Adaptive Fuzzy Dynamic Surface Control for Ball and Beam System

Yeong-Hwa Chang, Wei-Shou Chan, Chia-Wen Chang, and C. W. Tao

Abstract

This paper mainly investigates the control for the ball and beam system. Combined with the conventional dynamic surface control, an adaptive fuzzy scheme is proposed for the equilibrium balance of the ball. First, with the proper defined tracking errors of state variables, an integrating of tracking errors is considered as the fuzzy input. The parameters of fuzzy controller can be optimized by using the proposed adaptive mechanism such that the control performance can be improved. For the proposed control scheme, the closed-loop stability is also analyzed by the Lyapunov theorem. Simulations are performed to evaluate the effectiveness of the proposed works. From simulation results, the proposed adaptive fuzzy dynamic surface control scheme can provide better tracking responses than the conventional dynamic surface control counterparts.

Keywords: Ball and Beam System, Adaptive Fuzzy Dynamic Surface Control, Dynamic Surface Control, Underactuated System.

1. Introduction

In the real world, physical systems are often under-actuated, where the degree of inputs is smaller than the degree of state space. The control of under-actuated systems is challengeable and attracts a worthy discussion. Ball and beam system, a typical under-actuated system, is a representative platform in academics due to the inherent non-linearity and instability.

In the literature, the fuzzy control theory is utilized for the beam and ball system. Based on the experts’ experience, fuzzy controllers can provide satisfactory responses without the knowledge of exactly mathematical models [1]-[3]. However, the parameters of conventional fuzzy controller are required to be manually tuned to get better performance. Also, lack of systematic procedure to analyze the stability of closed-loop system is another task under taken. Backstepping control, typically addressed in nonlinear systems, is a systematic design procedure, where the stability analysis relies on the Lyapunov stability theorem [4]-[9]. Nevertheless, as the iterative procedure going on, the formulation of the stabilizing control command will be more complex with the increasing of system order. In [10]-[15], to relieve the possible derivation burden, a so-called dynamic surface control (DSC) was proposed. Similar to the backstepping control, DSC is also an iterative design procedure. First, through the Lyapunov stability analysis, a virtual desired trajectory of each state is defined. Then a stabilizing function is determined and a low-pass filter is utilized to overcome the derivation burden mentioned in the backstepping control. As the tracking error of virtual objective function converges, the required stabilizing controller can be iteratively derived.

In this paper, the last step of the conventional DSC is replaced with an adaptive fuzzy framework. The adaptive tuning mechanism can improve the tracking performance by optimizing the fuzzy parameters. The proposed adaptive fuzzy DSC (AFDSC) is utilized for a ball-and-beam system to balance the ball. The organization of this paper is as follows. In Sec. 2, the dynamic characteristics of the ball-beam system are discussed. The derivation procedures of the AFDSC are investigated in Sec. 3. The closed-loop stability of the controlled ball-and-beam system with the AFDSC is addressed in Sec. 4. In Sec. 5, simulation results and the comparisons with conventional DSC are provided. The concluding remarks are given in Sec. 6.

2. Ball and Beam System

The ball and beam system is a classic dynamic system discussed in academics. It is inherently non-linear and unstable, and despite its simple nature. The ball and beam system with a central pivot point housing a servo motor is shown in Fig. 1, where balancing the ball on the beam is the main task of concern. To derive the dynamic model of the ball and beam system, the following Euler-Lagrange dynamic equation is considered.

$$\frac{d}{dt} \left[ \frac{\partial L}{\partial q} \right] \frac{\partial L}{\partial \dot{q}} = Q$$  \hspace{1cm} (1)
where \( L = K - P \), \( K \) is the kinetic energy, \( P \) is the potential energy, \( Q \) is the generalized force, and \( q \) is the generalized coordinate. The kinetic energy and potential energy of the ball and beam system can be determined as follows

\[
K = 0.5(m_b + \frac{J_b}{R})r^2 + 0.5(J_b + J_b + m_br^2)\dot{\theta}_b^2
\]

\[
P = m_bgr\sin\theta_b
\]

where \( J_b \) is the inertia of the beam, \( J_b \) is the inertia of the ball, \( m_b \) is the mass of the ball, \( g \) is the gravity, \( r \) is the position of ball and \( \theta_b \) is the angle of beam.

