Study of Reliable Design Using T-S Fuzzy Modeling and Integral Sliding Mode Control Schemes

Yew-Wen Liang, Chih-Chiang Chen, and Sendren Sheng-Dong Xu

Abstract

This paper investigates robust active reliable control issues using a combination of Takagi-Sugeno (T-S) fuzzy system modeling and Integral Sliding Mode Control (ISMC) schemes. The presented reliable scheme is shown to retain the benefits of both T-S modeling and ISMC design. It not only alleviates the online computational burden due to the use of the T-S model but also preserves the many advantages of the ISMC scheme. Using the presented reliable scheme, the control mission can be successfully achieved without prompt external support even when some of the actuators fail to operate. Moreover, because the ISMC design allows the engineer to predict the performance of the uncertain system from that of the nominal control system, the presented reliable scheme also possesses a degree of freedom in the design of the nominal controller for better system performance of normally operated and different faulty, uncertain situations. An illustrative example is also given to demonstrate the benefits of the proposed scheme.

Keywords: Takagi-Sugeno fuzzy models, Reliable control, Integral sliding mode control, Nonlinear systems, Sliding mode control.

1. Introduction

The study of reliable (or fault-tolerance) control has attracted a considerable amount of attention and has become important (see, e.g., [1]-[9]) because repair and maintenance services generally cannot be provided instantly. The main objective of reliable control is to design an appropriate controller such that the closed-loop system can tolerate abnormal operation of specific control components and retain the overall system stability with acceptable system performance. The existing reliable designs include the coprime factorization-based approach [5], the algebraic Riccati equation-based approach [6], the linear matrix inequality-based approach [7], the Hamilton-Jacobi (HJ)-based approach [2], [8], and the Sliding Mode Control (SMC)-based approach [1]-[4]. Among these reliable control studies, only the HJ-based and the SMC-based approaches explore the reliability issue for nonlinear systems. However, because the HJ-based approach was studied using an optimal approach, its reliable controller is unavoidably dependent on the solution of an associated HJ equation that is, in general, difficult to solve. Though a power series method [10] may alleviate the difficulty via computer-based calculations, the obtained solution is only an approximation, and the computational burden grows quickly when the system is complicated. In contrast, the SMC-based reliable designs [3]-[4] do not require information about the solution of the HJ equation, and they retain the advantages of SMC schemes, including rapid response, easy implementation, and small sensitivity to model uncertainties and/or external disturbances [1], [3], [4], [11]-[13].

On the other hand, the fuzzy set theory and its applications have attracted considerable attention because of their efficiency and benefits [1], [12]-[30]. Among these, the so-called Takagi-Sugeno (T-S) fuzzy modeling method has many remarkable advantages (see, e.g., [1], [12]-[13], [16]-[30]), including 1) clearness of concept, 2) being justified as a universal approximator [21], 3) ease of construction, and 4) allowance of offline computing for most of the system parameters. These advantages make the T-S fuzzy modeling approach particularly useful when the nonlinear system model is complicated. The idea behind the T-S fuzzy modeling approach is to decompose the nonlinear system into several linear models according to different cases in which the associated linear model best fits the nonlinear one and then to aggregate each individual linear model into a single nonlinear model in terms of the membership functions of each model. Though a nonlinear system can be well approxi-
mated by a suitably selected T-S fuzzy model, it creates additional model uncertainties when the T-S fuzzy model is used to implement the original nonlinear system. To effectively compensate for the additional model uncertainty and possible disturbances, a set of schemes that combine T-S fuzzy modeling and SMC techniques have been presented [1], [12]-[13]. These schemes not only alleviate the online computational burden because they use the T-S model to approximate the original nonlinear system and most of the system parameters of the T-S model can be computed offline, they also preserve the advantage of the SMC design, as mentioned earlier.

Although the combination of the T-S modeling and the SMC schemes possess the aforementioned advantages, it was reported that the closed-loop system resulting from the SMC design might be sensitive to uncertainties and/or external disturbances during the period of time in which the system states have not yet reached the sliding manifold [31]. To avoid the reaching phase problem, an Integral Sliding Mode Control (ISMС) design and its applications have been presented recently [31]-[32]. In addition to the benefit of system states starting from the sliding manifold, the ISМС scheme also maintains the advantages of SMC and the following three characteristics: 1) the maximum control magnitude required for ISМС is usually smaller than those of SMC because the maximum control magnitude of SMC design usually occurs at the beginning of the reaching phase period; 2) the effect of mismatched uncertainty can be minimized through the setting of sliding manifold parameters [31]; and 3) the state responses of the nominal system and the matched-type uncertain system are identical if the system states remain on the sliding manifold. The last two properties provide the engineer an extra degree of freedom to design a suitable controller for the nominal system, creating a desired system trajectory for the states of the uncertain system to follow. As a result, the ISМС design enables the engineer to predict the performance of the uncertain system, which is generally not easily implemented by other nonlinear control techniques. In light of the benefits of the T-S modeling and the ISМС schemes mentioned above, this paper will investigate the reliability issues arising from the combination of these two schemes. With this approach, the engineer is capable of providing better system performance under normally operated and different faulty situations.

