Hybrid Intelligent Output-Feedback Control for Trajectory Tracking of Uncertain Nonlinear Multivariable Dynamical Systems

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Abstract

Output-feedback control for trajectory tracking is an important research topic of various engineering systems. In this paper, a novel online hybrid direct/indirect adaptive Petri fuzzy neural network (PFNN) controller with state observer for uncertain nonlinear multivariable dynamical systems using generalized projection-update laws is presented. This new approach consists of control objectives determination, approximator configuration design, system dynamics modeling, online control algorithm development, and system stability analysis. According to the importance and viability of plant knowledge and control knowledge, a weighting factor is utilized to sum together the direct and indirect adaptive PFNN controllers. Therefore, the controller design methodology is more flexible during the design process. Besides, an improved generalized projection-update law is utilized to tune the adjustable parameters to prevent parameter drift. To illustrate the effectiveness of the proposed online hybrid PFNN controller and observer-design methodology, numerical simulation results for inverted pendulum systems and rigid robot manipulators are given in this paper.

Keywords: output-feedback control, trajectory tracking, uncertain nonlinear systems.

1. Introduction

The conventional adaptive fuzzy neural network (FNN) control has direct and indirect FNN adaptive control categories [1, 2]. The direct adaptive FNN control using fuzzy logic systems as controllers has been proposed in [3-6]. Therefore, linguistic fuzzy control rules can be directly incorporated into the controller. Also, the indirect adaptive FNN control using fuzzy descriptions to model the plant has been developed in [7-9]. Then, fuzzy IF–THEN rules describing the plant can be directly incorporated into the indirect FNN controller. Recently, it is an important issue [10, 11] to choose a weighting factor to sum together the direct adaptive PFNN controller and indirect adaptive PFNN controller. In this paper, a hybrid direct/indirect adaptive PFNN control scheme is constructed by using the weighting factor adjusted by the tradeoff between plant knowledge and control knowledge. In addition, the free parameters can be flexibly tuned by the adaptive laws.

In recent years, fuzzy neural network has been developed into a powerful tool for modeling, analysis, and control of various engineering systems [12-17]. In [18-20], the authors investigated a T-S fuzzy neural approach for only considering the stabilization problem. Wang et al. [3, 7] developed an adaptive fuzzy-neural controller for single-input-single-output (SISO) nonlinear systems and so is hardly practical in real applications. Although Hwang and Hu [21] proposed a robust fuzzy-neural learning controller for multiple-input-single-output (MIMO) manipulators, the state feedback control scheme does not always hold in practical applications, because models of those systems are not always known. Also, more inputs (linguistic terms) and membership functions of the FNN are required for higher-order complex systems [22]. Adjusting the vast numbers of parameters will aggravate the already heavy computational burden. Besides, the magnitude of the derived adjustable parameters is generally too large to apply in a practical design. Thus, further improvement for the design algorithm is required, not only to alleviate the computation burden of parameter learning but also to reduce the magnitude of adjustable parameters demanded by practical applications as well. To solve the aforementioned problems, a T-S fuzzy inference system constructed from a Petri neural network structure [23, 24], which incorporates an improved generalized projection-update law, is developed in this paper.
2. Problem Formulation and Preliminary

Consider the nth-order MIMO uncertain nonlinear systems of the form [11]
\[
\dot{\chi}_i = A_i \chi_i + B_i \{f_i(x) + \sum_{j=1}^p g_{ij}(x)u_j + d_i\} \quad y_i = C_i \chi_i, i = 1, 2, \ldots, p
\]
(1)
where
\[
A_i = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
1 & 0 & 0 & \cdots & 0
\end{bmatrix},
B_i = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1
\end{bmatrix}, C_i = \begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]
(2)
and \(u = [u_1, u_2, \ldots, u_p]^T\) and \(y = [y_1, y_2, \ldots, y_p]^T\) are vectors of control inputs and system outputs, respectively. \(\chi_i = [x_{i1}, x_{i2}, \ldots, x_{in_i}]^T, \chi_2 = [x_{i1}, x_{i2}, \ldots, x_{in_i}]^T, \ldots\), \(\chi_p = [x_{i1}, x_{i2}, \ldots, x_{in_i}]^T\) and \(x = [\chi_1, \chi_2, \ldots, \chi_p]^T\) denote state vectors where not all \(x_i\) are assumed to be available for measurement. Only the system outputs \(y\) are assumed to be measurable. \(r_1 + r_2 + \ldots + r_p = n\). \(d_i = [d_{i1}, d_{i2}, \ldots, d_{ip}]^T\) is a vector of external disturbances. \(f_i\) and \(g_{ij}\) are unknown smooth functions.

Define the reference vectors \(y_{w_i} = [y_{w1}, \hat{y}_{w1}, \hat{y}_{w1}, \ldots, y_{w2}, \hat{y}_{w2}, \hat{y}_{w2}, \ldots]^{\top}\), the tracking error vectors \(e_i = y_{w_i} - \hat{\chi}_i\), and the estimated tracking error vectors \(\hat{e}_i = y_{w_i} - \hat{\chi}_i\) where \(\hat{e}_i\) and \(\hat{\chi}_i\) denote the estimations of \(e_i\) and \(\chi_i\), respectively. Based on the certainty equivalence approach, the optimal control law is
\[
u^* = \frac{1}{G(x)}[-F(x) + [y_{w1}, y_{w2}, \ldots, y_{w2}](y_{w2})]^T + [K_{1y_e}, K_{2y_e}, \ldots, K_{py_e}]^T
\]
(3)
where
\[
F(x) = [f_1, f_2, \ldots, f_p]^T,
G(x) = \begin{bmatrix}
g_{11} & g_{12} & \cdots & g_{1p} \\
g_{21} & g_{22} & \cdots & g_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
g_{p1} & g_{p2} & \cdots & g_{pp}
\end{bmatrix}
\]
and \(K_{iy_e} = [k_{i1y_e}, k_{i2y_e}, \ldots, k_{ipy_e}]^T\) are the feedback gain vectors, chosen such that the characteristic polynomials of \(A_i - B_iK_{iy_e}\) are Hurwitz because \((A_i, B_i)\) are controllable. However, \(F(x)\) and \(G(x)\) are unknown, the ideal controller (3) cannot be implemented, and not all system states can be measured. Therefore, we design an observer to estimate the state vector in the following context.

