Adaptive Fuzzy Backstepping Dynamic Surface Control of Uncertain Nonlinear Systems Based on Filter Observer

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Abstract

In this paper, a novel adaptive fuzzy backstepping output feedback control scheme is proposed for a class of single-input single-output (SISO) uncertain nonlinear systems without measurements of states. Fuzzy logic systems (FLS) are used to tackle unknown nonlinear functions, and the adaptive fuzzy output feedback controller is constructed by combining fuzzy filters observer design and the dynamic surface control (DSC) technique. It is proved that the proposed adaptive fuzzy control approach can guarantee that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB) and the tracking error converges to a small neighborhood of origin. Two key advantages of our scheme are that (i) the proposed control method does not require that all the states of the system be measured directly, and (ii) the problem of “explosion of complexity” is avoided. Finally, two simulations examples are included to illustrate the effectiveness of the proposed approach.

Keywords: Uncertain nonlinear systems, adaptive fuzzy control, dynamic surface control (DSC), backstepping technique, stability analysis.

1. Introduction

Fuzzy logic systems have been widely used to model nonlinearities. A fuzzy logic system is a universal approximator which, with the increased size of fuzzy rules, can approximate any nonlinearities with arbitrary precision [1]. Based on this capability, fuzzy logic systems are vastly adopted for nonlinear systems identification and control [2-5]. Most of them use fuzzy logic systems as nonlinear models for the underlying nonlinearity. The stability issues for adaptive fuzzy controllers are addressed by Lyapunov functions. However, these adaptive fuzzy controllers are only applied to a relatively simple class of nonlinear systems. The key requirement is that the unknown nonlinearities appear on the same equation as the control input in a state space representation. Such restrictions on the location of the uncertain nonlinear functions are usually referred to as matching conditions. If the unknown nonlinearities do not satisfy the matching conditions, the adaptive fuzzy controllers mentioned above cannot be implemented.

To handle the control problem of uncertain nonlinear systems without satisfying matching conditions, in recent years, many backstepping-based adaptive fuzzy controllers have been developed for uncertain nonlinear systems in strict-feedback form [6-13]. There is a limitation in these literatures. The limitation is the so-called “explosion of complexity,” which is inherent in the conventional backstepping technique [6-13]. This problem is caused by the repeated differentiations of virtual controllers and inevitably leads to a complicated algorithm with heavy computation burden. Especially, the complexity of controller grows drastically as the order of the system increases. Fortunately, Swaroop et al. [14] and Yip and Hedrick [15] proposed a dynamic-surface control (DSC) technique to eliminate this problem by introducing a first-order low-pass filter of the synthetic input at each step of the conventional backstepping-design procedure for a class of parametric strict-feedback SISO systems. Wang and Huang [16] extended the DSC approach to neural networks adaptive tracking control for a class of strict-feedback SISO uncertain nonlinear systems. However, the existing fuzzy adaptive backstepping control methods are all based on the assumption that the states of the systems are directly measured. As we know, in many control problems, state variables may be partially unavailable, in such cases, observer-based output feedback control schemes should be applied. It should be mentioned that unlike in the case of linear systems, separation principle does not hold for nonlinear systems. Perhaps for this reason, the problem of the output feedback control for nonlinear systems is very challenging and much more difficult than that of state feedback control. To handle the problem of states unmeasured, more recently, several papers have investigated semi-global stabilization of the uncertain nonlinear systems without satisfying the matching conditions and obtained some
interesting results [17-22]. Adaptive fuzzy adaptive output feedback backstepping control approaches are developed by [17-22] for a class of SISO or large-scale uncertain nonlinear systems, respectively. However, these literatures do not solve the problem of “explosion of complexity”.

Motivated by the above observations, in this paper, an adaptive fuzzy output feedback control approach, which is performed by incorporating the DSC technique, is developed for a class of SISO uncertain nonlinear strict-feedback systems. Two main advantages of the scheme can be summarized as follows: 1) The proposed control method does not require that all the states of the system be measured directly and 2) the problem of “explosion of complexity” is avoided, which leads to a much simpler controller with less computational burden. In particular, the number of parameters updated online is reduced.