The ball and beam system is an under-actuated system, where the driving torque is directly coupled on the variation of the beam angular. In (2) and (3), \( r \) and \( \theta_b \) are two dynamic variables. Thus, from (1), the Euler-Lagrange dynamic equation of the ball-and-beam system can be derived as

\[
\begin{bmatrix}
\frac{\partial L}{\partial r} \\
\frac{\partial L}{\partial \dot{r}} \\
\frac{\partial L}{\partial \theta_b} \\
\frac{\partial L}{\partial \dot{\theta}_b}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
m_bgr\sin\theta_b \\
m_bgr\cos\theta_b
\end{bmatrix} + \begin{bmatrix}
\ddot{r} \\
\ddot{\theta}_b
\end{bmatrix}
\]

\[
E = \int (\frac{1}{2}m_b\dot{r}^2 + \frac{1}{2}J_b\dot{\theta}_b^2 + m_bgr\sin\theta_b)\,dt
\]

\[
= \int \frac{1}{2}m_b\dot{r}^2 + \frac{1}{2}J_b\dot{\theta}_b^2 + m_bgr\sin\theta_b - \frac{1}{2}m_br^2\dot{\theta}_b^2 \,dt
\]

where \( \tau \) is the driving torque. Substituting (2) and (3) into (4), equivalently, we have

\[
\begin{bmatrix}
\frac{\partial L}{\partial r} \\
\frac{\partial L}{\partial \dot{r}} \\
\frac{\partial L}{\partial \theta_b} \\
\frac{\partial L}{\partial \dot{\theta}_b}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
m_bgr\sin\theta_b \\
m_bgr\cos\theta_b
\end{bmatrix} + \begin{bmatrix}
\ddot{r} \\
\ddot{\theta}_b
\end{bmatrix}
\]

Let the state variables be defined as \( x_1 = r \), \( x_2 = \dot{r} \), \( x_3 = \theta_b \), and \( x_4 = \dot{\theta}_b \). Thus, the state space representation of the ball-and-beam system can be described in the following (4)

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= bx_1x_3^2 - bg \sin x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= a(\tau - 2m_bx_1x_2x_4 - m_bgrx_1 \cos x_3)
\end{align*}
\]

where \( a = (J_b + J_b + m_br^2)^{-1} \), \( b = m_b(J_b + J_bR^2)^{-1} \).

It is noted that the driving torque is provided with a DC motor, shown in Fig. 2. The characteristics of the DC motor are given as

\[
e = L_a \frac{di_a}{dt} + r_a i_a + e_b
\]

\[
\tau = k_e i_a = J_a \frac{d\dot{\theta}_m}{dt} + B_e \dot{\theta}_m
\]

where \( \dot{\theta}_m \) is the angular speed, \( L_a \) is armature inductance, \( r_a \) is the armature resistance, \( i_a \) is the armature exciting current, \( k_e \) is the torque constant, and \( e_b \) is the back EMF voltage. According to the energy conservation, the following energy equivalence holds

\[
e_b \cdot i_a = r_a \cdot \dot{\theta}_m
\]

It is known that \( e_b \) is proportional to the angular speed, \( e_b = k_e \dot{\theta}_m \) for some constant \( k_e \). From (8) and (9), it can be obtained that \( k_e = k_e \).

