Robust Fuzzy Congestion Control of TCP/AQM Router via Perturbed Takagi-Sugeno Fuzzy Models

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Abstract

Because the number of transmission control protocol sessions usually varies from time to time, a perturbed Takagi-Sugeno fuzzy model is used in this paper to represent the dynamic transmission control protocol network systems. According to the proposed perturbed Takagi-Sugeno fuzzy model, a robust fuzzy controller design approach is investigated to achieve the congestion avoidance for the transmission control protocol network systems. In order to accomplish the above mission, some sufficient conditions are derived based on the Lyapunov stability theory. By solving these sufficient stability conditions, an active queue management router can be obtained via the proposed robust fuzzy congestion control technique.

Keywords: Perturbed Takagi-Sugeno fuzzy model, TCP network system, congestion control and robust fuzzy control.

1. Introduction

Fuzzy systems based on the Takagi-Sugeno (T-S) fuzzy model [1] have attracted great interests from scientific and engineering communities. The T-S fuzzy model provides an efficient method to represent complex nonlinear systems via fuzzy sets and fuzzy reasoning. By extending the linear control and constructing Parallel Distributed Compensator (PDC), the T-S fuzzy model based control has been applied to cope with complex nonlinear systems. The analysis and synthesis problems of T-S fuzzy systems have been extensively studied [2-12]. The T-S fuzzy systems can be separated into the homogeneous fuzzy systems [2-6] and the affine fuzzy systems [7-12]. The homogeneous fuzzy system is referred to the T-S fuzzy system of which consequent part is linear and which does not have a constant bias term. In contrast, the affine fuzzy system means the T-S fuzzy system of which consequent part is affine with a constant bias term.

In this paper, an Active Queue Management (AQM) router design problem for the Transmission Control Protocol (TCP) network system is discussed. The dynamic model of TCP network system has been proposed such that three key parameters, number of TCP sessions, link capacity and round-trip time are related to a feedback control problem. In order to understand the behavior of a dynamic TCP network, modeling and analysis of such network are very important. In [13], the authors used jump process driven stochastic differential equations [14] to model the behavior of a TCP network. In [15], the authors exploited fluid modeling to present a general methodology for the analysis of a network of routers supporting AQM with TCP flows. They modeled the data traffic as a fluid and specifically used Poisson counter driven stochastic differential equations to model sample path description of TCP traffic. By employing the small signal linearization about an operating point, the authors of [16] approximated the nonlinear fluid-based TCP dynamic model in [15] as a linear constant system. It can be found that the TCP network system is a nonlinear perturbed affine time-delay system because it has time delay, constant link capacity and varying sessions. Thus, the linear constant system will sacrifice the original nonlinear properties for applying the linearized technique. Therefore, the motivation of this paper is to use the fuzzy modeling technique to replace the linear modeling method such that the nonlinear TCP network system can be represented by a perturbed time-delay T-S fuzzy model.

The AQM design problems are important in TCP network system because AQM is embedded in the router and has much information about circumstances of current networks. Some kinds of AQM schemes have been proposed, such as Random Early Detection (RED) [16-17], Random Exponential Marking (REM) [18], Adaptive Virtual Queue (AVQ) [19], Proportional Integral (PI) controller [20-22], predictive controller [23] and fuzzy controller [24-25]. By applying the linearized model, references [18, 20-22] applied control theory to address congestion avoidance issues. However, the control approaches developed in [18, 20-22] sacrifice the original nonlinear properties because they applied the linear modeling technique to implement the TCP networks. To overcome the problem, the authors of [25]
used a time-delay affine T-S fuzzy model to reconstruct the TCP networks. However, the number of TCP sessions was assumed to be constant in [25]. In fact, the number of TCP sessions usually varies with time. In order to obtain a better approximation to the original nonlinear TCP network system, a perturbed time-delay affine T-S fuzzy model is constructed in this paper. According to this perturbed T-S fuzzy model, the contribution of this paper is to investigate the robust fuzzy congestion control for the TCP/AQM router design.

Employing the Linear Matrix Inequalities (LMI) technique [26], one can find a suitable common positive definite matrix for the stability conditions, and then to obtain a stable fuzzy controller for the closed-loop homogeneous T-S fuzzy models. However, the fuzzy controller design of the perturbed time-delay affine T-S fuzzy models is a challenging problem for the designers because the closed-loop stability conditions are not LMI formulations but Bilinear Matrix Inequalities (BMI) ones. The BMI conditions cannot be easily solved via a convex optimization algorithm [26]. In order to solve the BMI problem, an Iterative Linear Matrix Inequality (ILMI) algorithm has been presented in [7-10]. Extending the control approach of [7-10], a modified ILMI algorithm is developed in this paper to find feasible solutions for the synthesis problem of fuzzy controller design for perturbed time-delay affine T-S fuzzy models. Therefore, a novel congestion control approach for the AQM router can be accomplished in this paper based on the robust fuzzy controller design for the perturbed time-delay affine T-S fuzzy models.

