

Intuitionistic fuzzy quotient Lie superalgebras

Wenjuan Chen

Abstract

In this paper, the intuitionistic fuzzy Lie sub-superalgebras and ideals of Lie superalgebras are studied further. We define the intuitionistic fuzzy quotient Lie superalgebra by an intuitionistic fuzzy Lie ideal and prove that it is a Lie superalgebra. The intuitionistic fuzzy set of quotient Lie superalgebra is also considered. Finally, solvable intuitionistic fuzzy Lie ideals and nilpotent intuitionistic fuzzy Lie ideals are defined and the related properties are discussed.

Keywords: *Intuitionistic fuzzy sets, Intuitionistic fuzzy Lie ideals, Nilpotent intuitionistic fuzzy Lie ideals, Solvable intuitionistic fuzzy Lie ideals*

1. Introduction

After the introduction of fuzzy sets by Zadeh [26], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1, 2] is one among them. The theory of intuitionistic fuzzy sets has been applied in different areas, such as logic programming, decision making problems and topological space, etc. Especially, many people are interested in the study of algebras in the intuitionistic fuzzy settings [3-6, 8, 9, 13, 15, 17, 18]. For example, Biswas applied the concepts of intuitionistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group in [9]; Banerjee et al studied intuitionistic fuzzy subrings and ideals of a ring in [8]; M. Akram introduced Lie algebras into the intuitionistic fuzzy sets and discussed the related properties [3-5].

The pioneer description of the mathematical theory of Lie superalgebras was due to Kac [19] as the generalizations of the theory of Lie algebras. And it is well known that Lie superalgebras are important in theoretical physics where they are used to describe the mathematics of supersymmetry [12]. Recently, Lie superalgebras are more and more applied in several branches of both pure and applied sciences [16]. So we introduced Lie super-

algebras into intuitionistic fuzzy sets in [11] as a new direction of research Lie superalgebras. In [11], we defined intuitionistic fuzzy Lie sub-superalgebras and ideals of Lie superalgebras and investigated the related properties. We also defined the intuitionistic fuzzy bracket product of Lie superalgebras. These works generalized the theory of fuzzy Lie algebras [7, 14, 20, 21, 24, 25] and intuitionistic fuzzy Lie algebras [3-5].

As a continuation of [11], we further discuss the intuitionistic fuzzy Lie sub-superalgebras and ideals of Lie superalgebras in this paper. In Section 3, we define an intuitionistic fuzzy quotient Lie superalgebra by an intuitionistic fuzzy Lie ideal and show that it is a Lie superalgebra. So we can consider the intuitionistic fuzzy set of it. These works are the generalizations of [10]. And since many properties of solvable and nilpotent ideals of Lie algebras had been successful in establishing the analogous results in the fuzzy settings and in the intuitionistic fuzzy settings [4, 24, 25], we can generalize these results in Lie superalgebras. It is our main aim of this paper. Moreover, we know that if I is a solvable ideal of Lie superalgebra \mathcal{G} such that \mathcal{G}/I is solvable, then \mathcal{G} is solvable. We also show that this result is true in intuitionistic fuzzy sets. In order to finish these works, we need discuss the intuitionistic fuzzy sets of quotient Lie superalgebras in Section 4. In Section 5, we define the solvable intuitionistic fuzzy ideals and nilpotent intuitionistic fuzzy ideals of Lie superalgebras and characterize their properties.

2. Preliminaries

In this section, some relevant definitions and notations are reproduced.

Definition 2.1: Let V be a k -vector space. An intuitionistic fuzzy set (IFS for short) of V defined as an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in V\}$, where the functions $\mu_A : V \rightarrow [0, 1]$ and $\nu_A : V \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in V$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in V$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the

Corresponding Author: Wenjuan Chen is with the School of Sciences, University of Jinan, Jinan, Shandong, 250022, P. R. China.

E-mail: wjchenmath@gmail.com

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intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in V\}$.

In this paper, we use the symbols $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$.

Definition 2.2: An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of a vector space V is called an intuitionistic fuzzy subspace of V , if it satisfies the following conditions: for any $x, y \in V$ and $\alpha \in k$,

- (1) $\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y)$,
- (2) $\mu_A(\alpha x) \geq \mu_A(x)$ and $\nu_A(\alpha x) \leq \nu_A(x)$.

From this definition, we know that for any $x \in V$, we have $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$. In this paper, we always assume that $\mu_A(0) = 1$ and $\nu_A(0) = 0$. We also denote that 1_0 is the fuzzy subset which takes the value 1 at 0 and 0 elsewhere and 1_0^c is the fuzzy subset which takes the value 0 at 0 and 1 elsewhere. For more details on intuitionistic fuzzy sets, we refer the readers to [1, 2].

Definition 2.3 [23]: Suppose that V is a vector space and V_0, V_1 are its vector subspaces. If V is the direct sum of these two subspaces, $V = V_0 \oplus V_1$, then V (with this decomposition) is called a \mathbb{Z}_2 -graded vector space. V_0 and V_1 are called the even part and odd part of V respectively.

An element v of a \mathbb{Z}_2 -graded vector space $V = V_0 \oplus V_1$ has a unique expression of the form $v = v_0 + v_1$ ($v_i \in V_i$). In particular, if v is an element of either V_0 or V_1 , v is said to be homogeneous. For $v \in V_i$, we set $|v| = i$, and this number (an element of \mathbb{Z}_2) is called the homogeneous degree or the parity.

Definition 2.4 [23]: A \mathbb{Z}_2 -graded vector space $\mathcal{G} = \mathcal{G}_0 \oplus \mathcal{G}_1$ possessing the operation called the bracket product,

$$[,] : \mathcal{G} \times \mathcal{G} \ni (x, y) \xrightarrow{\text{bilinear}} [x, y] \in \mathcal{G}$$

is called Lie superalgebra, if it satisfies the following conditions:

- (1) $[\mathcal{G}_i, \mathcal{G}_j] \subseteq \mathcal{G}_{i+j}$ ($i, j \in \mathbb{Z}_2$),
- (2) $[y, x] = -(-1)^{|x||y|} [x, y]$,
- (3) $[x, [y, z]] - (-1)^{|x||y|} [y, [x, z]] = [[x, y], z]$.

In the above definition, x, y, z are elements of \mathcal{G} . The above condition (2) and (3) are written for homoge-

neous elements and the relations among general elements of \mathcal{G} can be computed by the bilinearity.

In [22], the definition of Lie superalgebra is given by **Definition 2.5 [22]:** Let $L = L_0 \oplus L_1$ be a superalgebra whose multiplication is denoted by a pointed bracket \langle, \rangle . This implies in particular that $\langle L_\alpha, L_\beta \rangle \subseteq L_{\alpha+\beta}$ for all $\alpha, \beta \in \mathbb{Z}_2$. We call L a Lie superalgebra if the multiplication satisfies the following identities:

$$\begin{aligned} \langle A, B \rangle &= -(-1)^{\alpha\beta} \langle B, A \rangle \\ (-1)^{\alpha\gamma} \langle A, \langle B, C \rangle \rangle + (-1)^{\alpha\beta} \langle B, \langle C, A \rangle \rangle + (-1)^{\beta\gamma} \langle C, \langle A, B \rangle \rangle &= 0 \end{aligned}$$

for all $A \in L_\alpha, B \in L_\beta, C \in L_\gamma, \alpha, \beta, \gamma \in \mathbb{Z}_2$.

In fact, the above definitions are equivalent. In this paper, we always denote a Lie superalgebra over field k by \mathcal{G} and adopt Definition 2.4 as the definition of Lie superalgebra.

Now, we recall the definitions of intuitionistic fuzzy Lie sub-superalgebras and ideals.

Definition 2.6 [11]: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of \mathcal{G} . Then $A = (\mu_A, \nu_A)$ is called an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} , if it satisfies the following conditions:

- (1) $A = (\mu_A, \nu_A)$ is a \mathbb{Z}_2 -graded intuitionistic fuzzy subspace,
- (2) $\mu_A([x, y]) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A([x, y]) \leq \nu_A(x) \vee \nu_A(y)$, for $x, y \in \mathcal{G}$.

