Adaptive Fuzzy Total Sliding-Mode Control of Unknown Nonlinear Systems

Chih-Min Lin, Chun-Fei Hsu, and Te-Yu Chen

Abstract

This paper proposes the adaptive fuzzy total sliding-mode control systems with integral (I-AFTSMC) and proportional-integral (PI-AFTSMC) learning algorithms for the unknown nonlinear systems. These AFTSMC systems are comprised of a fuzzy total sliding-mode controller and a robust controller. The fuzzy total sliding-mode controller is utilized to approximate an ideal controller and the robust controller is designed to cover the approximation error between the fuzzy total sliding-mode controller and the ideal controller. In these designs the fuzzy rules are on-line tuned by the derived learning algorithm in the sense of Lyapunov function, so that the stability of the system can be guaranteed. The proposed AFTSMC systems are applied to the fault accommodation control of a Van der Pol oscillator and trajectory tracking control of a linear piezoelectric ceramic motor. The simulation result of Van der Pol oscillator and the experimental result of linear piezoelectric ceramic motor demonstrate that the effectiveness of the proposed AFTSMC systems for achieving favorable tracking performance. Comparing to the integral learning algorithm, the proportional-integral learning algorithm can achieve faster convergence of the tracking error; this comparison is also illustrated by the simulations and experiments.

Keywords: Adaptive control, fuzzy system, total sliding-mode control, adaptive learning algorithm.

1. Introduction

In the control system design, if the exact model of the uncertain system is well known, there exists an ideal controller to achieve favorable control performance by possibly canceling all the system uncertainties [1]. However, since the parameters of control system and the external disturbances may be perturbed or unknown, the ideal controller is always unobtainable. To overcome this problem, a sliding-mode controller has been presented to confront these uncertainties if all the uncertainties including the parameter uncertainties and external disturbances are bounded [2, 3]. Moreover, by defining a time varying total sliding surface, a total sliding-mode control has been proposed for PM synchronous servo motor [4]. However, in order to satisfy the existence condition, sliding-mode control suffers from large control chattering that may excite the unmodeled high frequency response of the systems due to the discontinuous switching and imperfect implementations. The chattering phenomena in control efforts will wear the bearing mechanism and excite unmodelled dynamics. In general, there is a trade-off between chattering and robustness for the sliding-mode control.

Fuzzy control using linguistic information can model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analyses. It possesses several advantages such as robustness, model-free, universal approximation theorem and rule-based algorithm [5, 6]. However, the huge amount of fuzzy rules for a high-order system makes the analysis complex. To overcome this problem, a fuzzy sliding-mode control (FSMC) system has been presented to reduce the fuzzy rules by defining a sliding surface as the input variable of the fuzzy rules [7-9]. However, the fuzzy control and FSMC systems, though they are the most effective methods using expert knowledge without knowing the parameters and structure of the control systems, both have a major drawback in the lack of systematic design technique for the fuzzy rules. To tackle this problem, some researchers have focused on the use of the Lyapunov synthesis approach to construct an adaptive fuzzy control (AFC) system [10, 11]. With this approach, the fuzzy rules can be automatically adjusted to achieve satisfactory system response. In recent years, some researchers have proposed AFC designs based on the sliding-mode control scheme. This type of controller is referred to as the adaptive fuzzy sliding-mode control (AFSMC) [12, 13]. AFSMC has the advantages that it
can adjust the fuzzy rules like AFC and reduce the fuzzy rules like FSMC. However, the learning algorithms proposed in [10-13] are an integral type, which only describes the parameter change rate of the fuzzy rules.

By combining the adaptive fuzzy control with total sliding-mode control, this study presents two kinds of adaptive fuzzy total sliding-mode control (AFTSMC) systems for the unknown nonlinear systems. One is called as integral-type AFTSMC (I-AFTSMC), in which the controller parameters are tuned by the integral learning algorithms [10-13]. The other is called as propositional-integral-type AFTSMC (PI-AFTSMC), in which some controller parameters are tuned by the proportional learning algorithms and some are tuned by the integral learning algorithms [14]. With PI-AFTSMC, the convergence of the tracking error will be faster than that of I-AFTSMC. In addition, an error estimation mechanism is investigated to estimate the bound of approximation error so that the chattering phenomenon of the control effort can be reduced. Finally, the proposed I-AFTSMC and PI-AFTSMC systems are applied to control a Van der Pol oscillator and trajectory tracking control of an LPCM to illustrate the effectiveness of the proposed AFTSMC systems.