The organization of this paper is presented as follows. In Section 2, the time-delay affine T-S fuzzy model of TCP network with AQM routers is introduced. Section 3 provides the stability conditions for the existence of fuzzy control based AQM routers. In order to find the feasible solutions of these stability conditions, an ILMI algorithm is developed in Section 3. The effectiveness of proposed fuzzy control on managing queue utilization is illustrated in Section 4 by a numerical simulation. Finally, a conclusion is given in Section 5.

2. Perturbed Time-Delay Affine T-S Fuzzy Model of TCP/AQM Routers

According to [15], the TCP network system can be formulated as the following stochastic differential equations:

$$W(t) = \frac{1}{R(t)} \left( W(t) - W(t) \frac{W(t-R(t))}{R(t)} \times p(t-R(t)) \right) \quad (1a)$$

$$\dot{q}(t) = \begin{cases} r(t) - C & \text{if } 0 < q(t) < q_m \\ \max \{0, r(t) - C\} & \text{if } q(t) = 0 \\ \min \{0, r(t) - C\} & \text{if } q(t) = q_m \end{cases} \quad (1b)$$

where $W(t) \in [0, w_max]$ is the TCP window size; $w_max$ is the maximum window size; $R(t)$ is the round-trip times; $p(t) \in [0, 1]$ is the probability of packet mark/drop; $r(t)$ is the aggregate incoming rate; $C$ is link capacity; $q_m$ is the finite buffer size. The queue length, $q(t)$, changes depending on the queue occupancy rate, which is the accumulated different between the incoming rate and the outgoing link capacity, i.e., $r(t) - C$. In addition, the round trip time, $R(t)$, is the sum of propagation delay $T_p$ and queuing delay $q(t)/C$. That is,

$$R(t) = T_p + q(t)/C \quad (2)$$

As well as, the aggregate incoming rate $r(t)$ can be represented as a function of $W(t)$ and $q(t)$ as follows:

$$r(t) = \frac{W(t)}{R(t)} N(t) = \frac{W(t)}{T_p + q(t)} N(t) \quad (3)$$

where $N(t)$ is the load factor, i.e., number of TCP sessions. As shown in Fig. 1, Equation (1) denotes the network topology which was assumed to be a single bottleneck with TCP sources that share the bottleneck link and the same round-trip time.

Figure 1. Structure of TCP network systems.

In order to construct the affine T-S fuzzy model for the TCP dynamic model (1), we ignore the dependence of the time-delay argument $t - R(t)$ on window size $W(t)$ and queue length $q(t)$, and assume it is fixed to $t - R_c$, where $R_c$ is a constant value. The delay in $p(t)$ is omitted for the same reason. As a result, a simplified dynamic of TCP control model (1) can be described as follows:

$$W(t) = \frac{1}{T_p + q(t)/C} \left( W(t) - W(t) \frac{W(t-R_c)}{T_p + q(t)} \times p(t) \right) \quad (4a)$$
\[ q(t) = \frac{W(t)}{T_\tau + q(t)} C N(t) - C \]  

The construction of the affine T-S fuzzy model is achieved by applying the small signal linearization method [27]. In system (4), let \((W, q)\) be the system state to be controlled, \(p\) be the input, and \((W^a, q^a, p^a)\) be the equilibrium point of the system. Then, the equilibrium point \((W^a, q^a, p^a)\) can be obtained as follows by solving \(W(t)=0\) and \(q(t)=0\) with the assumption \(W(t-R) = W(t-R^*) = W^a\) and \(q(t-R) = q(t-R^*) = q^a\). Note that \(\lim_{{i \to \infty}} q(t-R^*) = 0\) and \(\lim_{{i \to \infty}} W(t-R^*) = W^a\), i.e., \(W^a = W^a\) and \(q^a = q^a\) when \(t \to \infty\). Hence, one can obtain the equilibrium point as follows:

\[ W^a = \frac{C}{N(T_\tau + q(t)/C)} \]  

(5a)

\[ p^a = \frac{2}{T_\tau + q^a/C} T_\tau + q^a/C = \frac{2}{W^a} (W^a) \]  

(5b)

\[ R^a = T_\tau + q^a/C \]  

(5c)

In order to shift the equilibrium point to origin, let us define new state variables as follows:

\[ x_i(t) = W(t) - W^a \]  

(6a)

\[ x_i(t) = q(t) - q^a \]  

(6b)

\[ x_i(t-R^*) = x_i(t) - W(t-R^*) - W^a \]  

(6c)

\[ x_i(t-R^*) = x_i(t) - q(t-R^*) - q^a \]  

(6d)

\[ u(t) = p(t) - p^a \]  

(6e)