Definition 2.7 [11]: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of \mathcal{G} . Then $A = (\mu_A, \nu_A)$ is called an intuitionistic fuzzy Lie ideal of \mathcal{G} , if it satisfies the following conditions:

- (1) $A = (\mu_A, \nu_A)$ is a \mathbb{Z}_2 -graded intuitionistic fuzzy subspace,
- (2) $\mu_A([x, y]) \geq \mu_A(x) \vee \mu_A(y)$ and $\nu_A([x, y]) \leq \nu_A(x) \wedge \nu_A(y)$, for $x, y \in \mathcal{G}$.

3. Intuitionistic fuzzy quotient Lie superalgebras

Definition 3.1: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . The intuitionistic fuzzy set $x + A : \mathcal{G} \rightarrow [0, 1]$ defined by $(x + \mu_A)(y) = \mu_A(y - x)$ and $(x + \nu_A)(y) = \nu_A(y - x)$ is called an intuitionistic fuzzy coset of the intuitionistic fuzzy Lie ideal $A = (\mu_A, \nu_A)$. The set of all such cosets is denoted by \mathcal{G} / A .

Lemma 3.1: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . Then for any $x, y \in \mathcal{G}$, $x + A = y + A \Leftrightarrow \mu_A(x - y) = \mu_A(0)$ and $\nu_A(x - y) = \nu_A(0)$.

Theorem 3.1: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . For any $x, y \in \mathcal{G}$ and $\alpha \in k$, define:

$$\begin{aligned} (x + A) + (y + A) &= (x + y) + A, \\ \alpha(x + A) &= \alpha x + A, \\ [(x + A), (y + A)] &= [x, y] + A. \end{aligned}$$

The operations are well defined and make \mathcal{G}/A into a Lie superalgebra called the intuitionistic fuzzy quotient Lie superalgebra of \mathcal{G} by A .

We give an example of intuitionistic fuzzy quotient Lie superalgebra.

Example 3.4: Let $N = N_{\bar{0}} \oplus N_{\bar{1}}$, where $N_{\bar{0}} = \langle e \rangle$, $N_{\bar{1}} = \langle a_1, a_2, b_1, b_2 \rangle$ and $[a_i, b_i] = e$, $i = 1, 2$, the remaining brackets are zero. Then N is a Lie superalgebra. (See [19])

Define $A_{\bar{0}} = (\mu_{A_{\bar{0}}}, \nu_{A_{\bar{0}}}) : N_{\bar{0}} \rightarrow [0, 1]$ by $\mu_{A_{\bar{0}}}(x) = 1$, $\nu_{A_{\bar{0}}}(x) = 0$ for all $x \in N_{\bar{0}}$. Then $A_{\bar{0}} = (\mu_{A_{\bar{0}}}, \nu_{A_{\bar{0}}})$ is an intuitionistic fuzzy subspace of $N_{\bar{0}}$.

Define $A_{\bar{1}} = (\mu_{A_{\bar{1}}}, \nu_{A_{\bar{1}}}) : N_{\bar{1}} \rightarrow [0, 1]$ as follows.

If $x = 0$, we define $\mu_{A_{\bar{1}}}(0) = 1$ and $\nu_{A_{\bar{1}}}(0) = 0$.

If $x \neq 0$, then $x = k_1 a_1 + k_2 a_2 + k_3 b_1 + k_4 b_2$, for $k_i \in k$,

(1) $k_i \neq 0$, for all i , we define

$$\mu_{A_{\bar{1}}}(x) = \mu_{A_{\bar{1}}}(a_1) \wedge \mu_{A_{\bar{1}}}(a_2) \wedge \mu_{A_{\bar{1}}}(b_1) \wedge \mu_{A_{\bar{1}}}(b_2)$$

and

$$\nu_{A_{\bar{1}}}(x) = \nu_{A_{\bar{1}}}(a_1) \vee \nu_{A_{\bar{1}}}(a_2) \vee \nu_{A_{\bar{1}}}(b_1) \vee \nu_{A_{\bar{1}}}(b_2),$$

(2) $k_i = 0$, for some i , we may assume $k_1 = 0$, then

$x = k_2 a_2 + k_3 b_1 + k_4 b_2$, we define

$$\mu_{A_{\bar{1}}}(x) = \mu_{A_{\bar{1}}}(a_2) \wedge \mu_{A_{\bar{1}}}(b_1) \wedge \mu_{A_{\bar{1}}}(b_2)$$

and

$$\nu_{A_{\bar{1}}}(x) = \nu_{A_{\bar{1}}}(a_2) \vee \nu_{A_{\bar{1}}}(b_1) \vee \nu_{A_{\bar{1}}}(b_2).$$

In any case $\mu_{A_{\bar{1}}}(a_1) = 1$, $\mu_{A_{\bar{1}}}(a_2) = 0.3$, $\mu_{A_{\bar{1}}}(b_1) = 1$, $\mu_{A_{\bar{1}}}(b_2) = 0.4$ and $\nu_{A_{\bar{1}}}(a_1) = 0$, $\nu_{A_{\bar{1}}}(a_2) = 0.6$, $\nu_{A_{\bar{1}}}(b_1) = 0$, $\nu_{A_{\bar{1}}}(b_2) = 0.5$. Then $A_{\bar{1}} = (\mu_{A_{\bar{1}}}, \nu_{A_{\bar{1}}})$ is an intuitionistic fuzzy subspace of $N_{\bar{1}}$.

Define $A = (\mu_A, \nu_A) : N \rightarrow [0, 1]$ as follows.

If $x = 0$, we define $\mu_A(0) = 1$ and $\nu_A(0) = 0$.

If $x \neq 0$, then $x = l e + k_1 a_1 + k_2 a_2 + k_3 b_1 + k_4 b_2$, for

$l, k_i \in k$.

(1) $l, k_i \neq 0$, for all i , we define

$$\mu_A(x) = \mu_A(e) \wedge \mu_A(a_1) \wedge \mu_A(a_2) \wedge \mu_A(b_1) \wedge \mu_A(b_2)$$

and

$$\nu_A(x) = \nu_A(e) \vee \nu_A(a_1) \vee \nu_A(a_2) \vee \nu_A(b_1) \vee \nu_A(b_2),$$

(2) l or $k_i = 0$, for some i , we may assume $k_1 = 0$, then

$x = l e + k_2 a_2 + k_3 b_1 + k_4 b_2$, define

$$\mu_A(x) = \mu_A(e) \wedge \mu_A(a_2) \wedge \mu_A(b_1) \wedge \mu_A(b_2)$$

and

$$\nu_A(x) = \nu_A(e) \vee \nu_A(a_2) \vee \nu_A(b_1) \vee \nu_A(b_2).$$

In any case $\mu_A(e) = 1$, $\mu_A(a_1) = 1$, $\mu_A(a_2) = 0.3$, $\mu_A(b_1) = 1$, $\mu_A(b_2) = 0.4$ and $\nu_A(e) = 0$, $\nu_A(a_1) = 0$, $\nu_A(a_2) = 0.6$, $\nu_A(b_1) = 0$, $\nu_A(b_2) = 0.5$. Then $\mu_A(x) = \mu_{A_{\bar{0}}}(x_{\bar{0}}) \wedge \mu_{A_{\bar{1}}}(x_{\bar{1}})$ and $\nu_A(x) = \nu_{A_{\bar{0}}}(x_{\bar{0}}) \vee \nu_{A_{\bar{1}}}(x_{\bar{1}})$.

Thus $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy Lie ideal of N .

Now since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy Lie ideal of N , we denote N/A the set of all the cosets. The operations are defined as Theorem 3.1, then N/A is an intuitionistic fuzzy quotient Lie superalgebra.

Theorem 3.2: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . Then $\mathcal{G}/A \cong \mathcal{G}/A_0$, where $A_0 = \{x \in \mathcal{G} \mid \mu_A(x) = \mu_A(0) \text{ and } \nu_A(x) = \nu_A(0)\}$ is an ideal of \mathcal{G} .

Proof: It is straightforward.

Since \mathcal{G}/A is a Lie superalgebra, we can consider the intuitionistic fuzzy set of it.

Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . Define $A^{(*)} = (\mu_{A^{(*)}}, \nu_{A^{(*)}}) : \mathcal{G}/A \rightarrow [0, 1]$ by $\mu_{A^{(*)}}(x + A) = \mu_A(x)$ and $\nu_{A^{(*)}}(x + A) = \nu_A(x)$, for any $x \in \mathcal{G}$.

