

# Decentralized Guaranteed Cost Control for Large-Scale T-S Fuzzy Systems

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## Abstract

**This paper studies the guaranteed cost control issues for the large-scale systems which are composed of Takagi-Sugeno (T-S) fuzzy subsystems with interconnections. A cost function is considered as the performance index for the system. By Lyapunov stability criterion and the parallel distributed compensation (PDC), some sufficient conditions are derived such that the large-scale fuzzy system is not only asymptotically stable but also cost-guaranteed. Furthermore, the interactive relations among the fuzzy subsystems are employed to relax previous results. The same ideas are also extended to the large-scale fuzzy system with uncertainty. Finally, two examples illustrate the effectiveness of the proposed criteria.**

**Keywords:** *Guaranteed cost control, T-S fuzzy model, large-scale system, uncertainty.*

## 1. Introduction

The so-called large-scale system is composed of a number of independent subsystems and a set of interconnections. The large-scale system is extensively applied in lots of fields, such as electric power system, computer networks, and urban traffic network. Hence, the stabilization issues of large-scale systems have been studied in lots of literature. [1-9].

During the last two decades, fuzzy systems have attracted great attention in academic research and industrial applications. Any nonlinear system can be represented by the famous Takagi-Sugeno (T-S) fuzzy model [10-11], and then the system can be systematically analyzed. The equivalent T-S fuzzy model is often effectively controlled by the parallel distributed compensation (PDC). There have been lots of studies explore the control problems by T-S fuzzy representations [12-17].

The large-scale systems represented by T-S fuzzy model are investigated recently [18-25]. Refs. [18-19] study the stability analysis of large-scale T-S fuzzy systems. By Lyapunov direct method and M-matrix, a decentralized control for large-scale T-S fuzzy systems is

proposed in [20]. The paper [21] studies the model reference tracking problem by decentralized fuzzy control. To overcome the effect of modelling error of a multiple time-delay large-scale system, a robust fuzzy control design is proposed in [22]. In [20-24], the PDC law is utilized to control the large-scale T-S fuzzy systems. In [25], the non-PDC control law is proposed to stabilize the uncertain fuzzy time-delay interconnected system.

In control design, it is often desirable to be not only asymptotically stable but also an adequate performance guaranteed, which is the so-called guaranteed cost control [26-28]. Herein, the cost-guaranteed problem for the large-scale T-S fuzzy system is considered. Moreover, the performance constraint often accompanies conservative stability conditions. Therefore, we also adopt some scheme to relax the conservative criterion.

Kim and Lee [29] consider the interactive relations among all subsystems to relax the stability criterion for T-S fuzzy system. Liu and Zhang [30] apply a similar idea to obtain a more relaxed stability criterion for T-S fuzzy system. Ref. [31] utilizes this idea to derive a relaxed stability criterion for large-scale T-S fuzzy systems. Although the interactive relations among all subsystems are considered to obtain more relaxed results in aforementioned studies, there is still no study employs this concept to guaranteed cost control for large-scale T-S fuzzy systems.

This paper focuses on the relaxed decentralized guaranteed cost control for large-scale T-S fuzzy systems. Firstly, the performance constraint is performed by a cost function. Secondly, the interactive relations among all fuzzy subsystems are adopted in the derivative of decentralized guaranteed cost control. Then the relaxed cost-guaranteed stabilization criterion for the large-scale T-S fuzzy systems is proposed. Also, relaxed guaranteed cost problem for the large-scale fuzzy systems with uncertainties is discussed. Finally, an application example and a numerical example are illustrated to show the effectiveness of proposed theorems. The way to choose an appropriate cost function according to the performance requirement is illustrated by the simulation results.

## 2. Problem formulation

Consider the nonlinear large-scale system represented by T-S fuzzy model as follows. Each subsystem  $S_i$  is composed of a set of fuzzy If-Then rules.

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$$S_i^l : \begin{cases} \text{If } z_{i1}(t) \text{ is } H_{i1}^l, \dots, \text{ and } z_{ig}(t) \text{ is } H_{ig}^l, \text{ then} \\ \dot{x}_i(t) = A_i^l x_i(t) + B_i^l u_i(t) + \sum_{j=1}^{\sigma} f_{ij}^l x_j(t) \end{cases} \quad (1)$$

where  $l=1, 2, \dots, r_i$ ;  $i=1, 2, \dots, \sigma$ ; and  $r_i$  denotes the number of fuzzy rules in subsystem  $S_i$ . The fuzzy sets of the rules  $S_i^l$  are denoted by  $H_{iq}^l (q=1, 2, \dots, g)$ . The premise variables in subsystem  $S_i$  are represented by  $z_i(t) = [z_{i1}(t), z_{i2}(t), \dots, z_{ig}(t)]$  which may be equal to  $x_i(t)$  or a function of  $x_i(t)$ . The system matrix and input matrix of the rule  $S_i^l$  are denoted by  $A_i^l$  and  $B_i^l$ , respectively. Moreover, the state vector  $x_i(t) \in \mathfrak{R}^{n_i}$ , the input vector  $u_i(t) \in \mathfrak{R}^{m_i}$ , and the interconnection  $f_{ij}^l$  with appropriate dimension between subsystem  $S_i$  and subsystem  $S_j$  in the rule  $l$  are all stated.

By the product fuzzy inference method and central-average defuzzifier, (1) is inferred as

$$\dot{x}_i(t) = \sum_{l=1}^{r_i} \mu_i^l(z_{iq}(t)) [A_i^l x_i(t) + B_i^l u_i(t) + \sum_{j=1}^{\sigma} f_{ij}^l x_j(t)] \quad (2)$$

where

$$\mu_i^l(z_{iq}(t)) = \frac{\omega_i^l(z_{iq}(t))}{\sum_{l=1}^{r_i} \omega_i^l(z_{iq}(t))}, \quad \omega_i^l(z_{iq}(t)) = \prod_q H_{iq}^l(z_{iq}(t)),$$

$H_{iq}^l(z_{iq}(t))$  is the grade of membership  $H_{iq}^l$  by  $z_{iq}(t)$ ;  $\omega_i^l(z_{iq}(t)) \geq 0$ , for  $l=1, 2, \dots, r_i$ ; and  $\sum_{l=1}^{r_i} \mu_i^l(z_{iq}(t)) = 1$ , for  $i=1, 2, \dots, \sigma$ . In the following,  $\mu_i^l(z_{iq}(t))$  is replaced by the abbreviation  $\mu_i^l$ .