2. Problem Formulation and Some Preliminaries

A. System Description

Some practical system can be defined by the following differential equations, for example, single-link flexible robot and electromechanical system

\[\dot{x}_i = x_{i+1} + f_i(x_i) + d_i(x_n), \quad i = 2, \ldots, n-1\]

\[\dot{x}_n = u + f_n(x_n) + d_n(x_n)\]

where \(x_i = [x_1, \ldots, x_n]^T \in R^i\) \((i = 1, 2, \ldots, n)\), \(u \in R\) and \(y \in R\) are the state, control input and output of system, respectively. \(f_i(x_i)\) \((i = 1, 2, \ldots, n)\) is an unknown smooth nonlinear function. \(d_i(x_n)\) \((i = 1, 2, \ldots, n)\) is a bounded perturbation. In this paper, it is assumed that only output \(y\) is available for measurement, and \(x_2(t), \ldots, x_n(t)\) are not accessible for measurement.

Assumption 1: There exists a set of known constants \(d_{i0}, i = 1, 2, \ldots, n\), such that \(|d_i(x_n)| \leq d_{i0}\).

For the given reference signal \(y_r(t)\) is a sufficiently smooth function of \(t\), \(y_r, \dot{y}_r, \ddot{y}_r\) are bounded, that is, there exists a known positive constant \(B_0\), such that \(|y_r, \dot{y}_r, \ddot{y}_r| \leq B_0\).

B. Fuzzy Logic Systems

A fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

\[R^i: \text{If } x_i \text{ is } F_{i}^{l} \text{ and } x_2 \text{ is } F_{2}^{l} \text{ and} \ldots \text{ and } x_n \text{ is } F_{n}^{l}, \text{ then } y \text{ is } G^{l}, l = 1, 2, \ldots, N\]

where \(x = [x_1, \ldots, x_n]^T\) and \(y\) are the fuzzy logic system input and output, respectively. Fuzzy sets \(F_{i}^{l}\) and \(G^{l}\), are associated with the fuzzy functions \(\mu_{F_{i}}(x_i)\) and \(\mu_{G^{l}}(y)\), respectively. \(N\) is the number of rules.

Through singleton function, center average defuzzification and product inference, the fuzzy logic system can be expressed as

\[y(x) = \frac{\sum_{i=1}^{N} \bar{y}_i \prod_{l=1}^{n} \mu_{F_{i}^{l}}(x_i)}{\sum_{i=1}^{N} \prod_{l=1}^{n} \mu_{F_{i}^{l}}(x_i)}\]

Define the fuzzy basis functions as

\[\varphi_{i} = \frac{\prod_{l=1}^{n} \mu_{F_{i}^{l}}(x_i)}{\sum_{i=1}^{N} \prod_{l=1}^{n} \mu_{F_{i}^{l}}(x_i)}\]

De-note \(\bar{y}_i = \max_{y \in R} \mu_{G^{l}}(y)\).

Lemma 1[1]: Let \(f(x)\) be a continuous function defined on a compact set \(\Omega\). Then for any given constant \(\delta > 0\), there exists a FLS (4) such as

\[\sup_{x \in \Omega} |f(x) - \theta^T \varphi(x)| \leq \delta\]

C. Fuzzy K-Filters Observer Design

By Lemma 1, the FLSs are universal approximators, i.e., they can approximate any smooth function on a compact space; therefore, we can assume that the nonlinear terms in (1) can be approximated as follows:

\[f_i(x_i) = \theta_i^T \varphi_i(x_i)\]

\[f_n(x_n) = \theta_n^T \varphi_n(x_n)\]

where \(\hat{x}_i = \hat{x}_i, \hat{x}_2, \ldots, \hat{x}_n\) and \(\hat{x} = \hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n\) are the estimate of state vectors \(x_i\) and \(x = [x_1, x_2, \ldots, x_n]^T\), respectively.
respectively.

According to Refs. [1, 18], define the optimal parameter vectors \( \hat{\theta}^* \) as

\[
\theta^* = \arg \min_{\theta \in \Theta_i} \sup_{\xi \in \Xi_1, \hat{\xi} \in \hat{\Xi}_2} \left[ f_i(\hat{\xi}, \theta_1) - f_i(x) \right]
\]

(6)

where \( \Omega_i, U_{i1} \) and \( U_{i2} \) are compact regions for \( \theta, \xi \) and \( \hat{\xi} \), respectively. In addition, the fuzzy approximation error \( \delta_i \) is defined as

\[
f_i(x) = f_i(\hat{\xi}, \theta^*_i) + \delta_i
\]

(7)

where \( |\delta_i| \leq \delta_{i0} \), and \( \delta_{i0} \) is a known bounded constant \((i = 1, 2, \ldots, n)\).