Lemma 3.2: $A^{(*)} = (\mu_{A^{(*)}}, \nu_{A^{(*)}})$ is well defined.

Theorem 3.3: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . Then $A^{(*)} = (\mu_{A^{(*)}}, \nu_{A^{(*)}})$ is an intuitionistic fuzzy Lie ideal of \mathcal{G}/A .

Proof: Let $x + A, y + A \in \mathcal{G}/A$. Since

$$\mu_{A^{(*)}}(x + A + y + A) = \mu_{A^{(*)}}(x + y + A)$$

$$\geq \mu_A(x) \wedge \mu_A(y) = \mu_{A^{(*)}}(x + A) \wedge \mu_{A^{(*)}}(y + A)$$

and $\nu_{A^{(*)}}(x + A + y + A) = \nu_{A^{(*)}}(x + y + A)$

$$\leq \nu_A(x) \vee \nu_A(y) = \nu_{A^{(*)}}(x+A) \vee \nu_{A^{(*)}}(y+A).$$

Also $\mu_{A^{(*)}}(\alpha(x+A)) = \mu_{A^{(*)}}(\alpha x + A)$

$$= \mu_A(\alpha x) \geq \mu_A(x) = \mu_{A^{(*)}}(x+A)$$

and $\nu_{A^{(*)}}(\alpha(x+A)) = \nu_{A^{(*)}}(\alpha x + A)$

$$= \nu_A(\alpha x) \leq \nu_A(x) = \nu_{A^{(*)}}(x+A),$$

we get that $A^{(*)}$ is an intuitionistic fuzzy subspace.

Next, we prove that $A^{(*)}$ has \mathbb{Z}_2 -graded structure.

Since A is an intuitionistic fuzzy Lie ideal of \mathcal{G} , we

have $A = A_0 \oplus A_1$, where $A_0 = (\mu_{A_0}, \nu_{A_0}) \in I^{\mathcal{G}_0}$,

$A_1 = (\mu_{A_1}, \nu_{A_1}) \in I^{\mathcal{G}_1}$. Define

$$A_0^{(*)} = (\mu_{A_0^{(*)}}, \nu_{A_0^{(*)}}): \mathcal{G}_0 / A \rightarrow [0,1] \text{ where}$$

$$\mu_{A_0^{(*)}}(x+A) = \mu_{A_0}(x) \text{ and } \nu_{A_0^{(*)}}(x+A) = \nu_{A_0}(x),$$

$$A_1^{(*)} = (\mu_{A_1^{(*)}}, \nu_{A_1^{(*)}}): \mathcal{G}_1 / A \rightarrow [0,1] \text{ where}$$

$$\mu_{A_1^{(*)}}(x+A) = \mu_{A_1}(x) \text{ and } \nu_{A_1^{(*)}}(x+A) = \nu_{A_1}(x).$$

It is obvious that $A_0^{(*)}, A_1^{(*)}$ are intuitionistic fuzzy subspaces. For any $x \in \mathcal{G}$, Define

$$A_0^{(*)'} = (\mu_{A_0^{(*)}'}, \nu_{A_0^{(*)}'}) : \mathcal{G} / A \rightarrow [0,1] \text{ where}$$

$$\mu_{A_0^{(*)}'}(x+A) = \mu_{A_0^c}(x) \text{ and } \nu_{A_0^{(*)}'}(x+A) = \nu_{A_0^c}(x),$$

$$A_1^{(*)'} = (\mu_{A_1^{(*)}'}, \nu_{A_1^{(*)}'}) : \mathcal{G} / A \rightarrow [0,1] \text{ where}$$

$$\mu_{A_1^{(*)}'}(x+A) = \mu_{A_1^c}(x) \text{ and } \nu_{A_1^{(*)}'}(x+A) = \nu_{A_1^c}(x).$$

For $x+A \in \mathcal{G}_0 / A$, $x \in \mathcal{G}_0$, we have

$$\mu_{A_0^{(*)}'}(x+A) = \mu_{A_0^c}(x) = \mu_{A_0}(x) = \mu_{A_0^{(*)}}(x+A)$$

and $\nu_{A_0^{(*)}'}(x+A) = \nu_{A_0^c}(x) = \nu_{A_0}(x) = \nu_{A_0^{(*)}}(x+A)$.

For $x+A \in \mathcal{G}_1 / A$, $x \in \mathcal{G}_1$, we have

$$\mu_{A_1^{(*)}'}(x+A) = \mu_{A_1^c}(x) = \mu_{A_1}(x) = \mu_{A_1^{(*)}}(x+A)$$

and $\nu_{A_1^{(*)}'}(x+A) = \nu_{A_1^c}(x) = \nu_{A_1}(x) = \nu_{A_1^{(*)}}(x+A)$.

So we have

$$\mu_{A_0^{(*)}'}(x+A) = \begin{cases} \mu_{A_0^{(*)}}(x+A), & x+A \in \mathcal{G}_0 / A \\ 0 & , x+A \notin \mathcal{G}_0 / A \end{cases},$$

$$\nu_{A_0^{(*)}'}(x+A) = \begin{cases} \nu_{A_1^{(*)}}(x+A), & x+A \in \mathcal{G}_0 / A \\ 1 & , x+A \notin \mathcal{G}_0 / A \end{cases},$$

$$\mu_{A_1^{(*)}'}(x+A) = \begin{cases} \mu_{A_1^{(*)}}(x+A), & x+A \in \mathcal{G}_1 / A \\ 0 & , x+A \notin \mathcal{G}_1 / A \end{cases},$$

$$\nu_{A_1^{(*)}'}(x+A) = \begin{cases} \nu_{A_1^{(*)}}(x+A), & x+A \in \mathcal{G}_1 / A \\ 1 & , x+A \notin \mathcal{G}_1 / A \end{cases}.$$

Then for any $x+A \in \mathcal{G} / A$, we have

$$\mu_{A^{(*)}}(x+A) = \mu_A(x) = \mu_{A_0}(x_0) \wedge \mu_{A_1}(x_1)$$

$$= \mu_{A_0^{(*)}}(x_0+A) \wedge \mu_{A_1^{(*)}}(x_1+A)$$

$$= \sup_{x+A=(a+A)+(b+A)} \{ \mu_{A_0^{(*)}}(a+A) \wedge \mu_{A_1^{(*)}}(b+A) \}$$

$$= (\mu_{A_0^{(*)}} + \mu_{A_1^{(*)}})(x+A) \text{ and}$$

$$\nu_{A^{(*)}}(x+A) = \nu_A(x) = \nu_{A_0}(x_0) \vee \nu_{A_1}(x_1)$$

$$= \nu_{A_0^{(*)}}(x_0+A) \vee \nu_{A_1^{(*)}}(x_1+A)$$

$$= \inf_{x+A=(a+A)+(b+A)} \{ \nu_{A_0^{(*)}}(a+A) \vee \nu_{A_1^{(*)}}(b+A) \}$$

$$= (\nu_{A_0^{(*)}} + \nu_{A_1^{(*)}})(x+A).$$

For $x \neq 0$, since $(A_0^c \cap A_1^c)(x) = (\mu_{A_0^c}(x) \wedge \mu_{A_1^c}(x),$

$\nu_{A_0^c}(x) \vee \nu_{A_1^c}(x)) = (0,1)$, we have $A_0^{(*)'} \cap A_1^{(*)'} = (0,1)$.

So $A^{(*)} = A_0^{(*)} \oplus A_1^{(*)}$. This implies $A^{(*)}$ is a \mathbb{Z}_2 -graded intuitionistic fuzzy subspace.

Let $x+A, y+A \in \mathcal{G} / A$. Then we have

$$\mu_{A^{(*)}}([x+A, y+A]) = \mu_{A^{(*)}}([x, y] + A)$$

$$= \mu_A([x, y]) \geq \mu_A(x) \vee \mu_A(y)$$

$$= \mu_{A^{(*)}}(x+A) \vee \mu_{A^{(*)}}(y+A),$$

$$\nu_{A^{(*)}}([x+A, y+A]) = \nu_{A^{(*)}}([x, y] + A)$$

$$= \nu_A([x, y]) \leq \nu_A(x) \wedge \nu_A(y)$$

$$= \nu_{A^{(*)}}(x+A) \wedge \nu_{A^{(*)}}(y+A).$$

Hence $A^{(*)} = (\mu_{A^{(*)}}, \nu_{A^{(*)}})$ is an intuitionistic fuzzy Lie ideal of \mathcal{G} / A .