Here, we will develop an observer-based hybrid direct/indirect adaptive controller. The overall control law is constructed as
\[
u = cu(x) + (1 - \alpha)u_p(\hat{x} | \theta_0) + u_1(\hat{x} | \theta_0)\]
(4)
where \(u_1 = [u_{11}, u_{12}, \ldots, u_{ip}]^T \in \mathbb{R}^p\) and \(u_D = [u_{D1}, u_{D2}, \ldots, u_{dp}]^T \in \mathbb{R}^p\) are indirect adaptive controller and direct adaptive controller, respectively. \(u_1 = [u_{11}, u_{12}, \ldots, u_{ip}]\) is the compensated control input vector.

\(\alpha \in [0, 1]\) is a weighting factor which can be adjusted by the tradeoff between plant knowledge and control knowledge. The indirect control law is described as
\[
u_1 = \frac{1}{G'(\hat{x})}[-\hat{F}(\hat{x}) + [y_{w1}(\hat{x}), y_{w2}(\hat{x}), \ldots, y_{w2}(\hat{x})]^T + [K_{1y_e}(\hat{x}), K_{2y_e}(\hat{x}), \ldots, K_{py_e}(\hat{x})]^T]
\]
(5)
However, one problem with the modeling approach is that the matrix \(\hat{G}\) may not be invertible. This can be overcome by using the following modification to the approximate matrix [26]:
\[
\hat{G} = \hat{G} + \mu I
\]
(6)
where I is an identity matrix, and μ is a positive constant. Suppose that the eigenvalues and eigenvectors of \( \mathbf{G} \) are \( \{\lambda_{g1}, \lambda_{g2}, \ldots, \lambda_{gp}\} \) and \( \{\mathbf{v}_{g1}, \mathbf{v}_{g2}, \ldots, \mathbf{v}_{gp}\} \), then
\[
\mathbf{Gv}_gi = [\mathbf{G} + \mu I]\mathbf{v}_gi = \mathbf{Gv}_gi + \mu \mathbf{v}_gi
\]
\[
= \lambda_{gi}\mathbf{v}_gi + \mu \mathbf{v}_gi = (\lambda_{gi} + \mu)\mathbf{v}_gi.
\]
Therefore the eigenvectors of \( \mathbf{G} \) are the same as the eigenvectors of \( \hat{\mathbf{G}} \), and the eigenvalues of \( \hat{\mathbf{G}} \) are \( (\lambda_{gi} + \mu) \). \( \hat{\mathbf{G}} \) can be made positive definite by increasing \( \mu \) until \( (\lambda_{gi} + \mu) > 0 \) for all \( i \), and therefore the matrix will be invertible.

Then we can rewrite (5) as follows:
\[
\mathbf{u}_j = (\hat{\mathbf{G}}(\hat{x})+k_G_\mu I)^{-1}(\hat{\mathbf{F}}(\hat{x})+\mathbf{y}_{i1}(\hat{x}),\mathbf{y}_{i2}(\hat{x}),\ldots,\mathbf{y}_{im}(\hat{x}))
\]
\[
+ [\mathbf{K}_{i1}^T \hat{\mathbf{e}}_1, \mathbf{K}_{i2}^T \hat{\mathbf{e}}_2, \ldots, \mathbf{K}_{ip}^T \hat{\mathbf{e}}_p]^T.
\]

The parameter \( k_G \) is chosen as
\[
k_G = \begin{cases} 
0, & \text{if } |\hat{\mathbf{G}}| > e_G \\
1, & \text{if } |\hat{\mathbf{G}}| \leq e_G
\end{cases}
\]
where \( e_G \) is a small positive constant.

Applying (4) and (8) to (1), we can obtain the error dynamic equation as
\[
\dot{\mathbf{e}}_i = \mathbf{A}_i \mathbf{e}_i - \mathbf{B}_i \mathbf{K}_i^T \hat{\mathbf{e}}_i + \mathbf{B}_i (\alpha f_i(\hat{x}) - f_i(x)) + \sum_{j=1}^{p} (\hat{g}_j(\hat{x}) - g_j(x))u_j
\]
\[
+ (1-\alpha)\sum_{j=1}^{p} g_j(x)(u^*-u_j) - \sum_{j=1}^{p} g_j(x)u_j + \alpha k_G d_{ai} - d_{ai}
\]
\[
\mathbf{e}_i = \mathbf{C}_i^T \hat{\mathbf{e}}_i
\]
where \( \mathbf{e}_i = y_{ai} - y_i \) denote the output tracking errors and
\[
\mathbf{d}_i = [d_{ai}, d_{2i}, \ldots, d_{di}]^T
\]
\[
= (\hat{\mathbf{G}}^{-1}(\hat{x}) - \hat{\mathbf{G}}^{-1}(\hat{x}))(-\hat{\mathbf{F}}(\hat{x})+\mathbf{y}_{i1}(\hat{x}),\mathbf{y}_{i2}(\hat{x}),\ldots,\mathbf{y}_{im}(\hat{x}))
\]
\[
+ [\mathbf{K}_j^T \hat{\mathbf{e}}_1, \mathbf{K}_{i2}^T \hat{\mathbf{e}}_2, \ldots, \mathbf{K}_{ip}^T \hat{\mathbf{e}}_p]^T.
\]

Next, consider the observers that estimate the vectors \( \mathbf{e}_i \) in (10).
\[
\hat{\mathbf{e}}_i = \mathbf{A}_i \hat{\mathbf{e}}_i - \mathbf{B}_i \mathbf{K}_i^T \hat{\mathbf{e}}_i + \mathbf{K}_i (\mathbf{e}_i - \hat{\mathbf{e}}_i)
\]
\[
\hat{\mathbf{e}}_i = \mathbf{C}_i^T \hat{\mathbf{e}}_i
\]
where \( \mathbf{K}_i = [k_{i1}^T, k_{i2}^T, \ldots, k_{ip}^T]^T \) are the observer gain vectors, chosen such that the characteristic polynomials of \( \mathbf{A}_i - \mathbf{K}_i \mathbf{C}_i^T \) are strictly Hurwitz because \( \mathbf{C}_i, \mathbf{A}_i \) are observable. The observation errors are defined as: \( \mathbf{e}_i = \mathbf{e}_i - \hat{\mathbf{e}}_i \) and \( \hat{\mathbf{e}}_i = \mathbf{e}_i - \hat{\mathbf{e}}_i \). Subtracting (11) from (10), the error dynamics are
\[
\dot{\mathbf{e}}_i = (\mathbf{A}_i - \mathbf{K}_i \mathbf{C}_i^T) \mathbf{e}_i + \mathbf{B}_i (\alpha f_i(\hat{x}) - f_i(x)) + \sum_{j=1}^{p} (\hat{g}_j(\hat{x}) - g_j(x))u_j
\]
\[
+ (1-\alpha)\sum_{j=1}^{p} g_j(x)(u^*-u_j) - \sum_{j=1}^{p} g_j(x)u_j + \alpha k_G d_{ai} - d_{ai}
\]
\[
\hat{\mathbf{e}}_i = \mathbf{C}_i^T \hat{\mathbf{e}}_i
\]
Besides, the output error dynamics of (12) can be given as
\[
\mathbf{e}_i = \mathbf{H}(s)(\alpha f_i(\hat{x}) - f_i(x)) + \sum_{j=1}^{p} (\hat{g}_j(\hat{x}) - g_j(x))u_j
\]
\[
+ (1-\alpha)\sum_{j=1}^{p} g_j(x)(u^*-u_j) - \sum_{j=1}^{p} g_j(x)u_j + \alpha k_G d_{ai} - d_{ai}
\]
where \( s \) is the Laplace variable, and \( H(s) = \mathbf{C}_i^T (\mathbf{I} - (\mathbf{A}_i - \mathbf{K}_i \mathbf{C}_i^T))^{-1} \mathbf{B}_i \) are the transfer functions of (12).

