Abstract

In this paper, a fuzzy neural network (FNN) based discrete adaptive iterative learning controller (AILC) is proposed for a class of discrete-time uncertain nonlinear plants which can repeat a given task over a finite time sequence. Compared with the existing discrete AILC schemes, the proposed strategy can be applied to the discrete-time uncertain nonlinear plants with not only initial resetting errors and iteration-varying desired trajectory, but also random bounded disturbances and unknown non-Lipschitz plant nonlinearities. Two FNNs are used as approximators to compensate for the unknown plant nonlinearities. To overcome the function approximation error and possibly large random bounded disturbance, a time-varying boundary layer is introduced to design an auxiliary error function. The auxiliary error function is then utilized to derive the adaptive laws since the optimal FNN parameters for a good function approximation and the optimal width of time-varying boundary layer are unavailable. By using a Lyapunov like analysis, we show that the closed-loop is stable in the sense that the adjustable parameters and internal signals are bounded for all the iterations. Furthermore, learning performance is guaranteed in the sense that the norm of output tracking error vector will asymptotically converge to a residual set which is bounded by the width of boundary layer.

Keywords: Fuzzy neural network, discrete adaptive iterative learning control, input disturbance, discrete-time nonlinear plant.

1. Introduction

Iterative learning control (ILC) is one of the most successful and effective control approaches to deal with the tasks of repeated tracking control or periodic disturbance rejection [1, 2, 3]. The traditional PID-type ILC algorithms are simple and easily implemented so that they have a lot of practical applications both for continuous-time [4, 5] and discrete-time systems [6, 7, 8]. But in general it has to assume that the plant’s nonlinearities satisfy global Lipschitz continuous condition for the design of PID-type ILC. Motivated by the concept of nonlinear adaptive control, the adaptive iterative learning control (AILC) [9, 10, 11, 12] schemes that tune the control parameters instead of control input itself between successive iterations have been successfully investigated for non-Lipschitz nonlinear plants. In addition to the extension to non-Lipschitz nonlinear plants, the most interesting part is that the AILC can also deal with the problems of iteration-varying reference trajectories, random bounded initial resetting error and random bounded disturbance. But it is noted that these AILC systems deeply depend on the fact that the unknown parameters are linearly parameterized with known nonlinear functions. If the system nonlinearity cannot be linearly parameterized, the FNN [13, 14, 15] is an effective strategy for AILC to deal with the unknown nonlinear systems [16, 17, 18]. Even though there were many successful results in the area of AILC for the past decade, we emphasize that these aforementioned AILC systems were designed and analyzed for continuous-time plants. Since it is necessary to store the data of desired trajectory, plant output and control parameters in memory when implementing an AILC system, it is more practical and important to design and analyze the discrete AILC for real implementation. Recently the discrete AILC schemes have been presented for SISO and MIMO nonlinear discrete-time systems in [19, 20, 21]. In [19], a discrete AILC was proposed for a class of discrete-time nonlinear systems with unknown time-varying parametric uncertainties. In [20], another discrete AILC was investigated for a class of discrete-time nonlinear systems with unknown time-varying parameters and time-varying uncertain nonlinear input gain. In [21], a discrete AILC was extended to MIMO discrete-time parametric nonlinear systems. In order to develop the adaptive laws for the unknown system parameters, it is required that the unknown parameters must be linear with respect to some known nonlinear vector-valued functions. If the nonlinear functions are unknown, a neural network based discrete AILC with dead zone scheme was proposed in [22] where two neural networks were utilized to
compensate for the plant unknown nonlinearities. However, the stability and convergence of the learning system was shown under a strict assumption that the network parameter errors and the function approximation errors are small enough. Furthermore, we can find that only the problems of iteration-varying reference trajectories and random bounded initial resetting error have been solved in [19, 20, 21, 22]. If there exists the uncertainty of random bounded disturbance, the closed-loop stability and tracking performance can not be guaranteed. Based on the above discussions, the motivation of this paper is now clear. We would like to present a new FNN based discrete AILC for unknown discrete-time nonlinear plants with not only iteration-varying reference trajectories and random bounded initial resetting error, but also random bounded disturbance. It should be emphasized that this extension is not a trivial work in the area of discrete AILC since all the proposed methods in the literature can not be applied directly to the case of nonlinear plants with random bounded disturbance. In this FNN based discrete AILC system, two FNNs are used as approximators to compensate for the plant unknown nonlinearities. It is noted that the bounds of the approximation errors and random disturbance are not required to be small enough. To overcome the uncertainties due to function approximation errors and random bounded disturbance, a time-varying boundary layer is introduced to design a dead-zone like auxiliary error function. This auxiliary error function is then utilized to construct the adaptive laws for adaptation of the network parameters and width of time-varying boundary layer along iteration axis. By using a Lyapunov like analysis, we show that the norm of output tracking error vector will asymptotically converge to a residual set bounded by the width of boundary layer as the number of iteration tends to infinity. Furthermore, the adjustable parameters as well as internal signals will remain bounded for all iterations. This paper is organized as follows. In section 2, a problem formulation is given. The fuzzy neural discrete AILC is presented in section 3. Based on the proposed discrete AILC and a derived error model, the analysis of closed-loop stability and learning performance will be studied extensively in section 4. Two simulation examples, including a practical application, will be given in section 5 to demonstrate the effectiveness of the proposed learning controller. Finally a conclusion is made in section 6.

