Delay-Dependent Fuzzy Control for Nonlinear Multiple Time-Delay Large-scale Systems by Dithers: Neural-Network-Based Approach

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Abstract

In this study, the stabilization problem for nonlinear multiple time-delay large-scale systems is considered. First, a neural-network (NN) model is employed to approximate each interconnected subsystem. Then, a linear differential inclusion (LDI) state-space representation is established for the dynamics of the NN model. Based on this LDI state-space representation, a robust fuzzy control design is proposed to overcome the effect of modeling errors between the nonlinear multiple time-delay large-scale systems and the NN models. Next, in terms of Lyapunov’s direct method, a delay-dependent criterion is derived in order to guarantee the stability (UUB) of nonlinear multiple time-delay large-scale systems. Subsequently, the stability conditions of this criterion are reformulated into linear matrix inequalities (LMIs). Based on the LMIs and the decentralized control scheme, a set of fuzzy controllers is synthesized to stabilize the nonlinear large-scale system and the $\infty$ control performance is achieved at the same time. If the designed fuzzy controllers cannot stabilize the nonlinear large-scale system, a batch of dithers (as the auxiliaries of fuzzy controllers) is introduced to stabilize the nonlinear large-scale system. The injection of high-frequency signals, commonly called dithers, into nonlinear systems may improve their performance. If the frequencies of the dithers are high enough, the outputs of the dithered large-scale system and those of its corresponding mathematical model—the relaxed large-scale system—can be made as close as desired. This makes it possible to obtain a rigorous prediction of the stability of the dithered large-scale system by establishing that of the relaxed large-scale system. Finally, a numerical example with simulations is provided to illustrate the feasibility of our approach.

Keywords: Large-scale systems, Dither, $\infty$ control, Delay-dependent stability criterion.

1. Introduction

A number of large-scale systems found in the real world are composed of a set of small interconnected subsystems, such as electric power systems, nuclear reactors, aerospace systems, economic systems, chemical and petroleum industries, and different types of social systems. In practice, due to the information transmission between subsystems, time delay naturally exists in large-scale systems. It often appears in various engineering systems [1], such as the structure control of tall buildings, large-scale structure systems, hydraulics and electronic networks. Notably, time delay is frequently the cause of poor performance and instability [2-3]. Consequently, the problem of stability analysis in time-delay systems remains a major focus of researchers wishing to inspect the properties of such systems. Numerous reports on this subject have been published [4-6]. The stability criteria of time-delay systems so far have been approached from two main directions according to the dependence on the size of delay. One method is to contrive stability conditions which do not include information on the delay, while the other method takes time delay into account. The former case is often referred to as delay-independent criteria and generally gives good algebraic conditions. However, abandonment of information on the size of time delay necessarily causes conservativeness of the criteria, especially when the delay is comparatively small. Hence, delay-dependent criteria are derived to handle the stability problem in this study.

Neural-network-based (NN-based) modeling has become an active research field in the past few years due to its unique merits in solving complex nonlinear system identification and control problems (see [7-11] and the references therein). A neural network can be trained to perform a particular function by adjusting the values of the connections (weights) between elements. Hence, the nonlinear system can be approximated as closely as desired by the NN models via repetitive training. There have been some successful applications
of NN in recent years [12-15]. Despite several promising empirical results and its nonlinear mapping approximation property, the rigorous closed-loop stability results for systems using NN-based controllers are still difficult to establish. Therefore, an LDI state-space representation was also introduced to deal with the stability analysis of NN models (for example, see [8-9]).

Fuzzy control has rapidly developed in both the academic and industrial communities over the past decade, and there have been many successful applications [1, 16-25]. In spite of the success, many basic problems remain to be solved. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems. Lately, significant research efforts have been devoted to these issues [17, 26-27]. All of them, however, neglected the modeling errors between nonlinear systems and fuzzy models. In fact, the existence of modeling error may be a potential source of instability for control designs based on the assumption that the fuzzy model exactly matches the nonlinear plant [28]. Recently, Kiriakidis [28], Chen et al. [29], Cao and Frank [30], and Cao and Lin [31] have proposed novel approaches to overcome the influence of modeling error in the field of model-based fuzzy control for nonlinear systems.

In real physical systems, some noises or disturbances always exist that may cause instability and thereby destroy the performance of control systems. Therefore, how to reduce the effect of external disturbances is an important issue for control design. The $H^\infty$ control problem for nonlinear systems has received considerable attention over the last few years [1, 22, 29-30, 32-33]; for this reason a fuzzy control design with guaranteed $H^\infty$ control performance was introduced for nonlinear multiple time-delay large-scale systems in this study.

Generally, almost all the existing researches of nonlinear control redesigned the fuzzy controller when the designed fuzzy controller cannot stabilize the nonlinear system. However, in this study, a batch of dithers (as the auxiliaries of the fuzzy controllers) is simultaneously introduced to stabilize the nonlinear multiple time-delay large-scale system instead of redesigning the fuzzy controllers. It has been long known that the injection of a high frequency signal, known as a dither, into a nonlinear system may improve its performance (see [34-42] and the references therein). Better performance is viewed as less distortion in the system output, augmented stability, and quenching of limit cycles as well as jump phenomena [38]. A rigorous analysis of stability in a general nonlinear system with a dither control was given in Steinberg and Kadushin [34]. On the basis of the relaxed method, the relaxed system may be stabilized by regulating appropriately the parameters of dither. Mossaheb [37] indicated that a dither of sufficiently high frequency may result in the output of the relaxed system and that of the dithered system being as close as desired. This makes it possible to obtain a rigorous prediction of the stability of the dithered system based on one of its corresponding relaxed system, provided that the dither has a high enough frequency.

The great majority of the works on nonlinear control have made use of fuzzy models to approximate nonlinear systems. Although using fuzzy models to approximate nonlinear systems is simpler than NNs, the NN models will approach nonlinear systems through iterative training and by adjusting the weights. That is to say, the modeling errors of NN models will be much fewer than those of fuzzy models. However, a literature search indicates that using dithers to stabilize nonlinear multiple time-delay large-scale systems via NN-based approach has not yet been discussed. Thus, a robust $H^\infty$ design is proposed in this study to synthesize a set of model-based fuzzy controllers and appropriate dithers to stabilize the nonlinear multiple time-delay large-scale system.

This study is organized as follows: The system description is provided in section II. In section III, a robust fuzzy control design is introduced. An NN-based approach to stabilizing nonlinear multiple time-delay large-scale systems via the composite of fuzzy controllers and dithers is proposed in section IV. The design algorithm is presented in section V. In section VI, the effectiveness of the proposed approach is illustrated through numerical simulation. Finally, conclusions are drawn in section VII.

2. System Description

Consider a nonlinear multiple time-delay large-scale system $N$ composed of $J$ interconnected subsystems $N_j$, $j=1, 2, \ldots, J$. The $j$th subsystem $N_j$ is described by the following equations:

$$\dot{X}_j(t) = y_j(X_j(t), U_j(t)) + \sum_{i=1}^{L_j} \rho_{ij}(X_i(t-\tau_{ij}^e)) + \sum_{i=1}^{L_j} b_{ij}(X_i(t)) + \tilde{e}_j(t)$$

(2.1)

where $y_j(\cdot)$ and $\rho_{ij}(\cdot)$ are the nonlinear vector-valued functions, $X_j(t)$ denotes the state vector, $U_j(t)$ is the input vector and $\tilde{e}_j(t)$ denotes the external disturbance with a known upper bound $\|\tilde{e}_j(t)\|$, $b_{uj}(\cdot)$ are the nonlinear interconnection between the $n$th and $j$th subsystems, $\tau_{ij}^e (k=1, 2, \ldots, L_j)$ are the
time delays in the \( j \)th subsystems and 
\[ \tau_{kj} \ (n = 1, 2, \cdots, J, \ n \neq j) \] are the time delays in the interconnections.

**Definition 2.1** [29]: The solutions of a dynamic system are said to be uniformly ultimately bounded (UUB) if there exist positive constants \( \Theta \) and \( \kappa \), and for every \( \eta \in (0, \kappa) \) there is a positive constant \( T = T(\eta) \), such that
\[
\| x(0) \| < \eta \Rightarrow \| x(t) \| \leq \Theta, \quad \forall t \geq T \tag{2.2}
\]

An example illustrating the stability of UUB is given in Figure 1, in which the brown curve indicates the geometrical implications of stability. A stable equilibrium point implies that the system’s trajectory starts somewhere within the ball \( B_\epsilon \) and eventually lies in the ball \( B_{\kappa} \).

![Figure 1. Uniformly ultimately bounded (UUB).](image)

In the following, each interconnected subsystem is approximated by an NN model. The dynamics of the NN models are then converted into linear differential inclusion (LDI) state-space representations. Subsequently, a set of model-based fuzzy controllers is synthesized to stabilize the nonlinear multiple time-delay large-scale system and the \( H^\infty \) control performance is achieved at the same time.

2.1. Neural Network (NN) Model

The \( j \)th subsystem of \( N \) can be approximated by an NN model, as shown in Figure 2, that has \( S_j \) \((\sigma = 1, 2, \cdots, S_j)\) layers with \( R_j^\sigma \) neurons for each layer, in which \( x_{1n}(t) - x_{\delta_n}(t) \ (n \neq j) \) are the interconnected state variables and \( u_{1j}(t) - u_{m_j}(t) \) are the input variables.

\[\hat{T} = T(\eta)\]
\[\| x(0) \| < \eta \Rightarrow \| x(t) \| \leq \Theta, \quad \forall t \geq T \tag{2.2}\]

An example illustrating the stability of UUB is given in Figure 1, in which the brown curve indicates the geometrical implications of stability. A stable equilibrium point implies that the system’s trajectory starts somewhere within the ball \( B_\epsilon \) and eventually lies in the ball \( B_{\kappa} \).