The organization of this paper is as follows. In Section 2, the control problem is presented. Section 3 describes the fuzzy total sliding-mode control. The AFTSMC design techniques with integral and proportional-integral learning algorithms are proposed in Section 4 and 5, respectively. The performance comparisons of a Van der Pol oscillator and an LPCM are made in Section 6. Finally, the conclusions are drawn in Section 7.

2. Problem Statement and Ideal Controller

Consider a class of nth-order nonlinear dynamic system expressed in the following form

$$x^{(n)}(t) = f(x,t) + g(x,t)u(t) + d(t)$$

where $f(x,t)$ and $g(x,t)$ are continuous nonlinear system dynamics, $u(t) \in \mathbb{R}$ is the control effort, $d(t)$ is the external disturbance, and $x = [x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)]^T \in \mathbb{R}^n$ is a state vector of the system which is assumed to be available. For (1) to be controllable, it is required that $g(x,t) \neq 0$ for all $x$.

The control objective is to find a control law so that the output state $x(t)$ can track a trajectory command $x_c(t)$. The tracking error is defined as

$$e(t) = x_c(t) - x(t).$$

If the controlled system dynamics and the external disturbance are exactly known (i.e., the functions $f(x,t)$, $g(x,t)$ and $d(t)$ are known), there exits an ideal controller

$$u^*(t) = \frac{1}{g(x,t)}[-f(x,t) - d(t) + x^{(n)}(t) + k'e]$$

where $e = [e(t), \dot{e}(t), \cdots, e^{(n-1)}(t)]^T \in \mathbb{R}^n$ and $k = [k_1, k_2, \cdots, k_n]^T \in \mathbb{R}^n$, in which $k_1, \cdots, k_n$ are positive constants. Applying the ideal control law (3) to system (1), results in the following error dynamics

$$e^{(n)} + k_{e}e^{(n-1)} + \cdots + k_be + k_ee = 0.$$ (4)

If $k_j$ are chosen such that all roots of the polynomial $h(s) = s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$ lie strictly in the open left half of the complex plane, then it implies that $\lim_{t \to \infty} e(t) = 0$ for any starting initial conditions. The error dynamics (4) can be rewritten in a vector form as

$$\dot{e} = A_n e$$

where $A_n = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -k_n & -k_{n-1} & \cdots & -k_1 \end{bmatrix} \in \mathbb{R}^{n \times n}$. Since the functions $f(x,t)$, $g(x,t)$ and $d(t)$ may be unknown or perturbed in practical applications, the ideal controller (3) can not be precisely obtained.

3. Design of Fuzzy Total Sliding-Mode Controller

The block diagram of the Fuzzy total SMC (FTSMC) system is shown in Fig. 1, where a time varying sliding surface is defined as [4]

$$s(t) = C(e) - C(e_0) - \int_0^t \dot{C}(e) A_n e \mathrm{d}\tau$$

in which $C(e)$ is a vector to be designed, and $e_0$ is the initial state of $e$. By defining the sliding surface as the input variable of fuzzy rules, the number of fuzzy rules for fuzzy SMC is smaller than that for fuzzy control which usually uses error and change-of-error as the input variables of fuzzy rules. This also can reduce the implementation complexity for real applications. Assume that there are $m$ rules in the fuzzy rule base of FTSMC, each of which has the following form

Rule $i$: IF $s$ is $\Theta_i$, THEN $u$ is $\alpha_i$, $i = 1, 2, \cdots, m$

where $\alpha_i$ is the singleton control action and $\Theta_i$ is the Gaussian membership function, which is represented by

$$\Theta_i = \exp(-\omega_i^2(s - c_i)^2)$$

where $c_i$ and $\omega_i$ are the center and inverse radius of Gaussian membership function, respectively. The fuzzy inference system performs the mapping according to
where $|\cdot|$ denotes the Euclidean norm. Define the parameter vectors $\alpha = [\alpha_1, \ldots, \alpha_m]^T$, $\Theta = [\Theta_1, \ldots, \Theta_n]^T$, $c = [c_1, \ldots, c_m]^T$, and $\omega = [\omega_1, \ldots, \omega_n]^T$, then (9) can be rewritten in a vector form as

$$u = \alpha^T \Theta(s, \omega, c).$$

(10)

In the FSMC design, the knowledge of human experts is needed to decide both of the Gaussian membership functions and fuzzy rules. The design of FSMC lacks for the systematic design procedure; thus, the adaptive FSMC (AFTSMC) will be proposed in the following sections.