2. Problem Formulation

In this paper, we consider a class of discrete-time nonlinear plants which can perform a given task repeatedly over a finite time sequence \( \forall t \in \{0,1,2,\cdots,N\} \) as follows:

\[ y'(t+1) = -f(X'(t)) + b(X'(t))u'(t) + d'(t) \quad (1) \]

where \( y'(t) \) is the plant output, \( d'(t) \) is a non-repeatable random disturbance, \( X'(t) = [y'(t), \cdots, y'(t-n+1), u'(t), \cdots, u'(t-m+1)]' \in R^{(n+m-1)} \) with \( n \) and \( m \) being the respective output and input delay orders, \( f(X'(t)) \) and \( b(X'(t)) \) are unknown real continuous nonlinear functions of \( X'(t) \). Here, \( j \) and \( t \) denote the index of iteration and time respectively. Given a specified desired trajectory \( y_j(t) \), \( \forall t \in \{0,1,2,\cdots,N+1\} \), the control objective is to force the output \( y'(t) \) to follow \( y_j(t) \) such that

\[ \lim_{t \to \infty} y_j(t) - y'(t) \leq \epsilon \]

for some small positive error tolerance bound \( \epsilon \) and for \( t \in \{1,2,\cdots,N+1\} \). In order to achieve this control objective, some assumptions on the nonlinear discrete-time system and desired trajectory are given as follows:

(A1) The discrete-time nonlinear plant is a relaxed system whose input \( u'(t) \) and output \( y'(t) \) are related by \( y'(t) = 0, t < 0 \).

(A2) There exists a positive known constant \( b_0 \) and unknown constant \( b_j \), such that \( b_j \leq b(X'(t)) \leq b_0 \) for all \( X'(t) \in R^{(n+m-1)} \) and \( j \geq 1 \).

(A3) There exists a positive unknown constant \( d_u \) such that \( |I'(t)| \leq d_u \) for all \( t \in \{0,1,\cdots,N\} \) and \( j \geq 1 \).

(A4) There exists a positive known constant \( y_j^* \) such that \( |y_j(t)| \leq y_j^* \) for all \( t \in \{0,1,\cdots,N\} \) and \( j \geq 1 \).

(A5) Let output tracking error be defined as \( e'(t) = y_j(t) - y'(t) \). The initial output error at each iteration \( e'(0) \) is not necessarily zero, small or fixed, but assumed to be bounded.

3. The FNN Based Discrete Adaptive ILC

In order to find the approach for controller design later, we first derive the output tracking error equation as

\[ e'(t+1) = y_j'(t+1) + f(X'(t)) - b(X'(t))u'(t) - d'(t) \quad (2) \]

\[ = b(X'(t))(g(X'(t))y_j'(t+1) + h(X'(t)) - u'(t) - d'(t) \]

where \( b(X'(t)) = f(X'(t))/b(X'(t)) \) and \( g(X'(t)) = 1/b(X'(t)) \). It is clear that if the nonlinear functions \( f(X'(t)) \) and \( b(X'(t)) \) are completely known and \( d'(t) = 0 \) , we can define the controller as

\[ u'(t) = h(X'(t)) + g(X'(t))y_j'(t+1) \]

such that \( e'(t+1) = 0 \) for all \( t \in \{0,1,\cdots,N\} \) and \( j \geq 1 \). Unfortunately, \( f(X'(t)) \) and \( b(X'(t)) \) are in general unknown or only partially known and \( d'(t) \neq 0 \). In order to overcome the
unknown nonlinear functions $f(X^j(t))$ and $b(X^j(t))$ (or equivalently, the unknown nonlinear functions $h(X^j(t))$ and $g(X^j(t))$), we apply the universal approximation technique to design the basic structure of our discrete AILC. The FNN structure is shown in Figure 1 (for detailed, please see [18]).