Using (5), (1) is equivalent to the following form:

\[
\dot{x} = Ax + \Psi^T \theta + \delta + d + e_u u
\]

(8)

\[
y = [1, 0, \ldots, 0]^T, \quad e_u = [0, \ldots, 0, 1]^T.
\]

Rewrite (8) as

\[
\dot{x} = A_0 x + ky + \Psi^T \theta + \delta + d + e_u u
\]

(9)

where \( k = [k_1, \ldots, k_n]^T \) and \( A_0 = A - ke_u^T \).

Choose vector \( k \) such that matrix \( A_0 \) is a strict Hurwitz, therefore, for any a given positive definite matrix \( Q = Q^T > 0 \), there exists a positive definite matrix \( P = P^T > 0 \) such that

\[
A_0^T P + PA_0 = -2Q
\]

(10)

Since the states \( x_1(t), \ldots, x_n(t) \) of the controlled system (1) cannot be measured directly, according to [21, 23], the fuzzy filters are constructed as

\[
\dot{\xi} = A_0 \xi + ky
\]

(11)

\[
\dot{\Omega} = A_0 \Omega + \Psi^T
\]

(12)

\[
\dot{\lambda} = A_0 \lambda + e_u u
\]

(13)

Define state estimation \( \hat{x} = \xi + \Omega \theta + \lambda \) and state estimation error

\[
\epsilon = x - \hat{x}
\]

(14)

From (9), (11)-(13) and (14), it can be shown that the state estimation error satisfies

\[
\dot{\epsilon} = A_0 \epsilon + \delta + d
\]

(15)

Remark 1: Note that \( \hat{x}(t) \) is unavailable because of the unknown parameter \( \theta \); therefore, \( \hat{x}(t) \) is a virtual estimate of \( x(t) \), which cannot be used in the latter controller design. Instead, it will be used for stability analysis. In fact, the actual state estimate is \( \hat{x}(t) = \xi + \Omega \dot{\theta} + \lambda \).

Consider the following Lyapunov function candidate for the error system (15) as

\[
V_0 = \frac{1}{2} \epsilon^T P \epsilon
\]

By using (8), Assumption 1 and Young’s inequality

\[
2xy \leq ax^2 + 1/y^2 (a > 0),
\]

the time derivative of \( V_0 \) along with (15) is

\[
\dot{V}_0 = \frac{1}{2} \epsilon^T (A_0^T P + PA_0) \epsilon + \epsilon^T P(\delta + d)
\]

\[
\leq -(\lambda_{\min}(Q) - 2 \epsilon)^T \epsilon + \frac{1}{4} \|P\| \|\delta\|^2 + \frac{1}{4} \|P\| \|d\|^2
\]

\[
\leq -(\lambda_{\min}(Q) - 2 \epsilon)^T \epsilon + D
\]

(17)

where \( \lambda_{\min}(Q) \) is the smallest eigenvalue of matrix \( Q \) and

\[
D = \frac{1}{4} \|P\| \sum_{i=1}^n \delta_{i0}^2 + \frac{1}{4} \|P\| \sum_{i=1}^n d_{i0}^2.
\]

3. Adaptive Fuzzy Dynamic Surface Control Design

In this section, we will incorporate the DSC technique proposed in [15] into a fuzzy based adaptive control design scheme for the system (1). Similar to the traditional backstepping design method, the recursive design procedure contains \( n \) steps.

From (9) and (13), we have

\[
\dot{y} = x_2 + \Psi^T \theta + \delta_{i1} + d_1
\]

\[
\dot{\lambda}_1 = -k_1 \lambda_1 + \lambda_2
\]

\[
\dot{\lambda}_2 = -k_1 \lambda_2 + \lambda_3
\]

(18)

\[
\vdots
\]

\[
\dot{\lambda}_{n-1} = -k_{n-1} \lambda_{n-1} + \lambda_n
\]

\[
\dot{\lambda}_n = -k_n \lambda_n + u
\]

where \( \Psi_i^T \) is the first row of \( \Psi^T \).