Figure 1. FTSMC for unknown nonlinear system.

4. Design of AFTSMC with Integral Learning Algorithm

To achieve a favorable tracking performance in FSMC design, both the Gaussian membership function and fuzzy rules base should be constructed by the time-consuming trial-and-error tuning procedures. To solve this drawback, an I-AFTSMC system is proposed first, which is comprised of a fuzzy total sliding-mode controller and a robust controller. The fuzzy total sliding-mode controller is utilized to approximate the ideal controller and the robust controller is designed to guarantee the system stability. By the universal approximation theorem, there exists an optimal fuzzy total sliding-mode controller $u^*_f$ such that [10]

$$u^* = u^*_{f} + \Delta = \alpha^T \Theta(s, \omega^*, c^*) + \Delta$$

(11)

where $\Delta$ denotes the approximation error and $\omega^*$, $c^*$ and $\alpha^*$ are the optimal parameter vectors of $\omega$, $c$ and $\alpha$, respectively. In fact, the optimal parameter vectors that are needed to best approximate the idea controller is unobtainable. Thus, an estimated fuzzy total sliding-mode controller is defined as

$$\hat{u}^*_f = \hat{\alpha}^T \hat{\Theta}(s, \hat{\omega}, \hat{c})$$

(12)

where $\hat{\alpha}$, $\hat{\omega}$ and $\hat{c}$ are the estimated parameter vectors of $\alpha^*$, $\omega^*$ and $c^*$, respectively. For notational convenience, denote $\hat{\Theta} = \Theta(s, \omega^*, c^*)$ and

$$\hat{\Theta} = \Theta(s, \hat{\omega}, \hat{c}).$$

Then the estimated error of fuzzy total sliding-mode controller can be obtained as

$$\tilde{u} = u^* - \hat{u}^*_{f} = -\hat{\alpha}^T \hat{\Theta} + \hat{\alpha}^T \hat{\Theta} + \hat{\alpha}^T \hat{\Theta} + \Delta$$

(13)

where $\hat{\alpha} = \alpha^* - \hat{\alpha}$ and $\hat{\Theta} = \Theta^* - \hat{\Theta}$. In the following, some learning algorithms will be derived to on-line tune the parameters of the estimated fuzzy total sliding-mode controller. To achieve this goal, the Taylor expansion linearization technique is employed to transform the nonlinear function into a partially linear form [10], i.e.

$$\hat{\Theta} = \begin{bmatrix} \hat{\Theta}_1 \\ \hat{\Theta}_2 \\ \vdots \\ \hat{\Theta}_m \end{bmatrix} = \begin{bmatrix} \frac{\partial \Theta}{\partial \omega} \\ \frac{\partial \Theta}{\partial c} \end{bmatrix} \bigg|_{\omega^*, c^*} + \begin{bmatrix} \hat{\Theta}_1 \\ \hat{\Theta}_2 \\ \vdots \\ \hat{\Theta}_m \end{bmatrix}_{c, \omega} \bigg|_{c^*} \tilde{c} + \hat{h}$$

(14)

or

$$\hat{\Theta} = A^T \hat{\omega} + B^T \hat{c} + \hat{h}$$

(15)

where $\hat{\omega} = \omega^* - \hat{\omega}$, $\hat{c} = c^* - \hat{c}$, $\hat{h}$ is a vector of higher-order terms,

$$A = \frac{\partial \Theta}{\partial \omega}, B = \frac{\partial \Theta}{\partial c}, \hat{h} = \frac{\partial \Theta}{\partial \omega}$$

are defined as

$$\frac{\partial \Theta}{\partial \omega} = \begin{bmatrix} \frac{\partial \Theta}{\partial \omega_1} \\ \frac{\partial \Theta}{\partial \omega_2} \\ \vdots \\ \frac{\partial \Theta}{\partial \omega_m} \end{bmatrix}_1$$