Two FNNs $O_b^{(4)}(X^j(t),W_b^j(t))$ and $O_g^{(4)}(X^j(t),W_g^j(t))$ which respectively perform as the approximators of $h(X^j(t))$ and $g(X^j(t))$ are described as follows

\[
O_b^{(4)}(X^j(t),W_b^j(t)) = W_b^j(t)^T O_b^{(3)}(X^j(t))
\]

\[
O_g^{(4)}(X^j(t),W_g^j(t)) = W_g^j(t)^T O_g^{(3)}(X^j(t))
\]

In the representation of (3) and (4), the FNNs are expressed as a series of radial basis functions expansion with the basis functions as $O_b^{(3)}(X^j(t)) \in \mathbb{R}^{|M_x|}$ and $O_g^{(3)}(X^j(t)) \in \mathbb{R}^{|M_x|}$. It is well known that the FNNs (3) and (4) can uniformly approximate real continuous nonlinear functions $h(X^j(t))$ and $g(X^j(t))$ respectively on a compact set $A \subset \mathbb{R}^{|x|}$. An important aspect of the above approximation property is that there exist optimal weights $W_b^*$ and $W_g^*$ such that the function approximation errors between the optimal FNNs $O_b^{(4)}(X^j(t),W_b^*)$ and $O_g^{(4)}(X^j(t),W_g^*)$ and functions $h(X^j(t))$, $g(X^j(t))$ can be bounded by prescribed constant $\varepsilon_b^*$ and $\varepsilon_g^*$ on the compact set $A$. Therefore, if we let

\[
h(X^j(t)) = O_b^{(4)}(X^j(t),W_b^*) + \varepsilon_b(X^j(t))
\]

\[
g(X^j(t)) = O_g^{(4)}(X^j(t),W_g^*) + \varepsilon_g(X^j(t))
\]

\[
= O_b^{(4)}(X^j(t),W_b^*) + \varepsilon_b(X^j(t)) + O_g^{(4)}(X^j(t),W_g^*) + \varepsilon_g(X^j(t))
\]

\[
\text{then the approximation errors will satisfy } |\varepsilon_b(X^j(t))| \leq \varepsilon_b^* \text{ and } |\varepsilon_g(X^j(t))| \leq \varepsilon_g^*, \forall X^j(t) \in A.
\]

In order to overcome the unknown approximation error and random input disturbance, we now define a dead-zone like auxiliary error function $e^j(t+1)$ as

\[
e^j(t+1) = e^j(t+1) - \phi^j(t+1) \text{sat} \left( \frac{e^j(t+1)}{\phi^j(t+1)} \right)
\]

for $t \in \{0,1,2,\ldots, N\}$. We don’t define $e^j(0)$ since it will not be utilized in our design of controller and adaptive laws. In (7), sat is the saturation function defined as

\[
sat \left( \frac{e^j(t+1)}{\phi^j(t+1)} \right) = \begin{cases} 1, & \text{if } e^j(t+1) > \phi^j(t+1) \\ e^j(t+1), & \text{if } |e^j(t+1)| \leq \phi^j(t+1) \\ -1, & \text{if } e^j(t+1) < -\phi^j(t+1) \\ \\ \end{cases}
\]

and $\phi^j(t+1)$ is the width of the time-varying boundary layer to be designed later. It is noted that $e^j(t+1)$ can be rewritten as

\[
e^j(t+1) = \begin{cases} e^j(t+1) - \phi^j(t+1), & \text{if } e^j(t+1) > \phi^j(t+1) \\ 0, & \text{if } |e^j(t+1)| \leq \phi^j(t+1) \\ e^j(t+1) + \phi^j(t+1), & \text{if } e^j(t+1) < -\phi^j(t+1) \\ \end{cases}
\]

and it can be easily shown that $e^j(t+1) \geq 1$. The auxiliary error function $e^j(t+1)$ is then utilized to design the adaptive laws for $W_b^j(t)$, $W_g^j(t)$ in (5) and $\phi^j(t+1)$ in (7) since the unknown optimal FNN parameter vectors $W_b^*$,
$w^*_g$ and the unknown uncertainty bound $\phi^* = h_u \epsilon_u^* + h_U \epsilon_U^*(y'_j(t))$ are unknown. The adaptive laws for $w^*_g(t)$, $w^*_g(t)$ and $\phi^*(t+1)$ at (next) $j+1$ th iteration to guarantee the convergence of learning error are given as follows:

$$w^{j+1}_g(t) = w^*_g(t) + \beta \epsilon'_g(t+1)O^{(3)}_g(X^j(t))$$

$$w^{j+1}_g(t) = w^*_g(t) + \beta \epsilon'_g(t+1)O^{(3)}_g(X^j(t))y'_j(t+1)$$

$$\phi^{j+1}(t+1) = \phi'(t+1) + \beta |\epsilon'_g(t+1)|$$

for $t \in [0,1,2,\cdots,N]$ , where $\beta_1, \beta_2, \beta_3 > 0$ are the adaptation gains. For the first iteration, we set $w^*_g(t) = w^*_g$, $w^*_g(t) = w^*_g$ to be any constant vector or simply as a zero vector $\forall t \in [0,1,2,\cdots,N]$ . On the other hand, we set $\phi'(t+1) = \phi^* > 0$ to be a small fixed value $\forall t \in [0,1,2,\cdots,N]$ . It is noted that $\phi^*(t+1) > 0$, $\forall t \in [0,1,2,\cdots,N]$ and $\forall j \geq 1$ . Detailed block diagram of the proposed fuzzy neural discrete AILC scheme is now shown in Figure 2.

