Practical Adaptive Fuzzy $H^\infty$ Tracking Control of Uncertain Nonlinear Systems

Yongping Pan, Meng Joo Er, Daoping Huang, and Tairen Sun

Abstract

This paper presents a practical direct adaptive fuzzy $H^\infty$ tracking control (AFHC) approach for a class of uncertain nonlinear systems with unknown control gain functions and external disturbances. A modified output tracking error is defined to eliminate high gain at the control input and to improve transient performance. An ideal control law is developed to eliminate the restriction that the control gain function must be known a priori under the direct adaptive scheme. A nonlinearly parameterized fuzzy system without redundant parameters is proposed to online approximate the ideal control law. The direct AFHC is derived by virtue of the Lyapunov synthesis, where both the antecedent and the consequent parameters of fuzzy rules are updated by the proposed adaptive law. Simulation studies have demonstrated that the proposed approach can outperform previous approaches with similar or less control efforts.

Keywords: Fuzzy control, adaptive control, $H^\infty$ tracking, nonlinear-in-parameters, uncertain nonlinear system.

1. Introduction

Recent years, adaptive approximation-based control using fuzzy systems and neural networks has been attracting widespread concern due to its effectiveness of dealing with both parametric and nonparametric uncertainties in nonlinear systems [1-37]. Adaptive fuzzy control (AFC) is typically classified into two categories, namely indirect and direct AFCs [1]. Generally speaking, uncertainties in AFC systems caused by fuzzy approximation errors (FAEs) and external disturbances are unavoidable. Adaptive fuzzy $H^\infty$ tracking control (AFHC) [3], which applies an additional robust control term to the basic AFC, is an effective approach for dealing with those uncertainties. The indirect AFHC has been well-developed in recent years [3-11]. Yet, the direct AFHC encounters more challenges than its indirect counterpart since it results in more constraints on the plant control gain functions. The first direct AFHC was proposed by Chen et al. in [3], and its several variations were subsequently developed in [12-15]. Yet, the control gain functions need to be known a priori in all these approaches. Latest approaches of the direct AFHC are to simply use some constants to replace the unknown control gain function so that its influence can be taken out of the control law [16-18]. Yet, the uncertainty of the closed-loop system would be increased in this type of approaches.

For nonlinear control using AFCs, if the explicit linear parameterization of the plant uncertainty is either known or possible, a linearly parameterized fuzzy system (LPFS) can be applied to obtain favorable control performance. Otherwise, a non-LPFS (NLPFS) needs to be applied to improve control performance [19]. With the characteristic of both the antecedent and the consequent parameters of fuzzy rules being adjustable, the NLPFS can not only reduce the number of fuzzy rules [1], but also enhance its capability of capturing fast changing dynamics [20]. By using the Taylor’s series, the FAE equation of the NLPFS was expanded into a partially linear form with a higher order term in [1]. Based on [1], a direct sliding mode AFC of induction and DC servomotors was proposed in [21], and a robust AFC of a dual-axis inverted pendulum was proposed in [22]. Yet, the control gain functions must be constants and monotonically positive adaptive laws are employed to update additional robust control terms in [21] and [22]. In [20], the FAE equation was further transformed into a linearly parameterizable form with a bounded residual term. Two NLPFS-based AFCs with residual term estimates were also presented in [20]. Based on [20], other variations of the NLPFS-based AFC were developed in [23-25]. Yet, the control gain functions must be known a priori in [20] and [23], monotonically positive adaptive laws are applied in [24], and the approaches are specialized for certain plants in
Consider the following affine nonlinear system [3]:
\[
\begin{align*}
\dot{x}(t) &= f(x) + g(x)u + d(t) \\
y &= x
\end{align*}
\]
where \( x = [x_1, x_2, ..., x_n]^T = [x_1, x_2, ..., x^{(p-1)}, x]^T \in \mathbb{R}^n \) is the state variable, \( u \in \mathbb{R} \) and \( y \in \mathbb{R} \) are the control input and system output, respectively, \( f(x) \) is the class \( C^1 \) nonlinear driving function, \( g(x) \) is the class \( C^1 \) control gain function, and \( d(t) \) is the external disturbance. Note that \( f(x), g(x) \) and \( d(t) \) are unknown in this study. Let \( y = [y_1, y_2, ..., y^{(p-1)}, y]^T \in \mathbb{R}^n \) and \( y_d = [y_{d1}, y_{d2}, ..., y_{dn}, y_{d}]^T \in \mathbb{R}^n \), where \( y_d \) denotes a bounded desired output which has all \( n \)th-order bounded derivatives. Define the output tracking error \( e := y_d - y \) and the error vector \( \epsilon := y_d - y = [\epsilon_1, \epsilon_2, ..., \epsilon_n]^T \).

\( e_1, e_2, ..., e^{(p-1)} \). Then, one makes the following reasonable assumptions.