![Figure 2. The jth NN model.](image)

To distinguish among these layers, superscripts are used to identify the layers. Specifically, we append the number of layers as a superscript to the names for each of these variables. Thus, the weight matrix for the \( \sigma \)th layer is written as \( W_j^\sigma \). Moreover, it is assumed that \( v(t) \) is the net input and \( T(v(t)) \) is the transfer function of the \( \sigma \)th layer. Subsequently, the transfer function vector of the \( \sigma \)th \((\sigma = 1, 2, \cdots, S_j)\) layer is defined as
\[
\Psi_j^\sigma(v(t)) = [T(v_1^\sigma(t)) \ T(v_2^\sigma(t)) \ \cdots \ T(v_{R_j^\sigma}(t))]^T \tag{2.3}
\]
where \( T(v_\zeta^\varphi(t)) \ (\zeta = 1, 2, \cdots, R_j^\varphi) \) is the transfer function of the \( \varphi \)th neuron. The final output of the \( j \)th NN model can then be inferred as follows:
\[
X_j(t) = \Psi_j^1(W_j^1)^{\Psi_j^2(W_j^1)^{\Psi_j^{S_j-1}(W_j^{S_j-1})}} \Psi_j^{S_j-2}(W_j^{S_j-2}) \cdots (W_j^1)^{\Psi_j^{S_j-1}(W_j^{S_j-1})}(W_j^1)^{A_j(t)}) \cdots)
\]
\[
\tag{2.4}
\]

where
\[
A_j^1(t) = \left( \cdots X_j^\varphi(t - \tau_{1n}) X_j^\varphi(t - \tau_{2n}) \cdots X_j^\varphi(t - \tau_{kn}) \right)
\]
\[
X_j^\varphi(t) = \left( \cdots x_{1n}(t - \tau_{1n}) x_{2n}(t - \tau_{2n}) \cdots x_{m_j}(t - \tau_{m_j}) \right)
\]
for \( n = 1, 2, \cdots, J, \ k = 1, 2, \cdots, L, \)

2.2. Linear Differential Inclusion (LDI)

To deal with the stability problem of the nonlinear multiple time-delay large-scale system \( N \), this study establishes the following LDI state-space representation for dynamics of the NN model, described as [43]:

† For simplicity of notation, we use \( S \) instead of \( S_j \) in the remainder of this paper.
\[ \dot{O}(t) = A(\phi(t))O(t), \quad A(\phi(t)) = \sum_{i=1}^{d} \theta_i(a(t)) \tilde{A}_i, \] (2.5)

where \( \phi \) is a positive integer, \( a(t) \) is a vector signifying the dependence of \( h_i(t) \) on its elements, \( \tilde{A}_i \) \( (i=1,2,\cdots,\phi) \) are constant matrices and \( O(t) = [a_1(t), a_2(t), \ldots, a_{\phi}(t)]^T \). Furthermore, assume that \( h_i(a(t)) \geq 0 \) and \( \sum_{i=1}^{\phi} \theta_i(a(t)) = 1 \). Based on the properties of the LDI, without loss of generality, we can use \( h_i(t) \) instead of \( h_i(a(t)) \). In the following, a procedure is taken to represent the dynamics of the \( j \)th NN model (2.4) by LDI state-space representation [09].

To begin with, notice that the output, \( T(v_\sigma^i(t)) \), satisfies
\[ g^{\sigma}_{v_\sigma^i(t)}(t) \leq T(v_\sigma^i(t)) \leq g^{\sigma}_{v_\sigma^i(t)}(t), \quad \forall v_\sigma^i(t) \geq 0 \]
\[ g^{\sigma}_{v_\sigma^i(t)}(t) \leq T(v_\sigma^i(t)) \leq g^{\sigma}_{v_\sigma^i(t)}(t), \quad \forall v_\sigma^i(t) < 0 \]
where \( g_{\sigma}^{i} \) and \( g_{\sigma}^{2} \) are the minimum and the maximum of the derivative of \( T(v_\sigma^i(t)) \), respectively:
\[ g_{\sigma}^{\sigma} = \begin{cases} \min_{v \in \sigma} \frac{dT(v_{\sigma}^i(t))}{dv_{\sigma}^i} & \text{when } \sigma = 0 \\ \max_{v \in \sigma} \frac{dT(v_{\sigma}^i(t))}{dv_{\sigma}^i} & \text{when } \sigma = 1 \end{cases} \quad (2.6) \]

Subsequently, the min-max matrix \( G_{\sigma} \) of the \( \sigma \)th layer is defined as follows:
\[ G_{\sigma} = \text{diag}[g_{\sigma}^{i}] = \begin{bmatrix} g_{v_\sigma^i}^{i} & 0 & 0 & \cdots & 0 \\ 0 & g_{v_\sigma^i}^{i} & 0 & \cdots & 0 \\ 0 & 0 & g_{v_\sigma^i}^{i} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & g_{v_\sigma^i}^{i} \end{bmatrix} \quad (2.7) \]

Moreover, based on the interpolation method, the transfer function \( T(v_\sigma^i(t)) \) can be represented as follows [9, 16]:
\[ T(v_\sigma^i(t)) = ( \sum_{\varphi=0}^{\phi} h_{v_\sigma^i}^{\varphi}(t) g_{v_\sigma^i}^{\varphi} + h_{v_\sigma^i}^{0}(t) g_{v_\sigma^i}^{0} ) v_\sigma^i(t), \]
(2.8)

where the interpolation coefficients \( h_{v_\sigma^i}^{\varphi}(t) \in [0,1] \) and \( \sum_{\varphi=0}^{\phi} h_{v_\sigma^i}^{\varphi}(t) = 1 \). (2.3) and (2.8), show that:
\[ \Psi_{v_\sigma^i}(t) = [T(v_\sigma^i(t)) \ T(v_\sigma^i(t)) \cdots \ T(v_\sigma^i(t))]^T \]
\[ = ( \sum_{\varphi=0}^{\phi} h_{v_\sigma^i}^{\varphi}(t) g_{v_\sigma^i}^{\varphi} + h_{v_\sigma^i}^{0}(t) g_{v_\sigma^i}^{0} ) v_\sigma^i(t), \]
\[ \cdots + \sum_{\varphi=0}^{\phi} h_{v_\sigma^i}^{\varphi}(t) g_{v_\sigma^i}^{\varphi} v_\sigma^i(t)]^T \quad (2.9) \]

Therefore, the final output of the NN model (2.4) can be reformulated as follows:
\[ X_j(t) = \sum_{p=0}^{s_j} h_{x_p}^{s_j}(t) G^s (W_j^s \cdots \sum_{m=0}^{s_j} h_{x_m}^{s_j}(t) G^s (W_j^s \cdots \sum_{r=0}^{s_j} h_{x_r}^{s_j}(t) G^s W_j^s)) \cdots (2.10) \]

where
\[ \sum_{r=0}^{s_j} h_{x_r}^{s_j}(t) = \sum_{r=0}^{s_j} h_{x_1}^{s_j}(t) \sum_{m=0}^{s_j} h_{x_2}^{s_j}(t) \cdots \sum_{r=0}^{s_j} h_{x_r}^{s_j}(t) \]
\[ \sum_{m=0}^{s_j} h_{x_2}^{s_j}(t) = \sum_{m=0}^{s_j} h_{x_1}^{s_j}(t) \sum_{m=0}^{s_j} h_{x_2}^{s_j}(t) \cdots \sum_{m=0}^{s_j} h_{x_m}^{s_j}(t) \]
\[ \sum_{m=0}^{s_j} h_{x_m}^{s_j}(t) = \sum_{m=0}^{s_j} h_{x_1}^{s_j}(t) \sum_{m=0}^{s_j} h_{x_2}^{s_j}(t) \cdots \sum_{m=0}^{s_j} h_{x_m}^{s_j}(t) \]
\[ \cdots \sum_{m=0}^{s_j} h_{x_m}^{s_j}(t) G^s W_j^s \sum_{m=0}^{s_j} h_{x_m}^{s_j}(t) G^s W_j^s \sum_{r=0}^{s_j} h_{x_r}^{s_j}(t) \]
\[ \zeta = 1,2,\cdots,R_j^\sigma, \quad E_j^\sigma = G^s W_j^s \cdots G^s W_j^s \sum_{r=0}^{s_j} h_{x_r}^{s_j}(t) \]
and \( r_j, m_j, p_j, (\zeta = 1,2,\cdots,R_j^\sigma) \) represent the variables \( \varphi \) of the \( \zeta \)th neuron of the first, second, and the \( R_j^\sigma \)th layer, respectively. Finally, according to (2.5), the dynamics of the \( j \)th \( (j=1,2,\cdots,J) \) NN model (2.10) can be rewritten as the following LDI state-space representation:
\[ \dot{X}_j(t) = \sum_{i=1}^{d} h_{x_i}^i(t) E_{ij} A_{ij} X_j(t) + B_{ij} U_j(t) \]
\[ + \sum_{k=1}^{L_j} \sum_{n=1}^{N} \hat{A}_{ikn} X_{ki}(t - \tau_{kn}) + \sum_{n=1}^{N} \hat{A}_{nk} X_{kn}(t - \tau_{kn}) \quad (2.12) \]

where \( \hat{A}_{ikn}, A_{ij}, B_{ij} \) and \( \hat{A}_{nk}, \) a is the partitions of \( E_{ij} \) corresponding to the partition \( N^f_j(t) \).