(16)

$$\frac{\partial \Theta}{\partial c} = \begin{bmatrix} \frac{\partial \Theta}{\partial c_1} \\ \frac{\partial \Theta}{\partial c_2} \\ \vdots \\ \frac{\partial \Theta}{\partial c_m} \end{bmatrix}_1$$

(17)

Substituting (15) into (13), gives

$$\tilde{u} = \hat{\alpha}^T (A^T \hat{\omega} + B^T \hat{c} + \hat{h}) + \hat{\alpha}^T (A^T \hat{\omega} + B^T \hat{c} + \hat{h}) + \hat{\alpha}^T \hat{\Theta} + \Delta$$

$$= \hat{\alpha}^T (A^T \hat{\omega} + \hat{c}) + \hat{\alpha}^T B^T \hat{c} + \hat{\alpha}^T \hat{h} + \hat{\alpha}^T A^T \hat{\omega}$$

$$+ \hat{\alpha}^T B^T \hat{c} + \hat{\alpha}^T \hat{h} + \hat{\alpha}^T \hat{\Theta} + \Delta$$

$$= \hat{\alpha}^T \Theta - A^T \hat{\omega} - B^T \hat{c} + \omega^T \bar{A} \hat{\omega} + \hat{c}^T \hat{B} \hat{c} + \epsilon_i$$

(18)

where $\omega^T \bar{A} \hat{\omega} = \hat{\alpha}^T A^T \hat{\omega}$ and $\hat{c}^T \hat{B} \hat{c} = \hat{\alpha}^T B^T \hat{c}$ are used since they are scales, and the sum of matching error $\epsilon_i = \hat{\alpha}^T A^T \hat{\omega} + \hat{\alpha}^T B^T \hat{c} + \hat{\alpha}^T \hat{h} + \Delta$ is assumed to be bounded by $|\epsilon_i| \leq E$ where $E$ is a positive constant. However, the approximation error bound $E$ is difficult to determine in practical applications, so that an estimation law of this approximation error bound will be derived in the following.

The block diagram of the I-AFTSMC system is shown in Fig. 2; the control system is comprised of a fuzzy total sliding-mode controller and a robust controller

$$u(t) = \hat{u}^* + u^*_{r}$$

(19)

Substituting (19) into (1) and using (3), the error dynamics becomes
\[ \dot{e} = A_m e + b_m g(x,t)(u' - \hat{u}_\mu - u_\mu) \]  
where \( b_m = [0, \ldots, 0, 1]^T \in \mathbb{R}^n \). Differentiating (6) with respect to time and using the error dynamics (20), gives
\[ \dot{s}(t) = \frac{\partial C(e)}{\partial e} \dot{e} + \frac{\partial C(e)}{\partial e} A_m e \]
where \( C(e) \) satisfies \( \frac{\partial C(e)}{\partial e} = [0, \ldots, 0, 1] \). Using (18), equation (21) can be rewritten as
\[ \dot{s}(t) = g(x,t)(\alpha^T (\Theta - A^T \hat{\omega} - B^T \hat{c}) + \hat{\omega}^T A \hat{u} + \hat{c}^T B \hat{u} + \epsilon_i - u_\mu) \]
Then, the following theorem can be stated and proved. 

**Theorem 1:** Consider a nonlinear dynamic system expressed by (1). The I-AFTSMC system is designed as (19), in which the fuzzy rules in (7) are on-line tuned by the integral learning algorithms
\[ \hat{a} = \eta_i s(t) g(x,t)(\Theta - A^T \hat{\omega} - B^T \hat{c}) \]
\[ \hat{\omega} = \eta_i s(t) g(x,t) A \hat{u} \]
\[ \hat{c} = \eta_i s(t) g(x,t) B \hat{u} \]
and the robust controller is designed as
\[ u_\mu = \hat{E}(t) \text{sgn}(s(t)) \text{sgn}(g(x,t)) \]
in which \( \text{sgn}() \) is a sign function and \( \hat{E}(t) \) is the estimation of the approximation error bound with the estimation algorithm given as
\[ \hat{E} = \eta_i |s(t)| |g(x,t)| \]
where \( \eta_i \) and \( \eta_\mu \) are positive learning-rates. Then, the stability of the I-AFTSMC system can be guaranteed.