The organization of this paper is as follows. The problem formulation and the main goal of this paper are given in Section 2. This is followed by the design of the T-S fuzzy model-based ISМС controller. In Section 4, the analytical results are applied to the attitude control of a spacecraft. Finally, Section 5 contains the conclusions.

2. Problem Statement

Consider a class of $n$ second-order nonlinear systems as given by

$$
\dot{x} = f(z, x) + Bu + d
$$

(1)

where $z, x \in \mathbb{R}^n$ denote the system states, $u \in \mathbb{R}^m$ are the control inputs, $d \in \mathbb{R}^n$ represent possible model uncertainties and/or external disturbances, $f(z, x) \in \mathbb{R}^n$ is a smooth function, $f(0,0) = 0$, $B \in \mathbb{R}^{m \times n}$ and $(\cdot)'$ denotes the transpose of a vector or a matrix. By letting $x_1 = z$ and $x_2 = \dot{z}$, Equation (1) can be rewritten as

$$
\dot{x}_1 = x_2
$$

(2)

and

$$
\dot{x}_2 = f(x_1) + Bu + d
$$

(3)

In this study, we will investigate the active reliable control issues for System (2)-(3), i.e., we assume that the actuator fault has been successfully detected and diagnosed by a Fault Detection and Diagnosis (FDD) mechanism. It should be noted that the success of an active reliable scheme mainly depends on the performance of the supplementary FDD method, including detection speed and accuracy [33]-[34]. Thus, it is important to select an appropriate FDD mechanism to meet the system requirements. In this study, the estimation error resulting from an FDD mechanism will be compensated by the robust controller designed later. In addition, the fault investigated in this study is allowed to be time varying, which includes degradation, amplification and outage [3].

Before the occurrence of faults, the engineer may adopt any type of control technique to fulfill the desired system performance. When the fault is detected and diagnosed, the control law is guided to switch to an active reliable control law for ensuring system performance. Thus, after the fault is detected and diagnosed, we may divide the actuators into two groups, $H$ and $F$, within which we assume that all of the actuators in $H$ (denoted by $u_H$) are healthy and those in $F$ (denoted by $u_F$) experience fault. This implies that System (2)-(3) can be rewritten as

$$
\dot{x}_1 = x_2
$$

(4)

and

$$
\dot{x}_2 = f(x_1) + B_H u_H + B_F u_F + d
$$

(5)

where $B = (B_H; B_F)$ and $u = (u^T_H; u^T_F)^T$. To successfully design a reliable controller, in the rest of this study, we assume that $u_H \in \mathbb{R}$ and $u_F \in \mathbb{R}^{m_F}$. The main goal of this paper is to organize an appropriate $u_F$ so that the origin of the closed-loop system is asymptotically stable (AS), even when actuators in $F$ are detected and diagnosed as experiencing faults by an FDD mechanism.
3. Fuzzy Model-Based ISMC Reliable Design

In light of the advantages of the T-S fuzzy modeling and the ISMC approaches, as mentioned in the Introduction section, this paper will combine the two schemes to design reliable controllers.

A. T-S Fuzzy Model Description

It is known that a nonlinear system can be well-approximated by a T-S fuzzy model [20], which is described by a combination of several linear models with suitable weightings. Suppose that there are $p$ rules with the corresponding linear models for System (4)-(5), as described by

$$
\dot{x}_i = A_i x + B_i u_i + B_i u_f,
$$

and

$$
\dot{x}_i = A_i x + B_i u_i + B_i u_f + \Delta f,
$$

where $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, and $B_i \in \mathbb{R}^{n \times m}$. Then, the T-S fuzzy model is constructed as

$$
\dot{x} = A(x) \hat{x} + B(x) \hat{u}_f + B(x) u_f,
$$

and

$$
\dot{x} = A(x) \hat{x} + B(x) \hat{u}_f + B(x) u_f + \Delta f,
$$

where $A(x) = \sum_{i=1}^{p} \alpha_i (x) A_i$ for some suitable nonnegative weightings $\alpha_i (x)$ with $\sum_i \alpha_i (x) = 1$.