2.3. Fuzzy Control

On the basis of the decentralized control scheme, a set of fuzzy controllers is synthetized to stabilize the nonlinear multiple time-delay large-scale system. The \( j \)th fuzzy controller is in the following form:

Rule \( \mathcal{E}_j : \) IF \( x_j(t) \) is \( M_{\mathcal{E}_j} \) and \( \cdots \) and \( x_j(t) \) is \( M_{\mathcal{E}_j} \), THEN \( U_j(t) = -C_{\mathcal{E}_j} X_j(t), \)
\( \varepsilon = 1, 2, \ldots, \mu_j \) and \( \mu_j \) is the number of IF-THEN rules of the fuzzy controller, and \( M_{\varepsilon \theta j} (\theta = 1, 2, \ldots, \delta_j) \) are the fuzzy sets. Hence, the final output of this fuzzy controller can be inferred as follows:

\[
U_j(t) = - \sum_{\varepsilon=1}^{\mu_j} \frac{w_{\varepsilon j}(t)}{\varepsilon} C_{\varepsilon j} X_j(t) = - \sum_{\varepsilon=1}^{\mu_j} \frac{w_{\varepsilon j}(t)}{\varepsilon} C_{\varepsilon j} X_j(t),
\]

(2.13)

with \( w_{\varepsilon j}(t) = \sum_{\varepsilon=1}^{\mu_j} M_{\varepsilon \theta j}(x_{\varepsilon j}(t)) \), \( h_{\varepsilon j}(t) = \frac{w_{\varepsilon j}(t)}{\sum_{\varepsilon=1}^{\mu_j} w_{\varepsilon j}(t)} \), in which \( M_{\varepsilon \theta j}(x_{\varepsilon j}(t)) \) is the grade of membership of \( x_{\varepsilon j}(t) \) in \( M_{\varepsilon \theta j} \). In this study, it is also assumed that \( w_{\varepsilon j}(t) \geq 0, \varepsilon = 1, 2, \ldots, \mu_j ; j=1, 2, \ldots, J \) and \( \sum_{\varepsilon=1}^{\mu_j} w_{\varepsilon j}(t) > 0 \) for all \( t \). Therefore, \( h_{\varepsilon j}(t) \geq 0 \), and \( \sum_{\varepsilon=1}^{\mu_j} h_{\varepsilon j}(t) = 1 \) for all \( t \).

2.4. \( H^\infty \) Control Design via Fuzzy Control

In general, some noises or disturbances always exist that may cause instability and worsen the performance of control systems. Therefore, how to attenuate the influence of the external disturbance is a significant problem of control design. In this work, not only the stability of uniformly ultimately bounded (UBB) but also the \( H^\infty \) control performance is guaranteed as follows:

\[
\int_0^{t_f} X_j^2(t) Z_j X_j(t) dt \leq \varrho_j^2 \int_0^{t_f} \hat{\varrho}_j^2(t) \bar{\varrho}_j(t) dt
\]

under zero initial conditions (i.e., \( X_j(t) = 0 \) for \( t \in [-\tau_{\text{max}}, 0] \)), in which \( \tau_{\text{max}} \) is the maximal value of \( \tau_j \)s) and \( t_f \) denotes the terminal time of the control, \( \varrho_j \) is a prescribed value which denotes the effect of \( \bar{\varrho}_j(t) \) on \( X_j(t) \). Moreover, \( Z_j \) is a positive definite weighting matrix. The physical meaning of (2.14) is that the effect of \( \bar{\varrho}_j(t) \) on \( X_j(t) \) must be attenuated below a desired level \( \varrho_j \) from the viewpoint of energy [29].

3. Robust Fuzzy Control Design and Stability Analysis

In this section, the stability of the nonlinear multiple time-delay large-scale system \( N \) is examined under the influence of modeling error.

3.1. Modeling Error

Substituting (2.13) into (2.1) and (2.12) yields the \( j \)th closed-loop subsystem \( \tilde{N}_j \) as follows:

\[
\begin{align*}
\dot{X}_j(t) &= \sum_{i=1}^{\mu_j} h_{ij}(t) [A_{ij} - B_{ij} C_{ij}] X_j(t) + \sum_{k=1}^{J} \sum_{l=1}^{J} \tilde{A}_{ijkl} X_k(t - \tau_{ij}) \\
&\quad + \sum_{k=1}^{J} \sum_{l=1}^{J} \tilde{A}_{ijkl} X_k(t - \tau_{ij}) + \sum_{l=1}^{J} \tilde{B}_{ijkl} X_k(t - \tau_{ij}) \\
&\quad + \sum_{l=1}^{J} \tilde{B}_{ijkl} X_k(t - \tau_{ij}) + \tilde{D}_j(t) \\
&\quad + \sum_{l=1}^{J} \sum_{m=1}^{J} \tilde{A}_{ijkl} X_k(t - \tau_{ij}) + \sum_{l=1}^{J} \sum_{m=1}^{J} \tilde{A}_{ijkl} X_k(t - \tau_{ij}) + \tilde{D}_j(t)
\end{align*}
\]

(3.1)

where \( y_j(X_j(t)), U_j(t) = \Gamma_j(X_j(t)) \) with

\[
U_j(t) = - \sum_{\varepsilon=1}^{\mu_j} \frac{w_{\varepsilon j}(t)}{\varepsilon} C_{\varepsilon j} X_j(t),
\]

\[
\Delta \Phi_j(t) = e_j(t) + \sum_{k=1}^{J} \sum_{l=1}^{J} \tilde{A}_{ijkl} X_k(t - \tau_{ij}) \tilde{\varrho}_j(t) \]

(3.2)

and \( \Delta \Phi_j(t) \) denotes the modeling error between the \( j \)th closed-loop nonlinear subsystem (3.1) and the \( j \)th closed-loop NN model \((2.12) \) and \((2.13) \).

Suppose that there exists the bounding matrix \( \Delta H_{i,j} \)

such that \( \| \Delta \Phi_j(t) \| \leq \| \sum_{\varepsilon=1}^{\mu_j} h_{ij}(t) X_j(t) \Delta H_{i,j} X_j(t) \| \)

(3.5)

for the trajectory \( X_j(t) \), and the bounding matrix \( \Delta H_{i,j} \) can be described as follows:

\[
\Delta H_{i,j} = \tilde{\varrho}_j(t) J_j
\]

(3.6)
where $H_j$ is the specified structured bounding matrix and $\|\mathcal{G}_{\text{inc}}\| \leq 1$, for $i = 1, 2, \ldots, \psi_j$; $e = 1, 2, \ldots, \mu_j$; $j = 1, 2, \ldots, J$. (3.5) and (3.6) show that:

$$\Delta \Phi_j(t) = \sum_{\tau_{k} \in \tau_{k}} h_{\tau_{k}}(t) = \sum_{\tau_{k} \in \tau_{k}} h_{\tau_{k}}(t)$$

Namely, the modeling error $\Delta \Phi_j(t)$ is bounded by the specified structured bounding matrix $H_j$.

**Remark 3.1 [29]**: The following simple example describes the procedures for determining $\mathcal{G}_{\text{inc}}$ and $H_j$. Assuming that the possible bounds for all elements in $\Delta H_{\text{inc}}$ are:

$$\Delta H_{\text{inc}} = \begin{bmatrix} \Delta l_{ij} & \Delta l_{ij}^2 \\ \Delta l_{ij} & \Delta l_{ij}^2 \\ \end{bmatrix}$$

where $-\gamma_{ij}^r \leq \Delta l_{ij}^r \leq \gamma_{ij}^r$ for some $\gamma_{ij}^r$, $r, s = 1, 2$ and $i = 1, 2, \ldots, \phi_j$, $e = 1, 2, \ldots, \mu_j$; $j = 1, 2, \ldots, J$.

A possible description for the bounding matrix $\Delta H_{\text{inc}}$ is:

$$\Delta H_{\text{inc}} = \begin{bmatrix} \mathcal{G}_{\text{inc}}^{11} & 0 \\ 0 & \mathcal{G}_{\text{inc}}^{12} \\ \end{bmatrix}$$

where $-1 \leq \mathcal{G}_{\text{inc}}^{1r} \leq 1$ for $r = 1, 2$. Notice that $\mathcal{G}_{\text{inc}}$ can be chosen by other forms as long as $\|\mathcal{G}_{\text{inc}}\| \leq 1$.

Then, check the validity of (3.5) in the simulation. If it is not satisfied, we can expand the bounds for all elements in $\Delta H_{\text{inc}}$ and repeat the design procedures until (3.5) holds.

### 3.2 Stability in the Presence of Modeling Error

In the following, a delay-dependent stability criterion is proposed to guarantee the stability of the closed-loop nonlinear multiple time-delay large-scale system $\hat{N}$, which consists of $J$ closed-loop subsystems described in (3.1). Before examining the stability of $\hat{N}$, a useful concept is given below.

**Lemma 1 [44]**: For the real matrices $A$ and $B$ with appropriate dimensions, we have:

$$A^TB + B^TA \leq \lambda A^TA + \lambda^{-1}B^TB$$

where $\lambda$ is a positive constant.