Since \( L_a \) is small enough to be neglected, (7) and (8) can be simplified as

\[
e = r_a i_a + k_e \dot{\theta}_m
\]

where \( i_a = k_e^{-1} \tau_m \). In Fig. 2, the gear ratio is denoted as \( n \), and the following relationships are known

\[
\frac{\dot{\theta}_m}{\dot{\theta}_u} = \frac{\tau}{\tau_u} = n
\]

By some manipulations, the torque to the ball and beam system can be derived as

\[
\tau = n^{-1} k_e r_a^{-1} \dot{\theta}_m = n r_a^{-1} k_e \dot{r}_a - n^{-1} k_e \dot{r}_a k_e^{-1} x_4
\]

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\]

\[
\begin{figure}[h]
\begin{center}
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Illustration of the ball-and-beam system.}
\end{center}
\end{figure}

\[
\begin{figure}[h]
\begin{center}
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Equivalent circuit of the armature-controlled DC motor.}
\end{center}
\end{figure}

\[
\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|}
\hline
Parameter & Value  \\
\hline
\hline
\( m_a \) & 0.01 kg \\
\( L_a \) & 0.026 H \\
\hline
\hline
\( m_b \) & 0.11 kg \\
\( k_e \) & 0.058 V/ rad/s \\
\hline
\hline
\( l \) & 1 m \\
\( r_a \) & 3.365 \Omega \\
\hline
\hline
\( R \) & 0.006 \Omega \\
\( J_a \) & 2.76 \times 10^{-4} \text{ kg m}^2 \\
\hline
\hline
\( J_b \) & 0.144 \times 10^{-4} \text{ kg m}^2 \\
\( B_e \) & 5.23 \times 10^{-2} \text{ Nm rad/s} \\
\hline
\hline
\( J_b \) & 9.6 \times 10^{-3} \text{ kg m}^2 \\
\( n \) & 15 \\
\hline
\hline
\( g \) & 9.8 \text{ m/s}^2 \\
\( k_i \) & 0.058 \text{ Nm/A} \\
\hline
\end{tabular}
\end{center}
\caption{Parameters of the ball-and-beam system.}
\end{table}

Substituting (12) into (5), we have the following equivalent state-space representation
3. Adaptive Fuzzy Dynamic Surface Control

An adaptive scheme of the dynamic surface control will be discussed in this section.

**Step 1:**
Define

$$S_i = x_i - y_d$$

(17)

where $x_i$ is the position of ball, and $y_d$ is the expected position or command. It is desired to track $y_d$ with any initial condition. Taking the derivative of (17), we have

$$\dot{S}_i = \dot{x}_i - \dot{y}_d.$$  

(18)

Suppose $\tau_i$ is a stabilizing function to be determined for (13). Choosing the Lyapunov function candidate $V_i(S_i) = 0.5S_i^2$, from (13) and (18), the relative derivative can be obtained as

$$\dot{V}_i(S_i) = S_i\dot{S}_i = S_i(x_i - \dot{y}_d)$$

(19)

It can be seen that (19) is negative definite if

$$x_i = \tau_i = x_i - \tau_i, k_i > 0.$$  

(20)

Thus, with the designated $x_i$ of (20), the state variable $x_i$ can stably track the $y_d$. To eliminate the phenomenon of oscillation, a low-pass filter is considered as follows,

$$\tau_i = \dot{x}_i = \dot{x}_i - \dot{y}_d = k_iS_1 + \dot{y}_d, k_1 > 0.$$  

(21)

With a proper chosen $\tau_i$, the smoothed $x_{2d}(t)$ can be equivalently considered as the required $x_2$.

**Step 2:**
Analogous to the discussion in Step 1, an error variable is defined as

$$S_2 = x_2 - x_{2d}.$$  

(22)

From (14), the derivative of (22) can be available as

$$\dot{S}_2 = \dot{x}_2 - \dot{x}_{2d} = bx_2 \dot{x}_2 - bg \sin x_3 - \dot{x}_{2d}.$$  

(23)

A Lyapunov function candidate is chosen as $V_2(S_2) = 0.5S_2^2$. From (23), the derivative of $V_2$ can be derived as follows

$$\dot{V}_2(S_2) = S_2\dot{S}_2 = S_2(bx_2 \dot{x}_2 + bx_1 \dot{x}_1 - bg \sin x_3 - \dot{x}_{2d})$$

(24)

It can be observed that (24) is negative definite if

$$x_1 = \tau_2 = \sin^{-1}(bx_1 \dot{x}_1 - x_{2d} - k_2S_2), k_2 > 0.$$  

(25)