The main purpose of this paper is to propose a criterion for the large-scale T-S fuzzy system (2) under which not only the system is stabilized by the fuzzy control but also the cost is guaranteed. Hence, we define a cost function  $J$  to present the performance.

$$J = \int_0^{\infty} \{x^T(t) Q x(t) + u^T(t) R u(t)\} dt \quad (3)$$

where  $x(t) = [x_1^T(t), x_2^T(t), \dots, x_{\sigma}^T(t)]^T$  and  $u(t) = [u_1^T(t), u_2^T(t), \dots, u_{\sigma}^T(t)]^T$  are the overall state vector and the overall input vector respectively;  $Q = \text{diag}[Q_1, Q_2, \dots, Q_{\sigma}]$  and  $R = \text{diag}[R_1, R_2, \dots, R_{\sigma}]$  are given positive definite symmetric matrices with appropriated dimensions. Associate with the cost function (3), a definition for the guaranteed cost fuzzy control is given as follows.

**Definition 1**[26]: Consider the T-S fuzzy system (2). If there exist a control  $u(t)$  and a scalar  $J_0 > 0$  such that the closed-loop system is asymptotically stable and the value of the cost function (3) satisfies  $J \leq J_0$ , then  $J_0$  is said to be a guaranteed cost and  $u(t)$  is said to be a guaranteed cost control law for the T-S fuzzy system (2).

Following Definition 1, then we could design a guaranteed cost PDC for the large-scale T-S fuzzy system in next section.

### 3. Guaranteed cost control design

Let the  $l$ -th rule of fuzzy PDC controller in the  $i$ -th subsystem be the following form

$$C_i^l : \begin{cases} \text{If } z_{i1}(t) \text{ is } H_{i1}^l, \dots, \text{ and } z_{ig}(t) \text{ is } H_{ig}^l, \text{ then} \\ u_i(t) = K_i^l x_i(t) \end{cases} \quad (4)$$

where  $l=1, 2, \dots, r_i$ ; and  $i=1, 2, \dots, \sigma$ . The PDC scheme shares the same membership functions of the fuzzy plant model. By the same inference engine and defuzzifier of (2), the weight of each PDC rule is the same as  $\mu_i^l$  of (2). The final output of the fuzzy controller  $C_i$  is

$$u_i(t) = \sum_{l=1}^{r_i} \mu_i^l K_i^l x_i(t) \quad (5)$$

Combining (2) and (5) yields the closed-loop fuzzy system as follows.

$$\dot{x}_i(t) = \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m [A_i^l + B_i^l K_i^m] x_i(t) + \sum_{l=1}^{r_i} \sum_{j=1}^{\sigma} \mu_i^l f_{ij}^l x_j(t) \quad (6)$$

where  $i=1, 2, \dots, \sigma$ . Now, the following theorem is derived to achieve the main objective of this paper.

**Theorem 1:** Consider the system (6) and the cost function (3) with given positive definite matrices  $Q$  and  $R$ . The PDC (5) not only can stabilize the large-scale fuzzy system (6) asymptotically but also ensure the cost function (3) satisfying  $J \leq J_0$ , if there exist matrices  $K_i^l$ ,  $T_i^{lm}$ , and positive definite matrices  $P_i$ , satisfying (7a), (7b), and (7c).

$$S_i^{ll} + \sum_{j=1}^{\sigma} P_i f_{ij}^l f_{ij}^{lT} P_i + \sigma \cdot I + Q_i + K_i^{lT} R_i K_i^l \leq T_i^{ll} \quad (7a)$$

$$S_i^{lm} + S_i^{ml} + \sum_{j=1}^{\sigma} P_i (f_{ij}^l f_{ij}^{lT} + f_{ij}^m f_{ij}^{mT}) P_i + 2\sigma \cdot I \quad (7b)$$

$$+ 2Q_i + K_i^{mT} R_i K_i^m + K_i^{lT} R_i K_i^l \leq T_i^{lm} + T_i^{ml}$$

$$T_i \equiv \begin{bmatrix} T_i^{11} & T_i^{12} & \dots & T_i^{1r_i} \\ T_i^{21} & T_i^{22} & \dots & T_i^{2r_i} \\ \vdots & \vdots & \ddots & \vdots \\ T_i^{r_i 1} & T_i^{r_i 2} & \dots & T_i^{r_i r_i} \end{bmatrix} < 0 \quad (7c)$$

where  $l=1, 2, \dots, r_i$ ;  $l < m \leq r_i$ ;  $i=1, 2, \dots, \sigma$ ;

$$S_i^{lm} = (G_i^{lm})^T P_i + P_i (G_i^{lm}); \quad G_i^{lm} = A_i^l + B_i^l K_i^m; \quad T_i^{lm} = T_i^{mlT}.$$

$$J_0 = x^T(0) P x(0) \quad (8)$$

where  $P = \text{diag}[P_1, P_2, \dots, P_{\sigma}]$ .

**Proof:** Let the Lyapunov function candidate be

$$V(x(t)) = \sum_{i=1}^{\sigma} v_i(x_i(t)) = \sum_{i=1}^{\sigma} x_i^T(t) P_i x_i(t) \quad (9)$$

Taking the derivative of  $v_i$  along (6) yields

$$\begin{aligned}
 \dot{V}(x(t)) &= \sum_{i=1}^{\sigma} \left[ \dot{x}_i^T(t) P_i x_i(t) + x_i^T(t) P_i \dot{x}_i(t) \right] \\
 &= \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \{ x_i^T(t) [(A_i^l + B_i^l K_i^m)^T P_i \\
 &+ P_i (A_i^l + B_i^l K_i^m)] x_i(t) + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t) \} \\
 &= \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} (\mu_i^l)^2 \{ x_i^T(t) [(G_i^{ll})^T P_i + P_i (G_i^{ll})] x_i(t) \\
 &+ 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t) \} \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m>l}^{r_i} \mu_i^l \mu_i^m \{ x_i^T(t) [(G_i^{lm})^T P_i + P_i (G_i^{lm})] x_i(t) \\
 &+ x_i^T(t) [(G_i^{ml})^T P_i + P_i (G_i^{ml})] x_i(t) \\
 &+ 2 \sum_{j=1}^{\sigma} x_j^T(t) (f_{ij}^{lT} + f_{ij}^{mT}) P_i x_i(t) \} \\
 &= \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} (\mu_i^l)^2 [x_i^T(t) S_i^{ll} x_i(t) + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t)] \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m>l}^{r_i} \mu_i^l \mu_i^m [x_i^T(t) (S_i^{lm} + S_i^{ml}) x_i(t) \\
 &+ 2 \sum_{j=1}^{\sigma} x_j^T(t) (f_{ij}^{lT} + f_{ij}^{mT}) P_i x_i(t)] \tag{10}
 \end{aligned}$$