![Diagram](image-url)

Figure 2. The block diagram of the fuzzy neural discrete AILC.

### 4. Analysis of Stability and Convergence

In this section, we will analyze the closed loop stability and learning error convergence. At first, we define the parameter errors as $\tilde{w}^*_g(t) = w^*_g(t) - w^*_g$ ,

$\tilde{w}^*_g(t) = w^*_g(t) - w^*_g$ . $\phi^*(t+1) = \phi'(t+1) - \phi^*$. Then, it is easy to show by subtracting the optimal control gains on both side of (8)-(10) that

$$\tilde{w}^{j+1}_g(t) = \tilde{w}^*_g(t) + \beta \epsilon'_g(t+1)O^{(3)}_g(X^j(t))$$

$$\tilde{w}^{j+1}_g(t) = \tilde{w}^*_g(t) + \beta \epsilon'_g(t+1)O^{(3)}_g(X^j(t))y'_j(t+1)$$

$$\tilde{\phi}^{j+1}(t+1) = \tilde{\phi}'(t+1) + \beta |\epsilon'_g(t+1)|$$

Now we are ready to state the main results in the following theorem.

**Main Theorem:** Consider the nonlinear discrete-time plant (1) and an iteration-dependent desired trajectory $y'_j(t)$ satisfying the assumptions (A1)-(A5). If the fuzzy neural discrete adaptive iterative learning controller as in (5), (7), (8), (9) and (10) is applied, with the following condition being satisfied

$$2 - b_U M_s \beta_1 - (y'_j)^2 b_U M_s \beta_2 - \beta_3 > 0$$

then the following facts will hold:

1. $e'_j(t+1)$ is bounded $\forall t \in [0,1,2,\cdots,N]$ , $\forall j \geq 1$ and $\lim_{t \to \infty} e'_j(t+1) = 0$, $\forall t \in [0,1,2,\cdots,N]$.

2. $e'(t+1)$ is bounded $\forall t \in [0,1,2,\cdots,N]$ , $\forall j \geq 1$ and $\lim_{t \to \infty} e'(t+1) = 0$, $\forall t \in [0,1,2,\cdots,N]$.

3. All adjustable control parameters $w^*_g(t)$ , $w^*_g(t)$ , $w^*_g(t)$ and control input $u^*(t)$ are bounded $\forall t \in [0,1,2,\cdots,N]$ and $\forall j \geq 1$.

**Proof:**

1. Define a positive function $V'(t)$ , $\forall t \in [0,1,2,\cdots,N]$ as

$$V'(t) = \frac{b_U}{\beta_1} \tilde{w}^*_g(t) \tilde{w}^*_g(t) + \frac{b_U}{\beta_2} \tilde{w}^*_g(t) \tilde{w}^*_g(t) + \frac{1}{\beta_3} (\tilde{\phi}'(t+1))^2$$

The difference between $V^{j+1}(t)$ and $V'(t)$ can be derived as follows:

$$V^{j+1}(t) - V'(t)$$

$$= \frac{b_U}{\beta_1} (\tilde{w}^{j+1}_g(t) \tilde{w}^{j+1}_g(t) - \tilde{w}^*_g(t) \tilde{w}^*_g(t))$$

$$+ \frac{b_U}{\beta_2} (\tilde{w}^{j+1}_g(t) \tilde{w}^{j+1}_g(t) - \tilde{w}^*_g(t) \tilde{w}^*_g(t))$$

$$+ \frac{1}{\beta_3} ((\tilde{\phi}^{j+1}(t+1))^2 - (\tilde{\phi}'(t+1))^2)$$

$$= 2b_U \epsilon'_g(t+1) \tilde{w}^*_g(t) O^{(3)}_g(X^j(t))$$

$$+ \beta b_U \epsilon'_g(t+1) \tilde{w}^*_g(t) O^{(3)}_g(X^j(t)) y'_j(t+1)$$

$$+ \beta b_U \epsilon'_g(t+1) \tilde{w}^*_g(t) O^{(3)}_g(X^j(t)) y'_j(t+1)$$

$$+ b_U \epsilon'_g(t+1) \tilde{\phi}'(t+1) + \beta |\epsilon'_g(t+1)|$$

Note that (6) can be rewritten as

$$-\tilde{w}^*_g(t) O^{(3)}_g(X^j(t)) - \tilde{w}^*_g(t) O^{(3)}_g(X^j(t)) y'_j(t+1)$$