B. ISMC Reliable Design

Using the T-S fuzzy model given by (8)-(9), the original nonlinear model (4)-(5) can be rewritten as

$$
\dot{\bar{x}} = \bar{x}_i,
$$

and

$$
\dot{\bar{x}} = \bar{A}(x) + B_i \bar{u}_i + B_i u_f + \Delta f,
$$

where $\bar{A}(x) = \sum_{i=1}^{p} \alpha_i (x) A_i$ denotes the extra model uncertainties due to the use of a T-S fuzzy model to approximate the original nonlinear system. To perform the active reliable design, we assume that the output values of the faulty actuators are successfully diagnosed by an FDD mechanism as

$$
\bar{u}_i = \hat{u}_i + \Delta \bar{u}_i,
$$

where $\hat{u}_i$, and $\Delta \bar{u}_i$ denote the estimated control value and the estimation error, respectively. Thus, (10)-(11) can be rewritten as

$$
\dot{\bar{x}} = \bar{x}_i,
$$

and

$$
\dot{\bar{x}} = \bar{A}(x) + B_i \hat{u}_i + B_i (\hat{u}_i + \Delta \bar{u}_i) + \Delta f
$$

In this faulty system, a matched uncertainty for a normally operated system might become mismatched because of actuator faults. Thus, we have to clarify the matched and the mismatched uncertainties for the faulty system. It is known that [31]

$$
L_i = B_i B_i^* B_i^* B_i^* + I,
$$

where $L_i \in \mathbb{R}^{n \times n}$ is the identity matrix, $B_i^*$ denotes the pseudo-inverse of $B_i$, which can be directly compensated by the healthy actuator $u_i$. Hence, the total uncertainty regarding the system given by (13)-(14) is

$$
\Delta \bar{f} = B_i \bar{u}_i + \Delta \bar{f} - B_i \bar{u}_i
$$

which can be further decomposed, using (15), into the matched and the mismatched parts as

$$
\Delta \bar{f} = B_i \bar{u}_i + \bar{d}_m + \bar{d}_u,
$$

where $\bar{d}_m = B_i \bar{d}_m$ and $\bar{d}_u = B_i \bar{d}_u$. To perform the ISMC reliable design, we impose the following three assumptions for the succeeding analysis:

**Assumption 3.1:** $B_i$ has full rank. That is, rank $(B_i) = k$ if $k < n$, and rank $(B_i) = n$ if $k \geq n$.

**Assumption 3.2:** There exist two nonnegative scalar functions $\rho_u(x,t)$ and $\rho_f(x,t)$ such that, $\forall x \in \mathbb{R}^n$ and $t \in \mathbb{R}^+$,

$$
\|\Delta f\| \leq \rho_u(x,t) \quad \text{and} \quad \|\bar{d}_u\| \leq \rho_f(x,t)
$$

where $\bar{d}_u$ and $\bar{d}_u$ are given in (17).

**Assumption 3.3:** There exists a controller $u_{as}$ such that the origin of the following T-S fuzzy model-based healthy subsystem

$$
\dot{x} = x,
$$

and

$$
\dot{x} = A(x) + B(x) u_{as},
$$

is globally AS. That is, there exists a continuously differentiable, positive definite and radially unbounded function $V(x)$ and a function $\gamma(\cdot): \mathbb{R}^n \to \mathbb{R}^n$ of class $K$ (for definition, see, e.g., [10]) such that

$$
V(x) = \frac{\partial V(x)}{\partial x} \left( A(x) + B(x) u_{as} \right) \leq -\gamma(x), \quad \forall x \in \mathbb{R}^n.
$$

It should be noted from (16) and Assumption 3.2 that although the upper bounds $\rho_u(x,t)$ and $\rho_f(x,t)$ require information about $\Delta f$, they can be easily obtained because $\|\bar{d}_u\|$ can be estimated offline. Moreover, a candidate of the healthy controller $u_{as}$ given in Assumption 3.3 can be derived from [21] and [35] by the so-called parallel distributed compensation (PDC) and linear matrix inequality (LMI) techniques. This PDC controller has the form $u_{as} = \sum_{i=1}^{p} \alpha_i (x) K_i x$, where $K_i$ is to be determined such that the conditions

$$
(A_i - B_i K_{ij} P + P (A_i - B_i K_{ij} P < 0, \quad i = 1, \ldots, p,
$$

are satisfied for some $P > 0$. Once $P$ has been obtained, we may set $V(x) = x^T P x$, and the function $\gamma(x)$ given by (21) can be obtained accordingly. Of course, the designer is allowed to adopt any other controller $u_{as}$ to meet their system performance requirements.