The \( n \) - step adaptive fuzzy output feedback backstepping design is based on the change of coordinates:
$z_i = y - y_r$  \hspace{1cm} (19)
$z_i = \lambda_i - \pi_i$  \hspace{1cm} (20)
$\chi_i = \pi_i - \alpha_{i-1}, \quad i = 2, \cdots, n$  \hspace{1cm} (21)

where $z_i$ is called the error surface, $\pi_i$ is a state variable, which is obtained through a first-order filter on intermediate function $\alpha_{i-1}$. Here $\chi_i$ is called the output error of the first-order filter.

**Step 1:** The time derivative of $z_1$ along with (18), (19) and (20) is

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r$$  
$$\dot{x}_1 = x_2 + \Psi^T_1(\dot{\theta} + \delta + d_1 - \dot{y}_r)$$  
$$\dot{\pi}_i = \pi_i - \alpha_{i-1}, \quad i = 2, \cdots, n$$

Step 1: The time derivative of $z_1$ along with (18), (19) and (20) is

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r$$  
$$\dot{x}_1 = x_2 + \Psi_1^T(\dot{\theta} + \delta + d_1 - \dot{y}_r)$$  
$$\dot{\pi}_i = \pi_i - \alpha_{i-1}, \quad i = 2, \cdots, n$$

where $\Omega_2$ is the second row of $\Omega$.

Consider the Lyapunov function candidate as

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \hat{\theta}^T \hat{\theta} + V_0$$  \hspace{1cm} (23)

where $r$ is a positive design constant. $\theta = \|\theta\|^2, \hat{\theta}$ is the estimate of $\theta$. $\hat{\theta} = \theta - \dot{\theta}$.

The time derivative of $V_1$ along with (22) is

$$\dot{V}_1 = [z_2 + \chi_2, \alpha_i + \xi_2 + \chi_2, \Omega_2 + \Psi^T_1(\dot{\theta} + \delta + d_1 + \dot{y}_r)] Z_1$$

and (24) can be rewritten as

$$\dot{V}_1 = [- (\lambda_{\min}(Q) - 3) e^T + (z_2 + \chi_2) z_1 + \frac{1}{4} z_1^2 + D + \hat{\theta}^T (\Omega_2 + \Psi^T_1) \dot{z}_1 - \frac{1}{4} z_1]$$

where $c_i > 0$ is a design constant.

Substituting (28) and (29) into (27) results in

$$\dot{V}_1 \leq - (\lambda_{\min}(Q) - 3) e^T e + (z_2 + \chi_2) z_1 + \frac{1}{4} z_1^2 + D + \hat{\theta}^T (\Omega_2 + \Psi^T_1) \dot{z}_1 - \frac{1}{4} z_1$$

where $D_i = z_2 - \dot{\pi}_i$ and

$$\dot{\pi}_i = \dot{\pi}_i - \alpha_i = - \frac{1}{2} \pi_i + B_1(\tau)$$

where $B_1(\tau)$ is a continuous function with the following expression

$$B_1(\tau) = c_i \dot{z}_1 + \frac{\tau}{\hat{\tau}} - \dot{\pi}_i$$

Step 2: From (18), (20) and (21), differentiating the second error variable $z_2$, we have

$$\dot{z}_2 = \hat{z}_2 - \dot{\pi}_i$$

Choose intermediate control function $\alpha_2$ as

$$\alpha_2 = -c_2 z_2 + k_2 \pi_3 - \dot{\pi}_2$$

where $c_2 > 0$ is a design constant.

Substituting (25) into (34) results in

$$\dot{\pi}_2 = \dot{\pi}_2 - \alpha_{\tau} = \frac{1}{2} \pi_2 + B_1(\tau)$$

Introduce a new state variable $\pi_3$ and let $\alpha_2$ pass through a first-order filter with the constant $\tau_3$ to obtain $\pi_3$, $\tau_3 \pi_3 + \pi_3 = \alpha_2$, $\pi_3(0) = \alpha_2(0)$
By defining the output error of this filter \( \chi = \pi - \alpha \), it yields \( \dot{\pi} = -\frac{\chi}{\tau} \) and
\[
\dot{\chi} = \ddot{\pi} - \dot{\alpha} = -\frac{\chi}{\tau} + B_1(\cdot)
\]
where \( B_1(\cdot) \) is a continuous function with the following expression
\[
B_1(\cdot) = c_2 \ddot{z} - k_2 \dot{\lambda} - \frac{\dot{\chi}}{\tau_2}
\]
(38)

**Step i** (*i = 3, ..., n−1*): A similar procedure in step 2 is employed recursively for step \( i = 3, ..., n−1 \). From (18), (20) and (21), the time derivative of the error variable \( z_i \) is
\[
\dot{z}_i = \dot{\lambda}_i - \dot{\pi}_i = -k_3 \dot{\lambda}_i + z_{i+1} + \chi_{i+1} + \alpha_i - \dot{\pi}_i
\]
(40)

Choose intermediate control function \( \alpha_i \) as
\[
\alpha_i = c_i z_i + k_i \dot{\lambda}_i + \dot{\pi}_i
\]
(41)

where \( c_i > 0 \) is a design constant.