Substituting (7a) and (7b) into (10) yields

$$\begin{aligned}
 \dot{V}(x(t)) &\leq \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} (\mu_i^l)^2 [x_i^T(t) (-Q_i - K_i^{lT} R_i K_i^l - \sum_{j=1}^{\sigma} P_i f_{ij}^l f_{ij}^{lT} P_i \\
 &- \sigma \cdot I + T_i^{ll}) x_i(t) + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t)] \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m>l}^{r_i} \mu_i^l \mu_i^m \{ x_i^T(t) [-2Q_i - K_i^{lT} R_i K_i^l - K_i^{mT} R_i K_i^m \\
 &+ T_i^{lm} + T_i^{ml} - \sum_{j=1}^{\sigma} P_i (f_{ij}^l f_{ij}^{lT} + f_{ij}^m f_{ij}^{mT}) P_i - 2\sigma \cdot I] x_i(t) \\
 &+ 2 \sum_{j=1}^{\sigma} x_j^T(t) (f_{ij}^{lT} P_i + f_{ij}^{mT} P_i) x_i(t) \}
 \end{aligned}$$

Since

$$-K_i^{lT} R_i K_i^l - K_i^{mT} R_i K_i^m \leq -K_i^{lT} R_i K_i^m - K_i^{mT} R_i K_i^l,$$

then we have

$$\begin{aligned}
 \dot{V}(x(t)) &\leq \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} (\mu_i^l)^2 [x_i^T(t) (-Q_i - K_i^{lT} R_i K_i^l - \sum_{j=1}^{\sigma} P_i f_{ij}^l f_{ij}^{lT} P_i - \sigma \cdot I \\
 &+ T_i^{ll}) x_i(t) + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t)] \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m>l}^{r_i} \mu_i^l \mu_i^m \{ x_i^T(t) [-2Q_i - K_i^{lT} R_i K_i^m - K_i^{mT} R_i K_i^l \\
 &+ T_i^{lm} + T_i^{ml} - \sum_{j=1}^{\sigma} P_i (f_{ij}^l f_{ij}^{lT} + f_{ij}^m f_{ij}^{mT}) P_i - 2\sigma \cdot I] x_i(t) \\
 &+ 2 \sum_{j=1}^{\sigma} x_j^T(t) (f_{ij}^{lT} P_i + f_{ij}^{mT} P_i) x_i(t) \}
 \end{aligned}$$

$$\begin{aligned}
 &\leq \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m x_i^T(t) (-Q_i - K_i^{lT} R_i K_i^m + T_i^{lm}) x_i(t) \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} (\mu_i^l)^2 \left[ - \sum_{j=1}^{\sigma} \left\| f_{ij}^{lT} P_i x_i(t) \right\|^2 - \sigma \|x_i(t)\|^2 \right. \\
 &+ 2 \sum_{j=1}^{\sigma} \|x_j(t)\| \cdot \left\| f_{ij}^{lT} P_i x_i(t) \right\| \left. \right] \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m>l}^{r_i} \mu_i^l \mu_i^m \left[ - \sum_{j=1}^{\sigma} \left\| f_{ij}^{lT} P_i x_i(t) \right\|^2 + 2 \sum_{j=1}^{\sigma} \|x_j(t)\| \cdot \left\| f_{ij}^{lT} P_i x_i(t) \right\| \right. \\
 &- \sigma \|x_i(t)\|^2 - \sum_{j=1}^{\sigma} \left\| f_{ij}^{mT} P_i x_i(t) \right\|^2 + 2 \sum_{j=1}^{\sigma} \|x_j(t)\| \cdot \left\| f_{ij}^{mT} P_i x_i(t) \right\| \\
 &- \sigma \|x_i(t)\|^2 \left. \right] \\
 &= \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m x_i^T(t) (-Q_i - K_i^{lT} R_i K_i^m + T_i^{lm}) x_i(t) \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} (\mu_i^l)^2 \left[ - \sum_{j=1}^{\sigma} \left( \left\| f_{ij}^{lT} P_i x_i(t) \right\| - \|x_j(t)\| \right)^2 + \sum_{j=1}^{\sigma} \|x_j(t)\|^2 \right. \\
 &- \sigma \|x_i(t)\|^2 \left. \right] \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m>l}^{r_i} \mu_i^l \mu_i^m \left[ - \sum_{j=1}^{\sigma} \left( \left\| f_{ij}^{lT} P_i x_i(t) \right\| - \|x_j(t)\| \right)^2 + \sum_{j=1}^{\sigma} \|x_j(t)\|^2 \right. \\
 &- \sigma \|x_i(t)\|^2 - \sum_{j=1}^{\sigma} \left( \left\| f_{ij}^{mT} P_i x_i(t) \right\| - \|x_j(t)\| \right)^2 \\
 &+ \sum_{j=1}^{\sigma} \|x_j(t)\|^2 - \sigma \|x_i(t)\|^2 \left. \right] \\
 &\leq \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m x_i^T(t) (-Q_i - K_i^{lT} R_i K_i^m + T_i^{lm}) x_i(t) \\
 &+ \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \left[ \sum_{j=1}^{\sigma} \|x_j(t)\|^2 - \sigma \|x_i(t)\|^2 \right] \\
 &= \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m x_i^T(t) (-Q_i - K_i^{lT} R_i K_i^m + T_i^{lm}) x_i(t) \\
 &+ \sum_{i=1}^{\sigma} \left[ \sum_{j=1}^{\sigma} \|x_j(t)\|^2 - \sigma \|x_i(t)\|^2 \right] \\
 &= \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m x_i^T(t) (-Q_i - K_i^{lT} R_i K_i^m + T_i^{lm}) x_i(t) \\
 &+ \sum_{i=1}^{\sigma} \left[ \sigma \|x_i(t)\|^2 - \sigma \|x_i(t)\|^2 \right] \\
 &= \sum_{i=1}^{\sigma} [-x_i^T(t) Q_i x_i(t) - u_i^T(t) R_i u_i(t)] \\
 &+ \sum_{i=1}^{\sigma} \begin{bmatrix} \mu_i^1 x_i(t) \\ \vdots \\ \mu_i^{r_i} x_i(t) \end{bmatrix}^T \begin{bmatrix} T_i^{11} & \cdots & T_i^{1r_i} \\ \vdots & \ddots & \vdots \\ T_i^{r_i 1} & \cdots & T_i^{r_i r_i} \end{bmatrix} \begin{bmatrix} \mu_i^1 x_i(t) \\ \vdots \\ \mu_i^{r_i} x_i(t) \end{bmatrix} \\
 &< -[x^T(t) Q x(t) + u^T(t) R u(t)] < 0 \tag{11}
 \end{aligned}$$

Then (6) is asymptotically stable. Integrating (11) from 0 to  $\infty$ , we have

$$\int_0^{\infty} \{x^T(t) Q x(t) + u^T(t) R u(t)\} dt < -V(x(\infty)) + x^T(0) P x(0)$$

Because  $V(x(t)) \geq 0$  and  $\dot{V}(x(t)) < 0$ , thus  $\lim_{t \rightarrow \infty} V(x(t)) = 0$ .