4. The intuitionistic fuzzy set of quotient Lie superalgebra

Let I be an ideal of \mathcal{G} . Quotient vector space \mathcal{G} / I is a Lie superalgebra, the bracket operation is defined by $[\bar{x}, \bar{y}] := \overline{[x, y]}$.

Theorem 4.1: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} and I be an ideal of \mathcal{G} . Define $B = (\mu_B, \nu_B)$ where

$$\mu_B(\bar{x}) = \sup_{z \in \bar{x}} \{ \mu_A(z) \} \text{ and } \nu_B(\bar{x}) = \inf_{z \in \bar{x}} \{ \nu_A(z) \},$$

for $\bar{x} = x+I$, $x \in \mathcal{G}$. Then $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} / I .

Proof: We first prove that $B = (\mu_B, \nu_B)$ is \mathbb{Z}_2 -graded intuitionistic fuzzy subspace. Since $A = (\mu_A, \nu_A)$ is a \mathbb{Z}_2 -graded intuitionistic fuzzy subspace, we have $A = A_0 \oplus A_1$, where

$$A_0 = (\mu_{A_0}, \nu_{A_0}) \in I^{\mathcal{G}_0}, A_1 = (\mu_{A_1}, \nu_{A_1}) \in I^{\mathcal{G}_1}.$$

Define $B_0 = (\mu_{B_0}, \nu_{B_0}) : \mathcal{G}_0 / I \rightarrow [0, 1]$, where

$$\mu_{B_0}(\bar{x}) = \sup_{m \in \bar{x}} \{\mu_{A_0}(m)\}, \nu_{B_0}(\bar{x}) = \inf_{m \in \bar{x}} \{\nu_{A_0}(m)\}$$

for $x \in \mathcal{G}_0$ and

define $B_1 = (\mu_{B_1}, \nu_{B_1}) : \mathcal{G}_1 / I \rightarrow [0, 1]$, where

$$\mu_{B_1}(\bar{x}) = \sup_{m \in \bar{x}} \{\mu_{A_1}(m)\}, \nu_{B_1}(\bar{x}) = \inf_{m \in \bar{x}} \{\nu_{A_1}(m)\}$$

for $x \in \mathcal{G}_1$. Then B_0, B_1 are intuitionistic fuzzy subspaces of $\mathcal{G}_0 / I, \mathcal{G}_1 / I$, respectively.

Define $B'_0 = (\mu_{B'_0}, \nu_{B'_0}) : \mathcal{G} / I \rightarrow [0, 1]$, where

$$\mu_{B'_0}(\bar{x}) = \sup_{m \in \bar{x}} \{\mu_{A'_0}(m)\}, \nu_{B'_0}(\bar{x}) = \inf_{m \in \bar{x}} \{\nu_{A'_0}(m)\}$$

for $x \in \mathcal{G}$ and

define $B'_1 = (\mu_{B'_1}, \nu_{B'_1}) : \mathcal{G} / I \rightarrow [0, 1]$, where

$$\mu_{B'_1}(\bar{x}) = \sup_{m \in \bar{x}} \{\mu_{A'_1}(m)\}, \nu_{B'_1}(\bar{x}) = \inf_{m \in \bar{x}} \{\nu_{A'_1}(m)\}$$

for $x \in \mathcal{G}$. Obviously, they are the intuitionistic fuzzy subspaces of \mathcal{G} / I as the extensions of B_0, B_1 .

Let $\bar{x} \in \mathcal{G} / I$. Then we have

$$\begin{aligned} \mu_B(\bar{x}) &= \sup_{z \in \bar{x}} \{\mu_A(z)\} = \sup_{z \in \bar{x}} \{\sup_{z=a+b} \{\mu_{A'_0}(a) \wedge \mu_{A'_1}(b)\}\} \\ &= \sup_{\bar{x}=\bar{m}+\bar{n}} \{\sup_{a \in \bar{m}} \{\mu_{A'_0}(a)\} \wedge \sup_{b \in \bar{n}} \{\mu_{A'_1}(b)\}\} \\ &= \sup_{\bar{x}=\bar{m}+\bar{n}} \{\mu_{B'_0}(\bar{m}) \wedge \mu_{B'_1}(\bar{n})\} = (\mu_{B'_0} + \mu_{B'_1})(\bar{x}), \end{aligned}$$

and

$$\begin{aligned} \nu_B(\bar{x}) &= \inf_{z \in \bar{x}} \{\nu_A(z)\} = \inf_{z \in \bar{x}} \{\inf_{z=a+b} \{\nu_{A'_0}(a) \wedge \nu_{A'_1}(b)\}\} \\ &= \inf_{\bar{x}=\bar{m}+\bar{n}} \{\inf_{a \in \bar{m}} \{\nu_{A'_0}(a)\} \vee \inf_{b \in \bar{n}} \{\nu_{A'_1}(b)\}\} \\ &= \inf_{\bar{x}=\bar{m}+\bar{n}} \{\nu_{B'_0}(\bar{m}) \vee \nu_{B'_1}(\bar{n})\} = (\nu_{B'_0} + \nu_{B'_1})(\bar{x}). \end{aligned}$$

Moreover, for $0 \neq x \in \mathcal{G}$, since $(A'_0 \cap A'_1)(x) =$

$$= (\mu_{A'_0 \cap A'_1}(x), \nu_{A'_0 \cap A'_1}(x)) = (0, 1),$$

$$(B'_0 \cap B'_1)(x) = (\mu_{B'_0 \cap B'_1}(x), \nu_{B'_0 \cap B'_1}(x))$$

$$= (\mu_{B'_0}(x) \wedge \mu_{B'_1}(x), \nu_{B'_0}(x) \vee \nu_{B'_1}(x)) = (0, 1).$$

So $B = B_0 \oplus B_1$, which shows that $B = (\mu_B, \nu_B)$ is a \mathbb{Z}_2 -graded intuitionistic fuzzy subspace.

Let $\bar{x}, \bar{y} \in \mathcal{G} / I$. Then we have

$$\mu_B([\bar{x}, \bar{y}]) = \mu_B(\overline{[x, y]}) = \sup_{z \in [x, y]} \{\mu_A(z)\}$$

$$\geq \sup_{z=[a, b] \in [x, y]} \{\mu_A([a, b])\} \geq \sup_{[a, b] \in [x, y]} \{\mu_A(a) \wedge \mu_A(b)\}$$

$$= \sup_{a \in \bar{x}} \{\mu_A(a)\} \wedge \sup_{b \in \bar{y}} \{\mu_A(b)\} = \mu_B(\bar{x}) \wedge \mu_B(\bar{y}),$$

and $\nu_B([\bar{x}, \bar{y}]) = \nu_B(\overline{[x, y]}) = \inf_{z \in [x, y]} \{\nu_A(z)\}$

$$\leq \inf_{z=[a, b] \in [x, y]} \{\nu_A([a, b])\} \leq \inf_{[a, b] \in [x, y]} \{\nu_A(a) \vee \nu_A(b)\}$$

$$= \inf_{a \in \bar{x}} \{\nu_A(a)\} \vee \inf_{b \in \bar{y}} \{\nu_A(b)\} = \nu_B(\bar{x}) \vee \nu_B(\bar{y}).$$

Hence $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} / I .

The intuitionistic fuzzy set $B = (\mu_B, \nu_B)$ defined in Theorem 4.1 is called the quotient intuitionistic fuzzy Lie sub-superalgebra of the intuitionistic fuzzy Lie ideal A relative to I and denoted by A / I .

Now, we define the intuitionistic fuzzy ideal of an intuitionistic fuzzy Lie sub-superalgebra.

Definition 4.1: Let $B = (\mu_B, \nu_B)$ be an intuitionistic fuzzy set of \mathcal{G} and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} with $B \subseteq A$. Then $B = (\mu_B, \nu_B)$ is called an intuitionistic fuzzy ideal of $A = (\mu_A, \nu_A)$, if it satisfies the following conditions:

(1) $B = (\mu_B, \nu_B)$ is a \mathbb{Z}_2 -graded intuitionistic fuzzy vector subspace of \mathcal{G} ,

$$(2) \mu_B([x, y]) \geq (\mu_B(x) \wedge \mu_A(y)) \vee (\mu_B(y) \wedge \mu_A(x))$$

and

$$\nu_B([x, y]) \leq (\nu_B(x) \vee \nu_A(y)) \wedge (\nu_B(y) \vee \nu_A(x)),$$

for any $x, y \in \mathcal{G}$.

