3)}{I_i} - \frac{9}{4} e_i^T e_i - r_i \bar{V}_{i,0} - r_i \bar{V}_{i,3} \right. \]
\[ - c_{i,3} z_{i,3}^2 + z_{i,3}^2 + \frac{1}{4} z_{i,3}^2 + \frac{1}{4} X_{i,3}^2 + \mu_i \bar{\beta} \bar{\beta}^T \]
\[ + \frac{\mu_i}{\gamma_i} \bar{\beta} \beta_i + D_{i,3} + \sum_{k=2}^{N} X_{i,k} \left( - \frac{X_{i,k}}{\bar{r}_{i,k}} + B_{i,k} \right) \]
\[ + \sum_{k=2}^{N} z_{i,k} \left( z_{i,k+1} + X_{i,k+1} - c_{i,k} z_{i,k} \right) - c_{i,r,n} z_{i,n}^2 \tag{68} \]

Since for any \( B_{i,0} > 0 \) and \( p > 0 \),

\[ \Pi_{i,0} := \left\{ (\gamma_i, r_i, \bar{V}_{i,0}, \bar{V}_{i,3}) : r_i^2 + \bar{V}_{i,0}^2 + \bar{V}_{i,3}^2 \leq B_{i,0} \right\} \]

and

\[ \Pi_{i,j} := \left\{ e_i^T P e_i + 1/\gamma_i \bar{\beta} \bar{\beta}^T + 1/\gamma_i \bar{\beta} \bar{\beta}^T + \sum_{k=2}^{N} X_{i,k}^2, \right. \]
\[ + \sum_{k=2}^{N} z_{i,k}^2 + \bar{V}_{i,0} + \bar{V}_{i,3} \leq 2 p \}

\[ i = 1, \ldots, N, \quad j = 1, \ldots, n_i \]

are compact in \( R^3 \) and \( R^{n_i+2+j+3} \), respectively, \( \Pi_{i,0} \times \Pi_{i,j} \) is also compact in \( R^{n_i+2+j+6} \). Therefore \( \bar{P}_{i,k} \) has a maximum \( M_{i,k} \) on \( \Pi_{i,0} \times \Pi_{i,j} \).

Using the facts

\[ z_{i,3} z_{i,3} \leq z_{i,3}^2 + \frac{1}{4} z_{i,3}^2 \]
\[ X_{i,3} z_{i,3} \leq z_{i,3}^2 + \frac{1}{4} X_{i,3}^2 \]
\[ z_{i,k} z_{i,k+1} \leq z_{i,k}^2 + \frac{1}{4} z_{i,k+1}^2 \]
\[ z_{i,k} X_{i,k+1} \leq z_{i,k}^2 + \frac{1}{4} X_{i,k+1}^2 \]

gives

\[ \dot{V} \leq \sum_{i=1}^{n} \left[ -\frac{2 \lambda_{\max} (I_i^3)}{I_i} - \frac{9}{4} e_i^T e_i - r_i \bar{V}_{i,0} - r_i \bar{V}_{i,3} \right. \]
\[ - c_{i,3}^2 z_{i,3}^2 + \frac{1}{4} z_{i,3}^2 + \frac{1}{4} X_{i,3}^2 + \frac{1}{4} X_{i,3}^2 + \mu_i \bar{\beta} \bar{\beta}^T \]
\[ + \frac{\mu_i}{\gamma_i} \bar{\beta} \beta_i + D_{i,3} + \sum_{k=2}^{N} X_{i,k} \left( - \frac{X_{i,k}}{\bar{r}_{i,k}} + B_{i,k} \right) \]
\[ + \sum_{k=2}^{N} z_{i,k} \left( z_{i,k+1} + X_{i,k+1} - c_{i,k} z_{i,k} \right) - c_{i,r,n} z_{i,n}^2 \]
\[ + \sum_{k=2}^{N} (2 z_{i,k}^2 + \frac{1}{4} z_{i,k+1}^2 + \frac{1}{4} \lambda_{\max} (I_i^3) - c_{i,k} z_{i,k}^2) \]
\[ - c_{i,r,n} z_{i,n}^2 \tag{69} \]

Choose

\[ c_{i,j} = 2 + c_j \]
\[ c_{i,k} > 2 + c_j, k = 2, \ldots, n_i - 1 \tag{70} \]
\[ c_{i,r,n} > 1 + c_j \]

where \( c_j \) is a positive constant.