Substituting (41) into (40) results in
\[
\dot{z}_i = z_{i+1} + \chi_{i+1} - c_i z_i
\]
(42)

Introduce a new state variable \( \pi_{i+1} \) and let \( \alpha_i \) pass through a first-order filter with the constant \( \tau_{i+1} \) to obtain
\[
\tau_{i+1} \ddot{\pi}_{i+1} + \pi_{i+1} = \alpha_i, \quad \pi_{i+1}(0) = \alpha_i(0)
\]
(43)

By defining the output error of the filter \( \chi_{i+1} = \pi_{i+1} - \alpha_i \), it yields \( \ddot{\pi}_{i+1} = -\frac{\pi_{i+1}}{\tau_{i+1}} \) and
\[
\dot{\chi}_{i+1} = \ddot{\pi}_{i+1} - \dot{\alpha}_i = -\frac{\pi_{i+1}}{\tau_{i+1}} + B_{i+1}(\cdot)
\]
(44)

where \( B_{i+1}(\cdot) \) is a continuous function with the following expression
\[
B_{i+1}(\cdot) = c_i \ddot{z} - k_i \dot{\lambda}_i - \frac{\dot{\chi}}{\tau_i}
\]
(45)

**Step n**: In the final step, define the error variable as \( z_n = \lambda_n - \pi_n \). The time derivative of the error variable \( z_n \) is
\[
\dot{z}_n = \dot{\lambda}_n - \dot{\pi}_n = -k_n \dot{\lambda}_n + u - \dot{\pi}_n
\]
(46)

Choose actual control function \( u \) as
\[
u = -c_n z_n + k_n \dot{\lambda}_n + \dot{\pi}_n
\]
(47)

where \( c_n > 0 \) is a design constant.

Substituting (47) into (46) results in
\[
\dot{z}_n = -c_n z_n
\]
(48)

### 4. The Stability Analysis of the Closed-Loop System

The goal of this section is to establish that the resulting closed-loop system possesses the semi-globally uniformly bounded property.

**Assumption 2**: For a given \( p > 0 \), for all initial conditions satisfying
\[
\varepsilon^T P e + 1/4 \tilde{\theta}^T \tilde{\theta} + \sum_{k=2}^n \chi_k^2 + \sum_{k=1}^n z_k^2 \leq 2 p.
\]

**Theorem 1**: Consider the closed-loop system (1). Under Assumptions 1-2, the fuzzy adaptive controller (45) with fuzzy K-filters (11), (12) and (13), the intermediate controls (28), (35) and (41) and parameter adaptive law (29) guarantees that all the signals in the resulting closed-loop system are SGUUB. Moreover, the tracking errors and the observer errors can be made arbitrarily small by choosing appropriate design parameters.

**Proof**: Consider the following Lyapunov function candidate
\[
V = V_1 + \frac{1}{2} \sum_{k=2}^n \dot{\chi}_k^2 + \frac{1}{2} \sum_{k=2}^n z_k^2
\]
(49)

The time derivative of \( V \) is
\[
\dot{V} = \dot{V}_1 + \sum_{k=2}^n \dot{\chi}_k \dot{\pi}_k + \sum_{k=2}^n \dot{z}_k \dot{z}_k
\]
(50)

Substituting (30), (42), (44) into (50) yields
\[
\dot{V} \leq (\lambda_{\min}(Q) - 3) \varepsilon^T P e + (z_2 + \chi_2) z_1 - c_i z_i^2 + D_1 + \mu \tilde{\theta}^2 \tilde{\theta} + \sum_{k=2}^n \chi_k \left[ -\frac{\dot{\chi}_k}{\tau_k} + B_k \right]
\]
(51)

\[
+ \sum_{k=2}^n \left[ z_{k+1} + \chi_{k+1} - c_k z_k \right] - c_n z_n^2
\]