**Assumption 1:** There exist unknown continuous functions \( \tilde{f}(x) \) and \( \tilde{g}(x) \), and unknown finite constants \( g_0 \) and \( \bar{d} \) such that \( f(x) \leq \tilde{f}(x), \quad 0 < g_0 \leq g(x) \leq \tilde{g}(x) \) and \( |d(t)| \leq \bar{d} \), hold, \( \forall x \in \mathbb{R}^n \).

**Assumption 2:** It has \( \partial g(x)/\partial x = 0, \quad \forall x \in \mathcal{D} \), where \( \mathcal{D} \subset \mathbb{R}^n \) is a controllable region.

In AFHC systems, high gain at the control input, which brings about large control efforts and possibly degrades transient behavior [2], is usually unavoidable while favorable \( H^\infty \) tracking performance is achieved [3-15]. To avoid this problem, one introduces the following modified tracking error (MTE) [38]:
\[
E(t) = \epsilon(t) - \eta(t)
\]
where \( \eta \in \mathbb{R} \) is designed to satisfy the following conditions: 1) To make \( \tilde{E} \) in (2) small enough at the onset of the motion \( t = 0 \); 2) should rapidly vanish as the motion evolves at \( t > 0 \). A suggested \( \eta \) is given by the following exponential form:
\[
\eta(t) = (\alpha_0 + \alpha_1 t + \cdots + \alpha_{n-1} t^{n-1}) \exp(-\lambda t)
\]
where \( \alpha_i \in \mathbb{R} \) with \( i = 0, 1, ..., n-1 \) are selected to satisfy Condition 1, and \( \lambda \in \mathbb{R}^n \) is selected to satisfy Condition 2. Expanding (2) by the Taylor’s series leads to
\[
E(t) = \sum_{i=1}^{n-1} ((\eta(t))^{(i)} \eta(t)) / i! + o(t^{n-1})
\]
where \( o(t^{n-1}) \) is an infinitesimal of higher order of \( t^{n-1} \). Thus, if one makes
\[
\eta^{(i)}(0) = \epsilon^{(i)}(0), \quad i = 0, 1, ..., n-1,
\]
then (4) becomes \( o(t^{n-1}) \), i.e., Condition 1 is satisfied. The values of \( \alpha_i \) can be obtained by resolving the equation set in (5). For the selection of \( \lambda \), one can follow the rule given in [38].

Now, for the system in (1) satisfying Assumption 1-2, the control objective is to determine a NLPFS-based AFC such that the closed-loop system achieves: 1) Stability in the sense that all involving variables are UUB; 2) the following \( H^\infty \) tracking performance [3]:
\[
\int_0^T E^T Q E dt \leq 2V_{v_0}(0) + \rho^2 \int_0^T \bar{V}^2 dt, \quad T \in [0, \infty)
\]
where \( E := [E_1, E_2, ..., E_n]^T = [E, \tilde{E}, ..., E^{(p-1)}]^T \), \( Q \) is a matrix satisfying \( Q = Q^T > 0, \quad \rho \in \mathbb{R} \) is a prescribed attenuation level, \( \bar{V} \in L_2[0, T] \) denotes a lumped uncertainty, \( V_{v_0} = V_{v_0} \tilde{G} \), and \( V_L \) is a Lyapunov function. Note that \( Q, V_L \) and \( \bar{V} \) will be defined in the following sections.

**Remark 1:** For the usage of the MTE in (2), there is a difference between the tracking and the regulation problems. For the tracking problem, since the desired output \( y_d \) is continuous, high control input gain usually only
occurs at $t = 0$. Thus, one only needs to set the parameters $\alpha_i$ in (3) at $t = 0$ once and for all. For the regulation problem, since $y_j$ is fast changed with the change of the set point, the current initial value of tracking errors must be obtained before the change of the set point, and $\alpha_i$ should be updated by re-solving (5).

3. Description of Fuzzy Systems

Consider the fuzzy system with $n$ inputs and a single output. Let $U_i = [-X_i, X_i]$ and $V = [-Y, Y]$ be universes of discourse of input variables $x_i$ and output variable $y$, respectively, where $i = 1, \ldots, n$. Let $G_i = \{G_i^j\}_{j=1,\ldots,M_i}$ be fuzzy partitions on $X_i$ and $Y$, respectively. Then, one constructs a fuzzy rule base with $M = n^p$ rules as follows:

$$R_i: \text{If } x_i \text{ is } G_i^j \text{ then } y = \sum_{i=1}^{M_i} \alpha_i(x_i) \cdot G_i^j \quad (7)$$

where $j = 1, \ldots, M_i$, and $\alpha_i$ is a per-

Let $\mathcal{G}$ denote the peak points of the membership functions (MFs) of $G_i^{j}$. For facilitating discussion, choose the MF of each $F_i^j$ as the Guassian function [20]:

$$\mu_{F_i^j}(x) = \exp\left(-\sigma_i^{j} (x - m_i^{j})^2 / 2 \right) \quad (9)$$

where $m_i^{j}$ is the mean, and $\sigma_i^{j}$ is the standard deviation. Using the product $t$-norm, one gets the $l_1 \ldots l_n$th fuzzy basic function as follows:

$$\hat{\mathcal{G}}_{l_1 \ldots l_n}(x, \hat{m}, \hat{\sigma}) = \prod_{i=1}^{n} \mu_{F_i^{l_i}}(x_i, m_i^{l_i}, \sigma_i^{l_i}) \quad (10)$$

where $\hat{m} := [m_1^{l_1}, \ldots, m_n^{l_n}]^T \in R^M$, $\hat{\sigma} := [\sigma_1^{l_1}, \ldots, \sigma_n^{l_n}]^T \in \mathcal{V}^n$, $M = n^p$. As indicated in [20], the defuzzifier can be simply defined as a weighted sum of each $\hat{\mathcal{G}}_{l_1 \ldots l_n}$. By the combination of the singleton fuzzifier, one obtains

$$y = \hat{F}(x) = \sum_{l_1}^{l_1} \cdots \sum_{l_n}^{l_n} \left( \prod_{i=1}^{n} \hat{\mathcal{G}}_{l_1 \ldots l_n}(x_i) \right) \quad (11)$$

Let $\hat{\Theta} := [\hat{\theta}, \hat{\theta}_1, \ldots, \hat{\theta}_M] \in \mathcal{X}^n$ and select $\hat{\theta}$, $\hat{m}$ and $\hat{\sigma}$ as the adjustable parameters. Then, one re-write (11) into a NLPFS form as follows:

$$y = \hat{F}(x | \hat{\theta}, \hat{m}, \hat{\sigma}) = \hat{\theta}^T \hat{\xi}(x, \hat{m}, \hat{\sigma}) \quad (12)$$

Define the compact sets $D = \{x \mid \|x\| \leq M_\alpha \}$, $\Omega_p = \{\dot{\theta} : \|\dot{\theta}\| \leq M_{\dot{\theta}} + \delta\}$, $\Omega_m = \{\hat{m} : \|m\| \leq M_m + \delta\}$, and $\Omega_\sigma = \{\hat{\sigma} : 0 < M_{\sigma} - \delta \leq \sigma_i^{j} \leq M_{\sigma} + \delta\}$, where $j = 1, \ldots, M_i$, $l_i = 1, \ldots, p$, $i = 1, \ldots, n$, $M_m, M_{\sigma}, M_{\theta}, M_{\dot{\theta}} \in \mathcal{R}^n$ are user-defined finite constants, and $\delta \in \mathcal{R}$ is a user-defined small constant for applying the smooth projection algorithm in [12]. Define the optimal FAE $w$ as follows:

$$w := F(x) - \hat{F}(x | \theta^*, m^*, \sigma^*) \quad (13)$$

where $\theta^*$, $m^*$ and $\sigma^*$ are $\mathcal{F}$ optimal adjustable parameters given by

$$(\theta^*, m^*, \sigma^*) := \arg \min_{\theta, m, \sigma} \sup_{x \in D} |w(x, \theta, m, \sigma)| \quad (14)$$

Accordingly, one has the following lemma.

Lemma 1 [20]: For any given continue real function $F(x)$ and arbitrarily small constant $\mathcal{E}_f \in \mathcal{R}$, there exist $\theta^*$, $m^*$ and $\sigma^*$ in (14) such that $\sup_{x \in D} |w| < \mathcal{E}_f$.

To facilitate derivations, define two new vectors:

$$\hat{\Theta} := [\hat{\theta}; \hat{\theta}] \quad (15)$$

$$\mathcal{X} := \left[ (\xi - \xi_a^m \hat{m} - \xi_a^\theta; \xi_a^\theta \hat{\theta}) ] \quad (16)$$

where $\hat{\Theta} = [\hat{\theta}, \hat{m}, \hat{\sigma}]$, $\hat{\Theta} = [\theta^*, m^*, \sigma^*]$, $\hat{\Theta} = \hat{\theta} - \theta^*$, $\hat{m} = \hat{m} - m^*$, $\hat{\sigma} = \hat{\sigma} - \sigma^*$, and $\xi_a^m \in \mathcal{R}^{M \times \mathcal{N}}$ and $\xi_a^\theta \in \mathcal{R}^{M \times \mathcal{N}}$ are partial derivative matrices given by

$$\xi_a^m = \left[ \left( \xi_a^m \xi_a^\theta \right) \xi_a^m \right], \xi_a^\theta = \left[ \left( \xi_a^m \xi_a^\theta \right) \xi_a^\theta \right] \quad (17)$$

$$\xi_a^\theta = \hat{\xi}_a^\theta, \xi_a^m = \hat{\xi}_a^m \quad (18)$$

Substituting (9) into (10) leads to

$$\hat{\xi}_a^m = \sum_{i=1}^{n} \exp\left(-\sigma_i^{j} (x - m_i^{j})^2 / 2 \right) \quad (19)$$

where $l_i = 1, \ldots, p$, $i = 1, \ldots, n$ and $j = 1, \ldots, M$. The calculations of $\xi_a^m$ and $\xi_a^\theta$ are given in Appendix. Finally, one gives the following lemma.