Then, the new equilibrium point can be stated as \(x_i(t) = 0\), \(x_i(t-R^*) = 0\), \(x_i(t-R^*) = 0\), \(x_i(t-R^*) = 0\), and \(u(t)=0\). Hence, the simplified TCP fluid-flow dynamical model (4) can be represented as

\[ \dot{x}_i(t) = -\frac{1}{T_\tau + [x_i(t) + q(t)/C]} \]  

(7a)

\[ \times \frac{[x_i(t) + q(t)] + W^a}{T_\tau + [x_i(t) + q(t)/C]} \]  

(7b)

According to the nonlinear dynamic model (7) with new state variables defined in (6), the corresponding perturbed time-delay affine T-S fuzzy model can be constructed of the following IF-THEN form by using the small signal linearization method [27].

**Rule i:** IF \(x_i(t)\) is \(\tilde{M}_{ii}\) and \(x_i(t)\) is \(\tilde{M}_{ii}\) THEN

\[ \dot{x}(t) = (A_i + \Delta A_i)x(t) + A_{ii}x(t-R^*) + B_iu(t) + (a_i + \Delta a_i) \]  

\[ x(t) = \psi(t), t \in [-R^*, 0^*], i = 1, 2, \ldots, r, x(t) \in \mathbb{R} \]  

(8)

where \(x(t) \in \mathbb{R}^2\) is the state vector and \(u(t) \in \mathbb{R}\) is the control input vector. The matrices \(A_i, a_i \in \mathbb{R}^{2 \times 2}\), \(B_i, \psi_i \in \mathbb{R}^{2 \times 1}\), and \(a_i \in \mathbb{R}^2\) are constant for \(i = 1, 2, \ldots, r\) and \(r\) is the number of IF-THEN fuzzy rules. Let us define \(f(x(t), x(t-R^*), u(t)) = \dot{x}(t) = \dot{x}_i(t) x_i(t)\), then the consequent system parameters can be obtained by [27] as follows:

\[ A_i = \frac{\partial f(x(t), x(t-R^*), u(t))}{\partial x(t)} = \frac{\partial \dot{x}(t)}{\partial x(t)} \]  

(9a)

\[ A_{ii} = \frac{\partial f(x(t), x(t-R^*), u(t))}{\partial x(t-R^*)} = \frac{\partial \dot{x}(t)}{\partial x(t-R^*)} \]  

(9b)

\[ B_i = \frac{\partial f(x(t), x(t-R^*), u(t))}{\partial u(t)} = \frac{\partial \dot{x}(t)}{\partial u(t)} \]  

(9c)

with the affine terms

\[ a_i = f(x', x', u') - A_i x_i - A_{ii} x_i - B_i u' \]  

(10)

where \((x', x', u')\) is the operating point of the \(i\)th fuzzy rule. Besides, the \(M_{ii}\) are fuzzy sets and \(\psi(t)\) is the initial condition of the state defined on \(-R^* \leq t \leq 0\). The region \(X_i \subseteq \mathbb{R}^2\) is assumed to be a fuzzy subspace and \(X_i\) is called as a cell. The set of cell indices is denoted as \(\tilde{I}\) and the union of all cells \(x(t) = \bigcup_{i \in \tilde{I}} X_i\) is referred to as the whole fuzzy space. Let \(\tilde{i}\) be the set of indices for the fuzzy rules that contain the origin and \(\tilde{i} \subseteq \tilde{I}\) be the set of indices for the fuzzy rules that does not contain the origin. The origin is an equilibrium point of the time-delay affine T-S fuzzy models and it is assumed that \(a_i = 0\) for \(i \in \tilde{i}\). The quantities \(A_i\), \(A_{ii}\), \(B_i\), and \(a_i\) are constant matrices. Besides, \(\Delta A_i\) and \(\Delta a_i\) are time-varying matrices with appropriate dimensions and they are structured in the following norm-bounded form:

\[ [\Delta A_i, \Delta a_i] = D_i[A_i(t) + Q_{ii}] \]  

(11)

where \(D_i, Q_{ii}\) and \(Q_{ii}\) are known real constant matrices of appropriate dimensions, and \(A_i(t)\) is an unknown matrix function with Lebesgue-measurable elements and satisfies \(A_i(t) A_i(t) \leq I\).

Given a pair of \((x(t), u(t))\), the final outputs of the fuzzy model (8) are inferred as follows:

\[ \\hat{x}(t) = \sum_{i=1}^{r} b_i(x(t)) \left[ (A_i + \Delta A_i)x(t) + A_{ii}x(t-R^*) + B_iu(t) + (a_i + \Delta a_i) \right] \]  

(12)

where

\[ x(t) = \left[ x_i(t) \ x_i(t) \right]^T \]  

(13)

\[ \psi_i(x(t)) = \left[ x_i(t) \ x_i(t) \right]^T \]  

(14)
\[ h_i(x(t)) = \frac{a_i(x(t))}{\sum a_i(x(t))} \quad (15a) \]

\[ h_i(x(t)) \geq 0 \quad (15b) \]

\[ \sum_i h_i(x(t)) = 1 \quad (15c) \]

The perturbed time-delay affine T-S fuzzy model of TCP network with AQM routers has been constructed in (8) or (12). In the next section, the stability conditions for the proposed fuzzy controller design are derived based on the above perturbed time-delay affine T-S fuzzy model.