**Theorem 1**: The trajectories of the closed-loop nonlinear multiple time-delay large-scale system $\hat{N}$ are uniformly ultimately bounded (UUB) and the $H^\infty$ control performance of (2.14) can be achieved for a prescribed $\gamma^2$, if there exist symmetric positive definite matrices $P_j$, $\psi_{\text{inc}}$, $Z_j$ and positive constants $a_j$, $\lambda_j$, $h_j$ and $f_j$ ($j = 1, 2, \ldots, J$) such that the following inequalities hold:

$$\Delta_{\text{inc},j} = \sum_{k=1}^{l_j} \tau_{k,j}Q_j + \psi_{\text{inc},j} + Z_j < 0$$

(3.10a)

$$\psi_{k,}\{\mathcal{A}_{\text{inc},j}(z_{k,}) - \psi_{k,}\} < 0$$

(3.10b)

for $i = 1, 2, \ldots, \psi_j$, $e = 1, 2, \ldots, \mu_j$, $j = 1, 2, \ldots, J$.

**Proof**: See Appendix.

**Remark 3.2.1**: Since the matrices $\psi_{\text{inc}}$ (in (3.11b)) are positive definite, the matrices $\hat{\psi}_{\text{inc}}$’s must be chosen to be negative definite to meet the stability conditions (3.10a). Hence, based on (3.11a), we have that the larger delay $\tau_{k,}$ will make Theorem 1 more difficult to satisfy.

**Remark 3.2.2**: According to Appendix, we have that $\hat{V}(t) < 0$ whenever (A7) holds. The larger external disturbance $\psi_{\text{ext}}$ results in the larger $Z_j$. Therefore, the larger external disturbance will make (3.10a) more difficult to satisfy.

**Remark 3.2.3**: Based on (3.7), the modeling error $\Delta \Phi_j(t)$ is assumed to be bounded by the specified structured bounding matrix $H_j$ and then a larger modeling error results in a larger $H_j$. According to the same corollary shown in Remark 3.2.1, the larger modeling error will make Theorem 1 more difficult to satisfy.

**Remark 3.2.4**: Equations. (3.10a) and (3.10b) can be reformulated into LMI via the following procedure. By introducing the new variables $Q_j = P_j^{-1}, Y_j = C_jQ_j$ and $\psi_{k,} = \psi_{k,}Q_j, \psi_{k,}Q_j$, (3.10a) and (3.10b) can be rewritten as follows:

$$\Delta_{\text{inc},j} = \sum_{k=1}^{l_j} \tau_{k,j}Q_j A_{ij}^T + \sum_{k=1}^{l_j} \tau_{k,j}Y_j B_{ij}^T + \sum_{k=1}^{l_j} \tau_{k,j}A_{ij}Q_j$$

(3.10a)

$$\psi_{k,}\{\mathcal{A}_{\text{inc},j}(z_{k,}) - \psi_{k,}\} < 0$$

(3.10b)
\[
+ \sum_{k=1}^{l_i} \tau_{k,n_j} A_{i,j} \hat{A}_{i,n_j} + k_{i,n_j} L_{i,n_j} Q_{i,n_j} + \sum_{k=1}^{l_i} \tau_{k,n_j} I
\]

\[
\text{for } i=1, 2, \ldots, \psi_j, \quad \varepsilon=1, 2, \ldots, \mu_j, \quad n, \quad j=1, 2, \ldots, J \]

where

\[
\Gamma = \sum_{k=1}^{l_i} \tau_{k,n_j} Q_j A_{i,j} - \sum_{k=1}^{l_i} \tau_{k,n_j} Y_j B_{i,j} + \sum_{k=1}^{l_i} \tau_{k,n_j} A_{i,j} Q_j
\]

From the above, the LMI constraint is always strictly feasible in \( X \), \( I \) and the original LMI (3.14a) is feasible if and only if the global minimum \( \text{tmn} \) of (3.14b) satisfies \( \text{tmn} < 0 \). In other words, if \( \text{tmn} < 0 \) will satisfy (3.13a) and (3.13b) and then the stability conditions (3.10a) and (3.10b) in Theorem 1 can be met.

Based on Theorem 1, a set of fuzzy controllers can be synthesized to stabilize the nonlinear multiple time-delay large-scale system \( N \). If the designed fuzzy controllers cannot stabilize the nonlinear multiple time-delay large-scale system, the fuzzy controllers and the dithers (as the auxiliaries of the fuzzy controllers) are simultaneously introduced to stabilize the nonlinear multiple time-delay large-scale system.

4. NN Relaxed System and Stability Analysis

4.1. Dithered Plant and Relaxed Model

A high frequency signal, commonly called dither \( d(t) \), with a finite switching number \( \eta \), is injected into the \( j \)th subsystem of the nonlinear multiple time-delay large-scale system \( N \). Thus, the \( j \)th dithered subsystem \( N_{d_j} \) is described as:

\[
\dot{X}_{d_j}(t) = Y_j(X_{d_j}(t), U_j(t), d(t))
\]

\[
+ \sum_{k=1}^{l_i} \rho_{k,n_j}(X_{d_k}(t - \tau_{k,n_j}), d(t))
\]

\[
+ \sum_{w=j} b_{w}(X_{w}(t), d(t)) + \delta_j(t).
\]

The algorithm for constructing the dither is given as follows [34]. The time interval \([0, T]\) is divided into an arbitrary number \( \eta \) of equal subintervals. The beginning of the first interval, the end of the first interval, the end of the second interval and the end of the \( \eta \)th interval are denoted as \( t_0, t_1, t_2 \) and \( t_\eta \), respectively. After dividing every interval \([t_q, t_{q+1}]\) for \( q = 0, 1, 2, \ldots, \eta - 1 \) into \( \ell \) subintervals, the length of the \( m \)th subinterval will be \( \alpha_m(t_q)[t_{q+1} - t_q] \) for \( m=1, 2, \ldots, \ell \) and the control \( \beta_m(t_q) \) is applied at the \( m \)th subinterval. Hence, the repetition frequency, shape and amplitude of the dither can be determined by regulating the parameters \( \eta, \alpha_m(t_q) \) and \( \beta_m(t_q) \). Figure 3 provides an example of constructing a dither to illustrate the proposed algorithm.

Remark 4.1.1: According to the algorithm above, the parameters \( \alpha_m(t_q) \) and \( \beta_m(t_q) \) are constant if the dither is set as a periodic signal. Hence, to reduce the computational burden, the dither is chosen to be a periodic signal and then \( \alpha_m(t) \) and \( \beta_m(t) \) are
respectively changed to $\alpha_m$ and $\beta_m$ in the remainder of this study.

Figure 3. Illustration of constructing a dither.

The corresponding relaxed subsystem $N_{ij}$ of the $j$th dithered subsystem (4.1a) is defined as [34]:

$$N_{ij} : \dot{X}_{ij}(t) = \sum_{m=1}^{l_j} \alpha_m \left( y_j(\bar{X}_{ij}(t)), U_j(t), \beta_m \right) + \sum_{k=1}^{l_j} \rho_{tk_j} \left( X_{r_n}(t-\tau_{tk_j}), \beta_m \right) + \sum_{n=1}^{\phi_j} b_{nj} \left( X_{r_n}(t), \beta_m \right) + \partial_j(t).$$  (4.1b)

where $\alpha_m$ is a non-negative constant that satisfies the following conditions: $0 \leq \alpha_m \leq 1$, $\sum_{m=1}^{l_j} \alpha_m = 1$ for $m = 1, 2, \ldots, l_j$.

Remark 4.1.2: The significance of the dithers’ frequencies lies in their effects on the deviation of the relaxed system from the dithered system and the deviation can be reduced as the dither’s frequency increases.

Remark 4.1.3: The curve $X_{ij}(t)$ satisfying (4.1b) is the uniform limit of curves $X_{ij}(t)$ that satisfies (4.1a).

That is to say, as the frequency of the dither approaches infinity, the trajectory $X_{ij}(t)$ described by the $j$th dithered subsystem $N_{ij}$ approaches that of the $j$th relaxed subsystem $N_{ij}$ by applying the averaging method to the high-frequency dithered term. Hence, $N_{ij}$ may be viewed as a mathematical model of $N_{ij}$ with a dither of sufficiently high frequency.

Based on Remark 4.1.3, if the switching number $\eta$ in $d(t)$ is sufficiently large, then the dithered large-scale system $N_j$ can be approximated by its corresponding mathematical model- the relaxed large-scale system $N_j$ and the approximation improves as $\eta$ increases. Consequently, the outputs of the dithered large-scale system $N_j$ and those of the relaxed large-scale system $N_j$ can be made as close as desired.