To effectively compensate for the additional uncertainties resulting from the use of the T-S model, actuator faults and the FDD estimation error, we will employ the
ISMC scheme for the design. The details are given below. Following the ISMC design procedure [31], [32], we introduce the sliding manifold as (23) below:

\[ s = D_u \{ x(t) - x(t_o) - \left[ A(x(t)) + B_u u_{m} \right] dt \} \tag{23} \]

where \( D_u \) has dimension \( D_u \in \mathbb{R}^{m \times m} \) if \( k < n \) or \( D_u \in \mathbb{R}^m \) if \( k \geq n \) and \( D_u B_u \) has full rank. From (13)-(14), (17) and (23), we have

\[ \dot{s} = D_u \{ x(t) - A(x) - B_u u_{m} \} \]

To keep the system states on the sliding manifold, we choose

\[ u_{eq} = \begin{cases} u_{m} - B_u \hat{u} - \rho(x, t) \left[ D_u B_u \right] s & \text{if } s \neq 0, \\ u_{m} - B_u \hat{u}, & \text{if } s = 0, \end{cases} \tag{25} \]

where

\[ \rho(x, t) > \rho_0(x, t) + \left\| D_u B_u \right\| D_u \rho_0(x, t) \tag{26} \]

It is noted that the reliable controller (25) involves information about the FDD mechanism. We have the next main result regarding system stability.

**Theorem 3.1:** Suppose that System (2)-(3) experiences actuator faults at control channels in F with estimated control values \( \hat{u} \) and estimated errors \( \Delta u \), as given by (12). Then, the origin of System (10)-(11), which is the same as System (2)-(3) using the T-S fuzzy model expression, under Assumptions 3.1-3.3 and the control law (25)-(26) is globally AS if

\[ \rho_0(x, t) + \frac{\partial V(x)}{\partial x} \Phi < \gamma|\Phi| \tag{27} \]

for all \( t \) and for all \( x \neq 0 \), in which

\[ \Phi = I - B_u \left[ D_u B_u \right] D_u \tag{28} \]

**Proof:** From the choice of \( D_u \) given by (23), we have \( \left[ D_u B_u \right] \left[ D_u B_u \right] \right] = I \), and \( \left[ D_u B_u \right] \left[ D_u B_u \right] = I \), when \( k < n \) and \( k \geq n \), respectively. It follows from (24)-(26) and Assumption 3.2 that

\[ s' = \begin{pmatrix} \rho(x, t) \left[ D_u B_u \right] s + d_u + [D_u B_u] D_u d_u \\ \left\| D_u B_u \right\| s \end{pmatrix} \]

whenever \( s \neq 0 \).

\[ \begin{cases} \rho_0(x, t) + \frac{\partial V(x)}{\partial x} \Phi < \gamma|\Phi|, & \text{if } s \neq 0, \\ \rho_0(x, t) + \frac{\partial V(x)}{\partial x} \Phi < 0, & \text{if } s = 0. \end{cases} \tag{29} \]

According to (29) and the fact that \( s(x(t_o)) = 0 \), we have \( s(x(t)) = 0 \) for all \( t \geq t_o \). That is, the states remain on the sliding manifold for all \( t \geq t_o \). Next, we will use the equivalent control method [31], [32] to determine the closed-loop dynamics on the sliding manifold (i.e., the sliding dynamics). The equivalent control, denoted by \( u_{eq}^* \), is known to solve the equation \( \dot{s} = 0 \) from (24). Because \( D_u B_u \) has full rank, we have

\[ u_{eq}^* = u_{m} - B_u \hat{u} - \rho(x, t) s \tag{30} \]

Substituting \( u_{eq}^* \) into (13)-(14) and using (17), we obtain the sliding dynamics

\[ \dot{x}_s = x_s \tag{31} \]

and

\[ \dot{x}_s = A(x) + B_u u_{m} + \Phi d \tag{32} \]

where \( \Phi \) is given by (28). From (31)-(32) and Assumption 3.3, we have

\[ V(x) = \frac{\partial V(x)}{\partial x} \left( A(x) + B_u u_{m} + \Phi d \right) \leq -\gamma|\Phi| + \rho_0(x, t) \frac{\partial V(x)}{\partial x} \Phi < 0 \]

for all \( x \neq 0 \). Hence, the origin is globally AS.