Therefore, the following inequality can be obtained.

$$\int_0^\infty \{x^T(t)Qx(t) + u^T(t)Ru(t)\}dt < x^T(0)Px(0) = J_0.$$

Q.E.D.

*Remark 1:* Notably, [29-31] have proved that the consideration of the interactive relations among all fuzzy subsystems can relax the stability criteria for T-S fuzzy systems. Hence, the interactive relations among all subsystems are considered in this study, which lies in (7c). The off-diagonal elements matrices of (7c), that is  $T_i^{lm}$  and  $T_i^{ml}$  in (7b), do not have to be positive definite, which admit more freedom while solving (7b).

*Remark 2:* In Theorem 1, our main task is to find matrices  $K_i^l$ ,  $T_i^{lm}$ , and  $P_i$ , satisfying (7a), (7b), and (7c). This problem will be solved efficiently by convex optimization method for LMIs. Multiplying  $P_i^{-1}$ , (i.e.,  $W_i$ ) on the left and right of (7a), (7b) and (7c) respectively, and using Schur complement, then (7a), (7b) and (7c) can be transformed into an LMI problem as (12a), (12b) and (12c) for seeking  $W_i$ ,  $Z_i^l$  and  $\bar{T}_i^{lm}$ . Thus the local state feedback gain is obtained from  $K_i^l = Z_i^l W_i^{-1}$ .

$$\begin{bmatrix} \Phi_i^{ll} & W_i & W_i & Z_i^{lT} \\ W_i & -(\sigma \cdot I)^{-1} & 0 & 0 \\ W_i & 0 & -Q_i^{-1} & 0 \\ Z_i^l & 0 & 0 & -R_i^{-1} \end{bmatrix} < 0 \quad (12a)$$

$$\begin{bmatrix} \Phi_i^{lm} + \Phi_i^{ml} & W_i & W_i & Z_i^{lT} & Z_i^{mT} \\ W_i & -(2\sigma \cdot I)^{-1} & 0 & 0 & 0 \\ W_i & 0 & -(2Q_i)^{-1} & 0 & 0 \\ Z_i^l & 0 & 0 & -R_i^{-1} & 0 \\ Z_i^m & 0 & 0 & 0 & -R_i^{-1} \end{bmatrix} < 0 \quad (12b)$$

$$\begin{bmatrix} \bar{T}_i^{11} & \bar{T}_i^{12} & \dots & \bar{T}_i^{1r_i} \\ \bar{T}_i^{21} & \bar{T}_i^{22} & \dots & \bar{T}_i^{2r_i} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{T}_i^{r_i 1} & \bar{T}_i^{r_i 2} & \dots & \bar{T}_i^{r_i r_i} \end{bmatrix} < 0 \quad (12c)$$

where  $l = 1, 2, \dots, r_i$ ;  $i = 1, 2, \dots, \sigma$ ;

$$\Phi_i^{lm} \equiv A_i^l W_i + W_i A_i^{lT} + B_i^l Z_i^m + Z_i^{mT} B_i^{lT} + \sum_{j=1}^{\sigma} f_{ij}^l f_{ij}^{lT} - \bar{T}_i^{lm},$$

$$W_i = P_i^{-1}, \quad Z_i^l = K_i^l W_i, \quad \bar{T}_i^{lm} = \bar{T}_i^{mlT} \quad \text{and} \quad \bar{T}_i^{lm} = W_i T_i^{lm} W_i.$$

After we finish the above problem, next, we will extend the above result to the large-scale system (1) with uncertainties  $\Delta A_i^l$  and  $\Delta B_i^l$  in (13)

$$S_i^l : \begin{cases} \text{If } z_{i1}(t) \text{ is } H_{i1}^l, \dots, \text{ and } z_{ig}(t) \text{ is } H_{ig}^l, \text{ then} \\ \dot{x}_i(t) = (A_i^l + \Delta A_i^l)x_i(t) + (B_i^l + \Delta B_i^l)u_i(t) + \sum_{j=1}^{\sigma} f_{ij}^l x_j(t) \end{cases} \quad (13)$$

where

$$[\Delta A_i^l \quad \Delta B_i^l] = \bar{D}_i^l \Theta_i^l(t) [\bar{E}_{1i}^l \quad \bar{E}_{2i}^l] \quad (14)$$

$\bar{D}_i^l$ ,  $\bar{E}_{1i}^l$ , and  $\bar{E}_{2i}^l$  are known constant matrices with appropriate dimensions, and  $\Theta_i^l(t)$  is assumed as a matrix with Lebesgue measurable elements bounded by

$$\Theta_i^{lT}(t)\Theta_i^l(t) \leq I$$

where  $I$  is the unity matrix. Applying the PDC (5) into the system (13), we can obtain the final closed-loop system as follows.

$$\begin{aligned} \dot{x}_i(t) = & \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m [(A_i^l + \Delta A_i^l) + (B_i^l + \Delta B_i^l)K_i^m] x_i(t) \\ & + \sum_{l=1}^{r_i} \sum_{j=1}^{\sigma} \mu_i^l f_{ij}^l x_j(t) \quad \text{for } i = 1, 2, \dots, \sigma. \end{aligned} \quad (15)$$

*Theorem 2:* Consider the system (15) and the cost function (3) with given positive definite matrices  $Q$  and  $R$ . The PDC (4) not only can stabilize the large-scale fuzzy system with uncertainties (14) robustly but also ensure the cost function (3) satisfying  $J \leq J_0$ , if there exist matrices  $K_i^l$ ,  $\bar{T}_i^{lm}$ , and positive definite matrices  $P_i$ , satisfying (16a), (16b) and (16c).