Since for any \( B_0 > 0 \) and \( p > 0 \),
\[
\Pi_0 := \{(y_r, \dot{y}_r, \ddot{y}_r) : \dot{y}_r^2 + \ddot{y}_r^2 + \dot{y}_r^2 \leq B_0\},
\]
and
\[
\Pi_i := \left\{ \varepsilon^T P e + (1/r) \tilde{\theta}^T \tilde{\theta} + \sum_{k=2}^i \dot{z}_k^2 + \sum_{k=1}^i z_k^2 \leq 2 p \right\},
\]

\[
i = 1, ..., n \text{ are compact sets in } R^3 \text{ and } R^{n+2i+3}, \text{ respectively, } \Pi_0 \times \Pi_i \text{ is also a compact set in } R^{n+2i+3}.
\]

Therefore \( \left| B_k \right| \) has a maximum \( M_k \) on \( \Pi_0 \times \Pi_i \).

By using the Young’s inequality, we have
\[
z_i z_i \leq z_i^2 + \frac{1}{4} z_i^2
\]
(52)
\begin{align}
\chi_2 z_1 &\leq z_1^2 + \frac{1}{4} \chi_2^2 \\
z_k z_{k+1} &\leq z_k^2 + \frac{1}{4} z_{k+1}^2 \\
z_k \chi_{k+1} &\leq z_k^2 + \frac{1}{4} \chi_{k+1}^2
\end{align}
(53)

By substituting (52)-(55) into (51), we obtain
\[
\dot{V} \leq - (\lambda_{\min}(Q) - 3) e^T e + 2 z_n^2 + \frac{1}{4} z_k^2 + \frac{1}{4} \chi_n^2
\]
(56)

Choose
\[
c_1 = 2 + c \\
c_k > \frac{1}{4} + c, k = 2, \ldots, n - 1
\]
(57)

where \( c \) is a positive constant.

By using \( \mu \tilde{\theta}^T \tilde{\theta} \leq - (\mu/2) \tilde{\theta}^T \tilde{\theta} + \mu/2 \theta^T \theta \), (56) can be rewritten as
\[
\dot{V} \leq - (\lambda_{\min}(Q) - 3) e^T e - c \sum_{k=1}^n z_k^2 - \frac{\mu}{2} \tilde{\theta}^T \tilde{\theta} + D_1
\]
(58)

By using \( |\chi_k M_k| \leq \chi_k^2 + 1/4 M_k^2 \), (38) becomes
\[
\dot{V} \leq - (\lambda_{\min}(Q) - 3) e^T e - c \sum_{k=1}^n z_k^2 - \frac{\mu}{2} \tilde{\theta}^T \tilde{\theta} - \sum_{k=1}^n m_k \chi_k^2 + D
\]
(59)

where
\[
D = D_1 + \mu/2 \theta^T \theta + \sum_{k=2}^n M_k^2 \quad \text{and} \quad m_k = 1/\tau_k - 1/2
\]
(\( \tau_k < 2 \)).

Defining
\[
C = \min \{2c, \mu, 2(\lambda_{\min}(Q) - 3)/\lambda_{\max}(P), 2m_2, \ldots, 2m_n\}
\]
where \( \lambda_{\max}(P) \) is the largest eigenvalue of matrix \( P \).

Then (59) can be further written as
\[
\dot{V} \leq -CV + D
\]
(60)

Integrating (60) over \([0, t]\) results in
\[
V(t) \leq S + e^{-Ct} V(0)
\]
(61)

where \( S = \bar{D}/C \).

By (61), and using the same arguments in [16, 17, 18], we can conclude that all the variables in the closed-loop system are bounded. Moreover, the tracking errors \( z_i = y - y_r \) can be made as small as desired by appropriate choice of the design parameters \( k, c_i, Q, \tau_i, \varsigma, r \) and \( \mu \).

The guidelines of the parameters selections in the control system are summarized as follows:
1. Given positive definite matrix \( Q \) and choose \( k \) such that matrix \( A_0 \) is a strict Hurwitz;
2. The design parameters \( c_i \) and \( r \) are required to satisfy that \( c_i > 0 \) and \( r > 0 \), respectively. The design parameters \( \varsigma \) and \( \tau_i \) are required to satisfy that \( 0 < \varsigma < 1 \) and \( 0 < \tau_i < 1 \), respectively.
3. The design parameter \( \mu \) in (29) is used for the \( \mu \)-modification, the appropriate choice of \( \mu \) can prevent the parameter \( \tilde{\theta} \) to drift. It is usually chosen as \( \mu > 0 \).