Lemma 2 [20]: For any given continue real function $F(x)$ and the NLPFS in (12), one has the following approximation equation:

$$\hat{F}(x | \theta, \hat{m}, \hat{\sigma}) - F(x) = \hat{\Theta}^T \mathcal{X} + d_{\omega} \quad (20)$$

in which $d_{\omega}$ is a bounded residual term given by

$$d_{\omega} = \hat{\omega}^T \left( \xi_a^m \mathcal{N} - \theta^* \right)^T \mathcal{N} \mathcal{N} - \hat{\xi}_a^\theta \mathcal{N} - w \quad (21)$$

where $\mathcal{N}(\cdot)$ is the sum of higher order arguments in the Taylor’s series expansions.

Remark 2: The adjustable parameters of all previous NLPFSs have redundancies if they do not work at self-organizing schemes [37], which would inevitably increase computational cost. From the definitions of $\hat{m}$ and $\hat{\sigma}$ under (10), one observes that the redundant parameters in the NLPFS of (12) are completely removed. Thus, comparing with matrices $\xi_a^m$, $\xi_a^\theta \in \mathcal{R}^{M \times (\mathcal{M} + \mathcal{N})}$ in [20] and $\xi_a^m$, $\xi_a^\theta \in \mathcal{R}^{M \times \mathcal{N}}$ in [1], one has $\xi_a^m$, $\xi_a^\theta \in \mathcal{R}^{M \times \mathcal{N}}$ in this study. Since $M = n^p$ and $N = n \cdot p$, both $(M \cdot n)$ and $M$ are usually much bigger than $N$. Thus, the sizes of $\xi_a^m$ and $\xi_a^\theta$ in this study are significantly reduced compared with the previous NLPFSs.
4. Practical Control Design

A. Lyapunov-Based Ideal Control Law

Consider the case that \( f(x) \) and \( g(x) \) are known and \( d = 0 \) in (1). Choose \( k = [k_0, \ldots, k_n]^T \in \mathbb{R}^n \) so that \( h(s) := s^n + k_1 s^{n-1} + \cdots + k_n \) is a Hurwitz polynomial, where \( s \) is a complex variable. From (1) and (2), one obtains
\[
\hat{E}_u = y^{(n)}_y - \eta^{(n)} - f(x) - g(x)u. 
\]
Design an ideal control law in the following form:
\[
u^* = \left( -f(x) + y^{(n)}_y - \eta^{(n)} + k^T E \right) / g(x) 
\quad - \hat{g}(x) E^T P E / (2E^T P bg^2(x)).
\]
Substituting (23) into (22) and making some transformations, one obtains
\[
\dot{E} = AE + b\left( \hat{g}(x) E^T P E / (2E^T P bg^2(x)) \right) \quad \text{(24)}
\]
where \( A = \begin{bmatrix} 0 & 1 & \vdots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \ldots & 1 \\ -k_n & -k_{n-1} & \ldots & -k_1 \end{bmatrix} \) and \( b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \).

From the selection of \( k \), there exists a unique positive definite symmetric matrix \( P \) for any given positive definite symmetric matrix \( Q \) such that
\[
A^T P + PA = -Q. \quad \text{(25)}
\]
Choose the Lyapunov function candidate for the system in (24) as follows:
\[
V_u = E^T P E / 2g(x). \quad \text{(26)}
\]
Differentiating (26) along (24) and using (25) yields
\[
\dot{V}_u = (E^T P E + E^T \dot{P} E) / 2g - \hat{g}E^T P E / 2g^2
\quad - E^T Q E / 2g - gE^T P E / 2g^2
\quad + \left( b \hat{g}E^T P E / (2E^T P bg^2) \right)^T E^T / g
\quad = - E^T Q E / 2g - gE^T P E / 2g^2
\quad + E^T P b \left( \hat{g}E^T P E / (2E^T P bg) \right)^T E^T / g
\quad = - E^T Q E / 2g.
\]
which implies that \( E \) asymptotically converges to zero. Combining with (2) and (3), one gets \( \lim_{\|x(t)\| \to 0} \|u(t)\| = 0 \).

B. Control Law Derivation

Since \( f(x) \) and \( g(x) \) are unknown and \( d \neq 0 \), some terms in \( u^* \) of (23) cannot be determined. Assumption 2 implies that \( \hat{g}(x) \) only depends on \( x \). Thus, one can employ the following NLPFS in the form of (12):
\[
\dot{u}(x | \hat{\theta}, \hat{\dot{m}}, \hat{\dot{\sigma}}) = \hat{\theta}^T \zeta(x, \hat{\dot{m}}, \hat{\dot{\sigma}}) \quad \text{(27)}
\]
to approximate \( u^* \). The definition of the optimal FAE is similar with (13). Then, design the control law:
\[
u = u(x | \hat{\theta}, \hat{\dot{m}}, \hat{\dot{\sigma}}) + u_b \quad \text{(28)}
\]
where \( u_b \) is the \( H^\infty \) control term given by
\[
u_b = E^T P b / 2g^2. \quad \text{(29)}
\]

Figure 1. The proposed overall control scheme.