Thus the state variable $x_2$ asymptotically tracks $x_{2d}$ with the designated $x_3$ of (25). Also, to eliminate the oscillation, a low pass filter is utilized as follows

$$\tau_3 = \dot{x}_3 = \dot{x}_3 - \dot{x}_{3d} = \tau_3, x_{3d}(0) = \tau_3(0).$$  

(26)

**Step 3:**
The following derivation is similar to the discussion in Step 2. First, an error variable is defined as

$$S_3 = x_3 - x_{3d}.$$  

(27)

From (15), the derivative of (27) can be obtained as

$$\dot{S}_3 = \dot{x}_3 - \dot{x}_{3d} = x_4 - \dot{x}_{3d}.$$  

(28)

A Lyapunov function candidate is chosen as $V_3(S_3) = 0.5S_3^2$. From (28), the derivative of $V_3$ can be obtained as

$$\dot{V}_3(S_3) = S_3\dot{S}_3 = S_3(x_4 - \dot{x}_{3d})$$

(29)

To ensure the negative definiteness of $\dot{V}_3(S_3)$, the required $x_4$ can be designed as

$$x_4 = \tau_4 = \dot{x}_4 - k_3S_3, k_3 > 0.$$  

(30)

It can be seen that $\dot{V}_3(S_3)$ is negative definite as expected.

$$\tau_4 = \dot{x}_4 = \dot{x}_4 - \dot{x}_{4d}(t) = \tau_4, x_{4d}(0) = \tau_4(0).$$  

(31)

**Step 4:**
Following from the Step 3, an error state is defined as

$$S_4 = x_4 - x_{4d}.$$  

(32)

Before proceeding to the next, it will be helpful to identify the directionality of state variables. In Fig. 2, if the ball is in the right of the central pivot, $x_1$ is positive. In addition, a positive $x_3$ means that the beam is in the counterclockwise direction from the horizontal equilibrium state. Consequently, it can be figured out that the ball is moving to the right with a positive $x_2$ and the beam is rotating counterclockwise with a positive $x_4$.

Referring to (17), (22), (27), and (32), $S_1$, $S_2$, $S_3$, and $S_4$ represent the tracking errors of ball position, ball velocity, beam angle and beam angular velocity, respectively. In order to speed the convergence rate, an integrated variable is defined as follows

$$S = S_1 - S_2 + S_3 + S_4.$$  

(33)

It is noted that the signs designated in (33) are based on the physical consistency. For example, if $S_1$ and $S_2$ are negative, and $S_3$ and $S_4$ are positive, the relative $S$ is positive. In this situation, referred to the signs defined before, negative $S_1$ and $S_2$ indicate that the ball is in the left to the central pivot, and the ball is further moving away to the left. Analogously, positive $S_3$ and $S_4$ means that the beam is rotating in the counterclockwise direction. Thus a clockwise torque is required for the purpose of ball balancing, i.e. a positive $S$ represents a counterclockwise torque.

When $S$ converges to zero, all of the subsystems guaranteed to be stable and the system states can
asymptotically track the respective virtual commands. In this paper, combined with the conventional DSC, an adaptive fuzzy controller is proposed, where \( S \) is considered as the input variable. Without loss of generality, the input and output membership functions are Gauss and singleton functions, respectively. The input membership functions are represented as follows

\[
\xi_s(S, \beta, c_i) = \exp(-\beta_i^2 (S - c_i)^2), \quad i = 1, \ldots, M
\]

where \( \beta_i \) and \( c_i \) respectively represent the width and center of the \( i \)th input membership function, \( \beta = [\beta_1, \beta_2, \ldots, \beta_M]^T \), \( C = [c_1, c_2, \ldots, c_M]^T \).