Using \( \mu_i \tilde{\beta} \tilde{\beta}^T \leq -\mu_i / 2 \tilde{\beta}^2 + \mu_i / 2 \tilde{\beta}^2 \) and \( \mu_i \bar{\beta} \beta_i \leq -\mu_i / 2 \beta_i^2 + \mu_i / 2 \beta_i^2 \), \( \mu_i \bar{\beta} \beta_i \leq -\mu_i / 2 \beta_i^2 + \mu_i / 2 \beta_i^2 \) gives

\[ \dot{V} \leq \sum_{i=1}^{n} \left[ -\frac{2 \lambda_{\max} (I_i^3)}{I_i} - \frac{9}{4} e_i^T e_i - r_i \bar{V}_{i,0} - r_i \bar{V}_{i,3} \right. \]
\[ - c_{i,3} z_{i,3}^2 + z_{i,3}^2 + \frac{1}{4} z_{i,3}^2 + \frac{1}{4} X_{i,3}^2 + \frac{1}{4} X_{i,3}^2 + \mu_i \bar{\beta} \bar{\beta}^T \]
\[ + \frac{\mu_i}{\gamma_i} \bar{\beta} \beta_i + D_{i,3} + \sum_{k=2}^{N} X_{i,k} \left( - \frac{X_{i,k}}{\bar{r}_{i,k}} + B_{i,k} \right) \]
\[ + \sum_{k=2}^{N} z_{i,k} \left( z_{i,k+1} + X_{i,k+1} - c_{i,k} z_{i,k} \right) - c_{i,r,n} z_{i,n}^2 \]
\[ + \sum_{k=2}^{N} (2 z_{i,k}^2 + \frac{1}{4} z_{i,k+1}^2 + \frac{1}{4} \lambda_{\max} (I_i^3) - c_{i,k} z_{i,k}^2) \]
\[ - c_{i,r,n} z_{i,n}^2 \]
\[ + \sum_{k=2}^{N} (2 z_{i,k}^2 + \frac{1}{4} z_{i,k+1}^2 + \frac{1}{4} \lambda_{\max} (I_i^3) - c_{i,k} z_{i,k}^2) \]
\[ - c_{i,r,n} z_{i,n}^2 \]

where \( D = \sum_{k=2}^{N} [D_{i,k} + \mu_i / 2 \tilde{\beta}^2 + \mu_i / 2 \beta_i^2 + \sum_{k=2}^{N} M_{i,k} ] \] and

\[ m_{i,k} = 1/\tau_{i,k} \leq 1/2 (\tau_{i,k} < 2) \.

Defining

\[ C = \min \left\{ 2c_{i,j}, \cdots, 2c_{i,j}, \mu_i, (2 \lambda_{\max} (I_i^3) / I_i - 2), \right. \]
\[ / \lambda_{\max} (\bar{P}), 2m_{i,2}, \cdots, 2m_{i,n}, \right\} \]

we have

\[ \dot{V} \leq -CV + D \tag{71} \]

Integrating (77) over \([0, t] \) results in

\[ V(t) \leq S + e^{-C} V(0) \tag{78} \]

where \( S = D / C \).

By (78), it can be proved that all the signals in the closed-loop system are SGUUB. Moreover, the tracking errors \( z_{i,i} = y_i - y_{i,v} \) can be made as small as desired by appropriate choice of the design parameters \( c_{i,j}, \cdots, c_{i,j}, \mu_i, I_i, \tau_{i,2}, \cdots, \tau_{i,n}, k_{i,3}, \cdots, k_{i,n}, \gamma_i, \) and \( \bar{P} \).
6. Simulation Studies

In this section, the feasibility of the proposed method and the control performances are illustrated by the following two examples.

Example 1: Consider the following large-scale system composed of two second-order subsystems:

\[
\begin{align*}
\Sigma_1: & \quad \dot{x}_{1,1} = x_{1,2} + f_{1,1}(y_1) + h_{1,1} + g_{1,1}(y_1, y_2) \\
& \quad \dot{x}_{1,2} = u_1 + f_{1,2}(y_1) + h_{1,2} + g_{1,2}(y_1, y_2) \\
& \quad y_1 = x_{1,1} \\
& \quad y_2 = x_{1,2} + f_{2,1}(y_2) + h_{2,1} + g_{2,1}(y_1, y_2) \\
\Sigma_2: & \quad \dot{x}_{2,1} = u_2 + f_{2,2}(y_2) + h_{2,2} + g_{2,2}(y_1, y_2) \\
& \quad y_1 = x_{2,1} + f_{2,1}(y_2) + h_{2,1} + g_{2,1}(y_1, y_2) \\
& \quad y_2 = x_{2,2} + f_{2,2}(y_2) + h_{2,2} + g_{2,2}(y_1, y_2)
\end{align*}
\]