3. Description of Petri Fuzzy Neural Network (PFNN) Systems

The basic configuration of the Petri fuzzy neural network (PFNN) consists of a typical T-S fuzzy inference system constructed from a Petri neural network structure. The fuzzy logic system can be divided into two parts: some fuzzy IF-THEN rules and a fuzzy inference engine. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector to an output linguistic variable. The ith fuzzy IF-THEN rule is written as
\[
R_i^{(i)}: \begin{cases} 
\mu_i = 1, & \text{if } \mathbf{z} \in \mathbf{F}_i \\
\mu_i = 0, & \text{if } \mathbf{z} \notin \mathbf{F}_i
\end{cases}
\]
where \( \mathbf{z} = [z_{1}, z_{2}, \ldots, z_{n+p}]^T \in \mathbf{R}^{n+p} \) is a vector of linguistic variables, \( \mathbf{y} \) represents the output of the fuzzy-neural network, \( F_i \) (\( i = 1, 2, \ldots, n \)) are fuzzy sets, and \( p_i \) (\( i = 1, 2, \ldots, n \)) are adjustable parameters which are called the weighting factors.

Fig. 1 shows the configuration of the Petri fuzzy neural function approximator. It has a total of seven layers. Nodes at layer I are input nodes (linguistic nodes) that represent input linguistic variables. Nodes at layer II are term nodes which act as membership functions to represent the terms of the respective linguistic variables. The layer III of the PFNN in this paper for producing tokens makes use of competition laws as follows to select suitable fired nodes:
\[
t_i = \begin{cases} 
1, & \mu_{f_i}(z_j) \geq d_i \\
0, & \mu_{f_i}(z_j) < d_i
\end{cases}
\]
where \( t_i \) is the transition and \( d_i \) is a dynamic threshold value varied with the corresponding tracking error to be tuned by the following equation [24]:

\[\text{(12)}\]
where $k_a$ and $k_s$ are positive constants. $E = \frac{1}{2} \sum e^2_i$ is the energy function and $e_i$ represent tracking errors. It means that if tracking errors become large, the threshold values will be decreased in order to fire more control rules for the present situation. In layer IV, the nodes perform the fuzzy rules. The links between layer IV and layer V are connected by the weighting factors. Each node of layer VI represents the product of the weight and input variable. In layer VII, outputs of PFNN stand for the values of outputs.

$$d^*_m = \frac{k_s \exp(-k_s E)}{1 + \exp(-k_s E)}$$

4. Hybrid Direct/Indirect Adaptive PFNN Controller with Observer

In this section, our primary task is to use the hybrid direct/indirect PFNN to approximate the uncertain nonlinear system (1) and the adaptive direct control. Besides, we will develop an improved generalized projection update law to adjust the parameters of the hybrid direct/indirect PFNN in order to achieve the control objective and to prevent parameter drift.

Then the observation error dynamic equation (12) can be rewritten as

$$\dot{\hat{e}}_i = (A_i - K_i C_i^T) \hat{e}_i + B_i(\alpha \hat{f}_i(\hat{x}|\theta^0_k) - \hat{f}_i(\hat{x}|\theta^0_k)$$

$$+ \sum_{j=1}^p (\hat{g}_{ij}(\hat{x}|\theta^0_{g_j}) - \hat{g}_{ij}(\hat{x}|\theta_{g_j})) u_{g_j})$$

$$- (1 - \alpha) \sum_{j=1}^p g_{ij}(x)(u_{D_j}(\hat{x}|\theta_{D_j}) - u_{D_j}(\hat{x}|\theta_{D_j}))$$

$$- B_i \sum_{j=1}^p g_{ij}(x)u_{g_j} + B_i w_{mi}$$

$$\tilde{e}_i = C_i^T \hat{e}_i$$

where

$$w_{mi} = \alpha(\hat{f}_i(\hat{x}|\theta^0_k) - f_i(x)) + \sum_{j=1}^p (\hat{g}_{ij}(\hat{x}|\theta^0_{g_j}) - g_{ij}(x)) u_{g_j}$$

$$+ (1 - \alpha) \sum_{j=1}^p g_{ij}(x)(u^* - u_{D_j}(\hat{x}|\theta_{D_j})) + P(k)$$

The optimal parameter estimations $\theta^*_f$, $\theta^*_g$, and $\theta^*_{D_k}$ are defined as

$$\theta^*_f = \arg\min_{\theta_f} \left[ \sup_{x \in \Omega_f} \left| f_i(x) - \hat{f}_i(\hat{x}|\theta^0_k) \right| \right]$$

$$\theta^*_g = \arg\min_{\theta_g} \left[ \sup_{x \in \Omega_g} \left| g_{ij}(x) - \hat{g}_{ij}(\hat{x}|\theta^0_{g_j}) \right| \right]$$

$$\theta^*_{D_k} = \arg\min_{\theta_{D_k}} \left[ \sup_{x \in \Omega_{D_k}} \left| u^* - u_{D_j}(\hat{x}|\theta_{D_k}) \right| \right]$$

where $\Omega_f$, $\Omega_g$, $\Omega_{D_k}$, $U_f$, and $U_k$ are compact sets of suitable bounds on $\theta_f$, $\theta_g$, $\theta_{D_k}$, $x$, and $\hat{x}$, respectively, and they are defined as