Substituting (28) into (22), one obtains the following tracking error dynamics:
\[
\dot{E}_u = y^{(n)}_y - \eta^{(n)} - f(x) - g(x) \left( \hat{u}(x | \hat{\theta}, \hat{\dot{m}}, \hat{\dot{\sigma}}) + u_b \right) - d. \quad \text{(30)}
\]
Adding and subtracting \( g(x)u^* \) on the right side of (30) and noting (23), one gets
\[
\dot{E}_u = -k^T E + g \left( u^* - \hat{u}(x | \hat{\theta}, \hat{\dot{m}}, \hat{\dot{\sigma}}) - u_b \right)
\quad + \left( \hat{g}E^T P E / (2E^T P bg) \right) - d. \quad \text{(31)}
\]
Applying Lemma 2 into (31) and making some transformations, one obtains
\[
\dot{E} = AE - bg(\hat{\theta}^T \Xi + w_L + u_b) + b \hat{g}E^T P E / (2E^T P bg) \quad \text{(32)}
\]
where \( w_L \) is a lump uncertainty given by
\[
w_L = d_{\alpha} + d / g(x). \quad \text{(33)}
\]
Noting Assumption 1 and \( d_{\alpha} \in L_{\alpha} \) shown in Lemma 2, one gets \( w_L \in L_{\alpha} \), i.e., there exists a finite constant \( \bar{w}_L \in R^+ \) such that \( \bar{w}_L = \sup_{x \in \mathbb{R}^n} |w_L| \).

Choose the Lyapunov function candidate for the system in (32) as follows:
\[
V_L = E^T P E / 2g(x) + \hat{\theta} \Gamma^{-1} \hat{\theta} / 2 \quad \text{(34)}
\]
in which \( \Gamma = \text{diag}(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5) = \text{diag}(\gamma_1, \ldots, \gamma_5) \in R^{m \times m} \),
\( \Gamma_1 = \text{diag}(\gamma_2, \ldots, \gamma_6) \in R^{n \times n} \),
\( \Gamma_2 = \text{diag}(\gamma_2, \ldots, \gamma_6) \in R^{n \times n} \),
and \( \gamma_1, \gamma_2, \gamma_3, \gamma_4 \in R^+ \) are learning rates. Now, one gives the following conclusion.

Theorem 1: For the system in (1), select (28) with the following parameter adaptive law:
\[
\hat{\theta} = \Gamma E^T P b \Xi(x, \hat{\dot{m}}, \hat{\dot{\sigma}}). \quad \text{(35)}
\]
as the control law. The overall control scheme is shown in Figure 1. Then, the closed-loop system not only guarantees stability in the sense that all involving variables are UUB, but also achieves the \( H^\infty \) tracking performance in (6), where \( w := (\hat{g})^{1/2} w_L \).

Proof: Differentiating (34) along (32) and using (25), one obtains
\[
\dot{V}_L = (E^T P E + E^T \dot{P} E) / 2g - \hat{g}E^T P E / 2g^2 + \hat{\theta} \Gamma^{-1} \hat{\theta}
\quad = - E^T Q E / 2g - \hat{g}E^T P E / 2g^2 + \hat{\theta} \Gamma^{-1} \hat{\theta}. \quad \text{(36)}
\]
where $x_1$ is the angular position of the pendulum, $x_2$ is the angular velocity, $g_v$ is the gravitational acceleration, $m_c$ is the mass of the cart, $m_p$ is the mass of the pendulum, and $l_p$ is the half-length of the pendulum. For simulation, select $m_c = 1$ kg, $m_p = 0.5 + 0.4\sin(t)$, $g_v = 9.8$ m/s², $l_p = 0.5$m, $d(t) = 6\cos(2t) + 4\sin((0.09t + 1) + t)$ at $t = 5$s, $x(0) = [\pi/12, 0]$, and $y_{cl} = 0.4\sin(0.3t) + 0.6\cos(0.8t)$.

The procedure of control parameter design is as follows: firstly, set $X_1 = X_2 = \pi/2$, and $\theta(0) = [0, \ldots, 0]^T$, and select the initial MFs of $F_i$ as follows:

$$
\mu_{F_i}(x) = \exp\left(-0.35(x_i - 0.5\pi(l_i - 3))^2/2\right)
$$

where $l_i = 1, \ldots, 5$ and $i = 1, 2$; secondly, let $k_1 = 2, k_2 = 1$, $Q = \text{diag}(10, 10)$ and $\rho = 0.1$ so that $P = [15, 5; 5, 5]$; finally, choose $\lambda = 7$ in (3), $M_\phi = 100$, $M_{\omega} = \pi/2$, $M_{\sigma_1} = 0.2$, $M_{\sigma_2} = 10$ and $\delta = 0.1$ in Section 3, and $\gamma_1 = 50$, $\gamma_2 = 10$ and $\gamma_3 = 10$ in (35).

Select the tracking indexes $J(\text{IAE})$ and $J(\text{IAE})$ and the control energy $E_c$ defined in [17] to evaluate control performance. To demonstrate the effectiveness of the proposed approach (denoted as Controller 4), choose the LPFS-based direct AFHC in [17] (denoted as Controller 1), the NLIFS-based direct AFC in [22] (denoted as Controller 2), and the proposed approach without MTE (denoted as Controller 3) as compared controllers. To make fair comparisons, all same parameters in these controllers are set to be the same values.