### 3. Designing Robust Fuzzy Congestion Controller

For the perturbed time-delay affine T-S fuzzy model represented by (12), a fuzzy controller is designed based on the PDC concept [2] as follows:

**Rule i**: If \( x_i(t) \) is \( M_{i_1} \) and \( x_i(t) \) is \( M_{i_2} \) THEN

\[ u(t) = -F_i x(t), \quad i = 1, 2, \ldots, r \text{ for } x(t) \in X_i, \quad i \in \hat{I} \quad (16) \]

where \( F_i \in \mathbb{R}^{1 \times 2} \) are constant. The output of the PDC-based fuzzy controller is determined by the summation as

\[ u(t) = -\sum h_i(x(t))\{F_i x(t)\} \quad (17) \]

Substituting (17) into (12), one can obtain corresponding closed-loop system as follows:

\[ \dot{x}(t) = \sum h_i(x(t))b_i(x(t))\{(A_i + \Delta A_i)x(t) + A_{it}(t^*-t)\} - BF_i x(t) + (a_i + \Delta a_i) \quad (18) \]

**Lemma 1 [28]**: For any two real matrices \( X \in \mathbb{R}^{n \times m} \) and \( Y \in \mathbb{R}^{p \times m} \), one has

\[ X^T Y + Y^T X \leq X^T N X + Y^T N^{-1} Y \quad (19) \]

where \( N > 0 \) is a constant matrix (or scalar).

The asymptotical stability analysis issue to closed-loop perturbed time-delay affine T-S fuzzy model (18) is discussed based on Lyapunov stability criterion. Besides, the Lemma 1 and \( S\)-procedure [27] are used to derive the stability conditions. In the following theorem, sufficient conditions for ensuring delay-independent stability of the perturbed time-delay affine T-S fuzzy model (18) are introduced.

**Theorem 1**: The perturbed time-delay affine T-S fuzzy model (18) is quadratically stable in the large if there exist common positive-definite matrices \( P > 0 \), \( S > 0 \), control gains \( F_i \) and scalars \( \xi_{i_0} \geq 0 \) and \( \alpha < 0 \) such that

\[
\begin{bmatrix}
\Lambda_{i_1} - \alpha P & P[D_i B_i] & P[D_i B_i]^T & F_i & F_i & P A_{i_2} & PA_{i_2} \\
\ast & -I/2 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & -I & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & -I & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & -S & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & -S & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & -S \\
\end{bmatrix} < 0
\]

for \( i \in \hat{I}_i \) \quad (20)

and \( P \cdot S > 0 \) for \( i \in \hat{I}_i \) and \( \hat{i}_i \) \quad (21)

where * denotes the transposed elements or matrices for symmetric position and

\[
\begin{bmatrix}
\Lambda_{i_1} - \alpha P & P[D_i B_i] & P[D_i B_i]^T & F_i & F_i & P A_{i_2} & PA_{i_2} \\
\ast & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & -I/2 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & -I & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & -I & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & -S & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & -S \\
\end{bmatrix} < 0
\]

for \( i \in \hat{I}_i \) \quad (22)

where \( \Pi \) denotes the transposed elements or matrices for symmetric position and

\[
\begin{align*}
\Lambda_{i_1} &= A_i^T P + P A_i + A_{i_2} + 2P \\
&\quad + E_i^T E_i + \sum_{q=1}^s E_i^T Q_i^q Q_i^q E_i \quad (23) \\
\Lambda_{i_2} &= P(a_i + a_2) - \sum_{q=1}^s E_i^T Q_i^q n_{q_1} \\
&\quad - \sum_{q=1}^s 2E_i^T n_{q_2} T_{q_1} \quad (24) \\
\Lambda_{i_2} &= [Q_i^q, Q_i^q]^T [Q_i^q, Q_i^q]^T - \sum_{q=1}^s 2E_i^T v_{q_1} \quad (25) \\
E_i &= B_i^T P + F_i \quad (26)
\end{align*}
\]

Besides, the \( S\)-procedure weighting parameter \( T_{q_1}, n_{q_1}, v_{q_1} \) are defined such that

\[
\sigma_{i_1}(x(t)) = x^T(t)T_{q_1}x(t) + 2n_{q_1}x(t) + v_{q_1} \leq 0, \quad q = 1, 2 \quad \text{and} \quad i = 1 \cdots r 
\]

for all \( x(t) \) which activates rule \( i \) (i.e., \( h_i(x(t)) > 0 \)).