$$\Omega_f = \{ \theta_f \in \mathbb{R}^n : \| \theta_f \| < m_\theta < \infty \}$$

$$\Omega_g = \{ \theta_g \in \mathbb{R}^n : \| \theta_g \| < m_\theta < \infty \}$$

$$\Omega_{D_k} = \{ \theta_{D_k} \in \mathbb{R}^n : \| \theta_{D_k} \| < m_\theta < \infty \}$$

$$U_f = \{ x \in \mathbb{R}^n : \| x \| < m_x < \infty \}$$

and

$$U_k = \{ \hat{x} \in \mathbb{R}^n : \| \hat{x} \| < m_x < \infty \}$$

By using $\hat{f}_i(\hat{x}|\theta^0_k) = \theta^*_f \psi(\hat{x})$, $\hat{g}_{ij}(\hat{x}|\theta^0_{g_j}) = \theta^*_g \psi(\hat{x})$, and $u_{D_j}(\hat{x}|\theta_{D_k}) = \theta^*_{D_k} \phi(\hat{x})$, (19) can be rewritten as

$$\dot{\hat{e}}_i = A_i \hat{e}_i + B_i(\alpha \hat{f}_i(\hat{x}) \psi(\hat{x}) + \sum_{j=1}^p \hat{g}_{ij}(\hat{x}|\theta^0_{g_j}) u_{g_j})$$

$$- (1 - \alpha) \sum_{j=1}^p g_{ij}(x)(u^* - u_{D_j}(\hat{x}|\theta_{D_j})) + B_i v_j + B_i w_{mi}$$

Fig. 1. Configuration of the Petri fuzzy neural approximator.
\[ \dot{e}_\alpha = C_i^T \hat{e}_i \]

where \( \hat{\theta}_f^* = \theta_f - \theta_{f^*} \), \( \hat{\theta}_{g_i} = \theta_{g_i} - \theta_{g_{i^*}} \), \( \hat{\theta}_{D_i} = \theta_{D_i} - \theta_{D_{i^*}} \),

\[ v_i = \sum_{j=1}^{p} g_{ij}(x) u_j, \quad \text{and} \quad A_i = A_i - K_{ai} C_i^T \]. By using the strictly-positive-real (SPR) Lyapunov design approach to analyze the stability of (21) and generate the adaptive output feedback update laws for \( \hat{\theta}_f^*, \hat{\theta}_{g_i} \), and \( \hat{\theta}_{D_i} \), (21) can be rewritten as

\[ \dot{e}_\alpha = H_i(s)(\alpha(\hat{\theta}_f^*) \psi(\hat{x}) + \sum_{j=1}^{p} \hat{\theta}_{g_{ij}} \psi(\hat{x}) u_j - v_i + w_i) \]

where

\[ H_i(s) = C_i^T (sI - (A_i - K_{ai} C_i^T))^{-1} B_i = \frac{1}{s^{\nu_i} + k_i s^{\nu_i-1} + \cdots + k^i} \] (23)

The transfer functions \( H_i(s) \) are known stable transfer functions. In order to be able to use the SPR-Lyapunov design approach, (22) can be rewritten as

\[ \dot{e}_\alpha = H_i(s) L_i(s)(\alpha(\hat{\theta}_f^*) \psi(\hat{x}) + \sum_{j=1}^{p} \hat{\theta}_{g_{ij}} \psi(\hat{x}) u_j) - (1 - \alpha) \hat{\theta}_{D_i} \psi(\hat{x}) - v_i + w_i \]

where

\[ v_{f_i} = L_i^{-1}(s)v_i, \quad w_i = L_i^{-1}(s)w_i, \quad w_i = u_{m_i} + e_i, \]

\[ e_i = \alpha(\hat{\theta}_f^*) \psi(\hat{x}) + \sum_{j=1}^{p} \hat{\theta}_{g_{ij}} \psi(\hat{x}) u_j - (1 - \alpha) \hat{\theta}_{D_i} \psi(\hat{x}) \]

\[ L_i(s) = (\alpha(\hat{\theta}_f^*) \psi(\hat{x}) + \sum_{j=1}^{p} \hat{\theta}_{g_{ij}} \psi(\hat{x}) u_j - (1 - \alpha) \hat{\theta}_{D_i} \psi(\hat{x}) \] and

\[ L_i(s) = \text{proper stable transfer functions and } H_i(s)L_i(s) = \text{proper SPR transfer functions} \]. \( w_i \) is assumed to satisfy \( \|w_i\| \leq k_{w_i} \) [3, 5], where \( k_{w_i} \) is a positive constant. Suppose that

\[ L_i(s) = s^{\nu_i-1} + h_{s^{\nu_i-2}} + h_{s^{\nu_i-3}} + \cdots + h_{s^{\nu_i-1}} \]

such that \( H_i(s)L_i(s) \) are proper SPR transfer functions. Then the state-space realization of (24) can be rewritten as

\[ \dot{e}_i = A_i \dot{e}_i + B_i (\alpha(\hat{\theta}_f^*) \psi(\hat{x}) + \sum_{j=1}^{p} \hat{\theta}_{g_{ij}} \psi(\hat{x}) u_j) - (1 - \alpha) \hat{\theta}_{D_i} \psi(\hat{x}) - v_i + w_i \]

where

\[ A_i = A_i - K_{ai} C_i^T \in \mathbb{R}^{\nu_i \times \nu_i}, \quad B_i = [h_1, h_2, \cdots, h_{\nu_i-1}] \in \mathbb{R}^{\nu_i}, \quad \text{and} \quad C_i = [1, 0, \cdots, 0] \in \mathbb{R}^{\nu_i} \].