**Proof:** Define a Lyapunov function in the following form
\[ V_i(s(t), \hat{a}, \hat{\omega}, \hat{c}, \hat{E}(t)) = \frac{\eta_i}{2} s(t)^2 + \frac{1}{2} a^T \hat{a} + \frac{1}{2} \omega^T \hat{\omega} + \frac{1}{2} \eta_i \hat{E}(t)^2 \]
where \( \hat{E}(t) = E - \hat{E}(t) \). Differentiating (28) with respect to time and using (22), gives
\[ V_i = \eta_i s(t) \dot{s}(t) + a^T \dot{\hat{a}} + \omega^T \dot{\hat{\omega}} + \hat{c}^T \dot{\hat{c}} + \frac{\eta_i}{2} \hat{E}(t) \dot{\hat{E}}(t) \]
\[ \dot{s}(t) = g(x,t)(\alpha^T (\Theta - A^T \hat{\omega} - B^T \hat{c}) + \hat{\omega}^T A \hat{u} + \hat{c}^T B \hat{u} + \epsilon_i - u_\mu) \]
\[ + \eta_i s(t) g(x,t)(\alpha^T (\Theta - A^T \hat{\omega} - B^T \hat{c}) + \hat{\omega}^T A \hat{u} + \hat{c}^T B \hat{u} + \epsilon_i - u_\mu) \]
\[ + \eta_i s(t) g(x,t)(\alpha^T (\Theta - A^T \hat{\omega} - B^T \hat{c}) + \hat{\omega}^T A \hat{u} + \hat{c}^T B \hat{u} + \epsilon_i - u_\mu) \]
Therefore, the learning algorithms for I-AFTSMC shown in (23)-(25) and (27) can be reconstructed as
\[ \dot{a} = r_i s(t) (\Theta - A^T \hat{\omega} - B^T \hat{c}) \text{sgn}(g(x,t)) \]
\[ \dot{\omega} = r_i s(t) A \hat{u} \text{sgn}(g(x,t)) \]
\[ \dot{c} = r_i s(t) B \hat{u} \text{sgn}(g(x,t)) \]
\[ \dot{\hat{E}} = \eta_i \dot{s}(t) \]
in which \( r_i = \eta_i |g(x,t)| \) and \( r_i = \eta_i |g(x,t)| \); they are taken as new learning-rates. In the following, the design algorithm of I-AFTSMC system is summarized as follows:

**Step 1:** The sliding surface \( s(t) \) is given as in (6).

**Step 2:** The fuzzy controller \( \hat{u}_\mu \) is given as
\[ \hat{a}^T \Theta(s, \hat{\Theta}, \dot{\hat{\Theta}}), \] where \( \hat{a}, \hat{\Theta} \) and \( \dot{\hat{\Theta}} \) are estimated by (33), (34) and (35), respectively.

Step 3: The robust controller \( u_{rob} \) is given as
\[ \dot{E}(t) \text{sgn}(s(t)) \text{sgn}(g(x, t)), \] where \( \dot{E}(t) \) is estimated by (36).

Step 4: The control law is given as \( u(t) = \hat{u}_p + u_{rob} \). Then, go back Step 1.

\[ \begin{align*}
\text{Trajectory Command} & \quad x_c \quad e \\
\text{Sliding Surface} (6) & \quad \hat{u}_p + \hat{u}_r \\
\text{Unknown Nonlinear System} (1) & \quad x \quad x
\end{align*} \]

Figure 2. 1-AFTSMC for unknown nonlinear system.

\[ \begin{align*}
\text{Trajectory Command} & \quad x_c \quad e \\
\text{Sliding Surface} (6) & \quad \hat{u}_p + \hat{u}_r \\
\text{Unknown Nonlinear System} (1) & \quad x \quad x
\end{align*} \]

Figure 3. PI-AFTSMC for unknown nonlinear system.