**Remark 1:** Suppose that the conditions stated in Theorem 3.1 and Assumptions 3.2-3.3 hold only for \( x \) in a neighborhood of the origin and that the functions \( V \) and \( \gamma \) given in Assumption 3.3 are locally positive definite. Then, the stability result in Theorem 3.1 is only a local result. That is, the origin is locally AS.

**Remark 2:** It is clear from (31)-(32) that under the presented reliable law, the matched uncertainties and/or disturbances can be completely rejected, which results in the state responses of the uncertain faulty system and the T-S fuzzy model-based healthy subsystem (19)-(20) being identical. Thus, if the number of healthy actuators is greater than or equal to \( n \) (i.e., \( k \geq n \)), then the uncertainties (including the model uncertainty resulting from the use of the T-S model, the actuator faults and the FDD estimation error) are clearly matched and can be completely compensated. In contrast, if \( k < n \), then the matched part of the uncertainties regarding the T-S fuzzy model-based healthy subsystem can be completely rejected, while the effect of the mismatched uncertainties remains minimal in the Euclidean norm sense if the projection matrix \( D_u \) given in (23) is selected to be \( D_u = B_u^{-1} \) [32]. This choice of \( D_u \) also simplifies the controller (25) as

\[ u_{eq} = \begin{cases} u_{m} - B_u \hat{u} - \rho(x, t) s & \text{if } s \neq 0, \\ u_{m} - B_u \hat{u}, & \text{if } s = 0. \end{cases} \]

As a result, the engineer is able to predict the uncertain faulty system performance from the performance of the nominal healthy subsystem by organizing a suitable controller \( u_{eq} \) to meet their system performance requirements.

### 4. Application to Spacecraft Attitude Stabilization

An attitude model for a spacecraft in a circular orbit can be described in the same form as System (2)-(3) with
The LQR feedback gain is denoted as \( K_{LQR} \) and almost all the eigenvalues or to stabilize due to excessive gain requirements [36], and the eigenvalues of \( A \) are \([0,0.0013,0.0009] \), in which almost all the eigenvalues are on the \( j\omega \)-axis. Hence, in this study, we only consider the case of a one-actuator outage. To demonstrate the application of the reliable design, we assume that \( \dot{u}_i \) fails at \( t=1 \).

\[
\begin{align*}
\mu(A, B_{u_\phi}) & = 0.3775 \\
\mu(A, B_{u_0}) & = 0.1021
\end{align*}
\]

To implement the presented reliable scheme, it remains to determine the nominal controller \( u_{\phi,0} \) stated in Assumption 3.3 for \( F = \phi \) and \( F = \{u_i\}_i \). We adopt the locally optimal T-S fuzzy controller design [35] for \( u_{\phi,0} \). This design adopts the LQR scheme to obtain the optimal feedback gains for every locally linear model and then employs the LMI to determine a common \( P \) satisfying (22). The performance weighting matrices of the LQR scheme are chosen to be \( Q = 0.5I_n \) and \( R = \text{diag}(0.4,0.3,0.3,1) \). The LQR feedback gain \( K_{LQR} \) for every locally linear model can then be computed offline. Two of them are given below:

(i) Before \( u_2 \) fails:

\[
K_{LQR} = \begin{bmatrix}
0.9159 & 0.9122 & 0.5736 & 1.3165 & 1.3024 & 0.9659 \\
0.8458 & -0.9785 & 1.0772 & 1.0873 & -1.3622 & 1.9636 \\
0.9756 & -0.8515 & -1.0754 & 1.3691 & -1.088 & -1.9626 \\
0.4053 & 0.403 & -0.4163 & 0.6111 & 0.6032 & -0.7913 \\
0.9158 & 0.9125 & 0.5733 & 1.3165 & 1.3019 & 0.9649 \\
0.8458 & -0.9787 & 1.0776 & 1.0878 & -1.3626 & 1.962 \\
0.9757 & -0.8518 & -1.0751 & 1.3687 & -1.0879 & -1.9583 \\
0.4053 & 0.4029 & -0.4165 & 0.6108 & 0.603 & -0.7901
\end{bmatrix}
\]

(ii) After \( u_2 \) fails:
Hence, the overall controller $u_{n_{0}}$ has the form

$$u_{n_{0}} = - \sum_{j_{1}, j_{2}, j_{3} = 0}^{1} \alpha(x)\alpha_{j_{1}}\alpha_{j_{2}}\alpha_{j_{3}} K_{i_{1}, j_{1}, j_{2}, j_{3}, k} x$$