4.2. NN Relaxed Model

In this subsection, the $j$th relaxed subsystem $N_{ij}$ is approximated by an NN model. The procedures of constructing the NN model for $N_{ij}$ are similar to those in Section II, and are therefore not repeated here. The final output of the $j$th closed-loop relaxed subsystem $\ddot{N}_{ij}$ can be described as follows:

$$\dot{X}_{ij}(t) = \bar{Y}_{ij}(X_{ij}(t)) + \sum_{n=1}^{\phi_j} \alpha_n \left[ \sum_{k=1}^{l_j} \rho_{kn_j} \left( X_{r_n}(t-\tau_{kn_j}), \beta_n \right) \right] + \dot{\partial}_j(t)$$

$$+ \sum_{n=1}^{\phi_j} b_{nj} \left( X_{r_n}(t), \beta_n \right) + \dot{\partial}_j(t) + \Delta \Phi_j(t)$$  (4.2)

for $i=1, 2, \ldots, \phi_j$, $\epsilon = 1, 2, \ldots, \mu_j$; $n_j = 1, 2, \ldots, J$.

where

$$\bar{Y}_{ij}(X_{ij}(t)) = \sum_{n=1}^{\phi_j} \alpha_n \left[ y_j(\bar{X}_{ij}(t)), U_j(t), \beta_n \right]$$

$$U_j(t) = \sum_{n=1}^{\phi_j} h_{nj} C_j X_{r_n}(t)$$

with $D_{rij} = A_{rij} - B_{rij} C_{\epsilon}$,

$$\Delta \Phi_j(t) = e_{ij}(t) + \sum_{k=1}^{l_j} \sum_{n=1}^{\phi_j} \bar{v}_{kn_j} (t-\tau_{kn_j}) + \hat{e}_{r_n}(t)$$,

in which

$$e_{ij}(t) = \Gamma_{rij} (X_{ij}(t)) = \frac{\bar{y}_{ij}}{l_j} \sum_{i=1}^{l_j} h_{ij} X_{ij}(t)$$

$$\bar{v}_{kn_j}(t-\tau_{kn_j}) = \sum_{i=1}^{l_j} \alpha_{kn_j} (X_{r_n}(t-\tau_{kn_j}), \beta_m)$$

$$- \sum_{i=1}^{l_j} \rho_{kn_j} \left( X_{r_n}(t-\tau_{kn_j}), \beta_m \right)$$

$$+ \sum_{i=1}^{l_j} h_{ij} \left( X_{r_n}(t-\tau_{kn_j}), \beta_m \right)$$

$$\hat{e}_{r_n}(t) = \sum_{n=1}^{\phi_j} \alpha_n b_{nj} (X_{r_n}(t), \beta_n) - \sum_{n=1}^{\phi_j} \sigma_n h_{nj} X_{r_n}(t)$$

4.3. Stability Analysis of the Closed-Loop Relaxed System

Hereafter, we are concerned with the stability of the closed-loop relaxed large-scale system $\ddot{N}_j$ instead of
discussing that of the closed-loop dithered large-scale system \( \tilde{N}_r \). Hence, the stability criterion of \( \tilde{N}_r \) is presented in the following.

**Theorem 2:** The trajectories of the closed-loop relaxed large-scale system \( \tilde{N}_r \) are UUB and the \( H^\infty \) control performance of (2.14) can be achieved for a prescribed \( \alpha_j \), if there exist symmetric positive definite matrices \( P_{i,j},~\psi_{k,j},~Z_{i,j} \) and positive constants \( a_{i,j},~\lambda_{i,j},~\gamma_{i,j} \) and \( f_{i,j} \) (\( j=1,2,\ldots,J \)) such that the following inequalities hold:

\[
\Delta_{i,k} = \sum_{k=1}^{l_i} \tau_{k,i} \xi_{k,i} + \sigma_{i,k} + Z_{i,j} < 0 \tag{4.3a}
\]

\[
\nabla_{i,k} = a_{i,j} L_i - \tilde{A}_{i,k} \tilde{A}_{i,k} - \psi_{i,j} < 0 \tag{4.3b}
\]

for \( i=1,2,\ldots,J \), \( e=1,2,\ldots,\mu_j \), \( k=1,2,\ldots,L_j \); \( n_j=1,2,\ldots,J \), where \( h_l L_j \rightarrow 3 \) and \( D_{i,j} = A_{i,j} B_{i,j} C_{i,j} \).

\[\xi_{i,k} = D_{i,k} P_{i,j} + P_{i,j} D_{i,k} + \lambda_{i,j} r_{i,k} H_{i,j}^T H_{i,j},\]

\[\sigma_{i,k} = a_{i,j} L_j \sum_{k=1}^{l_i} \tau_{k,i} P_{i,j} + L_j \lambda_{i,j} P_{i,j} + \sum_{k=1}^{l_i} \psi_{i,j} \]

\[+ f_{i,j} \sum_{k=1}^{l_i} \tau_{k,i} P_{i,j} \tilde{A}_{i,k} \tilde{A}_{i,k} \tilde{P}_{i,j}^T \]

\[+ \kappa_{i,k} L_i I + h_{i} - \sum_{k=1}^{l_i} \tau_{k,i} P_{i,j}^2 \]

with \( \kappa_{i,k} = \frac{h_l(t)}{h_l(t)} \).

**Proof:** The proof of Theorem 2 can be similarly derived by following the same procedure for Theorem 1 but with extra tuning parameters \( \alpha_m \) and \( \beta_m \). The proof is lengthy, so it is not repeated here.

**Remark 4.3.1:** By the same procedures as those in Remark 3.2.4, (4.3a) and (4.3b) can be rewritten as the following LMIs:

\[
\begin{bmatrix}
\Gamma_r & H_{i,j} Q_{i,j} & Q_{i,j} \\
(H_{i,j} Q_{i,j})^T & -\sum_{k=1}^{l_i} \tau_{k,i} \xi_{k,i} & 0 < 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sigma_{i,k} & Q_{i,j} \tilde{A}_{i,k} \\
\tilde{A}_{i,k} & \alpha_{i,j} I
\end{bmatrix} < 0
\] 

where

\[
\Gamma_r = \sum_{k=1}^{l_i} \tau_{k,i} \{Q_{i,j} \tilde{A}_{i,k} - (B_{i,j} Y_{i,j})^T + A_{i,j} Q_{i,j} - B_{i,j} Y_{i,j}\}
\]

For \( i=1,2,\ldots,J \), \( e=1,2,\ldots,\mu_j \), and \( k=1,2,\ldots,L_j \), \( e=1,2,\ldots,\mu_j \).

The representation of \( H_{i,j} \) is the same as that of the structured bounding matrix \( H_j \) in (3.6).
large-scale system \( \bar{N}_i \) is stable in the sense of Lyapunov, provided that \( \eta \) is sufficiently large.

**Proof:** The algorithm for constructing a dither \( d(t) \) given in Subsection 4.1 provides a means by which the solutions \( X_d(t) \) of the closed-loop dithered large-scale system \( \bar{N}_d \) and \( X_r(t) \) of the closed-loop relaxed large-scale system \( \bar{N}_r \) satisfy

\[
\lim_{\eta \to \infty} \| X_d(t) - X_r(t) \| = 0 .
\]

Thus, for a certain \( \eta \) we have

\[
\| X_r(t) \| < \gamma \text{ for } t > -\tau_{\text{max}} .
\]

Thus from (4.5):

\[
\| X_r(t) \| = \| X_r(t - \tau) + X_r(t) \| < \| X_r(t - \tau) \| + \| X_r(t) \| < \gamma_1 + \gamma \text{ for } -\tau_{\text{max}} < t \leq T .
\]

By taking \( \rho_1 = \varphi_1 \), \( \rho_2 = \gamma_1 + \gamma \) stability with respect to \( \{ \rho_1, \rho_2, -\tau_{\text{max}}, T, \| \| \} \) is proven.

**Theorem 4:** The state vector \( X_d(t) \) of the closed-loop dithered large-scale system \( \bar{N}_d \) is contractively stable with respect to \( \{ \rho_1, \rho_2, -\tau_{\text{max}}, T, \| \| \} \), if the trajectories of the closed-loop relaxed large-scale system \( \bar{N}_r \) are UUB, provided that \( \eta \) is sufficiently large.

**Proof:** Let the trajectories of the closed-loop relaxed large-scale system \( \bar{N}_r \) be UUB. One may select a \( T \) large enough so that for a time \( T \in (0, T] \), we have in addition to the stability properties proven in Theorem 3, \( \| X_r(t) \| < \gamma_2 \) for \( t \in [T, T] \), where \( \gamma_2 \) is arbitrarily small. Choosing

\[
\rho_1 = \varphi_1, \rho_2 = \gamma_1 + \gamma \text{ and } \rho_3 = \gamma_1 + \gamma_2,
\]

it follows that the closed-loop dithered large-scale system \( \bar{N}_d \) is contractively stable with respect to \( \{ \rho_1, \rho_2, \rho_3, -\tau_{\text{max}}, T, \| \| \} \).

5. **Algorithm**

The complete design procedure can be summarized in the following algorithm.

**Problem:** Given a nonlinear multiple time-delay large-scale system \( N \) composed of \( J \) interconnected subsystems \( N_j, j = 1, 2, \ldots, J \), how can we synthesize a set of \( H^\infty \) fuzzy controllers and find appropriate dithers to stabilize the large-scale system \( N \)?

This problem can be solved using the following steps.

**Step 1:** Construct the neural-network (NN) model for each interconnected subsystem of \( N \), and then convert the dynamics of the NN model into an LDI state-space representation.

**Step 2:** According to the state-feedback control scheme, a set of fuzzy controllers is synthesized to stabilize the closed-loop large-scale system \( \bar{N} \) and the \( H^\infty \) control performance of (2.14) is achieved at the same time. Subsequently, adjust the feedback gains \( C_j \) and verify the robust stability condition of \( \bar{N} \) by means of Theorem 1 and Remark 3.2.4 and Remark 3.2.5.

**Step 3:** If the stability conditions of Theorem 1 cannot be satisfied by regulating the feedback gains \( C_j \), a batch of dithers, as the auxiliaries of the fuzzy controllers, is injected into the nonlinear multiple time-delay large-scale system \( N \).

**Step 4:** Make use of the relaxed method to establish the relaxed large-scale system \( \bar{N}_r \).

**Step 5:** Reconstruct the NN model for each interconnected subsystem of \( N \).

**Step 6:** According to Theorems 2-4, we can regulate the dithers’ parameters \( (\alpha_\infty, \beta_\infty) \) to stabilize the closed-loop dithered large-scale system \( \bar{N}_d \).

6. **Numerical Example**

The following example illustrates the effectiveness of the proposed algorithm.