where

\[
\begin{align*}
f_{1,1}(y_1) &= -y_1 e^{-y_1^2}, \\
f_{1,2}(y_1) &= y_1^2, \\
f_{2,1}(y_2) &= \sin(y_2), \\
f_{2,2}(y_2) &= y_2^2, \\
g_{1,1}(y_1, y_2) &= y_1 + y_2, \\
g_{1,2}(y_1, y_2) &= 2y_1 y_2, \\
g_{2,1}(y_1, y_2) &= y_1 + 4y_2.
\end{align*}
\]

The reference signals are chosen as

\[
y_{1,r} = \sin(t) \quad \text{and} \quad y_{2,r} = \cos(t).
\]

From Eq. (2), by selecting the functions

\[
H_{1,2} = 2z_{1,1}, \\
H_{2,2} = 2z_{2,1}, \\
\bar{H}_{1,2} = 2\sin^2(t), \\
\bar{H}_{2,2} = 2\sin^2(t), \\
\kappa_{1,2} = 0, \quad \kappa_{2,2} = 0.
\]

Assumption 2 is satisfied. Choose fuzzy membership as

\[
\mu_i(y_i) = \exp[-\frac{(y_i - 6 + 2l)^2}{4}].
\]

Functions \(f_{i,j}(y)(i=1,2; j=1,2)\) can be approximated as

\[
\begin{align*}
f_{1,1}(y_1) &= \phi_{1,1}(y_1) + \delta_{1,1}(y_1), \\
f_{1,2}(y_1) &= \phi_{1,2}(y_1) + \delta_{1,2}(y_1), \\
f_{2,1}(y_2) &= \phi_{2,1}(y_2) + \delta_{2,1}(y_2), \\
f_{2,2}(y_2) &= \phi_{2,2}(y_2) + \delta_{2,2}(y_2).
\end{align*}
\]

Parameters in controller and adaptive laws are chosen as

\[
\begin{align*}
c_{1,1} &= 50, \\
c_{1,2} &= 20, \\
c_{2,1} &= 70, \\
c_{2,2} &= 30, \\
l_1 &= 0.5, \\
l_2 &= 0.4, \\
\gamma_1 &= 0.3, \\
\gamma_2 &= 0.2, \\
\bar{\gamma}_1 &= 0.5, \\
\bar{\gamma}_2 &= 0.3, \\
k_{1,1} &= 2, \\
k_{1,2} &= 4, \\
k_{2,1} &= 5, \\
k_{2,2} &= 8, \\
\mu_i &= \mu_i = 0.1, \\
\tau_{1,1} &= \tau_{2,2} = 0.005, \\
\tau_{1,2} &= \tau_{2,2} = 1.1, \\
\tau &= 0.6.
\end{align*}
\]

If the initial conditions are given as \(x_{1,1}(0) = 0.1, \quad x_{1,2}(0) = -0.1, \quad x_{2,1}(0) = 0.2, \quad x_{2,2}(0) = 0, \quad \hat{\delta}_1(0) = 0.45, \quad \hat{\delta}_2(0) = 0.5, \quad \hat{\beta}_1(0) = 1.5, \quad \hat{\beta}_2(0) = 1, \quad \text{and the step time is} \quad 0.01s, \quad \text{then we obtain the simulation results, which are shown by Figures 1-4.}
Example 2: Consider two inverted pendulums connected by a spring. Denoting \( x_{1,1} = \theta_1 \) (angular position), \( x_{1,2} = \dot{\theta}_1 \) (angular rate), \( x_{2,1} = \theta_2 \) and \( x_{2,2} = \dot{\theta}_2 \). Angular rates \( x_{1,2} = \dot{\theta}_1 \) and \( x_{2,2} = \dot{\theta}_2 \) are unavailable.