**Proof**:

Select a Lyapunov function as

\[ V(x(t)) = x^T(t) P x(t) \quad (28) \]

The derivative of the Lyapunov function \( V(x(t)) \) along the trajectories of (18) is

\[ \dot{V}(x(t)) = \sum_{i=1}^r h_i(x(t)) h_i(x(t)) x(t) \quad (29) \]

where
Based on the Lyapunov-Razumikhin theorem [29], if the condition (22) holds, the following relationship holds for all time, the stability condition is undoubtedly asymptotically stable. So, it is necessary to check the stability for the case of \( V(x(t-\tau)) < V(x(t)) \) only. Hence, if there exits a real number \( \delta > 1 \) such that \( V(x(t-\theta)) < \delta V(x(t)) \) for \( \theta \in [0, R^n] \), then (34) can be replaced as follows due to (35).

\[
V_{\delta} \leq x^T(t) \left( \frac{G_{\delta} + G_{\delta}^T}{2} + P \left( \frac{G_{\delta} + G_{\delta}^T}{2} \right) \right) x(t) + \delta x^T(t) \left( \frac{a_{ij} + a_{ij}^T}{2} \right) x(t)
\]

and if the condition (22) holds, the following relationship can be obtained.

\[
V(x(t-\tau)) = x^T(t-\tau) P x(t-\tau) \geq x^T(t-\tau) S x(t-\tau)
\]

Based on the Lyapunov-Razumikhin theorem [29], if the inequality \( V(x(t)) < V(x(t-\tau)) \) holds for all time, the
following matrix.

\[ L_0 = 2\delta(t) = x^T(t) \left( \begin{bmatrix} G_i \end{bmatrix} + \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) \right) P + P \left( \begin{bmatrix} G_i \end{bmatrix} + \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) \right) \]

\[ + 2P \left( \begin{bmatrix} D_i D_j \end{bmatrix} + D_i D_j \right) \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) \]

\[ + x^T(t) \left( \begin{bmatrix} a_i + a_j \end{bmatrix} + \left( \begin{bmatrix} a_i + a_j \end{bmatrix} \right)^T \right) P x(t) + \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) Q_{ij} \]

\[ + \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) x(t) \]

From Lemma 1, one can obtain

\[ L_0 \leq x^T(t) \left( \begin{bmatrix} A_i^T \end{bmatrix} P + A_i^T \end{bmatrix} + P A_i + P A_j \right) \]

\[ + 2P \left( \begin{bmatrix} D_i D_j \end{bmatrix} + D_i D_j \right) \left( \begin{bmatrix} A_i^T \end{bmatrix} P + A_i^T \end{bmatrix} + P A_i + P A_j \right) \]

\[ + 2P \left( \begin{bmatrix} D_i D_j \end{bmatrix} + D_i D_j \right) \left( \begin{bmatrix} A_i^T \end{bmatrix} P + A_i^T \end{bmatrix} + P A_i + P A_j \right) \]

It is obvious that if the following inequality is satisfied, then \( L_0 < 0 \) and \( \delta(t) < 0 \) can be obtained.

\[ \Gamma_{ij} - \begin{bmatrix} a_i + a_j \end{bmatrix} \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) X(t) \]

\[ + \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) Q_{ij} x(t) \]

\[ + \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) Q_{ij} \]

\[ \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) Q_{ij} \]

(39)

\[ \begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} D_i & D_j \end{bmatrix} \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} + F_i^T F_j^T + P A_i \]

\[ - P A_j \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) Q_{ij} \]

\[ - \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) Q_{ij} \]

\[ - \sum_{i,j} \left( \sum_{k=1}^{n_i} \Lambda_k \right) Q_{ij} \]

\[ + \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} \]

\[ - \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} \]

\[ + \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} \]

\[ - \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} \begin{bmatrix} \hat{B}_i & B_j \end{bmatrix} \]

(40)

where \( A_{11}, A_{12} \) and \( A_{22} \) are defined in (23), (24) and (25), respectively.

If the condition (21) is satisfied for a scalar \( \alpha < 0 \), then the inequality (43) can be obtained. In this case, one has \( \delta(t) < 0 \) and \( \nu(x(t)) < 0 \). The proof of stability condition (21) is completed for all \( x(t) \in X_i \), \( i \in I \).

Moreover, for the case of \( x(t) \in X_i \), \( i \in \hat{I} \), the stability condition (20) can be obtained by ignoring the S-procedure from the similar proof procedure.

Q.E.D.