5. Design of AFTSMC with PI Learning Algorithm

To speed up the convergence of the controller parameters and tracking error, a PI-AFTSMC system is developed and is shown in Fig. 3, in which the control system is comprised of a proportional fuzzy controller, an integral fuzzy controller and a robust controller, i.e.
\[ u(t) = \hat{u}_p + \hat{u}_r + u_{rob} = \hat{a}_p^T \Theta + \hat{a}_r^T \hat{\Theta} + u_{rob} \]  
where \( \hat{a}_p \) and \( \hat{a}_r \) are the proportional term and integral term of estimated parameter vector \( \hat{a} \), respectively. The optimal parameter vector \( \alpha^* \) is divided into two parts as [14]
\[ \alpha^* = \alpha_p^* + \alpha_r^* \]  
where \( \alpha_p^* \) and \( \alpha_r^* \) are the proportional term and integral term of the optimal parameter vector \( \alpha^* \), respectively. By the universal approximation theorem, (11) can be rewritten as [14]
\[ u^* = (\alpha_p^* + \alpha_r^*) \Theta^* + \Delta \]  
Substituting (37) into (1) and using (3), the error dynamics can be obtained as
\[ \hat{e} = A_{a} e + b_{m} g(x, t)(u^* - \hat{u}_p - \hat{u}_r - u_{rob}) \]  
Differentiating (6) with respect to time and using the error dynamics (40), yields
\[ \dot{\hat{s}}(t) = g(x, t)(u^* - \hat{u}_p - \hat{u}_r - u_{rob}) \]  
From (37) and (39), it is obtained that
\[ \dot{\hat{s}}(t) = g(x, t)(\alpha_p^* \Theta^* + \alpha_r^* \Theta^* - \hat{a}_p^T \hat{\Theta} - \hat{a}_r^T \hat{\Theta} + \Delta - u_{rob}) \]  
where \( \hat{a}_p = \alpha_p^* - \hat{a}_r \). Using the Taylor expansion linearization technique in (18) and substituting (15) into (42), gives
\[ \dot{\hat{s}}(t) = g(x, t)(\alpha_p^* \Theta^* + \alpha_r^* \Theta^* + \Delta - u_{rob}) \]  
where \( \alpha^0 A\hat{a}_r = \alpha_r^* A^\tau \hat{\Theta} \) and \( c^\tau \hat{B}_a r = \hat{a}_r^* B^\tau \hat{\Theta} \) are used since they are scales, and the sum of matching error \( \epsilon_r = \alpha_r^* \Theta^* + \alpha_r^* B^\tau \hat{\Theta} + \alpha_r^* \Theta^* + \Delta \) is assumed to be bounded by \( |\epsilon_r| \leq E \) where \( E \) is a positive constant.

Then, the following theorem can be stated and proved.

Theorem 2: Consider a nonlinear dynamic system expressed by (1). The PI-AFTSMC system is designed as (37), in which the fuzzy rules in (7) are on-line tuned by the learning algorithms (23)-(25) with \( \hat{a} \) replaced by \( \hat{a}_r \) and an additional proportional learning algorithm
\[ \hat{a}_p = \eta_p \dot{s}(t) g(x, t) \hat{\Theta} \]  
where \( \eta_p \) is a positive learning-rate; and the robust controller is designed as (26) with the approximation error bound estimated by (27). Then, the stability of the PI-AFTSMC system can be guaranteed.

Proof:
Define a Lyapunov function in the following form
Differentiating (45) with respect to time and using (43), gives
\[
\dot{V}_2 = \eta_1 s(t) \dot{s}(t) + \frac{1}{2} \dot{a}_r \dot{a}_r + \frac{1}{2} \dot{a}_t \dot{a}_t + \frac{1}{2} \dot{c} \dot{c} + \frac{\eta_1}{\eta_2} \tilde{E}(t) \tilde{E}(t)
\]
From the simulation results, the system fault occurs at 12.6 \text{sec}. (47)

6. Simulations and Experiments

In this section, the proposed I-AFTSMC and PI-AFTSMC systems are applied to control two nonlinear systems: a Van der Pol oscillator and an LPCM. It should be emphasized that the developments of the proposed AFTSMC methods do not need to know the system dynamics of the control system, they only need to know the sign of the control gain. For comparison, the FTSMC introduced in Section 3 is also applied for these systems.