$$= e'(t+1) - e'_g(X^j(t)) - e'_g(X^j(t)) y'_j(t+1) + \frac{d}{b(X^j(t))}$$

Substituting (16) and (7) into (15), it yields

$$2 - b_U M_s \beta_1 - (y'_j)^2 b_U M_s \beta_2 - \beta_3 > 0$$
\[ V^{i+1}(t) - V^i(t) = 2b_e e_{\phi}^i(t+1)\left(-\frac{e_{\phi}^i(t+1)}{b(X^i(t))} + e_{\phi}(X^i(t)) + e_{\phi}(X^i(t)) \right) \]

\[ + e_{\phi}(X^i(t))y_{\phi}^i(t+1) - \frac{d^i(t)}{b(X^i(t))} \]

\[ + \beta b_e (e_{\phi}^i(t+1))^2 \left\| O^i_s(X^i(t)) \right\|^2 \]

\[ + \beta b_e (e_{\phi}^i(t+1))^2 \left( y_{\phi}^i(t+1) \right)^2 \left\| O^i_s(X^i(t)) \right\|^2 \]

\[ + 2\left[e_{\phi}^i(t+1) \tilde{\phi}^i(t+1) + \beta (e_{\phi}^i(t+1))^2 \right] \]

\[ \leq -2e_{\phi}^i(t+1) e_{\phi}^i(t+1) + 2b_e (e_{\phi}^i(t+1)) e_{\phi}(X^i(t)) \]

\[ + \beta b_e (e_{\phi}^i(t+1))^2 \left\| O^i_s(X^i(t)) \right\|^2 \]

\[ + 2\left[e_{\phi}^i(t+1) \tilde{\phi}^i(t+1) + \beta (e_{\phi}^i(t+1))^2 \right] \]

\[ \leq -2e_{\phi}^i(t+1) e_{\phi}^i(t+1) + 2b_e (e_{\phi}^i(t+1)) e_{\phi}(X^i(t)) \]

\[ + \beta b_e (e_{\phi}^i(t+1))^2 \left\| O^i_s(X^i(t)) \right\|^2 \]

\[ + 2\left[e_{\phi}^i(t+1) \tilde{\phi}^i(t+1) + \beta (e_{\phi}^i(t+1))^2 \right] \]

\[ \leq -2e_{\phi}^i(t+1) e_{\phi}^i(t+1) + 2b_e (e_{\phi}^i(t+1)) e_{\phi}(X^i(t)) \]

\[ + \beta b_e (e_{\phi}^i(t+1))^2 \left\| O^i_s(X^i(t)) \right\|^2 \]

\[ + 2\left[e_{\phi}^i(t+1) \tilde{\phi}^i(t+1) + \beta (e_{\phi}^i(t+1))^2 \right] \]

\[ \leq -(2 + b_e M_{x} \beta_1 + (y_{\phi}^i)^2 b_e M_{x} \beta_1 + \beta_3) (e_{\phi}^i(t+1))^2 \]

\[ = -k(e_{\phi}^i(t+1))^2 \]

In general, we can design \( \beta_1, \beta_2, \beta_3 \) such that

\[ k = 2 - b_e M_{x} \beta_1 - (y_{\phi}^i)^2 b_e M_{x} \beta_1 - \beta_3 > 0, \]

and hence

\[ k(e_{\phi}^i(t+1))^2 \leq V^i(t) - V^{i+1}(t) \leq V^i(t) \]

for all \( j \geq 1 \). Note that \( V^i(t) \) is bounded \( \forall t \in \{0,1,2,\ldots,N\} \) since the fact that \( \hat{W}^i(t) = W^i(t) - W_0^i = W_0^i - W_0^i = \hat{W}^i \), \( \tilde{V}^i(t) = V_0^i(t) - W_0^i = W_0^i - W_0^i = \tilde{V}^i \), and \( \tilde{\phi}^i(t+1) = \phi^i(t+1) - \phi = \phi - \phi^* \) are bounded \( \forall t \in \{0,1,2,\ldots,N\} \). This implies that \( (e_{\phi}^i(t+1))^2 \) is bounded \( \forall t \in \{0,1,2,\ldots,N\} \). On the other hand, \( V^i(t) \) will converge to some positive function since \( V^i(t) \) is positive definite and monotonically decreasing by the fact of (17). Hence, \( V^{i+1}(t) - V^i(t) \) converges to zero and

\[ \lim_{j \to \infty} e_{\phi}^i(t+1) = e_{\phi}^i(t+1) = 0, \forall t \in \{0,1,2,\ldots,N\} \]

This proves (11) of the main theorem.