Theorem 1 provided the stability conditions for the closed-loop perturbed time-delay affine T-S fuzzy model (18). However, these conditions are of the BMI formulas which cannot be solved by the LMI technique [26]. In order to solve this problem, an ILMI algorithm is proposed to find the feasible solutions for conditions (20)-(22). The purpose of this algorithm is to interactively search for \( P, S, F_i, \xi_{ij}, \alpha \) and to update the auxiliary variables until \( \alpha \) becomes negative. Based on the stability conditions of Theorem 1, the solutions of the robust fuzzy control problem for TCP/AQM routers are solved via the following ILMI algorithm.

**Fuzzy Controller Design Algorithm-ILMI Algorithm**

Step 1:

Solve the initial matrix \( P^{(0)} \) from the following Ricatti equation.

\[ A^T P^{(0)} + P^{(0)} A - P^{(0)} B \hat{B}^T P^{(0)} + Q = 0 \]

where \( \hat{A} = \begin{bmatrix} 1 \end{bmatrix} A_i + \begin{bmatrix} 1 \end{bmatrix} B_i \), \( \hat{B} = \begin{bmatrix} 1 \end{bmatrix} B_i \) and \( Q > 0 \). The matrix \( Q \) is assigned by the designers. Besides, choose the ei-
genvalues of $A - BF(0)$ and solve the initial gains $F(0)$ by standard pole placement technique. Denote $k$ as the iteration index and set $k = 1$ for the initial conditions.

Step 2:
Find the iterative auxiliary variables $E_{ij}^{(k)}$ by the following equation:

$$E_{ij}^{(k)} = B_{i}^{T}P_{ij}^{(k)} + F_{ij}^{(k)}$$  \hspace{1cm} (45)

Using the auxiliary variables $E_{ij}^{(k)}$ to solve the optimization problem for $P_{ij}^{(k)}$, $S_{ij}^{(k)}$, $F_{ij}^{(k)}$, $z_{ij}^{(k)}$ from (20)-(22) subject to minimizing $\alpha_{ij}^{(k)}$. If $\alpha_{ij}^{(k)} < 0$ then $P_{ij}^{(k)}$, $S_{ij}^{(k)}$, $F_{ij}^{(k)}$ and $z_{ij}^{(k)}$ obtained in Step 2 are the feasible solutions and stop the iterative manner. Otherwise, if $\alpha_{ij}^{(k)} \geq 0$ then go to Step 3.

Step 3:
Resolve the optimization problem for $P_{ij}^{(k)}$, $S_{ij}^{(k)}$, $F_{ij}^{(k)}$ and $z_{ij}^{(k)}$ from (20)-(22) subject to minimizing $\text{trace}(P_{ij}^{(k)})$ by using $\alpha_{ij}^{(k)}$ and the corresponding auxiliary variables $E_{ij}^{(k)}$ obtained in Step 2. Given a predetermined small value $v$, if $\sum_{ij} \left\| P_{ij}^{(k)} - (B_{i}^{T}P_{ij}^{(k)} + F_{ij}^{(k)}) \right\| < v$, then the conditions (20)-(22) may not be feasible and stop the iterative manner. Otherwise, one can set $k = k + 1$ and go back to Step 2 to update the auxiliary variables $E_{ij}^{(k)}$ using $P_{ij}^{(k)}$, $F_{ij}^{(k)}$, where $P_{ij}^{(k)}$ and $F_{ij}^{(k)}$ are determined in Step 3.

By applying the above ILMI algorithm, one can find suitable matrices $P_{ij}^{(k)} > 0$, $S_{ij}^{(k)} > 0$, $F_{ij}^{(k)}$, and scalars $z_{ij}^{(k)} \geq 0$ to satisfy stability conditions (20)-(22). The ILMI algorithm provides a useful scheme to find the feasible solutions of robust fuzzy controller for the TCP network with AQM routers which is modeled by the perturbed time-delay affine T-S fuzzy model (18). In the next section, a numerical example is provided to illustrate the applicability and effectiveness of proposed fuzzy controller design procedure that can be efficiently employed to design AQM router for the TCP network systems.

4. Numerical Simulations

In order to verify the feasibility, usefulness and applicability of proposed fuzzy controller design approach, this section looks at an experiment with computer simulations. Let us consider a TCP network made up of a router connected to a single link with some sources and one destination. Referring to the simplified TCP fluid-flow dynamic model (4), one can obtain its time-delay affine T-S fuzzy model described in (8). Based on this time-delay affine T-S fuzzy model, one can find a stable T-S fuzzy controller by using the design procedure developed in Section 3.