To ensure $u_{n_{0}}$ is a stabilizer, the PDC design determines $P > 0$ satisfying

$$\left( A_{i, j, k} - B_{H} K_{i, j, k} \right)^{T} P + P \left( A_{i, j, k} - B_{H} K_{i, j, k} \right) < 0$$

for all $1 \leq i, j, k \leq 5$. With the aid of the MATLAB LMI tool box, one of the solutions of (22) is found to be

$$P \approx \begin{bmatrix}
1.5267 & 0.0445 & -0.0344 & 0.3045 & 0.0503 & -0.043 \\
0.0445 & 1.5186 & -0.0337 & 0.0505 & 0.2945 & -0.0421 \\
-0.0344 & -0.0337 & 1.8581 & -0.043 & -0.0421 & 0.7139 \\
0.3045 & 0.0505 & -0.43 & 0.3239 & 0.0583 & -0.0557 \\
0.0505 & 0.2954 & -0.0421 & 0.0583 & 0.3135 & -0.0543 \\
-0.043 & -0.0421 & 0.7139 & -0.0557 & -0.0543 & 1.2942
\end{bmatrix}$$

before $u_{k}$ fails, and

$$P \approx \begin{bmatrix}
1.5792 & -0.0249 & 0.0847 & 0.3662 & -0.0352 & 0.1186 \\
-0.0249 & 1.6137 & -0.2088 & -0.0352 & 0.419 & -0.2912 \\
0.0847 & -0.2088 & 2.2234 & 0.1186 & -0.2912 & 1.2696 \\
0.3662 & -0.0352 & 0.1186 & 0.3991 & -0.0527 & 0.1762 \\
-0.0352 & 0.419 & -0.2912 & -0.0527 & 0.484 & -0.4308 \\
0.1186 & -0.2912 & 1.2696 & 0.1762 & -0.4308 & 1.7425
\end{bmatrix}$$

after $u_{l}$ fails.

Finally, we need an FDD mechanism to perform the active reliable task. In this study, we adopt the observer and the observer parameters from [3] (10) and (11) on page 335 for the demonstration of the application because the adopted observer has been shown to be able to reflect the fault of any single actuator at an exponential rate and has the ability of attenuating high-frequency noises.

Numerical results are summarized in Figs. 1-4. Among these figures, we consider the following two cases: the first adopts the presented reliable scheme (labeled TS+ISMC), while the other employs the TS fuzzy model-based SMC reliable scheme from [1] (labeled TS+SMC). To emphasize the closeness of the system states between the uncertain system and the nominal system, the responding curves of the nominal system (19)-(20) are also presented (labeled TS0). The first two schemes assume $H = [u_{i}, u_{j}, u_{s}, u_{t}]$ before the detection of a fault and are guided to switch to their reliable laws in accordance with the FDD information, while TS0 switches $u_{n_{0}}$ from the normally operated situation to $H = [u_{i}, u_{j}, u_{s}, u_{t}]$ directly at $t = 1$. The parameters of the TS+SMC scheme are adopted from [1] as $M = 2I_{4}$ and $\eta = 0.5$, while the sliding surface parameter of the TS+ISMC scheme is chosen to be $D_{s} = I_{4}$. In this study, the disturbances and the initial states are assumed to be $d = 0.5 (\sin(t), \cos(t), \cos(3t))$ and $x(0) = (-0.7, -0.07, 1.5, 0.31, -0.2)$ [3]. The upper bound $\rho_{s}(t_{i})$ in Assumption 3.2 is set to be $||p_{s}|| + ||f||$, where $||f|| \approx \sup |x|$. To alleviate the chattering effect, the TS+ISMC controller given by (25) is replaced by $u_{n} = u_{n_{0}} - B_{H} B_{s} x - \rho(x, t) [D_{s} B_{s}] s / \varepsilon(s)$, where $\varepsilon(s) = 0.02$ and $\varepsilon(s) \approx \sup |D_{s} B_{s}] s | \geq 0.02$ and $\sup |D_{s} B_{s}] s | < 0.02$, respectively. Finally, the threshold for the alarm is set to be 0.2, i.e., the alarm is triggered if the magnitude of any of the residual signals from the observer is greater than 0.2.