$$S_i^{ll} + \Gamma_i^{ll} + \sum_{j=1}^{\sigma} P_i f_{ij}^l f_{ij}^{lT} P_i + \sigma \cdot I + Q_i + K_i^{lT} R_i K_i^l \leq \bar{T}_i^{ll} \quad (16a)$$

$$\begin{aligned} S_i^{lm} + S_i^{ml} + \Gamma_i^{lm} + \Gamma_i^{ml} + \sum_{j=1}^{\sigma} P_i (f_{ij}^l f_{ij}^{lT} + f_{ij}^m f_{ij}^{mT}) P_i \\ + 2\sigma \cdot I + 2Q_i + K_i^{lT} R_i K_i^l + K_i^{mT} R_i K_i^m \leq \bar{T}_i^{lm} + \bar{T}_i^{ml} \end{aligned} \quad (16b)$$

$$\bar{T}_i \equiv \begin{bmatrix} \bar{T}_i^{11} & \bar{T}_i^{12} & \dots & \bar{T}_i^{1r_i} \\ \bar{T}_i^{21} & \bar{T}_i^{22} & \dots & \bar{T}_i^{2r_i} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{T}_i^{r_i 1} & \bar{T}_i^{r_i 2} & \dots & \bar{T}_i^{r_i r_i} \end{bmatrix} < 0, \quad (16c)$$

where  $i = 1, 2, \dots, \sigma$ ;  $l = 1, 2, \dots, r_i$ ;  $l < m \leq r_i$ ;

$$G_i^{lm} = A_i^l + B_i^l K_i^m; \quad S_i^{lm} = (G_i^{lm})^T P_i + P_i (G_i^{lm}); \quad \bar{T}_i^{lm} = \bar{T}_i^{mlT};$$

$$\Gamma_i^{lm} = P_i \bar{D}_i^l \bar{D}_i^{lT} P_i + (\bar{E}_{1i}^l + \bar{E}_{2i}^l K_i^m)^T \cdot (\bar{E}_{1i}^l + \bar{E}_{2i}^l K_i^m);$$

and  $J_0$  is defined as (8).

*Proof:* Choose the Lyapunov function candidate as (9) and taking the derivative of  $v_i$  along (15) yields

$$\begin{aligned} \dot{V}(x(t)) = & \sum_{i=1}^{\sigma} \dot{v}_i(x_i(t)) = \sum_{i=1}^{\sigma} \left[ \dot{x}_i^T(t) P_i x_i(t) + x_i^T(t) P_i \dot{x}_i(t) \right] \\ = & \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \{ x_i^T(t) [(A_i^l + B_i^l K_i^m)^T P_i + P_i (A_i^l + B_i^l K_i^m) \\ & + (\Delta A_i^l + \Delta B_i^l K_i^m)^T P_i + P_i (\Delta A_i^l + \Delta B_i^l K_i^m)] x_i(t) \\ & + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^l P_i x_i(t) \}. \end{aligned}$$

According to (14), we get

$$\begin{aligned} \dot{V}(x(t)) \\ = & \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \{ x_i^T(t) [(A_i^l + B_i^l K_i^m)^T P_i + P_i (A_i^l + B_i^l K_i^m) \end{aligned}$$

$$\begin{aligned}
 & + (\bar{D}_i^l \Theta_i^l(t) (\bar{E}_{1i}^l + \bar{E}_{2i}^l K_i^m))^T P_i \\
 & + P_i (\bar{D}_i^l \Theta_i^l(t) (\bar{E}_{1i}^l + \bar{E}_{2i}^l K_i^m)) x_i(t) + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t) \\
 & \leq \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \{x_i^T(t) [(A_i^l + B_i^l K_i^m)^T P_i + P_i (A_i^l + B_i^l K_i^m) \\
 & + P_i \bar{D}_i^l \bar{D}_i^{lT} P_i + (\bar{E}_{1i}^l + \bar{E}_{2i}^l K_i^m)^T (\bar{E}_{1i}^l + \bar{E}_{2i}^l K_i^m)] x_i(t) \\
 & + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t)\} \\
 & = \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} (\mu_i^l)^2 [x_i^T(t) (S_i^{ll} + \Gamma_i^{ll}) x_i(t) + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t)] \\
 & + \sum_{i=1}^{\sigma} \sum_{l=1}^{r_i} \sum_{m>l}^{r_i} \mu_i^l \mu_i^m [x_i^T(t) (S_i^{lm} + \Gamma_i^{lm}) x_i(t) + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{lT} P_i x_i(t) \\
 & + x_i^T(t) (S_i^{ml} + \Gamma_i^{ml}) x_i(t) + 2 \sum_{j=1}^{\sigma} x_j^T(t) f_{ij}^{mT} P_i x_i(t)].
 \end{aligned}$$

The rest of the proof is similar to the corresponding part of the proof of Theorem 1.  $\square$

*Remark 3:* By the same procedures as Remark 2, (16a), (16b) and (16c) can be easily transformed into the following LMIs for seeking  $W_i$ ,  $Z_i^l$ , and  $\hat{T}_i^{lm}$ . Thus, the local state feedback gains are obtained from  $K_i^l = Z_i^l W_i^{-1}$ , for  $l = 1, 2, \dots, r_i$  and  $i = 1, 2, \dots, \sigma$ .

$$\begin{bmatrix}
 \bar{\Phi}_i^{ll} & * & * & * & * \\
 W_i & -(\sigma \cdot I)^{-1} & 0 & 0 & 0 \\
 W_i & 0 & -Q_i^{-1} & 0 & 0 \\
 Z_i^l & 0 & 0 & -R_i^{-1} & 0 \\
 \bar{E}_{1i}^l W_i + \bar{E}_{2i}^l Z_i^l & 0 & 0 & 0 & -I
 \end{bmatrix} < 0 \quad (18a)$$

$$\begin{bmatrix}
 \bar{\Phi}_i^{lm} + \bar{\Phi}_i^{ml} & * & * & * & * & * & * \\
 W_i & -(2\sigma \cdot I)^{-1} & 0 & 0 & 0 & 0 & 0 \\
 W_i & 0 & -(2Q_i)^{-1} & 0 & 0 & 0 & 0 \\
 Z_i^l & 0 & 0 & -R_i^{-1} & 0 & 0 & 0 \\
 Z_i^m & 0 & 0 & 0 & -R_i^{-1} & 0 & 0 \\
 \bar{E}_{1i}^l W_i + \bar{E}_{2i}^l Z_i^m & 0 & 0 & 0 & 0 & -I & 0 \\
 \bar{E}_{1i}^m W_i + \bar{E}_{2i}^m Z_i^l & 0 & 0 & 0 & 0 & 0 & -I
 \end{bmatrix} < 0 \quad (18b)$$