The tracking trajectories, control input comparison and transient process comparison in Example 1 are depicted in Figure 2, 3 and 4, respectively. One observes that the proposed approach with MTE makes both the angular position $x_1$ and the angular velocity $x_2$ successfully track their corresponding desired trajectories [see Figure 2 (a), (b)], and obtains the fastest tracking speed with the smallest tracking errors [see Figure 2 (c), (d)]; the MTE not only speedups transient process [see Figure 3], but also eliminates both high gain and chattering at the control input [see Figure 4 (d)]. One also sees that Controller 1 and Controller 3 cannot avoid high gain at the control input [see Figure 4 (a), (c)], and Controller 2 cannot avoid chattering at the control input [see Figure 4 (b)]. The performance comparisons of all controllers in Example 1 of Table 1 demonstrate that the proposed approach with MTE outperforms all compared controllers with similar or less control efforts.
Figure 2. Tracking trajectories in Example 1.

(a) Position tracking

(b) Velocity tracking

(c) Tracking error $e_1$

(d) Tracking error $e_2$

Figure 3. Transient process comparison in Example 1.

(a) Controller 1

(b) Controller 2

(c) Controller 3

(d) Controller 4

Figure 4. Control input Comparison in Example 1.
B. Example 2: Aircraft Wing Rock Suppression

Consider an aircraft wing rock model in the form of (1) with $n = 2$ and [39]:

$$\begin{align*}
f(x) &= \omega^2 x_1 + \mu_1 x_1^3 + b_1 x_2 + b_2 x_1 x_2^2 \\
g(x) &= 1 + 0.2 \sin(x_1)
\end{align*}$$  \hspace{1cm} (38)

where $x_1$ is the aircraft roll angle, $x_2$ is the roll rate, $\omega^2 = c_1 a_1$, $\mu_1 = c_2 a_2 - c_2$, $b_1 = c_3 a_3$, $b_2 = c_4 a_4$, $c_1$, $c_2$, $c_3$, $c_4$ are certain constants, and $a_i \in R$ with $i = 1, \ldots, 5$ are time-varying parameters to be relative to free-to-roll experiment conditions. The wing rock phenomenon is a limit cycle roll oscillation which may lead to serious danger of the aircraft. Note that $g(x)$ in (38) is set to be slightly different from that in [39] so that it is more suitable for verifying the proposed approach. The parameters $a_i$ with $i = 1, \ldots, 5$ varying with the high angle of attack $a_0$ at 80° swept back wing are given in Table 2. The time-varying wing rock model can be constructed by applying a cubic interpolation function to the data in Table 2. For simulation, let $c_1 = 0.354$, $c_2 = 0.001$, $x(0) = [\pi/3, 0]$, and $d(t) = 3 \sin(2t) + 5 \cos(5t)$ at $t = 10s$. The control objective is to suppress the output $y$ to be zero. The control parameters and compared controllers are selected to be the same as those in Example 1.

The tracking trajectories in Example 2 are shown in Figure 5. One observes that the proposed approach with MTE achieves favorable regulation performance [see Figure 5 (a), (b)] with the best transient process [see Figure 5 (c), (d)]. For the sake of saving space, the figures of other trajectories are omitted since they are essentially similar with those in Example 1. The preference comparisons of various controllers in Example 2 of Table 1 demonstrate that the proposed approach with MTE also outperforms all compared controllers with similar or less control efforts.

C. Example 3: Robotic Manipulator Tracking

Consider a single-link robotic manipulator model in the form of (1) with $n = 2$ and [17]:

$$\begin{align*}
f(x) &= -(d_1 x_1 + m g_1 l_1 \cos(x_1))/J \\
g(x) &= 1/J
\end{align*}$$  \hspace{1cm} (39)

where $x_1$ is the angular position of the manipulator, $x_2$ is the angular velocity, $m_1$ is the mass of the payload, $l_1$ is the length of the manipulator, $J = 1.33 m l_1^2$ is the inertia coefficient, and $d_1$ is the damping factor. For simulation, choose $m_1 = 5 + 4 \sin(t)$, $l_1 = 0.25$ m, $d_1 = 2$ kg m$^2$/s, $c_1 = 1.5$, $v_c = 0.3$, $x(0) = [\pi/12, 0]$, $d(t) = -\text{sgn}(x_1(t)) c_1 + v_c x_1(t)/J$, and $y_c$ to be a filtered output of $y_c$ under the filter $G(s) = 25/(s^2 + 10s + 25)$, where $y_c$ is a square signal that belongs to $[-\pi/6, \pi/6]$ with the period equaling to 10s. The control parameters and compared controllers are selected to be the same as in Example 1.