The adaptive laws to adjust parameter vectors \( \theta_f^* \), \( \theta_{g_i} \), and \( \theta_{D_i} \) are defined as

\[ \dot{\theta}_f = -\gamma_f \alpha(\hat{\theta}_f^*) \psi(\hat{x}) - \gamma_f \sigma_f \theta_f \]

\[ \dot{\theta}_{g_i} = -\gamma_{g_i} \alpha(\hat{\theta}_f^*) \psi(\hat{x}) u_i - \gamma_{g_i} \sigma_{g_i} \theta_{g_i} \]

\[ \dot{\theta}_{D_i} = -\gamma_{D_i} \alpha(\hat{\theta}_f^*) \psi(\hat{x}) - \gamma_{D_i} \sigma_{D_i} \theta_{D_i} \]

where \( \gamma_f > 0 \) (k=1,2,3) are learning rates and

\[ \sigma_f = \left\{ \begin{array}{ll}
\alpha(\hat{\theta}_f^*) \psi(\hat{x}) - \gamma_f \theta_f & \text{if } m_{\theta_f} \leq \theta_f \leq \zeta m_{\theta_f} \\
0, & \text{if } \|\theta_f\| \leq \zeta m_{\theta_f} \\
\sigma_f \phi, & \text{if } \|\theta_f\| > \zeta m_{\theta_f}
\end{array} \right. \] (29)

\[ \sigma_{g_i} = \left\{ \begin{array}{ll}
\alpha(\hat{\theta}_f^*) \psi(\hat{x}) u_i - \gamma_{g_i} \theta_{g_i} & \text{if } m_{\theta_{g_i}} \leq \theta_{g_i} \leq \zeta m_{\theta_{g_i}} \\
0, & \text{if } \|\theta_{g_i}\| \leq \zeta m_{\theta_{g_i}} \\
\sigma_{g_i} \phi, & \text{if } \|\theta_{g_i}\| > \zeta m_{\theta_{g_i}}
\end{array} \right. \] (30)

\[ \sigma_{D_i} = \left\{ \begin{array}{ll}
\alpha(\hat{\theta}_f^*) \psi(\hat{x}) - \gamma_{D_i} \theta_{D_i} & \text{if } m_{\theta_{D_i}} \leq \theta_{D_i} \leq \zeta m_{\theta_{D_i}} \\
0, & \text{if } \|\theta_{D_i}\| \leq \zeta m_{\theta_{D_i}} \\
\sigma_{D_i} \phi, & \text{if } \|\theta_{D_i}\| > \zeta m_{\theta_{D_i}}
\end{array} \right. \] (31)

where \( \zeta_1, \zeta_2, \zeta_3 \in [1,2] \) are scalars specified by the designer and

\[ \sigma_f = \frac{\|e_i\|}{\|\theta_f\|} \]

\[ \sigma_{g_i} = \frac{\|e_i^T \theta_{g_i} \psi(\hat{x}) u_i\|}{\|\theta_{g_i}\|} \]

\[ \sigma_{D_i} = \frac{\|e_i^T \theta_{D_i} \psi(\hat{x})\|}{\|\theta_{D_i}\|} \] (34)

Figure 2 illustrates a two-dimension example for \( \theta_f^* \). If the parameter vector is in the region \( \Omega_\alpha \) or on the boundary of the constraint set \( \Omega_\alpha \), but moving toward the inside of the region \( \Omega_\alpha \), then project the gradient vector \( \dot{\theta}_f \) onto the tangent of \( \Omega_\alpha \). If the parameter vector is in the region \( \Omega_\alpha \), then use the certain percentage of the projection. If the parameter vector is in
the region $\Omega$ or on the boundary of the constraint set $\Omega_0$ but moving toward the inside of the constraint set $\Omega$, then do not project the gradient vector $\hat{e}_i$.

**Lemma 1** [27]: The parameter projection algorithms (26)-(34) guarantee that the parameter vectors $\theta_j$, $\theta_{gj}$, and $\theta_{di}$ remain insider their respective regions $\Omega_j$, $\Omega_{gj}$, and $\Omega_{di}$.

**Theorem 1**: Consider the nonlinear systems (1) with the adaptive laws (26)-(34) and suppose that the compensated control inputs are chosen as

$$u_i = \begin{cases} 
\rho_i, & \text{if } \hat{e}_i \geq 0 \text{ and } \hat{e}_i > \sigma_i, \\
-\rho_i, & \text{if } \hat{e}_i < 0 \text{ and } \hat{e}_i > \sigma_i, \\
\rho_i \hat{e}_i / \sigma_i, & \text{if } \hat{e}_i < \sigma_i, i = 1, 2, \ldots, p
\end{cases}$$

(35)

where $\sigma_i$ are positive constants. Then $\hat{e}_i$ converge to zero as $t \to \infty$.

**Proof**: Given in the Appendix.

**Theorem 2**: Consider the nonlinear systems (1) with the adaptive laws (26)-(34). The control law is chosen as

$$u = \alpha (\hat{G}(\hat{x}) + k_{ci} u_i) + (\hat{F}(\hat{x}) + [y_m^{(1)}, y_m^{(2)}, \ldots, y_m^{(n_p)})^T + [K'_{c1} \hat{e}_1, K'_{c2} \hat{e}_2, \ldots, K'_{c_p} \hat{e}_p])^T + (1 - \alpha)u_q(\hat{x}) + u_s$$

(36)

where

$$\hat{F}(\hat{x}) = \begin{bmatrix}
0_{g_1} \phi(\hat{x}) \\
0_{g_2} \phi(\hat{x}) \\
\vdots \\
0_{g_p} \phi(\hat{x})
\end{bmatrix}, \hat{G}(\hat{x}) = \begin{bmatrix}
0_{g_1} \psi(\hat{x}) \\
0_{g_2} \psi(\hat{x}) \\
\vdots \\
0_{g_p} \psi(\hat{x})
\end{bmatrix}$$

(37)

and

$$u_q(\hat{x}) = [0_{g_{11}} \phi(\hat{x}), 0_{g_{12}} \phi(\hat{x}), \ldots, 0_{g_{p1}} \phi(\hat{x})]^T.$$  

(38)

Then, the closed-loop system is robust stable and $e_i$ converge to zero as $t \to \infty$.

**Proof**: Given in the Appendix.

The steps of the proposed hybrid direct/indirect adaptive Petri fuzzy neural network controller design are summarized in the following.

**Design Algorithm**:

**Step 1** Select the feedback and observer gain vectors $K_{ci}$, $K_{gi}$ such that the matrices $A_i - B_i K_{ci}^T$ and $A_i - K_{gi} C_i^T$ are Hurwitz matrices, respectively.

**Step 2** Choose appropriate values $\rho_i$ in (35), $\gamma_{di}$ in (26)-(28), $m_{gi}$, $m_{gdi}$, $m_{di}$, and $\hat{e}_i$ in (29)-(31).

**Step 3** From (16), select positive constants $k_u$ and $k_e$. Obtain the dynamic threshold value $d_{th}$.

**Step 4** Solve the state observer in (51).

**Step 5** Construct fuzzy sets for $\hat{x}_i$. From (18), compute the fuzzy basis vectors $\psi'$.

**Step 6** Select a weighting factor $\alpha \in [0,1]$ in (36). Then obtain the control law (36) and the update laws (26)-(28).