By the definition, it is immediately clear that

Remark 4.3: (1) $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy Lie ideal of \mathcal{G} if and only if $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy ideal of the intuitionistic fuzzy Lie sub-superalgebra $I_{\mathcal{G}} = (1_{\mathcal{G}}, 0_{\mathcal{G}})$.

(2) If $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy Lie ideal of \mathcal{G} and $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} such that $B \subseteq A$, then $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy ideal of $A = (\mu_A, \nu_A)$.

(3) If $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy ideal of intuitionistic fuzzy Lie sub-superalgebra $A = (\mu_A, \nu_A)$, then $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} .

The proofs of Theorem 4.2 and 4.3 are straightforward.

Theorem 4.2: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} and $B = (\mu_B, \nu_B)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . Then $A \cap B = (\mu_{A \cap B}, \nu_{A \cap B})$ is an intuitionistic fuzzy ideal of $A = (\mu_A, \nu_A)$.

Theorem 4.3: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie sub-superalgebra of \mathcal{G} and $B = (\mu_B, \nu_B)$ and $C = (\mu_C, \nu_C)$ be intuitionistic fuzzy ideals of $A = (\mu_A, \nu_A)$. Then $B \cap C = (\mu_{B \cap C}, \nu_{B \cap C})$ is also an intuitionistic fuzzy ideal of $A = (\mu_A, \nu_A)$.

5. Solvable intuitionistic fuzzy Lie ideals and nilpotent intuitionistic fuzzy Lie ideals

In this section, we give the definitions of solvable intuitionistic fuzzy Lie ideals and nilpotent intuitionistic fuzzy Lie ideals of Lie superalgebras, the corresponding theorems are parallel with Lie superalgebras.

Definition 5.1 [11]: For any intuitionistic fuzzy sets $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ of \mathcal{G} , we define the intuitionistic fuzzy bracket product $[A, B] = (\mu_{[A,B]}, \nu_{[A,B]})$ where

$$\mu_{[A,B]}(x) = \begin{cases} \sup_{x = \sum_{i \in N} \alpha_i [x_i, y_i]} \{ \min_{i \in N} \{ \mu_A(x_i) \wedge \mu_B(y_i) \} \}, \alpha_i \in k, x_i, y_i \in \mathcal{G} \\ 0, & x \neq \sum_{i \in N} \alpha_i [x_i, y_i] \end{cases}$$

and

$$\nu_{[A,B]}(x) = \begin{cases} \inf_{x = \sum_{i \in N} \alpha_i [x_i, y_i]} \{ \max_{i \in N} \{ \nu_A(x_i) \vee \nu_B(y_i) \} \}, \alpha_i \in k, x_i, y_i \in \mathcal{G} \\ 1, & x \neq \sum_{i \in N} \alpha_i [x_i, y_i] \end{cases}$$

In [11], we proved if $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ are two intuitionistic fuzzy Lie ideals of \mathcal{G} , then $[A, B]$ is also an intuitionistic fuzzy Lie ideal of \mathcal{G} . So we can give the following definitions.

Definition 5.2: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . Define inductively a sequence of intuitionistic fuzzy ideals of \mathcal{G} by

$$A^{(0)} = A, \quad A^{(1)} = [A^{(0)}, A^{(0)}], \quad A^{(2)} = [A^{(1)}, A^{(1)}], \quad \dots, \\ A^{(n)} = [A^{(n-1)}, A^{(n-1)}], \text{ then } A^{(n)} \text{ is called the } n \text{ th derived intuitionistic fuzzy Lie ideal of } \mathcal{G}. \text{ In which,}$$

$A^{(i+1)} = (\mu_{A^{(i+1)}}, \nu_{A^{(i+1)}})$ where

$$\mu_{A^{(i+1)}}(x) = \begin{cases} \sup_{x = \sum_{j \in N} \alpha_j [x_j, y_j]} \{ \min_{j \in N} \{ \mu_{A^{(i)}}(x_j) \wedge \mu_{A^{(i)}}(y_j) \} \}, \alpha_j \in k, x_j, y_j \in \mathcal{G} \\ 0, & x \neq \sum_{j \in N} \alpha_j [x_j, y_j] \end{cases}$$

$$\nu_{A^{(i+1)}}(x) = \begin{cases} \inf_{x = \sum_{j \in N} \alpha_j [x_j, y_j]} \{ \max_{j \in N} \{ \nu_{A^{(i)}}(x_j) \vee \nu_{A^{(i)}}(y_j) \} \}, \alpha_j \in k, x_j, y_j \in \mathcal{G} \\ 1, & x \neq \sum_{j \in N} \alpha_j [x_j, y_j] \end{cases}$$

From the definition, we can get

$$\mu_{A^{(0)}} \supseteq \mu_{A^{(1)}} \supseteq \mu_{A^{(2)}} \supseteq \dots \supseteq \mu_{A^{(n)}} \supseteq \dots$$

and

$$\nu_{A^{(0)}} \subseteq \nu_{A^{(1)}} \subseteq \nu_{A^{(2)}} \subseteq \dots \subseteq \nu_{A^{(n)}} \subseteq \dots$$

Definition 5.3: Let $A^{(n)}$ be as above. Define:

$$\eta^{(n)} = \sup \{ \mu_{A^{(n)}}(x) : 0 \neq x \in \mathcal{G} \}$$

and $\kappa^{(n)} = \inf \{ \nu_{A^{(n)}}(x) : 0 \neq x \in \mathcal{G} \}$.

Then it is clear that $\eta^{(0)} \geq \eta^{(1)} \geq \eta^{(2)} \geq \dots \geq \eta^{(n)} \geq \dots$

and $\kappa^{(0)} \leq \kappa^{(1)} \leq \kappa^{(2)} \leq \dots \leq \kappa^{(n)} \leq \dots$.

Definition 5.4: An intuitionistic fuzzy Lie ideal $A = (\mu_A, \nu_A)$ of \mathcal{G} is called a solvable intuitionistic fuzzy Lie ideal, if there is a positive integer n such that $\eta^{(n)} = 0$ and $\kappa^{(n)} = 1$. So it is a solvable intuitionistic fuzzy Lie ideal, then there is positive integer n such that $\mu_{A^{(n)}} = 1_0$ and $\nu_{A^{(n)}} = 1_0^c$.

Example 5.1: The Lie superalgebra N is in Example 3.1.

Define $A_{\bar{0}} = (\mu_{A_{\bar{0}}}, \nu_{A_{\bar{0}}}) : N_{\bar{0}} \rightarrow [0, 1]$ where $\mu_{A_{\bar{0}}}(x) = 1$, $\nu_{A_{\bar{0}}}(x) = 0$ for all $x \in N_{\bar{0}}$. Then it is an intuitionistic fuzzy subspace of $N_{\bar{0}}$.

Define $A_{\bar{1}} = (\mu_{A_{\bar{1}}}, \nu_{A_{\bar{1}}}) : N_{\bar{1}} \rightarrow [0, 1]$ as follows.

If $x = 0$, we define $\mu_{A_{\bar{1}}}(0) = 1$ and $\nu_{A_{\bar{1}}}(0) = 0$.