The tracking trajectories and performance comparisons of various controllers in Example 3 are shown in Figure 6 and Example 3 of Table 1, respectively. The qualitative analysis of these results is very similar with those in Examples 1 and 2. A major difference is that Controller 2 performs worst at this example.
6. Conclusions

This paper has successfully developed a practical direct AFHC for a class of uncertain affine nonlinear systems with unknown control gain functions and external disturbances. A modified output tracking error is introduced to eliminate high control input gain and to improve transient performance. An ideal control law is developed to eliminate the restriction that the control gain function must be known a priori under the adaptive direct scheme. A nonlinearly parameterized fuzzy system without redundant parameters is presented to online approximate the ideal control law. It is proved that the closed-loop system achieves $J^*$ tracking performance in the sense that all involving variables are UUB. Simulation applications have demonstrated that under parameter variations and external disturbances, the proposed approach can outperform previous approaches with similar or less control efforts. Further studies would focus on the output feedback design of the proposed approach.

<table>
<thead>
<tr>
<th>Cases</th>
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<th>Control-</th>
<th>Control-</th>
<th>Control-</th>
<th>Control-</th>
</tr>
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<tbody>
<tr>
<td>Example 1</td>
<td>$J$(ITAE)</td>
<td>6.7872</td>
<td>1.9334</td>
<td>1.6601</td>
<td><strong>1.0973</strong></td>
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<tr>
<td></td>
<td>$J$(IAE)</td>
<td>0.8243</td>
<td>0.5603</td>
<td>0.5478</td>
<td><strong>0.4295</strong></td>
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<tr>
<td></td>
<td>$E_c$</td>
<td>2774.0</td>
<td>2757.0</td>
<td><strong>2754.0</strong></td>
<td>2793.0</td>
</tr>
<tr>
<td>Example 2</td>
<td>$J$(ITAE)</td>
<td>3.5122</td>
<td>1.8391</td>
<td>3.1291</td>
<td><strong>1.3752</strong></td>
</tr>
<tr>
<td></td>
<td>$J$(IAE)</td>
<td>1.6183</td>
<td>1.5284</td>
<td>1.5935</td>
<td><strong>1.1806</strong></td>
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<tr>
<td></td>
<td>$E_c$</td>
<td>288.70</td>
<td><strong>212.50</strong></td>
<td>289.70</td>
<td>239.50</td>
</tr>
<tr>
<td>Example 3</td>
<td>$J$(ITAE)</td>
<td>1.1640</td>
<td>3.2070</td>
<td>0.8066</td>
<td><strong>0.4264</strong></td>
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<tr>
<td></td>
<td>$J$(IAE)</td>
<td>0.4251</td>
<td>0.5913</td>
<td>0.4043</td>
<td><strong>0.3167</strong></td>
</tr>
<tr>
<td></td>
<td>$E_c$</td>
<td>2746.0</td>
<td>2788.0</td>
<td><strong>2743.0</strong></td>
<td>2777.0</td>
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</table>

Table 2. Variation of wing rock parameters.

<table>
<thead>
<tr>
<th>$a_1$ (°)</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
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<tr>
<td>15.0</td>
<td>-0.0103</td>
<td>-0.0212</td>
<td>-0.1418</td>
<td>0.9974</td>
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<tr>
<td>21.5</td>
<td>-0.0421</td>
<td>-0.0146</td>
<td>0.0471</td>
<td>-0.1858</td>
</tr>
<tr>
<td>22.5</td>
<td>-0.0468</td>
<td>0.0195</td>
<td>0.0567</td>
<td>-0.2269</td>
</tr>
<tr>
<td>25.0</td>
<td>-0.0569</td>
<td>0.0325</td>
<td>0.0733</td>
<td>-0.3597</td>
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</table>

Appendix

Calculating Partial Derivative Matrices

From (19) one can directly obtain the expression of $\tilde{\xi}(x, m', \sigma')$. Let $\tilde{\xi}' := \tilde{\xi}(x, m', \sigma')$, $\theta' := [\theta_1', \ldots, \theta_m']$ and $\sigma' := [\sigma_1', \ldots, \sigma_m']$. Then, taking derivative of $\tilde{\xi}(x, m', \sigma')$ with respect to $m'$ at $(\tilde{m}, \tilde{\sigma})$, one gets

$$
\tilde{\xi}' = \left[ \frac{\partial \xi}{\partial m_1} \frac{\partial \xi}{\partial m_2} \ldots \frac{\partial \xi}{\partial m_m} \right]_{m' \to \tilde{m}} =
\begin{bmatrix}
\frac{\partial \xi_1}{\partial m_1} & \frac{\partial \xi_2}{\partial m_1} & \ldots & \frac{\partial \xi_1}{\partial m_m} \\
\frac{\partial \xi_2}{\partial m_1} & \frac{\partial \xi_2}{\partial m_2} & \ldots & \frac{\partial \xi_2}{\partial m_m} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \xi_1}{\partial m_1} & \frac{\partial \xi_2}{\partial m_1} & \ldots & \frac{\partial \xi_m}{\partial m_m}
\end{bmatrix}
$$
\[ \frac{\partial \xi_j}{\partial m^i} = \frac{\partial \xi_j^*}{\partial m^i} \bigg|_{m^i = \hat{m}^i} = \left\{ \begin{array}{cl} \xi^* (\sigma^* y (x - m^i)) & \text{if } l_j \text{ is in (9)} \\ 0 & \text{otherwise} \end{array} \right. \]