It is observed from Figure 1 that the stabilization performance is, as expected, achieved for all of the control schemes, and the state trajectories of TS+ISMC and those of TS0 are found to be almost identical, which agrees with the theoretical conclusion. In Figure 2, the sliding variables of the TS+ISMC are found to be zero all the time, which also agrees with the main results. In Figure 3(a), the actuator fault is observed to be successfully detected by the two reliable schemes. The time instants for the three sliding variables of SMC reaching zero are $t_{s} = 1.06$ sec and $t_{s} = 1.026$ sec. These can also be recognized from the alarm signals given in Figure 3(b), in which the alarm value 2 denotes the fault of the second actuator. After the fault is successfully detected and diagnosed, the associated active reliable controllers are activated, and the magnitude of the residual signals soon decreases, as shown in Figure 3(a). The persistent oscillation of the residual signals results from the effect of the disturbances. Fig. 4 shows the control curves of the three designs. It is noted that there are several peaks for the control curves of the TS+SMC design, which correspond to the state system reaching the sliding manifold and the switch of control due to the detection of a fault. These can be observed from Figure 2 and Figure 3(b), in which the time instants for the three sliding variables of SMC reaching zero are $t_{s} = 0.33$ sec, $t_{s} = 0.31$ sec and $t_{s} = 0.78$ sec. Finally, the presented TS+ISMC reliable design is found to perform better than the TS+SMC reliable design [1] in terms of the quadratic cost $\int x^{T} Q x + u^{T} Ru$, energy consumption $\int u^{T} Ru$, required maximum control magnitude $\|u\| \approx \sup |u|$, and CPU computing time according to the following relations:$\langle \int x^{T} Q x + u^{T} Ru \rangle_{TS+SMC} = 9.89 < \langle \int x^{T} Q x + u^{T} Ru \rangle_{TS+ISMC} = 23.1$.
\[
\begin{align*}
(\|u^T R u\|)_{\text{TS-ISMCC}} &= 8.03 < (\|u^T R u\|)_{\text{TS-SMCC}} = 21.3, \quad (\|u\|)_{\text{TS-ISMCC}} = 3.2 \\
< (\|u\|)_{\text{TS-SMCC}} &= 10.3 \quad \text{and} \quad (\text{CPU})_{\text{TS-ISMCC}} = 5.0633 \text{ sec} < \\
(\text{CPU})_{\text{TS-SMCC}} &= 5.2902 \text{ sec},
\end{align*}
\]
where the CPU time is obtained by repeatedly computing the controller (including the determination of membership weightings) \(5 \times 10^7\) times.

Though the TS+SMC consumes much more energy than the TS+ISMCC, it attains a larger convergence speed of the system states, which can be recognized from Figure 1. From this simulation, it is concluded that the presented reliable scheme preserves the advantages of both T-S fuzzy modeling and ISMC strategies. It not only achieves a stable performance when some of the actuators experience faults but also maintains the same state trajectories as those of the nominal system whenever the uncertainties are matched.

5. Conclusion

A T-S model-based ISMC reliable scheme has been presented in this paper for a class of second-order nonlinear uncertain systems. Due to the actuator faults, part of the matched uncertainties may become mismatched; the matched uncertainties can be completely rejected and the effect of mismatched uncertainties is not amplified. Thus, in contrast to the T-S modeling-based SMC design, the presented scheme possesses an additionally remarkable benefit of allowing the uncertain system performance to be addressed in advance based on the nominal system performance, which is not easy to implement using other nonlinear control techniques. As a result, the engineer can achieve a better system performance under normally operated and different faulty situations. Simulation results have demonstrated the
benefits of the proposed scheme.

Appendix

Here, we recall the T-S model construction and the upper bound estimation of $\Delta$ from [1] as follows. First, the drift term is expressed in the form of $f(x) = F(x)$ with

$$(F(x))_{i,j} = \frac{1}{l_i} \left[ a_0 e^{x_j} x_i \frac{d(x_j)}{x_i} - a_0 s^j x_i s^i x_j \frac{d(x_j)}{x_i} - 3 a_0 e^{x_j} x_i \frac{s^j(x_i)}{2x_i} \right],$$

$$(F(x))_{i,j} = \frac{1}{l_i} \left[ \frac{1}{4} a_0 e^{x_j} x_i \frac{d(x_j)}{x_i} - \frac{1}{4} a_0 s^j x_i x_j \frac{d(x_j)}{x_i} - \frac{1}{4} a_0 s^j x_i x_j \frac{s^j(x_i)}{2x_i} \right],$$

$$(F(x))_{i,j} = \frac{1}{l_i} \left[ \frac{1}{2} a_0 e^{x_j} x_i x_j \frac{s^j(x_i)}{x_i} - \frac{1}{2} a_0 s^j x_i x_j \frac{s^j(x_i)}{2x_i} \right],$$