5. Simulation Examples

In this section, two simulation examples are presented to show effectiveness of the proposed adaptive fuzzy backstepping control approach.

Example 1: Consider a second-order system as
\[
\dot{x}_1 = x_2 + f_1(x_1, x_2) \\
\dot{x}_2 = u + f_2(x_1, x_2) + d_1(x_1, x_2)
\]
(62)

\[
y = x_1
\]

where unknown functions are \( f_1(x_1, x_2) = -x_1e^{-x_1} \) and \( f_2(x_1, x_2) = \sin(1/1 + x_1^2) x_2 \) ; \( d_1(x_1, x_2) = \sin(x_1 x_2)/10 \)

and \( d_2(x_1, x_2) = 0.1/1 + x_1^2 + x_2^2 \).

The desired reference signal is \( y_r = \sin t \). The fuzzy membership functions are defined as
\[
\mu_{\hat{x}_1}(\hat{x}_1) = \exp \left[ - \frac{(-\hat{x}_1 + 3 - l)^2}{6} \right],
\]
\[
\mu_{\hat{x}_2}(\hat{x}_1, \hat{x}_2) = \exp \left[ - \frac{(-\hat{x}_1 + 3 - l)^2}{4} \right], \quad l = 1, \ldots, 5.
\]

\[
\times \exp \left[ - \frac{(-\hat{x}_2 + 3 - l)^2}{4} \right], \quad l = 1, \ldots, 5.
\]
The fuzzy basis functions are
\[ \varphi_j(\hat{x}_1) = \frac{\mu_{F_j}}{\sum_{i=1}^{5} \mu_{F_i}}, \quad \varphi_j(\hat{x}_2) = \frac{\mu_{F_j}}{\sum_{i=1}^{5} \mu_{F_i}}, \]

The design parameters in the control scheme are chosen as

\[ c_1 = 70, \quad c_2 = 10, \quad k_1 = 2, \quad k_2 = 4, \quad \zeta = 0.01, \quad \mu = 0.02, \quad \tau_2 = 0.01, \quad r = 0.5. \]

The initial conditions are chosen as
\[ x_1(0) = 1, \quad x_2(0) = -1, \] and others initial condition are 0.

The simulation results are shown in Figs. 1-3, where Fig. 1 is the trajectories of output \( y \) and reference signal \( y_r \); and Fig. 2 is the trajectory of \( x_2 \), and Fig. 3 is the trajectory of the control \( u \).

Example 2 [24]: Consider the electromechanical system shown by Figure 4.

The dynamics of the electromechanical system is described by the following equation.

\[ \begin{cases}
    M\ddot{q} + B\dot{q} + N\sin(q) = I \\
    L\dot{I} = V_c - RI - K_B\dot{q}
\end{cases} \tag{63} \]

where
\[ M = \frac{J}{K_r} + \frac{mL_0^2}{3K_r} + \frac{M_0L_0^2}{K_r} + \frac{2M_0R_0^2}{5K_r}, \]
\[ N = \frac{mLG}{2K_r} + \frac{M_0L_0G}{K_r}, \quad B = \frac{B_0}{K_r}. \]

\( J \) is the rotor inertia, \( m \) is the link mass, \( M_0 \) is the load mass, \( L_0 \) is the link length, \( R_0 \) is the radius of the load, \( G \) is the gravity coefficient, \( B_0 \) is the coefficient of viscous friction at the joint, \( q(t) \) is the angular motor position (and hence the position of the load), \( I(t) \) is the motor armature current, and \( K_B \) is the coefficient which characterizes the electromechanical conversion of armature current to torque. \( L \) is the armature inductance, \( R \) is the armature resistance, \( K_B \) is the back-emf coefficient, and \( V_c \) is the input control voltage. The values of the parameters are chosen as