Let us consider a TCP network with system parameters: link capacity $C = 3750$ packets/sec., propagation delay $T_p = 0.2$ seconds. In order to obtain the time-delay affine T-S fuzzy model, it is necessary to determine the numbers of IF-THEN fuzzy rules and the coordinate operating points. Here, we use three fuzzy rules to construct the time-delay affine T-S fuzzy model. First, we fix $N = 120$ to choose three queue length values, i.e., $q^* = 575$, $q^* = 175$, $q^* = 0$ packets, to be the operating points. Substituting $q^*$, $q^*$ and $q^*$ into (5), one can obtain the operating points as follows:

$$\left[ \begin{array}{ccc} W & q^* & q^* \\ W & q^* & q^* \\ W & q^* & q^* \end{array} \right], \left[ \begin{array}{c} p^* \\ p^* \end{array} \right]$$


Now, we choose $oper2$ in (46b) as the new equilibrium point for the state transformation. That is, $q^0 = q^* = 175$, $W^0 = W^* = 7.7083$, $p^0 = p^* = 0.0337$ and $R^0 = 0.2467$. Then, the coordinate new operating points for (46) can be obtained from (6) as follows:

$$\left( x^*, x^*, u^* \right)_{oper1} = \left[ \begin{array}{c} [3.3, 400]^T \\ [3.3, 400]^T \\ [-0.01725] \end{array} \right]$$

$$\left( x^*, x^*, u^* \right)_{oper2} = \left[ \begin{array}{c} [0, 0]^T \\ [0, 0]^T \\ [0] \end{array} \right]$$

$$\left( x^*, x^*, u^* \right)_{oper3} = \left[ \begin{array}{c} [-1.458, -175]^T \\ [-1.458, -175]^T \\ [0.01754] \end{array} \right]$$

Note that $\left( x^*, x^*, u^* \right)_{oper2}$ is the new equilibrium point for state variables. Applying the small signal linearization method to linearize nonlinear dynamic system (8) for three operating points described in (47), one can obtain three coordinate linear subsystems. Define the membership functions for state variable $x_i(t)$ as Fig. 2.

Then, the time-delay affine T-S fuzzy model can be obtained by blending the three linear subsystems as follows:

![Figure 2. Membership functions of $x_i(t)$](image-url)
Rule 1: IF $x_i(t)$ is $M_{i1}$, THEN
\[
\dot{x}(t) = (A_i + \Delta A_i)x(t) + A_\omega x(t-\tau(t)) + B_i u(t) + (a_i + \Delta a_i)
\]

Rule 2: IF $x_i(t)$ is $M_{i2}$, THEN
\[
\dot{x}(t) = (A_i + \Delta A_i)x(t) + A_\omega x(t-\tau(t)) + B_i u(t) + (a_i + \Delta a_i)
\]

Rule 3: IF $x_i(t)$ is $M_{i3}$, THEN
\[
\dot{x}(t) = (A_i + \Delta A_i)x(t) + A_\omega x(t-\tau(t)) + B_i u(t) + (a_i + \Delta a_i)
\]

where $A_i = \begin{bmatrix} 0 & 0.5259 \\ -0.5259 & 0.0044 \end{bmatrix}$, $A_\omega = \begin{bmatrix} -0.8 & -0.0067 \\ 0 & 600 \end{bmatrix}$, $\Delta A_i = \begin{bmatrix} 2.83 \times (N(t) - 120) \\ -0.0235 \times (N(t) - 120) \end{bmatrix}$, $\Delta A_\omega = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\Delta a_i = \begin{bmatrix} -0.2563 \\ -0.0021 \end{bmatrix}$, $\Delta a_\omega = \begin{bmatrix} 0.0075 \\ -0.0059 \end{bmatrix}$.

Note that the notation Rule $s_{ij}$ means the correlation between Rule $i$ and Rule $j$ of the plant part bounding region.

According to the membership functions defined in Fig. 2, the $S$-procedure is presented as follows. For Rule 11, i.e., $x_i(t) \geq 5$, the matrices of $S$-procedure are given as follows:
\[
T_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad n_{11} = \begin{bmatrix} 0 \\ -1 \frac{1}{2} \end{bmatrix}, \quad \nu_{11} = 5
\]

For Rule 33, i.e., $x_i(t) \leq -5$, the matrices of $S$-procedure are given as follows:
\[
T_{33} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad n_{33} = \begin{bmatrix} 0 \\ 1 \frac{1}{2} \end{bmatrix}, \quad \nu_{33} = 5
\]

For Rule 12, i.e., $5 \leq x_i(t) \leq 400$, the matrices of $S$-procedure are given as follows:
\[
T_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad n_{12} = \begin{bmatrix} 0 \\ -1 \frac{5}{2} \end{bmatrix}, \quad \nu_{12} = 5 \times 400
\]

For Rule 23, i.e., $-175 \leq x_i(t) \leq -5$, the matrices of $S$-procedure are given as follows:
\[
T_{23} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad n_{23} = \begin{bmatrix} 0 \\ -1 \frac{-175}{2} \end{bmatrix}, \quad \nu_{23} = (-175) \times (-5)
\]

The bounding region of $112$ iterations of the ILMI fuzzy controller design procedure on the membership function is shown in Fig. 2.