Example 1: Consider a Van der Pol oscillator with a system fault in the following [15, 16]: 
\[
\ddot{x}(t) = -2\sigma \zeta (\mu x^2(t) - 1) \dot{x}(t) - \sigma^2 \dot{x}(t) + u(t) + \beta(t - T) \eta(x(t))
\]
where \( \sigma = 0.9 \), \( \zeta = 0.6 \), \( \mu = 0.95 \), \( \beta(t - T) \eta(x(t)) \) is an abrupt fault occurring at an unknown time \( T \), and \( u(t) \) and \( x(t) \) are the system input and output, respectively. It can also be regarded as an RLC electrical circuit with a nonlinear resistor. In the simulations, the system fault \( \eta(x(t)) = 25 \sin(x(t)) \cos(2x(t)) \) occurs at \( T = 12 \text{ sec} \). The parameters in the proposed control systems are selected as \( k_1 = 2 \), \( k_2 = 1 \), \( \eta_1 = 2 \), \( \eta_2 = 0.01 \) and \( \eta_3 = 0.5 \). The simulation results of FTSMC are shown in Fig. 4, where the fuzzy rules are summarized in Table 1, in which the fuzzy labels are negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM) and positive big (PB). The simulation results show that the favorable tracking responses can be achieved before the occurrence of the fault; however, the fuzzy rules should be assigned prior through trial-and-error procedure. Moreover, as the system failure occurs, the tracking performance is deteriorated. To improve this drawback, the I-AFTSMC system is implemented. The simulation results of I-AFTSMC are shown in Fig. 5. After the Gaussian membership function and fuzzy rules having been learned, the favorable tracking responses can be obtained even in the presence of system failure; however, the convergence speed is not satisfactory for this I-AFTSMC system. Finally, the PI-AFTSMC system is implemented. The simulation results of PI-AFTSMC are shown in Fig. 6. From the simulation results, it can be seen that not only the favorable tracking responses can be obtained but also the convergence of the tracking error can be speeded up.
by the developed PI learning algorithm for this fault accommodation problem.

Table 1. Fuzzy rules of FTSMC for the Van der Pol oscillator.

<table>
<thead>
<tr>
<th>s</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>ZO</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_i)</td>
<td>-10</td>
<td>-8</td>
<td>-5</td>
<td>0</td>
<td>5</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Example 2: The driving principles of LPCMs are based on the ultrasonic vibration force of piezoelectric ceramic elements and mechanical frictional force [17]. Therefore, their mathematical models are complex and the motor parameters are time varying due to increasing in temperature and changing in motor drive operating condition [18, 19]. The experiment setup for the LPCM position control is shown in Fig. 7. This LPCM is a HR4 motor manufactured by Nanomotion with 15W 270Vrms 320mArms 14N. A servo control card is installed in the control PC, which includes multi-channels of D/A, A/D, PIO and encoder interface circuits. The position of the moving table is fed back using a linear scale. The dynamic motion equation of LPCM can be described by the Newton’s Law as

\[
[M + m(t)]\ddot{x}(t) = F(x, t) + G(x, t)u(t) \tag{54}
\]

where \(M\) is the mass of the moving table, \(m(t)\) is the mass of the payload, \(x = [x(t), \dot{x}(t)]^T\) represents the position and velocity of the moving table, \(F(x, t)\) is a nonlinear dynamic function including friction, ripple force and external disturbance, \(G(x, t)\) is the control gain of the LPCM resonant inverter, and \(u(t)\) is the input force to LPCM. Rewriting (54), the system dynamics of LPCM can be obtained as

\[
\ddot{x}(t) = \frac{F(x, t)}{M + m(t)} + \frac{G(x, t)}{M + m(t)}u(t)
\]

\[= f(x, t) + g(x, t)u(t) \tag{55}\]

where \(f(x, t) = \frac{F(x, t)}{M + m(t)}\) and \(g(x, t) = \frac{G(x, t)}{M + m(t)}\).