Let the singleton output membership function be

\[
\tilde{\xi} = \xi - \hat{\xi}
\]

and

\[
\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_M]^T
\]

Let the singleton output membership function be represented as

\[
\theta = [\theta_1, \theta_2, \ldots, \theta_M]^T
\]

It is known that the performance of fuzzy control can be improved by tuning the parameters of membership functions. Let \( u_{\text{fuzzy}}(S, \xi, \theta) \) be denoted as the fuzzy output corresponding to the fuzzy input \( S \) and membership functions \( \xi(S, \beta, C) \). In this paper, an adaptive fuzzy control scheme is adopted as follows

\[
u_{\text{fuzzy}} = u_{\text{fuzzy}}(S, \xi, \theta) - \tilde{u}_{\text{fuzzy}}(S, \hat{\xi}, \hat{\theta})
\]

where \( \hat{\xi} \) and \( \hat{\theta} \) are the optimized estimation parameters and \( \tilde{u}_{\text{fuzzy}}(S, \hat{\xi}, \hat{\theta}) \) is the associated defuzzified output, and \( \tilde{u}_{\text{fuzzy}} \) is the estimation deviation.

The estimation errors are defined as

\[
\tilde{\xi} = \xi - \hat{\xi}
\]

\[
\tilde{\theta} = \theta - \hat{\theta}
\]

The fuzzy IF-THEN rules are expressed as

\[
R^l : \text{IF } S \text{ IS } F^l, \quad \text{THEN } \hat{u}_{\text{fuzzy}} = G^l, \quad l = 1, \ldots, M
\]

Thus the defuzzified output can be obtained as

\[
u_{\text{fuzzy}} = \sum_{i=1}^{M} \hat{\theta}_i \xi_i(S, \hat{\beta}_i, \hat{c}_i) = \hat{\beta}^T \hat{\xi}(S, \hat{\beta}, \hat{C})
\]

in which

\[
\hat{\beta} = [\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_M]^T,
\]

\[
\hat{C} = [c_1, c_2, \ldots, c_M]^T
\]

\[
\hat{\xi}(S, \hat{\beta}, \hat{C}) = [\hat{\xi}_1(S, \hat{\beta}_1, \hat{c}_1), \hat{\xi}_2(S, \hat{\beta}_2, \hat{c}_2), \ldots, \hat{\xi}_M(S, \hat{\beta}_M, \hat{c}_M)]^T
\]

\[
\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_M]^T
\]

The proposed controller is described as

\[
u(t) = \tilde{u}_{\text{fuzzy}}(S, \hat{\theta}, \hat{\beta}, \hat{C}) + u_c(t)
\]

where \( u_c(t) \) is the compensation controller. Taking the power series expansion, we have the following linearization equation

\[
\tilde{\xi} = \begin{bmatrix}
\xi_1
\xi_2
\vdots
\xi_M
\end{bmatrix}
\]

\[
\tilde{\theta} = \begin{bmatrix}
\frac{\partial \xi_1}{\partial \beta_1}
\frac{\partial \xi_2}{\partial \beta_2}
\vdots
\frac{\partial \xi_M}{\partial \beta_M}
\end{bmatrix}
\]

\[
\tilde{C} + \begin{bmatrix}
\frac{\partial \xi_1}{\partial \theta_1}
\frac{\partial \xi_2}{\partial \theta_2}
\vdots
\frac{\partial \xi_M}{\partial \theta_M}
\end{bmatrix}_{\beta=\hat{\beta}}
\]

\[
= G \tilde{\xi} + H \tilde{\theta} + h.o.t.
\]

where \( h.o.t. \) means the high-order terms, \( \tilde{C} = C - \hat{C} \), \( \tilde{\theta} = \theta - \hat{\theta} \).

From (35)-(37) and (40)-(41), we have

\[
u(t) = u_{\text{fuzzy}} - \tilde{u}_{\text{fuzzy}} + u_c(t)
\]

\[
= \theta^T \tilde{\xi} - \hat{\theta}^T \hat{\xi} + u_c(t)
\]

\[
= (\hat{\theta} + \hat{\theta})^T (\hat{\xi} + \hat{\xi}) - \hat{\theta}^T \hat{\xi} + u_c(t)
\]

\[
= \hat{\theta}^T \hat{\xi} + \hat{\theta}^T \hat{\xi} + \hat{\theta}^T \hat{\xi} + u_c(t)
\]

\[
= \hat{\theta}^T \hat{\xi} + \hat{\theta}^T (G \tilde{\xi} + H \tilde{\theta} + h.o.t.) + \hat{\theta}^T \tilde{\xi} + u_c(t)
\]