The equations which describe the motion of the pendulums are defined by

\[
\begin{align*}
\dot{x}_{1,1} &= x_{1,2}, \\
\dot{x}_{1,2} &= \left( \frac{m_1 g r}{J_1} - \frac{kr^2}{4J_1} \right) \sin(x_{1,1}) + \frac{kr}{2J_1} (l - b) + h_{1,2} + \frac{u_1}{J_1} + \frac{kr^2}{4J_1} \sin(x_{1,1}), \\
y_1 &= x_{1,1}, \\
\dot{x}_{2,1} &= x_{2,2}, \\
\dot{x}_{2,2} &= \left( \frac{m_2 g r}{J_2} - \frac{kr^2}{4J_2} \right) \sin(x_{2,1}) + \frac{kr}{2J_2} (l - b) + h_{2,2} + \frac{u_2}{J_2} + \frac{kr^2}{4J_2} \sin(x_{2,1}), \\
y_2 &= x_{2,1}, \\
\end{align*}
\]

The parameter \( m_1 = 2 \) kg and \( m_2 = 2.5 \) kg are the pendulum end masses, \( J_1 = 1 \) kg and \( J_2 = 1 \) kg are the moments of inertia, \( k = 10 \) N/m is the spring constant of the connecting spring, \( r = 0.1 \) m is the pendulum height, \( l = 0.5 \) m is the natural length of the spring, and \( g = 9.8 \) m/s\(^2\) is gravitational acceleration. The distance between the pendulum hinges is defined as \( b = 0.5 \) m.

\[
h_{1,2} = \frac{x_{1,1}(t - \tau_{1,2}(t))}{1 + x_{1,2}^2(t - \tau_{1,2}(t))}, \quad h_{2,2} = \frac{x_{2,2}(t - \tau_{2,2}(t))}{1 + x_{2,2}^2(t - \tau_{2,2}(t))}, \quad \tau_{1,2} = \tau_{2,2} = 0.4(1 + \sin^2(t)).
\]

From Eq. (2), by selecting the functions \( H_{1,2} = z_{1,1}; H_{2,2} = z_{2,1}; \bar{h}_{1,2} = 0; \bar{h}_{2,2} = 0; \kappa_{1,2} = 0 \) and \( \kappa_{2,2} = 0 \), Assumption 2 is satisfied.

The reference signals \( y_{1,r} = 0 \) and \( y_{2,r} = 0 \). Choose fuzzy membership as

\[
\mu_{i,j}(y_i) = \exp\left[\frac{-|y_i - 3 + l|^2}{6}\right], \quad i = 1, 2; \quad l = 1, \ldots, 5.
\]

Parameters in controller and adaptive laws are chosen as

\[
c_{1,1} = 80, \quad c_{1,2} = 40, \quad c_{2,1} = 70, \quad c_{2,2} = 50, \quad l_i = 0.6, \quad l = 0.5, \quad \gamma_1 = 0.4, \quad \gamma_2 = 0.5, \quad \bar{r}_1 = 0.6, \quad \bar{r}_2 = 0.6, \quad k_{1,1} = 3, \quad k_{1,2} = 3, \quad k_{2,1} = 2, \quad k_{2,2} = 4, \quad \mu_1 = \mu_2 = 0.01, \quad \bar{r}_{1,2} = \bar{r}_{2,2} = 0.005, \quad \tau_{1,2} = \tau_{2,2} = 0.8, \quad \hat{\tau}_{1,2} = 0.8, \quad \tau = 0.9.
\]

If the initial conditions are given as \( x_{1,1}(0) = 0.314, \quad x_{2,1}(0) = -0.314, \) and others initial condition are 0. Then we obtain the simulation results, which are shown by Figures 5-8.
7. Conclusions

In this paper, the problem of adaptive fuzzy dynamic surface control has been investigated for a class of nonlinear time-delay large-scale systems with unknown time-varying delays based on fuzzy approximation approach. The problem of “explosion of complexity” has been avoided by using the DSC technique. In addition, the numbers of the on-line adaptive parameters are only N, the computation burden can be reduced accordingly, therefore it is convenient to implement this algorithm in practical systems.

Acknowledgment

This work was supported in part by the National Natural Science Foundation of China (Nos. 61203008, 61074014, 51179019), the Program for Liaoning Innovative Research Team in University (No. LT2012013), the Program for Liaoning Excellent Talents in University (No.LR2012016), the Natural Science Foundation of Liaoning Province (No. 20112012), and the Applied Basic Research Program of Ministry of Transport of P. R. China (Nos. 2011-329-225-390 and 2013-329-225-270).

References