If $x \neq 0$, then $x = k_1 a_1 + k_2 a_2 + k_3 b_1 + k_4 b_2$, for $k_i \in k$,

(1) $k_i \neq 0$, for all i , we define

$$\mu_{A_{\bar{1}}}(x) = \mu_{A_{\bar{1}}}(a_1) \wedge \mu_{A_{\bar{1}}}(a_2) \wedge \mu_{A_{\bar{1}}}(b_1) \wedge \mu_{A_{\bar{1}}}(b_2)$$

and

$$\nu_{A_{\bar{1}}}(x) = \nu_{A_{\bar{1}}}(a_1) \vee \nu_{A_{\bar{1}}}(a_2) \vee \nu_{A_{\bar{1}}}(b_1) \vee \nu_{A_{\bar{1}}}(b_2),$$

(2) $k_i = 0$, for some i , we may assume $k_1 = 0$, then

$x = k_2 a_2 + k_3 b_1 + k_4 b_2$, we define

$$\mu_{A_{\bar{1}}}(x) = \mu_{A_{\bar{1}}}(a_2) \wedge \mu_{A_{\bar{1}}}(b_1) \wedge \mu_{A_{\bar{1}}}(b_2)$$

and

$$\nu_{A_{\bar{1}}}(x) = \nu_{A_{\bar{1}}}(a_2) \vee \nu_{A_{\bar{1}}}(b_1) \vee \nu_{A_{\bar{1}}}(b_2).$$

In any case $\mu_{A_{\top}}(a_1) = 0.2$, $\mu_{A_{\top}}(a_2) = 1$, $\mu_{A_{\top}}(b_1) = 0.1$, $\mu_{A_{\top}}(b_2) = 0.1$, $\mu_{A_{\top}}(b_2) = 1$ and $\nu_{A_{\top}}(a_1) = 0.7$, $\nu_{A_{\top}}(a_2) = 0$, $\nu_{A_{\top}}(b_1) = 0.9$, $\nu_{A_{\top}}(b_2) = 0$. Then it is an intuitionistic fuzzy subspace of N_{\top} .

Define $A = (\mu_A, \nu_A) : N \rightarrow [0, 1]$ as follows.

If $x = 0$, we define $\mu_A(0) = 1$ and $\nu_A(0) = 0$.

If $x \neq 0$, then $x = le + k_1a_1 + k_2a_2 + k_3b_1 + k_4b_2$, for $l, k_i \in k$.

(1) $l, k_i \neq 0$, for all i , we define

$$\mu_A(x) = \mu_A(e) \wedge \mu_A(a_1) \wedge \mu_A(a_2) \wedge \mu_A(b_1) \wedge \mu_A(b_2)$$

and

$$\nu_A(x) = \nu_A(e) \vee \nu_A(a_1) \vee \nu_A(a_2) \vee \nu_A(b_1) \vee \nu_A(b_2),$$

(2) l or $k_i = 0$, for some i , we may assume $k_1 = 0$, then

$x = le + k_2a_2 + k_3b_1 + k_4b_2$, define

$$\mu_A(x) = \mu_A(e) \wedge \mu_A(a_2) \wedge \mu_A(b_1) \wedge \mu_A(b_2)$$

and

$$\nu_A(x) = \nu_A(e) \vee \nu_A(a_2) \vee \nu_A(b_1) \vee \nu_A(b_2).$$

In which $\mu_A(e) = 1$, $\mu_A(a_1) = 0.2$, $\mu_A(a_2) = 1$, $\mu_A(b_1) = 0.1$, $\mu_A(b_2) = 1$ and $\nu_A(e) = 0$, $\nu_A(a_1) = 0.7$, $\nu_A(a_2) = 0$, $\nu_A(b_1) = 0.9$, $\nu_A(b_2) = 0$.

Then $A = A_{\bar{0}} \oplus A_{\top}$ is an intuitionistic fuzzy Lie ideal of N .

Let $A^{(0)} = A$ and define $A^{(1)} = [A^{(0)}, A^{(0)}]$. Note that $[a_i, b_i] = e$ and the other brackets are zero.

For $0 \neq x \in N$, $x = le + k_1a_1 + k_2a_2 + k_3b_1 + k_4b_2$.

If $x \in N_{\top}$, then x can not be expressed as $x = \sum \alpha_i [x_i, y_i]$, so $\mu_{A^{(1)}}(x) = 0$, $\nu_{A^{(1)}}(x) = 1$.

If $x \in N_{\bar{0}}$, then x can be expressed as $x = \alpha_1 [a_1, b_1] + \alpha_2 [a_2, b_2]$, $\alpha_1, \alpha_2 \in k$. We calculate

$$\mu_{A^{(1)}}(x) = \sup_{x = \sum_{i=1,2} \alpha_i [a_i, b_i]} \{ \min_{i=1,2} \{ \mu_{A^{(0)}}(a_i) \wedge \mu_{A^{(0)}}(b_i) \} \} = 0.1,$$

$$\nu_{A^{(1)}}(x) = \inf_{x = \sum_{i=1,2} \alpha_i [a_i, b_i]} \{ \max_{i=1,2} \{ \nu_{A^{(0)}}(a_i) \vee \nu_{A^{(0)}}(b_i) \} \} = 0.$$

Define $A^{(2)} = [A^{(1)}, A^{(1)}]$, we calculate for

$$\mu_{A^{(2)}}(x) = \sup_{x = \sum_{i=1,2} \alpha_i [a_i, b_i]} \{ \min_{i=1,2} \{ \mu_{A^{(1)}}(a_i) \wedge \mu_{A^{(1)}}(b_i) \} \} = 0,$$

$$\nu_{A^{(2)}}(x) = \inf_{x = \sum_{i=1,2} \alpha_i [a_i, b_i]} \{ \max_{i=1,2} \{ \nu_{A^{(1)}}(a_i) \vee \nu_{A^{(1)}}(b_i) \} \} = 1.$$

So $\eta^{(0)} \geq \eta^{(1)} \geq \eta^{(2)} = 0$ and $\kappa^{(0)} \leq \kappa^{(1)} \leq \kappa^{(2)} = 1$,

which imply that $A = (\mu_A, \nu_A)$ is a solvable intuitionistic fuzzy Lie ideal of N .

Following from the definition of a solvable intuitionistic fuzzy Lie ideal, we can easily get

Lemma 5.1: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . Then $A = (\mu_A, \nu_A)$ is a solvable intuitionistic fuzzy Lie ideal if and only if there is a positive integer n such that $\mu_{A^{(n)}} = 1_0$ and $\nu_{A^{(n)}} = 1_0^c$ for all $m \geq n$.

Theorem 5.1: Homomorphic images of solvable intuitionistic fuzzy Lie ideals are also solvable intuitionistic fuzzy Lie ideals.

Proof: Let $\varphi : \mathcal{G} \rightarrow \mathcal{G}'$ be a homomorphism of Lie superalgebra and assume that $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy Lie ideal of \mathcal{G} . Let $\varphi(A) = B$, i.e., $\mu_B = \mu_{\varphi(A)}$, $\nu_B = \nu_{\varphi(A)}$. We prove $\mu_{\varphi(A^{(n)})} = \mu_{B^{(n)}}$ and $\nu_{\varphi(A^{(n)})} = \nu_{B^{(n)}}$ by induction on n , where n is any positive integer. Indeed, let $y \in \mathcal{G}'$. Consider $n = 1$,

$$\begin{aligned} \mu_{\varphi(A^{(1)})}(y) &= \mu_{\varphi([A, A])}(y) = \sup_{y = \varphi(x)} \{ \mu_{[A, A]}(x) \} \\ &= \sup_{y = \varphi(x)} \{ \sup_{x = \sum_{i \in N} \alpha_i [x_i, y_i]} \{ \min_{i \in N} \{ \mu_A(x_i) \wedge \mu_A(y_i) \} \} \} \\ &= \sup_{y = \sum_{i \in N} \alpha_i \varphi[x_i, y_i]} \{ \min_{i \in N} \{ \mu_A(x_i) \wedge \mu_A(y_i) \} \} \\ &= \sup_{y = \sum_{i \in N} \alpha_i [a_i, b_i]} \{ \min_{i \in N} \{ \mu_A(x_i) \wedge \mu_A(y_i) \} : \varphi(x_i) = a_i, \varphi(y_i) = b_i \} \\ &= \sup_{y = \sum_{i \in N} \alpha_i [a_i, b_i]} \{ \min_{i \in N} \{ \mu_B(a_i) \wedge \mu_B(b_i) \} \} \\ &= \mu_{[B, B]}(y) = \mu_{B^{(1)}}(y), \text{ and} \\ \nu_{\varphi(A^{(1)})}(y) &= \nu_{\varphi([A, A])}(y) = \inf_{y = \varphi(x)} \{ \nu_{[A, A]}(x) \} \\ &= \inf_{y = \varphi(x)} \{ \inf_{x = \sum_{i \in N} \alpha_i [x_i, y_i]} \{ \max_{i \in N} \{ \nu_A(x_i) \vee \nu_A(y_i) \} \} \} \\ &= \inf_{y = \sum_{i \in N} \alpha_i \varphi[x_i, y_i]} \{ \max_{i \in N} \{ \nu_A(x_i) \vee \nu_A(y_i) \} \} \\ &= \inf_{y = \sum_{i \in N} \alpha_i [a_i, b_i]} \{ \max_{i \in N} \{ \nu_A(x_i) \vee \nu_A(y_i) \} : \varphi(x_i) = a_i, \varphi(y_i) = b_i \} \\ &= \inf_{y = \sum_{i \in N} \alpha_i [a_i, b_i]} \{ \max_{i \in N} \{ \nu_B(a_i) \vee \nu_B(b_i) \} \} \\ &= \nu_{[B, B]}(y) = \nu_{B^{(1)}}(y). \end{aligned}$$