(12) The boundedness of \( V^i(t) \) at each iteration over \( \{0,1,2,\ldots,N\} \) also ensures the boundedness of \( \hat{W}^i(t), \tilde{V}^i(t), \tilde{\phi}^i(t+1) \) for all \( j \geq 1 \). The boundedness of \( e_{\phi}^i(t+1) \) at each iteration over \( \{0,1,2,\ldots,N\} \) can be concluded from equation (7) because \( \phi(t+1) \) is always bounded. Furthermore, the bound of \( e_{\phi}^i(t+1) \) will satisfy

\[ \lim_{j \to \infty} e_{\phi}^i(t+1) = e_{\phi}^i(t+1) = 0, \forall t \in \{0,1,2,\ldots,N\} \]

This proves (12) of the main theorem.

(13) It is noted that boundedness of \( e_{\phi}^i(t+1) \) is shown in (12) for all \( t \in \{0,1,2,\ldots,N\} \) and \( j \geq 1 \). Together with the fact that all the adjustable parameters are bounded due to the boundedness of \( V^i(t) \) at each iteration over \( \{0,1,2,\ldots,N\} \), we can conclude that the control input \( u^i(t) \) is bounded according to (5). Thus, (13) of the main theorem is established.

**Remark 1:** Since the output tracking error \( e_{\phi}^i(t+1) \) can be shown to converge to a residual set which is bounded by the boundary layer \( \phi^* = \phi(t+1) \), it is necessary to make \( \phi^*(t+1) \) as small as possible for all \( t \in \{0,1,2,\ldots,N\} \). This is why we set the initial value of the boundary layer as a small constant \( \phi^0 \), i.e., \( \phi^0(t+1) = \phi^0 > 0 \), \( t \in \{0,1,2,\ldots,N\} \). The adaptation gain \( \beta_i \) in (10) should also be chosen as small as possible such that \( \phi^*(t+1) \), \( t \in \{0,1,2,\ldots,N\} \) will remain in a reasonable small value for all \( j \geq 1 \). Fortunately, the adaptation gain \( \beta_i \) can be chosen as small as possible since it is required to satisfy the convergent condition (14).

**Remark 2:** In addition to \( \beta_i \), it is necessary to choose the adaptation gains \( \beta_1, \beta_2 \) such that the convergent condition (14) can be satisfied. In general, it is suggested to firstly set \( \beta_1 \) as small as possible due to the reason discussed in Remark 1. Then, small values of \( \beta_1 \) and \( \beta_2 \) will also be chosen in order to make the value of \( k = 2 - b_e M_{x} \beta_1 - (y_{\phi}^i)^2 b_e M_{x} \beta_1 - \beta_3 \) in (17) as large as possible. It is interesting to note from (17) that the con-
vergent speed will be faster if \( k \) is larger. We also observe this behavior from our experiences. However, the learning performance will decline if \( \beta_1 \) and \( \beta_2 \) are too small since the adaptation processes (11) and (12) will almost stop. It seems that there exists an optimal adaptation gain \( \beta \) for the proposed discrete AILC to get the fastest convergent speed. How to find the optimal adaptation gain is a possible future work for this paper, especially when the upper bound \( b_c \) is unknown.

5. Simulation Example

Example 1: In this example, we use the proposed fuzzy-neural discrete adaptive iterative learning controller to iteratively control an unknown non-BIBO nonlinear dynamic plant [23]. The difference equation of the nonlinear dynamic plant is given as

\[
\begin{align*}
J(t) &= m_x(t) + b_c(t) + g(x(t)), \\
J(t) &= m_x(t) + b_c(t) + g(x(t)) + \epsilon(t),
\end{align*}
\]

where \( J(t) \) is the plant output, \( u(t) \) is the control input, and \( \epsilon(t) \) is a bounded random disturbance. The parameters \( m_x(t) \) and \( b_c(t) \) are partially known. However, we often arbitrarily choose the initial parameters. In addition, it is noted that \( \phi(t+1) \) should be as small as possible for all \( t \in [0,1,2,\cdots,200] \) so that the output tracking error \( e^i(t+1) \) can converge to a residual set which is bounded by the small boundary layer \( \phi(t+1) \) for all \( t \in [0,1,2,\cdots,200] \).