Let \( \varepsilon = \hat{\theta}^T (h.o.t.) + \hat{\theta}^T \tilde{\xi} \). Eq. (46) can be rewritten as

\[
u(t) = \hat{\theta}^T (G \tilde{\xi} + H \tilde{\theta}) + \hat{\theta}^T \tilde{\xi} + \varepsilon + u_c(t)
\]

From (16), (32) and (47), the derivative of \( S_4 \) can be represented as

\[
\dot{S}_4 = n(\tilde{\xi}^T (G \tilde{\xi} + H \tilde{\theta}) + \tilde{\theta}^T \tilde{\xi} + \varepsilon + u_c(t))
\]

\[
= n\tilde{\xi}^T \tilde{\xi} - 2amg_kx_2x_4 - amg_kx_1 \cos x_3 - \dot{x}_{ad}
\]

It is denoted that

\[
\eta = n\tilde{\xi}^T \tilde{\xi} - n\tilde{\xi}^T \tilde{\xi} - 2amg_kx_2x_4 - amg_kx_1 \cos x_3 - \dot{x}_{ad}
\]

Equivalently, from (48), we have

\[
\dot{S}_4 = \tilde{\xi}^T (G \tilde{\xi} + H \tilde{\theta}) + \tilde{\theta}^T \tilde{\xi} + u_c(t) + \eta
\]

It is known that \( \eta \) is an uncertain term. Suppose \( \eta \) is bounded such that \( |\eta| \leq E \) for a constant \( E \). However, since \( E \) is unavailable, it is sufficient to replace \( E \) by the estimation value \( \hat{E} \), and the estimation error is defined as

\[
\hat{E} = E - \hat{E}
\]

With the following adaptive law and \( u_c(t) \), the associated tracking errors of the closed-loop system will converge to zeros.
\[ \dot{C} = nr_S r_a^{-1} ak G^T \dot{\theta} \]  
(51)

\[ \dot{\beta} = nr_S r_a^{-1} ak H^T \dot{\theta} \]  
(52)

\[ \dot{\hat{\theta}} = nr_S r_a^{-1} ak \dot{\zeta} \]  
(53)

\[ \dot{E} = r_i |S| \]  
(54)

\[ u_c = -r_i (nak)_C^{-1} \dot{E} \text{sgn} (S) \]  
(55)

The proposed controller is given as

\[ u(t) = \theta^T \dot{\zeta} (S, \beta, \dot{C}) - r_i (nak)_C^{-1} \dot{E} \text{sgn}(S) \]  
(56)

### 4. Stability Analysis

In this section, the stability of the AFDCS controller ball-and-beam system will be analyzed. First, a Lyapunov function candidate is chosen as

\[ V = 0.5 \sum_{i=1}^{3} V_i + 0.5 S_i^2 + \frac{E^2}{2r_1} + \frac{G^C i C}{2r_2} + \frac{\dot{\beta}^T \ddot{\beta}}{2r_3} + \frac{\dot{\theta}^T \ddot{\theta}}{2r_4} \]  
(57)

From (49), the derivative of (57) can be obtained as

\[ \dot{V} = -k_i S_i^2 - k_2 S_2^2 - k_3 S_3^2 + \dot{C}^T (nS_i r_a^{-1} ak G^T \dot{\theta} \] 

\[ - \frac{1}{r_2} \dot{\dot{C}} + \beta^T (nS_i r_a^{-1} ak H^T \dot{\theta} - \frac{1}{r_3} \dot{\beta}) + \ddot{\theta}^T \] 

\[ (nS_i r_a^{-1} ak \dot{\zeta} - \frac{1}{r_4} \dot{E} \dot{\zeta}) + S_i (nr_a^{-1} ak u_i (t) + \eta) - \frac{1}{r_1} \dot{\dot{E}} \]  
(58)