To summarize, Fig. 3 illustrates the overall scheme of the observer-based adaptive Petri fuzzy neural control proposed in this paper.

**5. Illustrative Examples**

This section presents the simulation results of the proposed observer-based hybrid Petri fuzzy neural network control for unknown nonlinear dynamical systems to illustrate that the tracking error of the closed-loop system can be made arbitrarily small. In addition, the simulation results confirm that the effect of
all the estimation errors and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

**Example 1:** Consider the trajectory-tracking problem of inverted pendulum system. Two cases corresponding to two different numbers of carts are simulated with a step size of 0.001. In case 1, we consider the problem of balancing of an inverted pendulum on a cart shown in Fig. 4. Let \( x_1 \) be the angle of the pendulum with respect to the vertical line. The dynamic equations of the inverted pendulum system \([10]\) are

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (f + gu + d_\alpha)
\]

\[y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\
x_2 \end{bmatrix}\]

(39)

where

\[
f = \frac{g_0 \sin(x_1) - mlx_2^2 \cos(x_1) \sin(x_1)}{M + m};
\]

\[g = \frac{\cos(x_1)}{l(M + m)} + \frac{\sin^2(x_1)}{l^2(M + m)} \frac{4l - m\cos^2(x_1)}{M + m};
\]

and \( M \) is the mass of the cart, \( m \) is the mass of the rod, \( g_0 = 9.8 \text{ m/s}^2 \) is the acceleration due to gravity, \( l \) is the half length of the rod, \( u \) is the control input, and \( d_\alpha \) is an external disturbance which is assumed to be a random value in the interval \([-0.03, 0.03]\). In this example, we assume that \( M = 1 \text{ kg}, m = 0.1 \text{ kg}, \) and \( l = 0.5 \text{ m} \).

Fig. 4. Inverted pendulum system.

The design parameters are selected as \( K_1 = [144, 24]^T \), \( K_2 = [60, 900]^T \), \( k_1 = 0.4 \), \( k_3 = 300 \), \( \gamma_1 = \gamma_2 = \gamma_3 = 5 \), and \( \rho = 20 \). We use the proposed control law in (36) to control the output \( y \) of the system to track the reference signal \( y_m(t) = 0.1 \sin(0.5t) + 0.1 \cos(t) \). The trajectories of system outputs \( y \) and reference signal \( y_m \) with \( \alpha = 0.3 \) are shown in Fig. 5. Fig. 6 illustrates that the curves of the states \( x_1 \) and \( \hat{x}_1 \) if \( \alpha = 0.3 \) is chosen. The response of control input \( u \) with \( \alpha = 0.3 \) is shown in Fig. 7. Applying the different weighting factor \( \alpha \), the tracking error performance of case 1 in example 1 is shown in Fig. 9. The simulation results indicate that the estimation state \( \hat{x}_1 \) takes very short time to catch up to the system state \( x_1 \). Moreover, the tracking performance is also very good.

Fig. 5. The trajectories of the system output \( y \) and the reference signal \( y_m \) (case 1) in Example 1. (a) Steady trajectories (0-20 sec). (b) Transient trajectories (0-1 sec).

Fig. 6. The trajectories of the state \( x_1 \) and the estimated state \( \hat{x}_1 \) (case 1) in Example 1. (a) Steady trajectories (0-20 sec). (b) Transient trajectories (0-0.3 sec).

Fig. 7. Response of control input \( u \) (case 1) in Example 1.

Fig. 8. Tracking performance with different \( \alpha \) (case 1) in Example 1.
In case 2, we consider a tracking problem of two inverted pendulums connected by a spring mounted on two carts, which is shown in Fig. 9. Define the state variables as \( x_{11} = \theta_1, x_{12} = \theta_1, x_{21} = \theta_2 \), and \( x_{22} = \dot{\theta}_2 \). The dynamic equations of the system [28] can be described as:

\[
\begin{align*}
\dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= [(g/c) - (ka(a - cl)/cm^2)]x_{11} - (m/M)\sin(x_{11})x_{12}^2 \\
&\quad + (ka(a - cl)/cm^2)x_{11}^2 + (1/cm^2)u_i + d_{ii} \\
y_i &= x_{11}, i, j = 1, 2 (i \neq j)
\end{align*}
\] (40)

where \( h_i(x, u), i = 1, 2 \) are unknown nonlinear functions. \( x_{1i} \) and \( x_{2i} \) are the angle and the angular velocity of the \( i \)th pendulum, respectively. \( u_i \) and \( y_i \) are the control force and the system output, respectively. The external disturbances \( d_{ii} \) are random values in the interval \([-0.03, 0.03]\). The parameters are assumed as follows: \( m = 1 \text{ kg} \) (mass of pendulum), \( M = 5 \text{ kg} \) (mass of cart), \( c = m/(m + M) \), \( a = 0.2 \), \( l = 1 \text{ m} \) (length of pendulum), \( k = 1 \text{ N/m} \) (spring constant), and \( g = 9.8 \text{ m/s}^2 \) (gravity constant). The control objective is to force the system output \( y_1 \) to track the reference signal \( y_{m1} = 0.09\pi \sin(0.5t) + 0.03\pi \sin(1.5t) \) and the system output \( y_2 \) to track the reference signal \( y_{m2} = 0.09\pi \cos(0.5t) + 0.03\pi \cos(1.5t) \). The design parameters are chosen as \( k_c = 0.4 \), \( k_i = 300 \), \( \gamma_{ki} = 5 \), and \( \rho_i = 15 \). The feedback and observer gain vectors are selected as \( K_c = [144, 24]^T \) and \( K_d = [60, 900]^T \), respectively.