$$\begin{bmatrix}
 \hat{T}_i^{11} & \hat{T}_i^{12} & \dots & \hat{T}_i^{1r_i} \\
 \hat{T}_i^{21} & \hat{T}_i^{22} & \dots & \hat{T}_i^{2r_i} \\
 \vdots & \vdots & \ddots & \vdots \\
 \hat{T}_i^{r_i 1} & \hat{T}_i^{r_i 2} & \dots & \hat{T}_i^{r_i r_i}
 \end{bmatrix} < 0 \quad (18c)$$

where  $W_i = P_i^{-1}$ ,  $Z_i^l = K_i^l W_i$ ,  $\bar{\Phi}_i^{lm} \equiv A_i^l W_i + W_i A_i^{lT} + B_i^l Z_i^m + Z_i^{mT} B_i^{lT} + \bar{D}_i^l \bar{D}_i^{lT} + \sum_{j=1}^{\sigma} f_{ij}^l f_{ij}^{lT} - \hat{T}_i^{lm}$ ,  $\hat{T}_i^{lm} = W_i \tilde{T}_i^{lm} W_i$ , and

the asterisk (\*) stands for the transpose elements (matrices) in the symmetric positions of a matrix.

### 4. An Illustrative Example

*Example 1:* First, we consider a large-scale system composed of two-machine subsystems  $S_i$  as follows [1]

$$S_i : \begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) \\ \dot{x}_{i2}(t) = -\frac{D_i}{M_i} x_{i2}(t) + \frac{1}{M_i} u_i(t) + \sum_{j=1}^2 \frac{E_i E_j Y_{ij}}{M_i} \\ \quad \times [\cos(\delta_{ij}^0 - \theta_{ij}) - \cos(x_{i1}(t) - x_{j1}(t) + \delta_{ij}^0 - \theta_{ij})] \end{cases} \quad (19)$$

for  $i = 1, 2$ ; where the absolute rotor angle and angular velocity of the machine in subsystem  $S_i$  are denoted by  $x_{i1}(t)$  and  $x_{i2}(t)$  respectively. The inertia coefficient  $M_i$ , the damping coefficient  $D_i$ , the internal voltage  $E_i$ , and the modulus of the transfer admittance  $Y_{ij}$  between the machine in subsystem  $S_i$  and in subsystem  $S_j$  are all stated. Let the parameters of the two-machine large-scale system be as follows.

$E_1 = 1.017$ ,  $E_2 = 1.005$ ,  $M_1 = 1.03$ ,  $M_2 = 1.25$ ,  $D_1 = 0.8$ ,  $D_2 = 1.2$ ,  $Y_{12} = Y_{21} = 1.98$ ,  $\theta_{12} = -\theta_{21} = 1.5$ ,  $\delta_{12}^0 = -\delta_{21}^0 = 1.2$ . According to the derivation in [21], the subsystem  $S_i$  for the large-scale system (19) can be presented by the T-S fuzzy system model as follows.

Rule 1 : If  $x_{i1}(t)$  is about  $-\pi/2$  and  $x_{i2}(t)$  is about  $-\pi/2$ ,

$$\text{then } \dot{x}_i = A_i^1 x_i(t) + B_i^1 u_i(t) + \sum_{j=1}^2 f_{ij}^1 x_j(t).$$

Rule 2 : If  $x_{i1}(t)$  is about  $-\pi/2$  and  $x_{i2}(t)$  is about 0,

$$\text{then } \dot{x}_i = A_i^2 x_i(t) + B_i^2 u_i(t) + \sum_{j=1}^2 f_{ij}^2 x_j(t).$$

Rule 3 : If  $x_{i1}(t)$  is about  $-\pi/2$  and  $x_{i2}(t)$  is about  $\pi/2$ ,

$$\text{then } \dot{x}_i = A_i^3 x_i(t) + B_i^3 u_i(t) + \sum_{j=1}^2 f_{ij}^3 x_j(t).$$

Rule 4 : If  $x_{i1}(t)$  is about 0 and  $x_{i2}(t)$  is about  $-\pi/2$ ,

$$\text{then } \dot{x}_i = A_i^4 x_i(t) + B_i^4 u_i(t) + \sum_{j=1}^2 f_{ij}^4 x_j(t).$$

Rule 5 : If  $x_{i1}(t)$  is about 0 and  $x_{i2}(t)$  is about 0,

$$\text{then } \dot{x}_i = A_i^5 x_i(t) + B_i^5 u_i(t) + \sum_{j=1}^2 f_{ij}^5 x_j(t).$$

Rule 6 : If  $x_{i1}(t)$  is about 0 and  $x_{i2}(t)$  is about  $\pi/2$ ,

$$\text{then } \dot{x}_i = A_i^6 x_i(t) + B_i^6 u_i(t) + \sum_{j=1}^2 f_{ij}^6 x_j(t).$$

Rule 7 : If  $x_{i1}(t)$  is about  $\pi/2$  and  $x_{i2}(t)$  is about  $-\pi/2$ ,

$$\text{then } \dot{x}_i = A_i^7 x_i(t) + B_i^7 u_i(t) + \sum_{j=1}^2 f_{ij}^7 x_j(t).$$

Rule 8 : If  $x_{i1}(t)$  is about  $\pi/2$  and  $x_{i2}(t)$  is about 0,

$$\text{then } \dot{x}_i = A_i^8 x_i(t) + B_i^8 u_i(t) + \sum_{j=1}^2 f_{ij}^8 x_j(t).$$