\[ J = 1.625 \times 10^{-3}\text{Kg} \cdot \text{m}^2, \quad m = 0.506\text{Kg}, \quad R_0 = 0.023\text{m}, \quad M_0 = 0.434\text{Kg}, \quad L_0 = 0.305\text{m}, \quad B_0 = 16.25 \times 10^{-3}\text{N} \cdot \text{m} / \text{rad}, \quad L = 25.0 \times 10^{-3}\text{H}, \quad R = 5.0\Omega, \quad K_r = K_B = 0.90\text{N} \cdot \text{m} / \text{A}. \]
By introducing the variable changes \( x_1 = q \), \( x_2 = \dot{q} \), \( x_3 = Ml \) and \( u = LVz \), the dynamics given by (63) is rewritten in the following form
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + f_2(x_1, x_2) + d_2(x_1, x_2) \\
\dot{x}_3 &= u + f_1(x_1, x_2, x_3) \\
y &= x_1
\end{align*}
\]  
(64)

where \( f_2(x_1, x_2) = -\frac{B}{M}x_2 \) and \( f_1(x_1, x_2, x_3) = -\frac{K_B}{L}x_2 - \frac{R}{LM}x_3 \) are unknown functions;
\( d_2 = -\frac{N}{M}\sin(x_1) \) is a bounded perturbation.

The desired reference signal is \( y_r = 0.8\cos(6t) \).

The fuzzy membership functions are defined as
\[
\begin{align*}
\mu_{F_1}(\hat{x}_2) &= \exp \left[-\frac{(\hat{x}_2 + 4 - 2l)^2}{4}\right], \\
\mu_{F_2}(\hat{x}_2, \hat{x}_3) &= \exp \left[-\frac{(\hat{x}_2 + 2 - l)^2}{2}\right] \times \exp \left[-\frac{(\hat{x}_3 + 2 - l)^2}{6}\right], & l = 1, 2, 3.
\end{align*}
\]

The fuzzy basis functions are
\[
\varphi_{2j}(\hat{x}_1) = \frac{\mu_{F_2}}{\sum_{j=1}^{3}\mu_{F_2}}, \quad \varphi_{3j}(\hat{x}_2, \hat{x}_3) = \frac{\mu_{F_3}}{\sum_{j=1}^{3}\mu_{F_3}},
\]
\( j = 1, 2, 3 \)

The controller and parameter adaptive laws are given as follows:
\[
\begin{align*}
\dot{\alpha}_1 &= -c_1z_1 - \xi_2 + \dot{\xi}_r - \delta_\alpha \tanh(\delta_\alpha z_1) \\
&\quad - (\Omega_2 + \Psi_1)^Tz_1 - \frac{1}{4}z_1 \\
\dot{\theta} &= r(\Omega_2 + \Psi_1)^Tz_1 - \mu \dot{\theta} \\
\alpha_2 &= -c_2z_2 + k_2\alpha_1 + \dot{\xi}_2 \\
u &= -c_3z_3 + k_3\alpha_2 + \dot{\xi}_3
\end{align*}
\]  
(65)

The design parameters in the control scheme are chosen as
\( c_1 = 100, c_2 = 10, c_3 = 10, k_1 = 10, k_2 = 12, k_3 = 20, \)
\( \varsigma = 0.1, \tau_2 = 0.1, \tau_3 = 0.1, \mu = 2, \delta_{30} = 0.2, \)
\( r = 0.6 \).

The initial conditions are chosen as \( x_1(0) = 1, \) and others initial condition are 0. The simulation results are shown in Figs. 5-8, where Fig. 5 is the trajectories of output \( y \) and reference signal \( y_r \); Fig. 6 and Fig. 7 are the trajectories of \( x_2 \) and \( x_3 \), respectively; and Fig. 8 is the trajectory of the control \( u \).

6. Conclusions

In this paper, the adaptive fuzzy output feedback control problem for a class of SISO uncertain nonlinear systems has been considered. Based on the backstepping and DSC techniques, a novel adaptive fuzzy output feedback control algorithm has been developed. It has been proved that the proposed control approach can guarantee that the closed-loop system is SGUUB. The proposed algorithm exhibits the following features. First, the problem of “explosion of complexity” existing in the conventional backstepping methods can be overcome, and second, the restrictive assumption in previous results that states are available for feedback can be relaxed. The proposed algorithm is in a much simpler form, thus is much easier to be implemented in applications. Simulation results have been presented to illustrate the effectiveness and good transient performance of the proposed algorithm.
Acknowledgment

This work was supported in part by the National Natural Science Foundation of China (Nos. 51179019, 60874056, 61074014), the Outstanding Youth Funds of Liaoning Province (No. 2005219001), the Natural Science Foundation of Liaoning Province (No. 20102012) and China Postdoctoral Special Science Foundation (No. 200902241).

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