![Fig. 9. Configuration of two inverted pendulums connected by a spring mounted on two carts.](image)

The initial states are chosen to be \( x(0) = [0.2, 0, 0, 0]^T \) and \( \dot{x}(0) = [0.1, 0, -0.1, 0]^T \). Figures 10 and 11 illustrate the outputs \( y_1 \) and \( y_2 \) of the system can quickly track the reference signal \( y_{m1} \) and \( y_{m2} \), respectively. Figures 12 and 13 illustrate that the proposed state observers can generate the estimated states \( \hat{x}_{11} \) and \( \hat{x}_{22} \) very quickly and accurately. The control inputs \( u_1 \) and \( u_2 \) are shown in Fig. 14. The simulation results indicate that the effect of all the modeling errors and the external disturbances on the tracking errors is attenuated efficiently by the proposed online adaptive controller.
Example 2: Let us consider a tracking problem of two-link robot manipulators, which is shown in Fig. 15. The dynamic equations of the system [5] can be described as:

\[
\mathbf{M} (\theta) \ddot{\theta} + \mathbf{C}(\theta, \dot{\theta}) \dot{\theta} + \mathbf{G}(\theta) = \mathbf{u} + \mathbf{u}_d
\]

where \( \theta = [\theta_1, \theta_2]^T \in \mathbb{R}^{2 \times 1} \) is the joint position vector, \( \dot{\theta} = [\dot{\theta}_1, \dot{\theta}_2]^T \in \mathbb{R}^{2 \times 1} \) is the joint velocity vector, \( \ddot{\theta} = [\ddot{\theta}_1, \ddot{\theta}_2]^T \in \mathbb{R}^{2 \times 1} \) is the joint acceleration vector, \( \mathbf{M}(\theta) \in \mathbb{R}^{2 \times 2} \) is the inertia matrix, \( \mathbf{C}(\theta, \dot{\theta}) \in \mathbb{R}^{2 \times 2} \) is the matrix of centripetal and Coriolis forces, \( \mathbf{G}(\theta) \in \mathbb{R}^{2 \times 2} \) is the gravity vector, \( \mathbf{u} = [u_1, u_2]^T \in \mathbb{R}^{2 \times 1} \) is the motor torque vector, and \( \mathbf{u}_d = [u_{d1}, u_{d2}]^T \in \mathbb{R}^{2 \times 1} \) is the vector of additive bounded disturbances. The matrices of the dynamic equations are expressed as follows:

\[
\mathbf{M}(\theta) = \begin{bmatrix}
\frac{1}{4} m_1 + m_2 & \frac{1}{4} m_2 l_1 \cos(\theta_1) & \frac{1}{4} m_2 l_2 \cos(\theta_1) \\
\frac{1}{4} m_2 l_1 \cos(\theta_1) & \frac{1}{2} m_2 l_1^2 + m_1 l_2 \cos(\theta_1) & \frac{1}{4} m_2 l_2 \cos(\theta_1) \\
\frac{1}{4} m_2 l_2 \cos(\theta_1) & \frac{1}{4} m_2 l_2 \cos(\theta_1) & \frac{1}{2} m_2 l_2^2
\end{bmatrix}
\]

\[
\mathbf{C}(\theta, \dot{\theta}) = \begin{bmatrix}
-\frac{1}{2} m_2 l_1 (2 \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1^2) \sin(\theta_1) \\
\frac{1}{2} m_2 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2)
\end{bmatrix}
\]

\[
\mathbf{G}(\theta) = \begin{bmatrix}
\frac{1}{2} m_1 g l_1 \cos(\theta_1) + \frac{1}{2} m_2 g l_2 \cos(\theta_1 + \theta_2) \\
\frac{1}{2} m_1 g l_1 \cos(\theta_2) + \frac{1}{2} m_2 g l_2 \cos(\theta_1 + \theta_2)
\end{bmatrix}
\]

where \( l_1 \) and \( l_2 \) are the lengths; \( m_1 \) and \( m_2 \) are the mass of the links, respectively. Define the state variables as \( x_{11} = \theta_1, x_{12} = \dot{\theta}_1, x_{21} = \theta_2, \) and \( x_{22} = \dot{\theta}_2 \). The parameter values are \( m_1 = 0.5 \text{kg}, \ m_2 = 0.5 \text{kg}, \ l_1 = 1 \text{m}, \ l_2 = 0.8 \text{m} \) and \( g = 9.8 \text{m/s}^2 \). \( \mathbf{d}_d = [d_{d1}, d_{d2}]^T = \mathbf{M}^{-1}(\theta) \mathbf{u}_d \) is a vector of the external disturbances which are assumed to be random values in the interval \([-0.03, 0.03]\). According to the initial states, four cases are simulated, as shown in Table I.

Table I. Four Cases of Initial States for Example 2.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Initial states</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>( \mathbf{x}(0) = [0.2, 0, 0, 0]^T, \ \mathbf{x}(0) = [0.1, 0, -0.1, 0]^T )</td>
</tr>
<tr>
<td>Case 2</td>
<td>( \mathbf{x}(0) = [0.25, 0, 0.15, 0]^T, \ \mathbf{x}(0) = [0, 0, 0, 0]^T )</td>
</tr>
<tr>
<td>Case 3</td>
<td>( \mathbf{x}(0) = [-0.1, 0, 0.45, 0]^T, \ \mathbf{x}(0) = [0, 0, 0.6, 0]^T )</td>
</tr>
<tr>
<td>Case 4</td>
<td>( \mathbf{x}(0) = [-0.15, 0, 0.4, 0]^T, \ \mathbf{x}(0) = [0.1, 0, 0.5, 0]^T )</td>
</tr>
</tbody>
</table>

The control objective is to force the angular displacements \( x_{11} = \dot{\theta}_1 \) and \( x_{21} = \dot{\theta}_2 \) of the two-link planar manipulator to track the desired trajectories \( \dot{y}_{m1} = 0.1 \pi \cos(t) \) and \( \dot{y}_{m2} = 0.1 \pi \sin(t) \), respectively. Figs. 16 and 17 show the tracking trajectories of the first angular displacement \( x_{11} \), the first reference signal \( \dot{y}_{m1} \), the second angular displacement \( x_{21} \), and the second reference signal \( \dot{y}_{m2} \). From Figs. 18 and 19, it is observed the state observers \( \hat{x}_{11} \) and \( \hat{x}_{21} \) can generate the estimated states very fast and correct. Besides, the control inputs \( u_1 \) and \( u_2 \) are shown in Fig. 20. The simulation results in this case shown in Figs. 18-20 demonstrate that the effectiveness and applicability of the proposed method.
6. Conclusions

Under the constraint that not all system states can be measured, an observer-based hybrid intelligent output-feedback controller design for trajectory tracking of uncertain nonlinear multivariable dynamical systems was proposed in this paper. By using the PFNN approximator and the observer-based hybrid robust control law, the computation burden can be efficiently shortened and the effect of all the modeling errors and the external disturbances on the tracking errors is attenuated efficiently. The proposed online intelligent controller design methodology is more flexible by using the weighting factor $\alpha$ adopted to combine direct adaptive PFNN controller with indirect adaptive PFNN controller. Moreover, improved generalized projection-update laws were utilized to tune the free parameters to prevent parameters drift. By using strictly-positive-real (SPR) Lyapunov theory, the proposed overall scheme guarantees that the closed-loop systems can achieve the successful system control, the valuable state observer, and the desired tracking performance. The effectiveness of the improved adaptive trajectory-tracking control approach is verified by computer simulation results of the inverted pendulum systems and the two-link robot manipulators. In the future, investigation on the adaptive tuning of the weighting factor $\alpha$ and designing uncertain nonaffine nonlinear systems will be interesting research topics in this field.