(D3) Since \( M_s = 4 \), \( M_g = 4 \) is given in step (D2) and \( b_c = 1.5 \), the adaptive laws (8), (9) and (10) with the adaptation gains \( \beta_1 = \beta_2 = 0.125 \), \( \beta_1 = 10^{-3} \) are adopted so that the learning condition \( 2 - b_c \leq b_c(\phi(t)) \) is satisfied. In general, the adaptation gain \( \beta_1 \) will be chosen as small as possible to ensure that \( \phi(t+1) \), \( t \in [0,1,2,\cdots,200] \) will remain in a reasonable small value for all \( j \geq 1 \).

To study the effect of learning performances by using the proposed fuzzy neural discrete AILC, Figure 3 (a) is used to show \( \max_{t \in [0,1,2,\cdots,200]} |\phi(t)| \) with respective to iteration \( j \). It is clear that the asymptotical convergence proves the technical result given in (11) of main theorem. Since the learning process is almost completed at the 40th iteration, the learning error \( e^i(t) \) is shown in Figure 3 (b). In Figure 3 (b), the trajectory of \( e^i(t) \) satisfies \( |e^i(t)| \leq \phi(t), t \in [1,2,\cdots,200] \) clearly proves (12) of the main theorem. Since the nice output tracking performance at the 40th iteration are achieved, we show the relation between plant output \( y^0(t) \) and desired trajectory \( y^d(t) \) in Figure 3 (c) for \( t \in [0,1,2,\cdots,200] \). To see the control behavior that \( y^0(t) \) is close to \( y^d(t) \) for \( t \in [1,2,\cdots,200] \) except the initial one \( y^0(0) \), the trajectories between \( y^0(t) \) and \( y^d(t) \) are shown again in Figure 3(d) but only for the time sequence \( t \in [0,1,2,\cdots,10] \). It is clear that \( y^0(t) \) converges to \( y^d(t) \) after \( t \geq 1 \). Finally, the bounded control force \( u^i(t) \) is shown in Figure 3 (e).
Figure 3. (a) max_{j=1,2,⋯,200} |y_j(t)| versus control iteration j; (b) e^{+}(t) (solid line) and \phi^{+}(t) (dotted lines) versus time t \in \{1,2,⋯,200\}; (c) y^{+}(t) (solid line) and y^{\ddagger}(t) (dotted line) versus time t \in \{0,1,⋯,200\} at the 40th control iteration; (d) y^{\ddagger}(t) (○○○) and y^\ddagger(t) (--) versus time t \in \{0,1,⋯,10\} at the 40th control iteration; (e) u^\ddagger(t) versus time t.

Example 2: In order to apply the proposed fuzzy neural discrete ILC for a more practical system with iteration-varying reference trajectories, random bounded initial resetting errors and random bounded disturbance, we adopt the following typical surge tank model that can be represented by the following difference equation [24]:

\[
y'(t+1) = y'(t) + T_s \left[ \frac{-\sqrt{2gy'(t)}}{A_c(y'(t))} + \frac{1}{A_c(y'(t))} u'(t) \right] + d^\ddagger(t),
\]

y'(0) = 0.5 + 0.5\text{randn}

where \(y'(t)\) denotes the liquid level (system output), \(u'(t)\) is the input flow (control input), \(A_c(y'(t))=\sqrt{ay'(t)^2+b}\) is the cross sectional area of the tank, \(g = 9.8\text{m/s}^2\) is the acceleration due to gravity, \(T_s\) is the sampling time and \(d^\ddagger(t) = 0.1\text{randn}\) is a non-repeatable random disturbance, \(\text{randn}\) is a generator of random number with normal distribution, mean= 0 and variance= 1. Here, we choose the system parameters as \(a = 1, b = 3\) and \(T_s = 0.1\text{s}\), respectively. The control objective is to force the system output \(y'(t)\) to track a desired iteration-varying trajectory \(y^+_j(t) = 2 + \sin(\pi t / 150) + 0.1 \cos(2\pi t / 50), t \in \{0,1,⋯,500\}\) as close as possible even there might exist random bounded initial resetting error and random bounded disturbance in each iteration. The design steps are summarized as:

(D1) Define the output error as \(e'(t) = y'_j(t) - y'(t)\), the initial output error \(e'(0)\) is given by \(|e'(0)| = |y'_j(0) - y'(0)| = 2 + 0.1\cos(2\pi t / 50) - (0.5 + 0.5\text{randn})\) for each iteration.