Equivalently, we have

\[ \dot{V} = -k_i S_i^2 - k_2 S_2^2 - k_3 S_3^2 + \dot{C}^T (nS_i r_a^{-1} ak G^T \dot{\theta} \] 

\[ - \frac{1}{r_2} \dot{\dot{C}} + \beta^T (nS_i r_a^{-1} ak H^T \dot{\theta} - \frac{1}{r_3} \dot{\beta}) + \ddot{\theta}^T \] 

\[ (nS_i r_a^{-1} ak \dot{\zeta} - \frac{1}{r_4} \dot{E} \dot{\zeta}) + S_i (nr_a^{-1} ak u_i (t) + \eta) - \frac{1}{r_1} \dot{\dot{E}} \]  
(59)

Substituting (51)-(53) into (59), we have

\[ \dot{V} = -k_i S_i^2 - k_2 S_2^2 - k_3 S_3^2 + S_i (nr_a^{-1} ak u_i (t) + \eta) \] 

\[ - \frac{1}{r_1} \dot{\dot{E}} \]  
(60)

Furthermore, substituting (54)-(55) into (60), it leads to

\[ \dot{V} = -k_i S_i^2 - k_2 S_2^2 - k_3 S_3^2 - \ddot{E} [S] + S \dot{\eta} -(E - \dot{E}) [S] \] 

\[ \dot{V} = -k_i S_i^2 - k_2 S_2^2 - k_3 S_3^2 + |S| \dot{\eta} - E [S] \] 

\[ \leq -k_i S_i^2 - k_2 S_2^2 - k_3 S_3^2 + |S| \dot{\eta} - E [S] \] 

\[ \leq -k_i S_i^2 - k_2 S_2^2 - k_3 S_3^2 + |S| \dot{\eta} - E \]  
(61)

From the assumption that \(|\dot{\eta}| \leq E\), it can be concluded that \(\dot{V}\) is negative definite and the closed-loop ball-and-beam system is stable with the proposed AFDSC approach.

### 5. Simulation Results

Referred to the system parameters in Table 1, the effectiveness of the proposed control scheme is evaluated. For the ball and beam system, both the conventional DSC and the proposed AFDSC method are considered. The required parameters are given as \(k_1 = 0.5, k_2 = 3, k_3 = 1, r_1 = 0.1, r_2 = 0.1, r_3 = 0.1, r_4 = 0.1\). Also, the parameters of input and output membership functions are given as

\[ [\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7] = [-1/3, 1/3, 1/3] \]

\[ [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7] = [-1, -1, 1, 1, 1, 1] \]

Typically, the control goal of the ball and beam system is that the ball can be asymptotically balanced at the center of beam regardless of the initial conditions. In this paper, two simulations with different initial conditions are addressed, where the initial setting are \(x_0 = [0.3 0 0 0]\) and \(x_0 = [0.3 0 -0.0524 0]\), respectively. For the case \(x_0 = [0.3 0 0 0]\), control response relative to the conventional DSC and AFDSC are shown in Fig. 3 and Fig. 4. It can be seen that with the proposed AFDSC the convergence rate of the ball is faster than the DSC counterpart. Similarly, with the initial setting \(x_0 = [0.3 0 -0.0524 0]\), the effectiveness of the proposed AFDSC can be observed from the control responses shown in Fig. 5 and Fig. 6. The control voltages of the previous simulations are shown in Fig. 7. It can be seen that the control histories are quite smooth for both the DSC and AFDSC.

![Fig. 3. Responses of the ball](image)
6. Conclusion

In this paper, combined with DSC, an adaptive fuzzy scheme is proposed for the balance control of a ball and beam system. In the design of AFDSC, the integrated tracking error of states is considered as the fuzzy input. With the adaptive learning mechanism, the parameters of fuzzy controller can be tuned to get better control performance. The closed-loop stability of the AFDSC controlled is analyzed by the Lyapunov’s Theorem. Simulation results illustrate that the proposed AFDSC can provide better equilibrium responses than the conventional DSC counterparts.

References


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