For the time-delay affine T-S fuzzy model (48), the T-S fuzzy controller can be designed by applying Theorem 1 and ILMI algorithm developed in Section 3. In Step 1 of ILMI algorithm, the eigenvalues of $A_i - B_i F_i^{(0)}$ are chosen as $(-1 \ 0)$. Then, the initial $F_i^{(0)}$ can be obtained by applying standard pole placement technique as follows:
\[
F_i^{(0)} = \begin{bmatrix} 0.0121 \\ -0.0001 \end{bmatrix}, \quad F_i^{(0)} = \begin{bmatrix} 0.0297 \\ -0.0002 \end{bmatrix}, \quad F_i^{(0)} = \begin{bmatrix} 0.0492 \\ -0.0003 \end{bmatrix}
\]

Let us assign the matrix $Q = I$. Then, the matrix $P^{(0)}$ can be obtained as follows by solving Riccati equation (44).
\[
P^{(0)} = \begin{bmatrix} 0.0219 \\ 0.0075 \end{bmatrix} \begin{bmatrix} 0.0075 \\ 0.0059 \end{bmatrix}
\]

In this example, we can get a feasible solution after 112 iterations of the ILMI fuzzy controller design procedure. The final decay rate $\alpha$ is $-0.3804$ and the feasible solutions are obtained as follows:
Thus, the T-S fuzzy controller has the following form:

**Rule 1:** IF \( x_i(t) \) is \( M_{1i} \) THEN
\[
 u(t) = [-27.9496\ -33.1886] x(t)
\]

**Rule 2:** IF \( x_i(t) \) is \( M_{2i} \) THEN
\[
 u(t) = [-88.7028\ -107.2815] x(t)
\]

**Rule 3:** IF \( x_i(t) \) is \( M_{3i} \) THEN
\[
 u(t) = [-75.6525\ -92.8659] x(t)
\]

In order to demonstrate the effectiveness and applicability of proposed design methodology, the fuzzy controller (56) is employed to control the TCP network system (4) for the simulations. In the simulations, the number of TCP sessions \( N(t) \) is assumed as a random integer function with range \( 80 \leq N(t) \leq 160 \). The simulation results of window size \( W(t) \) and queue length \( q(t) \) are shown in Figs. 3-4. From the simulation results of Figs. 3-4, one can find that the present designed fuzzy controller (56) can drive the TCP network system (4) to successfully achieve the desired equilibrium point \( q^* = 175 \) and \( W^* = 7.083 \). In general, the TCP flows are varying in the practical network; hence, the T-S fuzzy controller design approach proposed in this paper provides a better choice for the designers in superintending the congestion control for the TCP networks.

To meet the crossover condition \( \|L(j\omega_g)\|=1 \), we insist that
\[
 K_{pi} = \frac{\omega_g}{C} \left[ \frac{1}{\omega_g} + \frac{1}{R_c} \right] \left[ \frac{C^2}{2N} \right] \quad (58)
\]

In [20-21], the authors set the unity gain crossover as \( \omega_g = 0.52 \text{ rad/s} \), and set the phase margin to about \( 80^\circ \). Thus, from (58) one can obtain \( K_{pi} = 9.64 \times 10^{-4} \). For obtaining a digital implementation, the z-domain transfer function of the above PI controller is constructed as follows.
\[
 p(z) = \frac{az - b}{z - 1} \quad (59)
\]

where \( \delta q = q - q_{ref} \) with \( q_{ref} \) being the desired queue length to which we want to regulate. The transfer function (57) can be converted into a difference equation of the variables yielding, at time \( t = kT \), where \( T = 1/f_s \),
\[
 p(kT) = a \cdot \delta q(kT) - b \cdot \delta q((k - 1)T) + p((k - 1)T) \quad (60)
\]

In this simulation, we implemented the PI controller with a sampling frequency of 160 Hz. The PI coefficients \( a \) and \( b \) that were implemented as \( 1.822 \times 10^{-5} \) and \( 1.816 \times 10^{-3} \), respectively. The \( q_{ref} \) for the PI controller was chosen to be 175 packets. The responses of the PI controller are shown in Figs. 5-6.
Figure 6. The responses of queue length $q(t)$ with PI controller [20-21].

Referring to Figs. 3-6, one can find that the responses of the proposed robust fuzzy control approach are better than PI control method introduced in [20-21].

5. Conclusions

Explosive growths in multi-media and end-to-end applications in the Internet have resulted in the traffic congestion characterized by packet losses and delays. Recently, considerable research has been undertaken on AQM router. However the number of TCP sessions was assumed to be constant in previous work. In this paper, a perturbed T-S fuzzy model was employed to simulate the behavior of varying numbers of session for nonlinear TCP network systems. According to the perturbed T-S fuzzy model, a robust fuzzy congestion controller design for the TCP/AQM router has been developed in this paper. The simulation results showed that the proposed fuzzy controller has successfully provided robust performance when the number of TCP sessions varies from time to time.

References