The LPCM drive system is assumed to be controllable and the sign of \(g(x, t)\) is known, i.e., \(g(x, t) > 0\). The parameters in the proposed control systems are selected as \(k_1 = 2, k_2 = 1, \eta_1 = 0.5, \eta_2 = 0.01\) and \(\eta_3 = 0.05\). Two test conditions are provided in the experiments, which are the nominal condition and the payload condition. The payload condition is the addition of one iron disk with 4.3 kg weight to the mass of the moving table which is with 0.9 kg weight. The control objective is to control the moving table to follows a \(m_0 = 7.0\) periodic step command. The experimental results of FTSMC are shown in Fig. 8 with the fuzzy rules summarized in Table 2. The tracking responses are shown in Figs. 8(a) and 8(c); and the associated control efforts are shown in Figs. 8(b) and 8(d) for nominal condition and payload condition, respectively. From the experimental results it is seen that FTSMC cannot achieve satisfactory tracking performance in the payload condition. The experimental results of I-AFTSMC are shown in Fig. 9. After the fuzzy rules having been learned, the favorable tracking responses can be obtained, even in the presence of payload. However, the convergence speed is not satisfactory.
Finally, the experimental results of PI-AFTSMC are shown in Fig. 10. From the experimental result it is seen that not only the favorable tracking responses can be obtained but also the convergence of the tracking error can be speeded up.

Table 2. Fuzzy rules of FTSMC for the LPCM.

<table>
<thead>
<tr>
<th>$s\alpha_i$</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>ZO</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>-4</td>
<td>-3.8</td>
<td>-1.8</td>
<td>0</td>
<td>1.8</td>
<td>3.8</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 7. Computer-controlled LPCM system.

Table 2. Fuzzy rules of FTSMC for the LPCM.

Figure 8. Experimental results of FTSMC for the LPCM.

Figure 9. Experimental results of I-AFTSMC for the LPCM.

Figure 10. Experimental results of PI-AFTSMC for the LPCM.
7. Conclusions

This paper has successfully developed an I-AFTSMC system and a PI-AFTSMC system for the unknown nonlinear systems. Simulation and experimental results demonstrate that the proposed PI-AFTSMC system achieves favorable tracking performance with fast convergence of the tracking error without knowing the dynamic function of the controlled system. The major contributions of this paper are (1) the successful derivation of the integral and proportional-integral learning algorithms for the AFTSMC systems and the stability analysis of these systems; (2) the successful implementation of the I-AFTSMC and PI-AFTSMC systems without knowing the controlled system; and (3) the successful applications for the accuracy control of a Van der Pol oscillator and a linear piezoelectric ceramic motor.

Acknowledgments

The authors appreciate the partial financial support from the National Science Council of Republic of China under the grant NSC 100-2628-E-032-003.

References

Chih-Min Lin received the B. S. and M. S. degrees in control engineering and a PhD degree in electronics engineering from National Chiao Tung University, Taiwan, Republic of China in 1981, 1983 and 1986, respectively. He joined the faculty of the Department of Electrical Engineering, Yuan Ze University, Taiwan, in 1993 and is currently the Dean and a Chair Professor of the College of Electrical and Communication Engineering. He is an IEEE Fellow and IET Fellow and serves as the Board of Governors of IEEE Systems, Man, and Cybernetics Society. His research interests include fuzzy neural systems, cerebellar model articulation controller, intelligent control, and systems engineering.

Chun-Fei Hsu received the B. S., M. S. and PhD degrees in electrical engineering from Yuan Ze University, Taiwan, Republic of China in 1997, 1999 and 2002, respectively. After graduation, he joined the Department of Electrical and Control Engineering, National Chiao Tung University, Taiwan, Republic of China. He joined the faculty of the Department of Electrical Engineering, Chung Hua University, Taiwan, in 2007. He joined the faculty of the Department of Electrical Engineering, Tamkang University, Taiwan, in 2011 and is currently an Associate Professor of Electrical Engineering. He received the Young Automatic Control Engineering Award in 2007 from the Chinese Automatic Control Society, R.O.C. His research interests include servomotor drives, adaptive control, flight control and intelligent control using fuzzy system and neural network technologies.

Te-Yu Chen was born in Taiwan, in 1978. He received the B.S. and M.S. degrees in Electrical Engineering from Yuan-Ze University, Taoyuan, Taiwan, in 2000 and 2002, respectively. During 2002–2004, he was with the Hon Hai Precision Industry Company, Ltd., as an Engineer. He received the PhD degree in Electrical Engineering from Yuan Ze University, Taiwan, Republic of China in 2008. His research interests include adaptive control, flight control and intelligent control using fuzzy system and neural network technologies, and signal processing and radar system.