(D2) Let \(X'(t) = y'(t) \in R\), then the fuzzy neural discrete AILC \(u'(t)\) is designed as in (5), (7), (8), (9) and (10) where two fuzzy neural networks with input \(X'(t)\) are given as \(O^{(4)}(X'(t),W^4_h(t))\) and \(O^{(4)}(X'(t),W^4_g(t))\). Since the working domain of the desired trajectory \(y^+_j(t)\) is within the interval \([0.35,5]\), the centers and variances of \(O^{(4)}(X'(t),W^4_h(t))\) and \(O^{(4)}(X'(t),W^4_g(t))\) are chosen as \([m^h_{1,\ell}, m^g_{1,\ell}, m^h_{2,\ell}, m^g_{2,\ell}] = [0.1,2,4,3,6]\) and \(\sigma^h_{\ell} = 0.6, \ell = 1,⋯,4\), respectively. For simplicity, the centers and variances of the centers and variances of \(O^{(4)}(X'(t),W^4_h(t))\) are also the same as those for \(O^{(4)}(X'(t),W^4_g(t))\). In addition, since there is little information about the unknown consequent parameters \(W^4_h(t), W^4_g(t)\) and the width of the boundary layer \(\phi'(t+1)\) at the first iteration, we simply set the initial values of the consequent parameters as \(W^4_h(t) = [0.1,0.1,0.1,0.1]^T\) and \(W^4_g(t) = [0.1,0.1,0.1,0.1]^T\) and \(\phi'(t+1) = 0.005\) for all \(t \in \{0,1,2,⋯,500\}\). In general, the initial consequent parameters can be roughly estimated if the nonlinear functions \(h(X'(t))\) and \(g(X'(t))\) are partially known, therefore we often arbitrarily choose the initial parameters. In addition, it is noted that \(\phi'(t+1)\) should be as small as possible for all \(t \in \{0,1,2,⋯,500\}\) so that the output tracking error \(e'(t+1)\) can converge to a residual set which is bounded by the small boundary layer \(\phi'(t+1)\) for all \(t \in \{0,1,2,⋯,500\}\).

(D3) Since \(M_h = 4\) and \(M_g = 4\) are given in step (D2),
5.3 $y^* = 3.5$ and $b_c = 0.0577$ (since $y^*(t) \geq 0$), the adaptive laws (8), (9) and (10) with the adaptation gains $\beta_1 = 0.6213$, $\beta_2 = 0.6213$, $\beta_3 = 10^{-3}$ are adopted so that learning condition $2 - b_c M \beta_1 + (y^j)^2 b_c M \beta_2 - \beta_3 = 0.1 > 0$ in (17) can be satisfied. It is noted that the small adaptation gain $\beta_3$ in (10) will be chosen in order to guarantee that $\phi^j(t + 1), t \in \{0, 1, 2, \ldots, 500\}$ will remain in a reasonable small value for all $j \geq 1$.

To demonstrate the technical result of (11) of main theorem, we first apply the proposed fuzzy neural discrete AILC to the surge tank plant and show the asymptotic convergence of $\max_{t=0,1,2,\ldots,500} |\hat{e}^j(t)|$ versus iteration $j$ in Figure 4 (a). In Figure 4 (b), the tracking error at the 50th iteration is shown and the fact of $|\hat{e}^0(t)| \leq \phi^0(t), t \in \{1, 2, \ldots, 500\}$ is observed. This proves (t2) of main theorem and also demonstrates the robustness of proposed fuzzy neural discrete AILC against the uncertainties caused by random bounded initial resetting error and random bounded disturbance. In Figure 4 (c) and Figure 4 (d), we show the tracking performance between $y^0(t)$ and $y^0_0(t)$. It is clear that the system output will converge to the iteration-varying reference trajectory except for the initial point. Finally, due to the boundary layer design, the smooth learned control input $u^0(t)$ is shown in Figure 4 (e).

![Figure 4](image-url)

6. Conclusion

This work successfully extends the typical fuzzy neural discrete-time adaptive control scheme to a new area of fuzzy neural discrete-time adaptive iterative learning control for repeatable discrete-time uncertain nonlinear systems with iteration-varying reference trajectory, random bounded initial resetting error and random bounded disturbance. In the main structure of the proposed discrete AILC, two FNNs are introduced as function approximators to compensate for the uncertainties from plant unknown nonlinearities. In addition to solving the problem of plant unknown nonlinearities, it is still necessary to deal with the uncertainties from the function approximation error and random bounded disturbance in the design of this discrete AILC. Hence, a dead-zone like auxiliary error is then constructed by using a time-varying boundary layer to overcome these uncertainties. This auxiliary error is utilized to design the adaptive laws which update the weights of FNN and the width of boundary layer. In order to ensure the closed-loop stability and guaranteed learning performance, the learning gains of adaptive laws should satisfy a sufficient learning condition. Under this learning condition, a Lyapunov-like analysis is presented to show that output tracking error will asymptotically converge to a residual set bounded by the boundary layer as iteration goes to infinity. Furthermore, all the adjustable parameters as well as internal signals remain bounded. Two simulation examples, including a practical application to repetitive tracking control of a surge tank plant, are shown to demonstrate the effect of the proposed fuzzy neural discrete AILC.

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References


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