Similarly, taking derivative of \( \xi(x, m^i, \sigma^*) \) with respect to \( \sigma^* \) at \((\hat{m}, \hat{\sigma})\), one gets

\[ \xi^* = \frac{\partial \xi(x, m^i, \sigma^*)}{\partial \sigma^* i} \bigg|_{\sigma^* = \hat{\sigma}} = \left[ \begin{array}{c} \frac{\partial \xi_1}{\partial \sigma_1} \\ \frac{\partial \xi_2}{\partial \sigma_2} \\ \vdots \\ \frac{\partial \xi_n}{\partial \sigma_n} \end{array} \right] \]

where

\[ \frac{\partial \xi_j}{\partial \sigma^* i} = \bigg| \frac{\partial \xi^*_j}{\partial \sigma^* i} \bigg|_{\sigma^* = \hat{\sigma}} = \left\{ \begin{array}{cl} -\xi^* \sigma^* y (x - m^i) & \text{if } l_j \text{ is in (9)} \\ 0 & \text{otherwise} \end{array} \right. \]

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### References


1994.


Yongping Pan received the B.Eng. degree in automation and M.Eng. degree in control theory and control engineering from the Guangdong University of Technology (GDUT), Guangzhou, China, in 2004 and 2007, respectively, and the Ph.D. degree in control theory and control engineering from the South China University of Technology (SCUT), Guangzhou, in 2011. From 2007 to 2008, he was a Control Software Engineer in Santak Electronic (Shenzhen) Co., Ltd., Eaton Co., and an R&D Engineer in Light Engineering (China) Co., Ltd., respectively. He is currently a Research Fellow of the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. He has authored about 30 referred papers, including more than 10 in international journals. He also serves as the Reviewer for several international journals. His research interests include adaptive approximation-based control, fuzzy logic and neural networks, machine condition prediction, and embedded control system.
Meng Joo Er received the B.Eng. and M.Eng. degrees in electrical engineering from the National University of Singapore, Singapore, in 1985 and 1988, respectively, and the Ph.D. degree in systems engineering from the Australian National University, Canberra, Australia, in 1992. From 1987 to 1989, he was an R&D Engineer with Chartered Electronics Industries Pte Ltd. and a Software Engineer with Telerate R&D Pte Ltd., respectively. He served as the Director of the Intelligent Systems Centre, Nanyang Technological University (NTU), from 2003 to 2006, and the Director of the Renaissance Engineering Program, NTU, from 2009 to 2012. He is currently a Full Professor with the School of Electrical and Electronic Engineering, NTU. He has authored five books, 16 book chapters, and more than 400 referred journal and conference papers. His research interests include control theory and applications, fuzzy logic and neural networks, cognitive systems, computational intelligence, robotics and automation, sensor networks, and biomedical engineering. Currently, Dr. Er is a Senior Member of IEEE, and a Fellow of Institution of Engineers, Singapore (IES). He was the Editor-in-Chief for the IES Journal B: Intelligent Devices and Systems from 2007 to 2012, and was invited as an Associate Editor for more than 15 international journals. Prof. Er was the winner of the IES Prestigious Publication (Application) Award in 1996, the IES Prestigious Publication (Theory) Award in 2001, and the IES Prestigious Engineering Achievement Award in 2011. He also received the Best Session Presentation Award at the World Congress on Computational Intelligence in 2006, and the Thomson Top 1% Citation (in its field) Award in 2007. Furthermore, together with his students, he has won more than 40 awards at international and local competitions.

Daoping Huang received the B.Eng. degree in chemical automation and instruments and the M.Eng. and Ph.D. degrees in automatic control theory and applications, all from the South China University of Technology (SCUT), Guangzhou, China, in 1982, 1986, and 1998, respectively. From 1995 to 1996, he was with the University of Gent, Belgium, as a government sponsored visiting scholar. In 1982, he started his teaching and research career with the Department of Automation, SCUT. Now, as a Full Professor and the Vice Dean of the School of Automation Science and Engineering, he has authored three academic books and more than 170 journal and conference papers. His research interests cover intelligent detection and control, soft-sensing technology, as well as fault diagnosis and accident prediction of industrial process. Dr. Huang serves as the Vice Director of the Education Committee and a member of the Process Control and Application Committees, Chinese Association of Automation. He has directed over 10 research projects. He was granted Third Prize from the National Education Committee in 1992 and Second Prize from the Guangdong Provincial Government in 2005 for his contributions to science and technology.

Tairen Sun received the M.S. degree in operations research and cybernetics from the Sun Yat-sen University, Guangzhou, China, and the Ph.D. degree in control theory and control engineering from the South China University of Technology, Guangzhou, in 2008 and 2011, respectively. Now he is a Lecturer in the School of Electrical and Information Engineering, Jiangsu University, Zhenjiang, China. His research interests include intelligent control, robot control, etc.