These prove the case of $n = 1$. Suppose that the case of $n - 1$ is true, then $\mu_{\varphi(A^{(n)})} = \mu_{\varphi([A^{(n-1)}, A^{(n-1)}])}$

$$= \mu_{[\varphi(A^{(n-1)}), \varphi(A^{(n-1)})]} = \mu_{[B^{(n-1)}, B^{(n-1)}]} = \mu_{B^{(n)}} \text{ and } \nu_{\varphi(A^{(n)})}$$

$$= \nu_{\varphi([A^{(n-1)}, A^{(n-1)})]} = \nu_{[\varphi(A^{(n-1)}), \varphi(A^{(n-1)})]} = \nu_{[B^{(n-1)}, B^{(n-1)}]} = \nu_{B^{(n)}}.$$

Let m be a positive integer such that $\mu_{A^{(m)}} = 1_0$ and $\nu_{A^{(m)}} = 1_0^c$. Then for any $0 \neq y \in \mathcal{G}'$, we get $\mu_{B^{(m)}}(y) = \mu_{\varphi(A^{(m)})}(y) = \sup_{y=\varphi(x)} \{1_0(x)\} = 0$, $\nu_{B^{(m)}}(y) = \varphi(\nu_{A^{(m)}})(y) = \inf_{y=\varphi(x)} \{1_0^c(x)\} = 1$. So $\mu_{B^{(m)}} = 1_0$ and $\nu_{B^{(m)}} = 1_0^c$.

Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} and I be an ideal of \mathcal{G} . We can prove that A/I is an intuitionistic fuzzy Lie ideal of \mathcal{G}/I imitating Theorem 4.1.

Theorem 5.2: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} and A/I be a solvable intuitionistic fuzzy Lie ideal of \mathcal{G}/I . If $B = (\mu_B, \nu_B)$ is a solvable intuitionistic fuzzy Lie ideal of \mathcal{G} and is also an intuitionistic fuzzy ideal of $A = (\mu_A, \nu_A)$ such that $B(I) = A(I)$, then $A = (\mu_A, \nu_A)$ is solvable.

Proof: Let φ be the canonical projection from \mathcal{G} to \mathcal{G}/I . From the proofs of Theorem 4.1 and 5.1, we can get $\mu_{\varphi(A^{(n)})} = \mu_{(A/I)^{(n)}}$ and $\nu_{\varphi(A^{(n)})} = \nu_{(A/I)^{(n)}}$. Since A/I is solvable, there exists n such that $\mu_{(A/I)^{(n)}} = 1_0$ and $\nu_{(A/I)^{(n)}} = 1_0^c$.

For $0 \neq \bar{y} \in \mathcal{G}/I$, we have

$$\sup_{m \in \varphi^{-1}(\bar{y})} \{\mu_{A^{(n)}}(m)\} = \mu_{\varphi(A^{(n)})}(\bar{y}) = \mu_{(A/I)^{(n)}}(\bar{y}) = 0$$

$$\text{and } \inf_{m \in \varphi^{-1}(\bar{y})} \{\nu_{A^{(n)}}(m)\} = \nu_{\varphi(A^{(n)})}(\bar{y}) = \nu_{(A/I)^{(n)}}(\bar{y}) = 1.$$

Notice that $m \in \mathcal{G}$ and $m \neq 0$, we get $\mu_{A^{(n)}}(m) = 0$ and $\nu_{A^{(n)}}(m) = 1$.

$$\text{For } \bar{y} = 0, \text{ we have } \sup_{m \in \varphi^{-1}(0)} \{\mu_{A^{(n)}}(m)\} = \mu_{\varphi(A^{(n)})}(0) = 1$$

$$\text{and } \inf_{m \in \varphi^{-1}(0)} \{\nu_{A^{(n)}}(m)\} = \nu_{\varphi(A^{(n)})}(0) = 0.$$

Since $\varphi^{-1}(0) = I$ and $B(I) = A(I)$, we have

$\mu_{B^{(n)}}(I) = \mu_{A^{(n)}}(I)$ and $\nu_{B^{(n)}}(I) = \nu_{A^{(n)}}(I)$. For any $x \in I$, B is solvable, then there exists n such that $\mu_{B^{(n)}} = 1_0$ and $\nu_{B^{(n)}} = 1_0^c$, we have $\mu_{A^{(n)}} = 1_0$ and $\nu_{A^{(n)}} = 1_0^c$. Hence for any $x \in \mathcal{G}$, we always have $\mu_{A^{(n)}} = 1_0$ and $\nu_{A^{(n)}} = 1_0^c$, which imply that $A = (\mu_A, \nu_A)$ is solvable.

Remark 5.1: By Remark 4.1(2), the result is also true if

the condition of Theorem 5.2 can be reduced to "If $B = (\mu_B, \nu_B)$ is a solvable intuitionistic fuzzy Lie ideal of \mathcal{G} and $B \subseteq A$ such that $B(I) = A(I)$ ".

Lemma 5.2: Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy Lie ideals of \mathcal{G} . Then $(A \oplus B)^{(n)} = A^{(n)} \oplus B^{(n)}$.

Proof: Let $0 \neq x \in \mathcal{G}$. Then we have $[A, B] = (\mu_{[A, B]}, \nu_{[A, B]})$ where

$$\begin{aligned} \mu_{[A, B]}(x) &= \sup_{x = \sum_{i \in N} \alpha_i [x_i, y_i]} \{\min(\mu_A(x_i) \wedge \mu_B(y_i))\} \\ &\leq \mu_A(x) \wedge \mu_B(x) = 0, \\ \nu_{[A, B]}(x) &= \inf_{x = \sum_{i \in N} \alpha_i [x_i, y_i]} \{\max(\nu_A(x_i) \vee \nu_B(y_i))\} \\ &\geq \nu_A(x) \vee \nu_B(x) = 1 \end{aligned}$$

So $\mu_{[A, B]} = 1_0$ and $\nu_{[A, B]} = 1_0^c$. Consequently, for any positive integer a, b , we have $\mu_{[A^{(a)}, B^{(b)}]} = 1_0$ and $\nu_{[A^{(a)}, B^{(b)}]} = 1_0^c$. Now we prove the lemma by induction on n . Let $n = 1$. Then by Theorem 5.1 of [11], we have $(A \oplus B)^{(1)} = [A \oplus B, A \oplus B]$

$$= [A, A] \oplus [A, B] \oplus [B, A] \oplus [B, B] = A^{(1)} \oplus B^{(1)}.$$

Suppose that the case of $n - 1$ is true, then

$$\begin{aligned} (A \oplus B)^{(n)} &= [(A \oplus B)^{(n-1)}, (A \oplus B)^{(n-1)}] \\ &= [A^{(n-1)} \oplus B^{(n-1)}, A^{(n-1)} \oplus B^{(n-1)}] = A^{(n)} \oplus B^{(n)}. \end{aligned}$$

Theorem 5.3: Direct sum of solvable intuitionistic fuzzy Lie ideals is also a solvable intuitionistic fuzzy Lie ideal.

Proof: Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be solvable intuitionistic fuzzy Lie ideals. Then there exist positive integers m, n such that $\mu_{A^{(m)}} = 1_0, \nu_{A^{(m)}} = 1_0^c$ and $\mu_{B^{(n)}} = 1_0, \nu_{B^{(n)}} = 1_0^c$. Since $(A \oplus B)^{(m+n)} = A^{(m+n)} \oplus B^{(m+n)}$, we have $\mu_{(A \oplus B)^{(m+n)}} = \mu_{A^{(m+n)} \oplus B^{(m+n)}} = 1_0$ and $\nu_{(A \oplus B)^{(m+n)}} = \nu_{A^{(m+n)} \oplus B^{(m+n)}} = 1_0^c$. So $A \oplus B$ is a solvable intuitionistic fuzzy Lie ideal.