**Example:** The purpose of this example is to synthesize a set of \( H^\infty \) fuzzy controllers such that the following nonlinear multiple time-delay large-scale system \( N \) composed of three interconnected subsystems, which are described as follows, can be stabilized.

**Subsystem 1:**

\[
\begin{align*}
\dot{x}_{11}(t) &= -14.5x_{11}(t) - 1.42x_{21}(t) + 0.02x_{31}(t) + 0.3x_{12}(t) \\
&\quad + 0.2x_{22}(t) + 1.3x_{13}(t) + 0.2x_{32}(t) - 0.1x_{13}(t) \\
&\quad + 0.1x_{11}(t - 0.1) + 0.0x_{12}(t - 0.1) \\
&\quad + 0.0x_{21}(t - 0.1) + 0.1x_{22}(t - 0.1) + 0.1x_{11}(t) \\
&\quad + 0.4x_{x2}(t) + 1.1x_{13}(t) + 0.4x_{33}(t) - 0.1x_{33}(t) \\
&\quad + 0.1x_{11}(t - 0.1) + 0.1x_{21}(t - 0.1) \\
&\quad + 0.0135x_{11}(t - 0.16) + 0.1x_{21}(t - 0.16) + u_{11}(t)
\end{align*}
\]

(6.1a)
Subsystem 2:
\[
\begin{align*}
\dot{x}_{12}(t) &= -15x_{12}(t) + 1.72x_{22}(t) - 0.02x_{22}(t) + 0.3x_{12}(t) \\
&\quad + 0.2x_{21}(t) + 1.1x_{11}(t) + 0.2x_{21}(t) - 0.1x_{11}(t) \\
&\quad + 0.01x_{12}(t - 0.12) + 0.01x_{22}(t - 0.12) \\
&\quad + 0.01x_{12}(t - 0.15) + 0.01x_{22}(t - 0.15) + u_{12}(t), \\
\dot{x}_{22}(t) &= -5x_{12}(t) + 0.7x_{22}(t) - 0.3x_{22}(t) + 0.1x_{12}(t) \\
&\quad + 0.8x_{21}(t) + 1.1x_{11}(t) + 0.4x_{21}(t) - 0.1x_{11}(t) \\
&\quad + 0.01x_{21}(t - 0.12) + 0.01x_{22}(t - 0.12) \\
&\quad + 0.01x_{21}(t - 0.15) + 0.01x_{22}(t - 0.15) + u_{12}(t).
\end{align*}
\] (6.1b)

Subsystem 3:
\[
\begin{align*}
\dot{x}_{13}(t) &= -13.5x_{13}(t) - 1.34x_{23}(t) + 0.04x_{23}(t) + 0.3x_{11}(t) \\
&\quad + 0.2x_{21}(t) + 1.13x_{12}(t) + 0.2x_{22}(t) - 0.1x_{12}(t) \\
&\quad + 0.01x_{13}(t - 0.13) + 0.01x_{22}(t - 0.13) + 0.01x_{11}(t - 0.14) \\
&\quad + 0.01x_{12}(t - 0.14) + 0.01x_{21}(t - 0.14) + u_{13}(t), \\
\dot{x}_{23}(t) &= -4x_{13}(t) + 1.2x_{23}(t) - 0.4x_{23}(t) + 0.1x_{11}(t) \\
&\quad + 0.4x_{21}(t) + 1.1x_{12}(t) + 0.4x_{22}(t) - 0.1x_{12}(t) \\
&\quad + 0.02x_{13}(t - 0.13) + 0.01x_{21}(t - 0.13) \\
&\quad + 0.01x_{13}(t - 0.14) + 0.01x_{21}(t - 0.14) + u_{13}(t).
\end{align*}
\] (6.1c)

How can we synthesize three fuzzy controllers to stabilize the nonlinear multiple time-delay large-scale system \( \mathcal{N} \) and achieve the \( H^\infty \) control performance at the same time?

Solution: Solve the above problem according to the following steps.

Step 1: Establish the NN model for each interconnected subsystem via back propagation algorithm, and then the dynamics of each NN model is converted into an LDI state-space representation.

Figure 4: The NN model of Subsystem 1.

NN model of Subsystem 1:
The first NN model to approximate Subsystem 1 is constructed by 10-2-2, as shown in Figure 4. The transfer functions of the hidden layer are chosen as follows:
\[
T(v_{1j}^\phi(t)) = 20 \left[ \frac{2}{1 + \exp(-v_{1j}^\phi(t)/10)} \right]^{-1}, \quad \text{for } \phi = 1, 2. \tag{6.2}
\]
The transfer functions of the output layer are chosen as follows:
\[
T(v_{2j}^\phi(t)) = v_{2j}^\phi(t), \quad \text{for } \phi = 1, 2. \tag{6.3}
\]

From Figure 4, we have
\[
v_{1j}^\phi = W_{10}^\phi x_{1j}(t-0.1) + W_{11}^\phi x_{1j}(t-0.16) + W_{12}^\phi x_{1j}(t-0.1) + W_{13}^\phi x_{1j}(t-0.16) + W_{14}^\phi x_{1j}(t-0.1) + W_{15}^\phi x_{1j}(t-0.16) + W_{13}^\phi x_{1j}(t-0.16) \tag{6.4a}
\]
\[
v_{2j}^\phi = W_{21}^\phi T(v_{1j}^\phi) + W_{22}^\phi T(v_{2j}^\phi), \quad \phi = 1, 2. \tag{6.4b}
\]

According to (2.6), the minimum and the maximum of the derivative of each transfer function shown in (6.2) and (6.3) can be obtained as follows:
\[
g_{21} = 0, \quad g_{22} = 1 \quad \text{and } g_{11} = g_{12} = 1, \quad \text{for } \phi = 1, 2.
\]

To simplify the notation, we let \( g_{j0} = g_j^1, \quad g_{j1} = g_j^2, \quad g_{j0}^* = g_j^0, \quad g_{j1}^* = g_j^1. \) Then, based on the interpolation method, we have
\[
\dot{x}_{1j}(t) = \sum_{\phi=0}^{1} h_{j\phi}^1(t) g_{j0} \sum_{\phi=0}^{1} W_{1\phi}^j T(v_{1\phi}^j(t)) \]
\[
= \sum_{\phi=0}^{1} h_{j\phi}^1(t) g_{j0} \sum_{\phi=0}^{1} W_{1\phi}^j \{ h_{1\phi}^1 g_{1\phi} + h_{2\phi}^1 g_{2\phi} \} v_{1\phi}^j \]
\[
= \sum_{\phi=1}^{2} h_{j\phi}^1(t) g_{j0} \sum_{p=1}^{2} \sum_{q=0}^{1} h_{j\phi}^1(t) h_{j\phi}^p(t) \{ g_{j\phi} W_{1\phi}^j v_{1\phi}^j + g_{j\phi} W_{1\phi}^j v_{1\phi}^j \}, \tag{6.6}
\]
\[
\dot{x}_{1j}(t) = \sum_{\phi=0}^{1} h_{j\phi}^2(t) g_{j0} \sum_{\phi=0}^{1} W_{2\phi}^j T(v_{2\phi}^j(t)) \]
\[
= \sum_{\phi=0}^{1} h_{j\phi}^2(t) g_{j0} \sum_{\phi=0}^{1} W_{2\phi}^j \{ h_{2\phi}^1 g_{2\phi} + h_{1\phi}^2 g_{1\phi} \} v_{1\phi}^j \]
\[
= \sum_{\phi=0}^{1} h_{j\phi}^2(t) g_{j0} \sum_{p=0}^{1} \sum_{q=0}^{1} h_{j\phi}^2(t) h_{j\phi}^p(t) \{ g_{j\phi} W_{2\phi}^j v_{1\phi}^j + g_{j\phi} W_{2\phi}^j v_{1\phi}^j \}. \tag{6.7}
\]

Inserting (6.4a, 6.4b) into (6.6-6.7), leads to:
\[
\dot{x}_{1j}(t) = \sum_{\phi=0}^{1} \sum_{\rho=0}^{1} \sum_{\sigma=0}^{1} h_{j\rho\phi}^1(t) h_{j\rho\phi}^2(t) h_{j\rho\phi}^1(t) \{ \lambda_{j\rho\phi} X_j(t) + BU(t) + D_{j\rho\phi} X_j(t) - 0.1) + D_{j\rho\phi} X_j(t) - 0.16
\]

* The indices in \( W_{j\phi\rho} \) state that the weight of the \( \sigma \)th layer in the \( j \)th NN model represents the connection to the \( \zeta \)th neuron from the \( \theta \)th source. Moreover, the symbol \( v_{j\phi}^\sigma \) denotes the net input of the \( \zeta \)th neuron of the \( \sigma \)th layer in the \( j \)th NN model, and the indices \( \sigma, \zeta \) and \( j \) shown in \( h_{j\phi}^\sigma \) \( (\phi = 0, 1) \) indicate the same thing.