Rule 9 : If  $x_{i1}(t)$  is about  $\pi/2$  and  $x_{i2}(t)$  is about  $\pi/2$ ,

$$\text{then } \dot{x}_i = A_i^9 x_i(t) + B_i^9 u_i(t) + \sum_{j=1}^2 f_{ij}^9 x_j(t).$$

for  $i = 1, 2$ , where

$$A_1^1 = \begin{bmatrix} 0 & 1 \\ -0.7046 & -0.7767 \end{bmatrix}; B_1^1 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^1 = \begin{bmatrix} 0 & 0 \\ 0.5086 & 0 \end{bmatrix};$$

$$A_1^2 = \begin{bmatrix} 0 & 1 \\ -1.4809 & -0.7767 \end{bmatrix}; B_1^2 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^2 = \begin{bmatrix} 0 & 0 \\ 1.5483 & 0 \end{bmatrix};$$

$$A_1^3 = \begin{bmatrix} 0 & 1 \\ -1.4536 & -0.7767 \end{bmatrix}; B_1^3 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^3 = \begin{bmatrix} 0 & 0 \\ 1.4556 & 0 \end{bmatrix};$$

$$A_1^4 = \begin{bmatrix} 0 & 1 \\ 1.0472 & -0.7767 \end{bmatrix}; B_1^4 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^4 = \begin{bmatrix} 0 & 0 \\ -0.7669 & 0 \end{bmatrix};$$

$$A_1^5 = \begin{bmatrix} 0 & 1 \\ -0.5139 & -0.7767 \end{bmatrix}; B_1^5 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^5 = \begin{bmatrix} 0 & 0 \\ 0.5249 & 0 \end{bmatrix};$$

$$A_1^6 = \begin{bmatrix} 0 & 1 \\ -1.5480 & -0.7767 \end{bmatrix}; B_1^6 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^6 = \begin{bmatrix} 0 & 0 \\ 1.4812 & 0 \end{bmatrix};$$

$$A_1^7 = \begin{bmatrix} 0 & 1 \\ 1.1261 & -0.7767 \end{bmatrix}; B_1^7 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^7 = \begin{bmatrix} 0 & 0 \\ -1.1295 & 0 \end{bmatrix};$$

$$A_1^8 = \begin{bmatrix} 0 & 1 \\ 0.7686 & -0.7767 \end{bmatrix}; B_1^8 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^8 = \begin{bmatrix} 0 & 0 \\ -1.0486 & 0 \end{bmatrix};$$

$$A_1^9 = \begin{bmatrix} 0 & 1 \\ -0.5066 & -0.7767 \end{bmatrix}; B_1^9 = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix}; f_{12}^9 = \begin{bmatrix} 0 & 0 \\ 0.7017 & 0 \end{bmatrix};$$

$$A_2^1 = \begin{bmatrix} 0 & 1 \\ 0.2776 & -0.96 \end{bmatrix}; B_2^1 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^1 = \begin{bmatrix} 0 & 0 \\ -0.4410 & 0 \end{bmatrix};$$

$$A_2^2 = \begin{bmatrix} 0 & 1 \\ -0.6478 & -0.96 \end{bmatrix}; B_2^2 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^2 = \begin{bmatrix} 0 & 0 \\ 0.9436 & 0 \end{bmatrix};$$

$$A_2^3 = \begin{bmatrix} 0 & 1 \\ -0.9528 & -0.96 \end{bmatrix}; B_2^3 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^3 = \begin{bmatrix} 0 & 0 \\ 0.9284 & 0 \end{bmatrix};$$

$$A_2^4 = \begin{bmatrix} 0 & 1 \\ 1.2532 & -0.96 \end{bmatrix}; B_2^4 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^4 = \begin{bmatrix} 0 & 0 \\ -1.2188 & 0 \end{bmatrix};$$

$$A_2^5 = \begin{bmatrix} 0 & 1 \\ 0.3200 & -0.96 \end{bmatrix}; B_2^5 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^5 = \begin{bmatrix} 0 & 0 \\ -0.2912 & 0 \end{bmatrix};$$

$$A_2^6 = \begin{bmatrix} 0 & 1 \\ -0.9129 & -0.96 \end{bmatrix}; B_2^6 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^6 = \begin{bmatrix} 0 & 0 \\ 0.6472 & 0 \end{bmatrix};$$

$$A_2^7 = \begin{bmatrix} 0 & 1 \\ 1.2135 & -0.96 \end{bmatrix}; B_2^7 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^7 = \begin{bmatrix} 0 & 0 \\ -1.2104 & 0 \end{bmatrix};$$

$$A_2^8 = \begin{bmatrix} 0 & 1 \\ 1.2206 & -0.96 \end{bmatrix}; B_2^8 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^8 = \begin{bmatrix} 0 & 0 \\ -1.2670 & 0 \end{bmatrix};$$

$$A_2^9 = \begin{bmatrix} 0 & 1 \\ -0.5133 & -0.96 \end{bmatrix}; B_2^9 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}; f_{21}^9 = \begin{bmatrix} 0 & 0 \\ -0.3509 & 0 \end{bmatrix};$$

and  $f_{ii}^l = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ,  $l = 1, 2, \dots, 9$ ,  $i = 1, 2$ .

For the convenience of design, triangle-type membership functions are chosen for all fuzzy rules. According to Theorem 1 and the LMIs in Remark 2 with  $Q_1 = Q_2$

$$= \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, R_1 = R_2 = 0.3, \text{ we obtain the positive definite}$$

matrices  $P_i$  and  $K_i^l$  for subsystem  $S_1$  and subsystem  $S_2$  as follows.

For subsystem  $S_1$ :

$$P_1 = \begin{bmatrix} 31.0584 & 16.6737 \\ 16.6737 & 11.7860 \end{bmatrix}, K_1^1 = \begin{bmatrix} -48.3300 & -34.0208 \end{bmatrix},$$

$$K_1^2 = \begin{bmatrix} -52.7347 & -37.2090 \end{bmatrix}, K_1^3 = \begin{bmatrix} -51.9817 & -36.6687 \end{bmatrix},$$

$$K_1^4 = \begin{bmatrix} -48.4941 & -34.0134 \end{bmatrix}, K_1^5 = \begin{bmatrix} -48.3145 & -33.9963 \end{bmatrix},$$

$$K_1^6 = \begin{bmatrix} -52.2048 & -36.8360 \end{bmatrix}, K_1^7 = \begin{bmatrix} -49.6547 & -34.8271 \end{bmatrix},$$

$$K_1^8 = \begin{bmatrix} -49.4251 & -34.6906 \end{bmatrix}, K_1^9 = \begin{bmatrix} -48.6764 & -34.2519 \end{bmatrix}.$$

For subsystem  $S_2$ :

$$P_2 = \begin{bmatrix} 33.4126 & 17.7828 \\ 17.7828 & 11.7019 \end{bmatrix}, K_2^1 = \begin{bmatrix} -44.0257 & -28.7854 \end{bmatrix},$$

$$K_2^2 = \begin{bmatrix} -45.3337 & -29.6977 \end{bmatrix}, K_2^3 = \begin{bmatrix} -45.3812 & -29.7473 \end{bmatrix},$$

$$K_2^4 = \begin{bmatrix} -45.7796 & -29.8802 \end{bmatrix}, K_2^5 = \begin{bmatrix} -43.8552 & -28.6711 \end{bmatrix},$$

$$K_2^6 = \begin{bmatrix} -44.6768 & -29.2801 \end{bmatrix}, K_2^7 = \begin{bmatrix} -45.7550 & -29.8664 \end{bmatrix},$$

$$K_2^8 = \begin{bmatrix} -46.0050 & -30.0292 \end{bmatrix}, K_2^9 = \begin{bmatrix} -44.1279 & -28.8961 \end{bmatrix}.$$

The complete simulation results with initial conditions  $x_1(0) = [-1 \ 1]^T$ , and  $x_2(0) = [1 \ -1]^T$  are shown in Fig. 1. It is obvious that they are stabilized asymptotically and a guaranteed cost of the closed-loop system is  $J_0 = 19.0459$  ( $J = 12.172 < J_0$ ).