Definition 5.5: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy Lie ideal of \mathcal{G} . Define inductively a sequence of intuitionistic fuzzy Lie ideals of \mathcal{G} by

$A^0 = A, A^1 = [A, A^0], A^2 = [A, A^1], \dots, A^n = [A, A^{n-1}]$, which is called the descending central series of an intuitionistic fuzzy Lie ideal $A = (\mu_A, \nu_A)$ of \mathcal{G} . We get

$$\mu_{A^0} \supseteq \mu_{A^1} \supseteq \mu_{A^2} \supseteq \dots \supseteq \mu_{A^n} \supseteq \dots$$

and $\nu_{A^0} \subseteq \nu_{A^1} \subseteq \nu_{A^2} \subseteq \dots \subseteq \nu_{A^n} \subseteq \dots$.

Definition 5.6: For an intuitionistic fuzzy Lie ideal $A = (\mu_A, \nu_A)$, define $\eta^n = \sup\{\mu_{A^n}(x) : 0 \neq x \in \mathcal{G}\}$ and $\kappa^n = \inf\{\nu_{A^n}(x) : 0 \neq x \in \mathcal{G}\}$, for any positive integer n . The intuitionistic fuzzy Lie ideal is called a nilpotent intuitionistic fuzzy Lie ideal, if there is a positive integer m such that $\eta^m = 0$ and $\kappa^m = 1$, or equivalently, $\mu_{A^m} = 1_0$ and $\nu_{A^m} = 1_0^c$.

Example 5.2: Let us take the basis h, e, f of $\mathfrak{sl}(1|1)$ as follows $h = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. Then h is an even element, and e and f are odd element. Their bracket products are as follows: $[e, f] = [f, e] = h$, the other brackets = 0. Then $\mathfrak{sl}(1|1)$ is a three-dimensional Lie superalgebra. (See [23])

Define $A_{\bar{0}} = (\mu_{A_{\bar{0}}}, \nu_{A_{\bar{0}}}) : \mathfrak{sl}(1|1)_{\bar{0}} \rightarrow [0, 1]$ where $\mu_{A_{\bar{0}}}(x) = \begin{cases} 0.6, & x = h \\ 1, & \text{otherwise} \end{cases}, \nu_{A_{\bar{0}}}(x) = \begin{cases} 0.4, & x = h \\ 0, & \text{otherwise} \end{cases}$. Then $A_{\bar{0}} = (\mu_{A_{\bar{0}}}, \nu_{A_{\bar{0}}})$ is an intuitionistic fuzzy subspace of $\mathfrak{sl}(1|1)_{\bar{0}}$.

Define $A_{\bar{1}} = (\mu_{A_{\bar{1}}}, \nu_{A_{\bar{1}}}) : \mathfrak{sl}(1|1)_{\bar{1}} \rightarrow [0, 1]$ where $\mu_{A_{\bar{1}}}(x) = \begin{cases} 0.3, & x = e \\ 0.5, & x = f \\ 1, & \text{otherwise} \end{cases}, \nu_{A_{\bar{1}}}(x) = \begin{cases} 0.7, & x = e \\ 0.5, & x = f \\ 0, & \text{otherwise} \end{cases}$. Then $A_{\bar{1}} = (\mu_{A_{\bar{1}}}, \nu_{A_{\bar{1}}})$ is an intuitionistic fuzzy subspace of $\mathfrak{sl}(1|1)_{\bar{1}}$.

Define $A = (\mu_A, \nu_A) : \mathfrak{sl}(1|1) \rightarrow [0, 1]$ where $\mu_A(x) = \mu_{A_{\bar{0}}}(x_{\bar{0}}) \wedge \mu_{A_{\bar{1}}}(x_{\bar{1}})$ and $\nu_A(x) = \nu_{A_{\bar{0}}}(x_{\bar{0}}) \vee \nu_{A_{\bar{1}}}(x_{\bar{1}})$. Then $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy Lie ideal of $\mathfrak{sl}(1|1)$.

Let $A^0 = A$ and $A^1 = [A, A^0]$. If $x \in \mathfrak{sl}(1|1)_{\bar{1}}$, x can not be expressed as $x = \sum \alpha_i [x_i, y_i]$, then $\mu_{A^1}(x) = 0, \nu_{A^1}(x) = 1$. If $x \in \mathfrak{sl}(1|1)_{\bar{0}}$, then $x = \alpha[e, f]$ or $x = \beta[f, e]$, $\alpha, \beta \in k$, we have

$$\mu_{A^1}(x) = \sup\{\mu_A(e) \wedge \mu_{A^0}(f)\} = \sup\{\mu_A(f) \wedge \mu_{A^0}(e)\} = 0.3$$

and

$$\nu_{A^1}(x) = \inf\{\nu_A(e) \vee \nu_{A^0}(f)\} = \inf\{\nu_A(f) \vee \nu_{A^0}(e)\} = 0.7.$$

Define $A^2 = [A, A^1]$, we calculate if $x \in \mathfrak{sl}(1|1)_{\bar{1}}$, $\mu_{A^2}(x) = 0, \nu_{A^2}(x) = 1$; if $x \in \mathfrak{sl}(1|1)_{\bar{0}}$, then

$$\mu_{A^2}(x) = \sup\{\mu_{A^1}(e) \wedge \mu_{A^1}(f)\} = \sup\{\mu_{A^1}(f) \wedge \mu_{A^1}(e)\} = 0$$

and

$$\nu_{A^1}(x) = \inf\{\nu_{A^1}(e) \vee \nu_{A^1}(f)\} = \inf\{\nu_{A^1}(f) \vee \nu_{A^1}(e)\} = 1.$$

Then we get $\eta^0 \geq \eta^1 \geq \eta^2 = 0$ and $\kappa^0 \leq \kappa^1 \leq \kappa^2 = 1$. So $A = (\mu_A, \nu_A)$ is a nilpotent intuitionistic fuzzy Lie ideal of $\mathfrak{sl}(1|1)$.

Theorem 5.4: Homomorphic images of nilpotent intuitionistic fuzzy Lie ideals are also nilpotent intuitionistic fuzzy Lie ideals. Direct sum of nilpotent intuitionistic fuzzy Lie ideals is also a nilpotent intuitionistic fuzzy Lie ideal.

Proof: It can be proved by imitating Theorem 5.1 and Theorem 5.3.

Theorem 5.5: If $A = (\mu_A, \nu_A)$ is a nilpotent intuitionistic fuzzy Lie ideal of \mathcal{G} , then it is solvable.

Proof: Since $A = (\mu_A, \nu_A)$ is a nilpotent intuitionistic fuzzy Lie ideal, there exists m such that $\mu_{A^m} = 1_0$ and $\nu_{A^m} = 1_0^c$. It is easy to prove that $\mu_{A^{(m)}} \subseteq \mu_{A^m}$ and $\nu_{A^{(m)}} \supseteq \nu_{A^m}$ by induction on m . So

$$\mu_{A^{(m)}} = \mu_{[A^{(m-1)}, A^{(m-1)}]} \subseteq \mu_{[A, A^{m-1}]} = \mu_{A^m} = 1_0$$

and

$$\nu_{A^{(m)}} = \nu_{[A^{(m-1)}, A^{(m-1)}]} \supseteq \nu_{[A, A^{m-1}]} = \nu_{A^m} = 1_0^c.$$

6. Conclusions

The aim of this paper is to introduce and study Lie superalgebras in intuitionistic fuzzy settings. We define an intuitionistic fuzzy quotient Lie superalgebra by an intuitionistic fuzzy Lie ideal of Lie superalgebra. Also, we consider the solvable and nilpotent intuitionistic fuzzy Lie ideals of Lie superalgebras and investigate related properties. These works generalized the related theory of fuzzy Lie algebras. Our future work on this topic will focus on studying of adjoint representation of intuitionistic fuzzy Lie sub-superalgebras and the Killing form of intuitionistic fuzzy Lie sub-superalgebras. In addition, we shall study: (1) Roughness based on fuzzy Lie ideals (2) Soft Lie superalgebras (3) Generalized fuzzy soft Lie superalgebras. Our obtained results probably can be applied in various fields such as artificial intelligence, signal processing, multi-agent systems, pattern recognition, robotics, expert systems, medical diagnosis and engineering.

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