† The symbol \( a \) denotes the number 10 throughout this example.
The third NN model to approximate Subsystem 3 is constructed by 10-2-2. The transfer functions of the hidden layer are chosen as follows:

\[ T(v_3^1(t)) = \frac{2}{1 + \exp(-v_3^1(t)/10)} - 1, \text{ for } \zeta = 1, 2. \]

The transfer functions of the output layer are chosen as follows:

\[ T(v_3^2(t)) = v_3^2(t), \text{ for } \zeta = 1, 2. \]

In similar fashion, we have the following LDI state-space representation:

\[ X(t) = \sum_{i=1}^{n_1} h_{i3}(t) A_{i3} X_i(t) + B_{i3} U_i(t) + \bar{A}_{i11} X_i(t - 0.11) + \frac{3}{2} \bar{A}_{i12} X_i(t) \]

`NN model of Subsystem 2:`

The second NN model to approximate Subsystem 2 is constructed by 10-2-2. The transfer functions of the hidden layer are chosen as follows:

\[ T(v_2^1(t)) = \frac{2}{1 + \exp(-v_2^1(t)/10)} - 1, \text{ for } \zeta = 1, 2. \]

The transfer functions of the output layer are chosen as follows:

\[ T(v_2^2(t)) = v_2^2(t), \text{ for } \zeta = 1, 2. \]

Using the same procedures as those in the first NN model, we obtain the following LDI state-space representation:

\[ X_2(t) = \sum_{i=1}^{n_1} h_{i2}(t) A_{i2} X_i(t) + B_{i2} U_i(t) + \bar{A}_{i12} X_i(t - 0.12) + \frac{3}{2} \bar{A}_{i12} X_i(t) \]

`NN model of Subsystem 3:`

\[ + \bar{A}_{i13} X_i(t - 0.16) + \frac{3}{2} \bar{A}_{i13} X_i(t) \]

\[ + \bar{A}_{i12} X_i(t - 0.15) + \frac{3}{2} \bar{A}_{i12} X_i(t) \]

\[ + \bar{A}_{i22} X_i(t - 0.15) + \frac{3}{2} \bar{A}_{i22} X_i(t) \]

In accordance with Remark 3.1, the specified structured bounding matrices are chosen as
The matrices $\Delta_{i\alpha j}$'s and $V_{k\alpha n}$'s described in (3.10a) and (3.10b) must be negative definite. Furthermore, for the purpose of guaranteeing the $H^\infty$ control performance of (2.14), the matrix $Z_j$ in (3.10a) is chosen to be positive definite.

At first, based on (6.14, 6.17, 6.20-6.24), we can get the common solutions $Q_j$, $Y_j$, and $\varphi_{k\alpha n}$ via the Matlab LMI toolbox with $a_1=1.8$, $a_2=1$, $a_3=6.5$, $\lambda_1=10$, $\lambda_2=2$, $\lambda_3=14$, $f_1=7$, $f_2=7$, $f_3=7$, $h_1=0.1$, $h_2=0.1$, $h_3=0.1$:

$Q_1 = \begin{bmatrix} 0.2130 & 0.2303 & 0.2123 \\ 0.2303 & 0.2123 & 0.7474 & 0.7690 \\ 0.7474 & 0.7690 & 0.7607 \\
\end{bmatrix}$, $Q_2 = \begin{bmatrix} 0.3598 & 0.3754 \\ 0.3754 & 0.3219 \\
\end{bmatrix}$, $Q_3 = \begin{bmatrix} 0.7474 & 0.7690 \\ 0.7690 & 0.7607 \\
\end{bmatrix}$.

From (6.24)

$\varphi_{11} = \varphi_{211} = \begin{bmatrix} 43.8226 & 43.5993 \\ 43.5993 & 43.6560 \\
\end{bmatrix}$, $\varphi_{12} = \varphi_{222} = \begin{bmatrix} 120.4329 & 113.9892 \\ 113.9892 & 108.9310 \\
\end{bmatrix}$, $\varphi_{13} = \varphi_{233} = \begin{bmatrix} 512.2929 & 516.6236 \\ 516.6236 & 521.1941 \\
\end{bmatrix}$.

$Y_{11} = \begin{bmatrix} 92.3233 & 92.3041 \\ 92.3041 & 92.2701 \\
\end{bmatrix}$, $Y_{12} = \begin{bmatrix} 435.4273 & 434.4386 \\ 434.4386 & 434.4387 \\
\end{bmatrix}$, $Y_{13} = \begin{bmatrix} 577.8809 & 577.5875 \\ 577.5875 & 577.5856 \\
\end{bmatrix}$.

Then, the feedback gains $C_{i\alpha j}$'s are obtained as follows:

$C_{11} = \begin{bmatrix} 213.1074 & 203.8430 \\ 203.8430 & 981.3698 \\
297.9230, C_{12} = \begin{bmatrix} 912.6456 & 285.2698 \\ 285.2698 & 912.6455 \\
285.2690, C_{13} = \begin{bmatrix} 201.7227 & 555.3780 \\ 555.3780 & 961.4459 \\
201.7227, C_{21} = \begin{bmatrix} 555.3789 \\
\end{bmatrix}$

and the best value $\min_{\alpha \in [1, 2]}$ of LMI Solver (Matlab) is 0.51. According to Remark 3.2.5, the synthesized fuzzy controller cannot stabilize the nonlinear multiple time-delay large-scale system $N$. Step 3: A set of periodic symmetrical square-wave dithers $d(t)$ with sufficiently high frequency is injected into the nonlinear multiple time-delay large-scale system $N$, and the dithered large-scale system $N_d$ can be written as follows:

Subsystem 1:
\[
\dot{x}_{11}(t) = -14.5 x_{11}(t) - 1.42 x_{21}(t) + 0.02 [x_{21}(t) + d(t)]^3 + 0.3 x_{12}(t) + 0.2 x_{22}(t) + 1.3 x_{13}(t) + 0.2 x_{23}(t) \\
-0.01 [x_{21}(t) + d(t)]^3 + 0.01 x_{11}(t - 0.1) + 0.01 x_{21}(t - 0.1) \\
+ 0.01 x_{21}(t - 0.16) + 0.1 \sin(5 t) + u_{11}(t) \\
\]

Subsystem 2:
\[
\dot{x}_{12}(t) = -15 x_{12}(t) + 1.72 x_{22}(t) - 0.02 [x_{22}(t) + d(t)]^3 + 0.3 x_{13}(t) + 0.2 x_{23}(t) + 1.3 x_{12}(t) + 0.2 x_{22}(t) \\
-0.01 [x_{22}(t) + d(t)]^3 + 0.01 x_{12}(t - 0.1) + 0.01 x_{22}(t - 0.12) \\
+ 0.01 x_{22}(t - 0.12) + 0.1 x_{22}(t - 0.15) \\
+ 0.01 x_{12}(t - 0.15) + 0.01 x_{22}(t - 0.15) + u_{12}(t) \\
\]

Subsystem 3:
\[
\dot{x}_{13}(t) = -13.5 x_{13}(t) - 1.34 x_{23}(t) + 0.04 [x_{23}(t) + d(t)]^3 + 0.3 x_{11}(t) + 0.2 x_{21}(t) + 1.3 x_{13}(t) + 0.2 x_{23}(t) \\
-0.01 [x_{21}(t) + d(t)]^3 + 0.01 x_{11}(t - 0.1) + 0.01 x_{13}(t - 0.13) \\
+ 0.01 x_{23}(t - 0.14) + 0.1 \sin(5 t) + u_{13}(t) \\
\]

Subsystem 4:
\[
\dot{x}_{23}(t) = -4 x_{13}(t) + 1.2 x_{23}(t) - 0.4 [x_{23}(t) + d(t)]^3 + 0.1 x_{11}(t) + 0.4 x_{21}(t) - 0.1 [x_{21}(t) + d(t)]^3 \\
+ 0.01 x_{12}(t - 0.13) + 0.01 x_{22}(t - 0.13) \\
+ 0.01 x_{13}(t - 0.14) + 0.01 x_{23}(t - 0.14) + u_{13}(t) \\
\]
Subsystem 1:

\[
\begin{align*}
    x_{11}(t) &= -14.5x_{11}(t) - 1.42x_{21}(t) + 0.02 \sum_{n=1}^{\ell} \alpha_n \left[ x_{21}(t) + \beta_n \right]^3 \\
    &\quad + 0.3x_{12}(t) + 0.2x_{22}(t) + 1.3x_{13}(t) + 0.2x_{23}(t) \\
    &\quad - 0.1 \sum_{n=1}^{\ell} \alpha_n \left[ x_{23}(t) + \beta_n \right]^3 + 0.01x_{11}(t) - 0.11 \\
    &\quad + 0.01x_{12}(t) + 0.01x_{13}(t) + 0.15x_{21}(t) + 0.16 \\
    &\quad + 0.01x_{23}(t) + 0.1 \sin(5t) + u_{11}(t) \\
    &\quad + 0.01x_{21}(t) - 0.16 + u_{11}(t) \\
    N_{e1} &= 1.1 (\gamma) + 0.4 (\eta) + 0.01 (\zeta)  \\
\end{align*}
\]