$$(F(x))_{i,j} = 0,$$

$$(F(x))_{i,j} = -a_0 e^{x_j} x_i x_j + \frac{1}{l_i} \left[ \frac{1}{2} e^{x_j} x_i + a_0 e^{x_j} x_i x_j + a_0 e^{x_j} x_i x_j \right],$$

$$(F(x))_{i,j} = a_0 e^{x_j} x_i x_j - \frac{1}{l_i} \left[ \frac{1}{2} e^{x_j} x_i + a_0 e^{x_j} x_i x_j - a_0 e^{x_j} x_i x_j \right],$$

$$(F(x))_{i,j} = \frac{1}{l_i} \left[ \frac{1}{4} a_0 e^{x_j} x_i x_j \frac{s^j(x_i)}{x_i} - \frac{1}{4} a_0 e^{x_j} x_i x_j \frac{s^j(x_i)}{2x_i} \right],$$

$$(F(x))_{i,j} = -a_0 e^{x_j} x_i x_j + \frac{1}{l_i} \left[ \frac{1}{2} e^{x_j} x_i + a_0 e^{x_j} x_i x_j \right],$$

$$(F(x))_{i,j} = a_0 e^{x_j} x_i x_j + \frac{1}{l_i} \left[ \frac{1}{2} e^{x_j} x_i + a_0 e^{x_j} x_i x_j \right],$$

$$(F(x))_{i,j} = 0.$$

Thus, we have $S_l = 125$ locally linear models. Define $D_{i,j,k} = \{x \mid x_{i,j} \leq x_i \leq x_{i+1} \leq x_{i,j} \leq x_i \leq x_{i,j} \leq x_i \leq x_{i,j}, \leq x_{i+1} \leq x_{i+1} \leq x_{i+1}, \leq x_{i+1}, \leq 1 \leq x_i \leq 1, l = 4, 5, 6 \}.$

Then, when $x \in D_{i,j,k}$, the T-S model given by (8)-(9) is constructed as

$$A(x) = \sum_{l,x_i \leq 0} a(x) \alpha_e(x) \alpha_e(x) \cdot A_{i,j,k} \cdot x,$$

where

$$\alpha_e(x) = \|x_i - x_{i+1}||x_i - x_{i+1}|,$$

and

$$\alpha_e(x) = \|x_i - x_{i+1}||x_i - x_{i+1}||x_i - x_{i+1}|,$$

Clearly, $\sum_{l,x_i \leq 0} a(x) \alpha_e(x) \alpha_e(x) = 1.$$

The upper bound of $\Delta$, denoted by $\|\Delta\| = \sup \|\Delta\|$, over the region $D_{i,j,k}$ can also be easily computed off-line. These associated upper bounds are listed in Table 2 below.

Figure 5. Membership functions for the states $x_i$, $x_j$, and $x_k$.

<table>
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Table 2. Estimated upper bounds \( \| M \| \) in the region \( D_{i,j} \).

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References


Yew-Wen Liang was born in Taiwan in 1960. He received the BS degree in Mathematics from the Tung Hai University, Taichung, Taiwan, Republic of China in 1982, the M.S. degree in Applied Mathematics in 1984 and the Ph.D. degree in Electrical and Control Engineering in 1998 from the National Chiao Tung University, Hsinchu, Taiwan, Republic of China. Since August 1987, he has been with the National Chiao Tung University, where he is currently an Associate Professor of Electrical and Computer Engineering. His research interests include linear and nonlinear systems control, reliable control, and fault detection and diagnosis issues.

Chih-Chiang Chen was born in Taiwan in 1987. He received the B.S. degree in Electrical Engineering from National Yulin University of Science and Technology, Douliou, Taiwan, R.O.C., in 2009, and M.S. degree in Electrical and Control Engineering from National Chiao Tung University, Hsinchu, Taiwan, R.O.C., in 2011. He is currently working toward the Ph.D. degree and his research interests include lin-
ear and nonlinear systems control, fault tolerant control, and
fault detection and diagnosis.

Sendren Sheng-Dong Xu received a Ph.D. degree in Electrical and Control
Engineering from National Chiao Tung University, Taiwan, in 2009. He is cur-
rently an assistant professor with the Graduate Institute of Automation and
Control, National Taiwan University of Science and Technology, Taiwan. His
research interests include intelligent control systems, signal processing, image processing, and em-
bedded systems.