Appendix

A. Proof of Theorem 1

Consider the Lyapunov-like function candidate

$$V = \sum_{i=1}^{n} V_i$$

where

$$V_i = \frac{1}{2} \sum_{i=1}^{n} \bar{e}_i^T \Gamma_i \bar{e}_i + \frac{\alpha}{2\gamma_{i1}} \bar{\theta}^T \bar{\theta} + \frac{\alpha}{2\gamma_{i2}} \sum_{j=1}^{n} \bar{\theta}_j^T \bar{\theta}_j + (1-\alpha) \sum_{j=1}^{n} \bar{\theta}_j^T \bar{\theta}_j$$

where $\Gamma_i = \Gamma_i^T > 0$. Differentiating (42) with respect to time, we get

$$\dot{V}_i = \frac{1}{2} \sum_{i=1}^{n} \bar{e}_{i1}^T \Gamma_i \bar{e}_{i1} + \frac{1}{2} \sum_{i=1}^{n} \bar{e}_{i2}^T \Gamma_i \bar{e}_{i2} + \frac{\alpha}{\gamma_{i1}} \bar{\theta}_{j1}^T \bar{\theta}_{j1} + \frac{\alpha}{\gamma_{i2}} \sum_{j=1}^{n} \bar{\theta}_{j1}^T \bar{\theta}_{j1}$$

$$+ (1-\alpha) \sum_{j=1}^{n} \bar{\theta}_{j1}^T \bar{\theta}_{j1}.$$
\[ + \sum_{j=1}^{p} \hat{\theta}_{gj}^\top \psi(\hat{x})u_j - (1 - \alpha) \hat{\theta}_{\alpha}^\top \psi(\hat{x}) - v_p + w_p \]
\[ + \frac{\alpha}{\gamma_1} \hat{\theta}_{\alpha}^\top \psi(\hat{x}) + \frac{\alpha}{\gamma_2} \sum_{j=1}^{p} \hat{\theta}_{gj}^\top \psi(\hat{x}) + \frac{(1 - \alpha)}{\gamma_3} \hat{\theta}_{\alpha}^\top \psi(\hat{x}). \]

(45)

Because \( H_i(s)L_i(s) \) are SPR, there exists \( \Gamma_i = \Gamma_i^T > 0 \) such that
\[ A_i^\top \Gamma_i + \Gamma_i A_i = -Q_i \]
\[ \Gamma_i B_i = C_i \]
where \( Q_i = Q_i^T > 0 \). By using (46), (45) becomes
\[ \dot{V}_i = -\frac{1}{2} \hat{e}_i^\top Q \hat{e}_i + \hat{e}_n (\alpha(\hat{\theta}_{\alpha}^\top \psi(\hat{x}) + \sum_{j=1}^{p} \hat{\theta}_{gj}^\top \psi(\hat{x})u_j)
\[ - (1 - \alpha) \hat{\theta}_{\alpha}^\top \psi(\hat{x}) + \frac{\alpha}{\gamma_1} \hat{\theta}_{\alpha}^\top \psi(\hat{x}) + \frac{\alpha}{\gamma_2} \sum_{j=1}^{p} \hat{\theta}_{gj}^\top \psi(\hat{x})u_j)
\[ + \frac{(1 - \alpha)}{\gamma_3} \hat{\theta}_{\alpha}^\top \psi(\hat{x}) \]

(47)

By using (35), and the fact \( \lambda_{\min}(Q_i) \| \hat{e}_i \|^2 \geq \lambda_{\min}(Q_i) \| \hat{e}_i \|^2 \), where \( \lambda_{\min}(Q_i) > 0 \), we have
\[ \dot{V}_i \leq -\frac{1}{2} \lambda_{\min}(Q_i) \| \hat{e}_i \|^2 + \hat{e}_n (\alpha(\hat{\theta}_{\alpha}^\top \psi(\hat{x}) + \sum_{j=1}^{p} \hat{\theta}_{gj}^\top \psi(\hat{x})u_j)
\[ - (1 - \alpha) \hat{\theta}_{\alpha}^\top \psi(\hat{x}) + \frac{\alpha}{\gamma_1} \hat{\theta}_{\alpha}^\top \psi(\hat{x}) + \frac{\alpha}{\gamma_2} \sum_{j=1}^{p} \hat{\theta}_{gj}^\top \psi(\hat{x})u_j)
\[ + \frac{(1 - \alpha)}{\gamma_3} \hat{\theta}_{\alpha}^\top \psi(\hat{x}) \]

(48)

Inserting (26)-(34) in (48) yields
\[ \dot{V} \leq -\frac{1}{2} \eta \sum_{i=1}^{p} \| \hat{e}_i \|^2 \]

(49)

where \( \eta = \min_{\lambda_{\min}(Q_i)} \). (42) and (49) only guarantee that \( \hat{e}_n(t) \in L_\infty \) and \( \hat{e}_n(t) \in L_\infty \), but not its convergence. Because all variables in the right-hand side of (49) yields
\[ \int_0^\infty \sum_{i=1}^{p} \| \hat{e}_i(t) \|^2 \, dt \leq \frac{1}{2} \eta \]

Since the right side of (50) is bounded, \( \hat{e}_n(t) \in L_\infty \). Using Barbalat’s lemma [25], we have \( \lim_{t \to \infty} \| \hat{e}_n(t) \| = 0 \). This completes the proof.

B. Proof of Theorem 2
First, from Theorem 1, we have \( \lim_{t \to \infty} \| \hat{e}_n(t) \| = 0 \) and \( \hat{e}_n(t) \in L_\infty \). Using (11), we can obtain the error dynamics as
\[ \hat{e}_e = (A_i - B_i K_i^F) \hat{e}_e + K_{\alpha} C_i^\top \hat{e}_e \]
\[ \hat{e}_n = C_i^\top \hat{e}_e. \]

(51)

Similarly, \( \hat{e}_e(t) \) is bounded because \( A_i - B_i K_i^F \) is a Hurwitz matrix and \( \hat{e}_n(t) \) converge to zero as \( t \to \infty \).

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