Subsystem 2:

\[
\begin{align*}
    x_{12}(t) &= -15x_{12}(t) + 1.72x_{22}(t) - 0.02 \sum_{n=1}^{\ell} \alpha_n \left[ x_{22}(t) + \beta_n \right]^3 \\
    &\quad + 0.3x_{11}(t) + 0.2x_{21}(t) + 1.3x_{13}(t) + 0.2x_{23}(t) \\
    &\quad - 0.1 \sum_{n=1}^{\ell} \alpha_n \left[ x_{23}(t) + \beta_n \right]^3 + 0.01x_{12}(t) - 0.12 \\
    &\quad + 0.01x_{22}(t) - 0.12 + 0.01x_{12}(t) - 0.15 \\
    &\quad + 0.02x_{22}(t) - 0.15 + 0.1 \sin(5t) + u_{12}(t) \\
    N_{e2} &= 5x_{12}(t) + 0.7x_{22}(t) - 0.3 \sum_{n=1}^{\ell} \alpha_n \left[ x_{22}(t) + \beta_n \right]^3 \\
    &\quad + 0.1x_{11}(t) + 0.8x_{21}(t) + 1.1x_{13}(t) + 0.4x_{23}(t) \\
    &\quad - 0.1 \sum_{n=1}^{\ell} \alpha_n \left[ x_{23}(t) + \beta_n \right]^3 + 0.01x_{12}(t) - 0.12 \\
    &\quad + 0.01x_{22}(t) - 0.12 + 0.01x_{12}(t) - 0.15 \\
    &\quad + 0.01x_{22}(t) - 0.15 + u_{12}(t) \\
    N_{e3} &= 15x_{12}(t) - 1.34x_{22}(t) + 0.04 \sum_{n=1}^{\ell} \alpha_n \left[ x_{22}(t) + \beta_n \right]^3 \\
    &\quad + 0.3x_{11}(t) + 0.2x_{21}(t) + 1.3x_{13}(t) + 0.2x_{23}(t) \\
    &\quad - 0.1 \sum_{n=1}^{\ell} \alpha_n \left[ x_{23}(t) + \beta_n \right]^3 + 0.01x_{12}(t) - 0.13 \\
    &\quad + 0.01x_{23}(t) - 0.13 + 0.1 \sin(5t) + u_{12}(t) \\
    &\quad + 0.01x_{23}(t) - 0.14 + u_{12}(t) \\
\end{align*}
\]

Step 5: In the following, the amplitudes of dithers are found to stabilize the closed-loop relaxed large-scale system $N_r$. The procedures of constructing the NN model for $N_r$ are similar to those in Step 1 and the feedback gains $C_{ij}$ are chosen to be the same as those in (6.25).

Step 6: In accordance with Remark 3.1, the specified structured bounding matrices $H_{i,j}, H_{2,i}, H_{3,j}$ and $B_{ij}$ are set to be

\[
\begin{bmatrix}
    700 & 0 \\
    0 & 700
\end{bmatrix}, \begin{bmatrix}
    200 & 0 \\
    0 & 200
\end{bmatrix}, \begin{bmatrix}
    1500 & 0 \\
    0 & 1500
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix}
\].

To satisfy the stability conditions of Theorem 2, all the matrices of $\Pi_{i,j}$ and $V_{i,k}$ depending on the dither’s amplitude $\xi$ must be negative definite. Hence, based on Remark 4.3.2, this study focuses on how to appropriately regulate the

Figure 5a. Time responses of the control input $u_{11}(t)$.

Figure 5b. Time responses of the control input $u_{12}(t)$.

Figure 5c. Time responses of the control input $u_{13}(t)$.

with $\ell = 2$, $\alpha_1 = 0.5$, $\alpha_2 = 1 - \alpha_1 = 0.5$ and $\beta_1 = - \beta_2 = \xi$, in which $\xi$ is a real constant. The trajectories of control inputs are shown in Figures 5a-5c.
dither’s amplitude \( \xi \) such that the best value \( t_{\text{min}} \) of LMI Solver (Matlab) is negative. Figure 6 shows the best value \( t_{\text{min}} \) with respect to the dither’s amplitude \( \xi \).

![Figure 6](image)

**Figure 6.** The best value \( t_{\text{min}} \) of LMI Solver with respect to the amplitude \( \xi \) of dither.

From Figure 6 we have that if the dither’s amplitude \( \xi \) is within 1.083 and 1.541, the inequalities in (4.3a) and (4.3b) are satisfied, i.e., the trajectories of the relaxed system \( \tilde{N}_r \) are UUB. Furthermore, the illustrations in Figures 7a-7c show that the assumption of

\[
\| \Delta \Phi_{j}(t) \| \leq \sum_{i=1}^{2} \sum_{j=1}^{3} h_{ij}(t) \Delta H_{r,ij} X_j(t) \|
\]

for \( j=1, 2, 3 \) is satisfied with \( \xi = 1.5 \) and initial conditions: \( x_{11}(0)=2.1 \), \( x_{12}(0)=-1.7 \), \( x_{13}(0)=-1.9 \), \( x_{21}(0)=2.5 \), \( x_{22}(0)=1.8 \) and \( x_{23}(0)=-1.2 \).

![Figure 7a](image)

**Figure 7a.** Plots of \( \| \Delta \Phi_{j}(t) \| \) (blue line) and \( \sum_{i=1}^{2} \sum_{j=1}^{3} h_{ij}(t) \Delta H_{r,ij} X_j(t) \| \) (red line).

![Figure 7b](image)

**Figure 7b.** Plots of \( \| \Delta \Phi_{j}(t) \| \) (blue line) and \( \sum_{i=1}^{2} \sum_{j=1}^{3} h_{ij}(t) \Delta H_{r,ij} X_j(t) \| \) (red line).

![Figure 7c](image)

**Figure 7c.** Plots of \( \| \Delta \Phi_{j}(t) \| \) (blue line) and \( \sum_{i=1}^{2} \sum_{j=1}^{3} h_{ij}(t) \Delta H_{r,ij} X_j(t) \| \) (red line).

According to Theorem 4, the closed-loop dithered large-scale system \( \tilde{N}_d \) is contractively stable as long as the switching number \( \eta \) is large enough. Moreover, the state responses of \( \tilde{N}_d \) (\( \xi = 1.5 \), \( \omega = 10 \) rad/s (green line) and \( \omega = 1500 \) rad/s (blue line)) and the closed-loop relaxed large-scale system \( \tilde{N}_r \) (red line) are shown in Figures 8a-8f. Obviously, the trajectories of \( \tilde{N}_d \) can be approximated by those of its corresponding mathematical model \( \tilde{N}_r \), and the approximation improves as the frequency of the dither increases. This makes it possible to rigorously predict the stability of \( \tilde{N}_d \) by establishing that of \( \tilde{N}_r \).

![Figure 8a](image)

**Figure 8a.** Time responses of the state \( x_{11}(t) \).

![Figure 8b](image)

**Figure 8b.** Time responses of the state \( x_{21}(t) \).
An effective approach via the composite of fuzzy controllers and dithers is presented in this work. On the basis of this approach, we can synthesize a set of fuzzy controllers and find appropriate dithers to stabilize the nonlinear multiple time-delay large-scale system. First, the NN model is employed to approximate each interconnected subsystem. Then, the dynamics of each NN model is converted into an LDI state-space representation. Next, a delay-dependent criterion is derived from Lyapunov's direct method to guarantee the stability (UUB) of nonlinear multiple time-delay large-scale systems. Subsequently, the stability conditions of this criterion are reformulated into linear matrix inequalities (LMIs). According to the LMIs and the decentralized control scheme, a set of fuzzy controllers is synthesized to stabilize the nonlinear large-scale system and the $H^\infty$ control performance is achieved at the same time. If the designed fuzzy controllers cannot stabilize the nonlinear large-scale system, a batch of dithers (as the auxiliaries of fuzzy controllers) is simultaneously introduced to stabilize it. The simulation results conclusively demonstrate that the proposed fuzzy controllers are able to stabilize the nonlinear multiple time-delay large-scale system by appropriately regulating the dithers' parameters.

### Appendix: Proof of Theorem 1

Let the Lyapunov function for $\bar{N}$ be defined as

$$V(t) = \sum_{j=1}^{J} V_j(t)$$

$$= \sum_{j=1}^{J} \sum_{i=1}^{I} \int_{-\pi}^{\pi} X_i(t) P_j(t) X_j(t) \psi_{k_{nj}}(t) \psi_{k_{nj}}(t) d\pi$$

(A1)

where the weighting matrices $P_j = P_j^T > 0$ and $\psi_{k_{nj}} = \psi_{k_{nj}}^T > 0$. We then evaluate the time derivative of $V(t)$ on the trajectories of (3.1) to get

$$\dot{V}(t) = \sum_{j=1}^{J} \sum_{i=1}^{I} \left[ \sum_{l=1}^{L} \sum_{k_{nj}} \int_{-\pi}^{\pi} \dot{X}_i(t) P_j(t) X_j(t) \psi_{k_{nj}}(t) \psi_{k_{nj}}(t) d\pi \right]$$

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\partial}{\partial t} \left[ \sum_{l=1}^{L} \int_{-\pi}^{\pi} \dot{X}_i(t) \left( \psi_{k_{nj}}(t) X_j(t) - X_j(t) \psi_{k_{nj}}(t) \right) X_j(t) \psi_{k_{nj}}(t) d\pi \right]$$

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\partial}{\partial t} \left[ \sum_{l=1}^{L} \int_{-\pi}^{\pi} \dot{X}_i(t) \left( \psi_{k_{nj}}(t) X_j(t) - X_j(t) \psi_{k_{nj}}(t) \right) X_j(t) \psi_{k_{nj}}(t) d\pi \right]$$

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\partial}{\partial t} \left[ \sum_{l=1}^{L} \int_{-\pi}^{\pi} \dot{X}_i(t) \left( \psi_{k_{nj}}(t) X_j(t) - X_j(t) \psi_{k_{nj}}(t) \right) X_j(t) \psi_{k_{nj}}(t) d\pi \right]$$

where $\dot{X}_i(t)$ is the time derivative of $X_i(t)$. The proof is completed by showing that $\dot{V}(t) \leq 0$ for all $t$. **\( \uparrow \)**

**\( \uparrow \)** Based on the concept of interconnection, the matrix $\dot{A}_{ijj}$ is set to be zero.
\[ V(t) \leq \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} h_{ij}(t) h_{ij}(t) X_{ij}(t) \left( (A_{ij} - B_{ij} C_{ij}) \right)^{T} \tau_{k_{ij}} P_{j} \]
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References


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