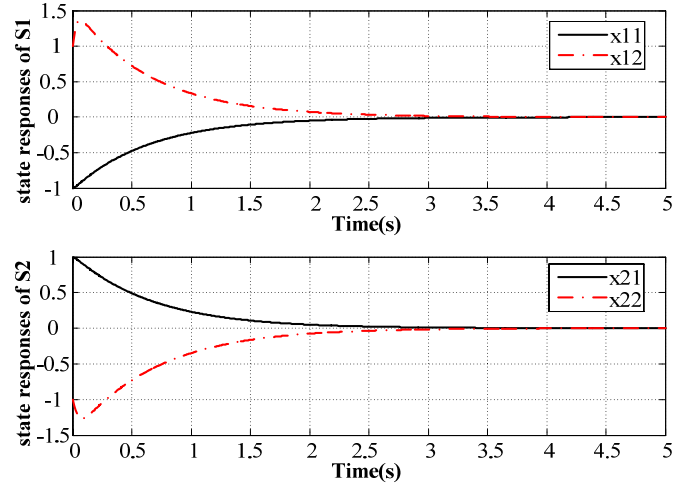


Fig. 1. The state responses of Example 1.

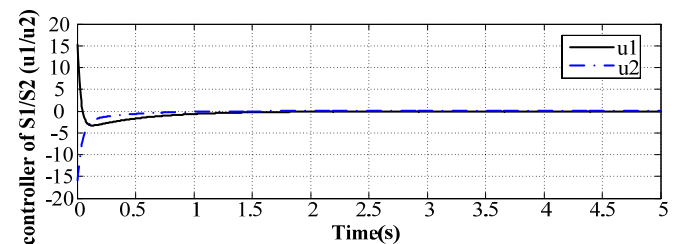


Fig. 2. The controller signals of Example 1.

*Example 2:* Now, we consider a large-scale system with uncertainties  $\Delta A_i^l$  and  $\Delta B_i^l$ , which is similar to the example in [19]. The fuzzy rules of this uncertain T-S fuzzy large-scale model are as follows.

For subsystem  $S_i$ :

$$S_i^l : \begin{cases} \text{If } x_{i1}(t) \text{ is } H_{i1}^l, \\ \dot{x}_i(t) = (A_i^l + \Delta A_i^l)x_i(t) + (B_i^l + \Delta B_i^l)u_i(t) + \sum_{j=1}^3 f_{ij}^l x_j(t) \end{cases}$$

where  $i = 1, 2, 3$ ;  $l = 1, 2$ ;  $x_i = [x_{i1} \ x_{i2}]^T$ ;

$[\Delta A_i^l \ \Delta B_i^l] = \bar{D}_i^l \Theta_i^l(t) [\bar{E}_{1i}^l \ \bar{E}_{2i}^l]$ ; and

$$A_1^1 = \begin{bmatrix} 2 & -3 \\ 0.2 & -11 \end{bmatrix}; \quad D_1^1 = \begin{bmatrix} -0.3 & 0.3 \\ 0.1 & 0.2 \end{bmatrix} = \bar{E}_{11}^1; \quad B_1^1 = \begin{bmatrix} 4 \\ -2 \end{bmatrix};$$

$$\bar{E}_{21}^1 = \begin{bmatrix} 0.6 \\ 0.5 \end{bmatrix}; \quad f_{12}^1 = \begin{bmatrix} -2 & 0 \\ 1.3 & -1 \end{bmatrix}; \quad f_{13}^1 = \begin{bmatrix} -1 & 2 \\ 0.25 & -1 \end{bmatrix};$$

$$A_1^2 = \begin{bmatrix} -6 & 2.3 \\ 1.1 & -11 \end{bmatrix}; \quad D_1^2 = \begin{bmatrix} 0.2 & 0.8 \\ 1.2 & -0.4 \end{bmatrix} = \bar{E}_{11}^2; \quad B_1^2 = \begin{bmatrix} 3 \\ -4 \end{bmatrix};$$

$$\bar{E}_{21}^2 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}; \quad f_{12}^2 = \begin{bmatrix} -1 & 0 \\ 1.3 & -1 \end{bmatrix}; \quad f_{13}^2 = \begin{bmatrix} -1 & 2 \\ 0.5 & -1 \end{bmatrix};$$

$$A_2^1 = \begin{bmatrix} 4 & 2.5 \\ 2.6 & -9 \end{bmatrix}; \quad D_2^1 = \begin{bmatrix} -0.5 & 0.5 \\ 1.5 & -0.7 \end{bmatrix} = \bar{E}_{12}^1; \quad B_2^1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix};$$

$$\bar{E}_{22}^1 = \begin{bmatrix} 0.3 \\ 0.6 \end{bmatrix}; \quad f_{21}^1 = \begin{bmatrix} -0.5 & 0.5 \\ 0 & -1 \end{bmatrix}; \quad f_{23}^1 = \begin{bmatrix} -1 & 0.9 \\ 1 & -1 \end{bmatrix};$$

$$A_2^2 = \begin{bmatrix} 3 & 3.5 \\ 1.2 & -10 \end{bmatrix}; \quad D_2^2 = \begin{bmatrix} -0.2 & 0 \\ 0.8 & 0.8 \end{bmatrix} = \bar{E}_{12}^2; \quad B_2^2 = \begin{bmatrix} 3 \\ -4 \end{bmatrix};$$

$$\bar{E}_{22}^2 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}; \quad f_{21}^2 = \begin{bmatrix} -0.5 & 0.6 \\ 0 & -0.8 \end{bmatrix}; \quad f_{23}^2 = \begin{bmatrix} -1 & 1 \\ 1 & -0.9 \end{bmatrix};$$

$$A_3^1 = \begin{bmatrix} 5 & 1 \\ 1 & -10 \end{bmatrix}; \quad D_3^1 = \begin{bmatrix} -1 & 0.4 \\ 0 & -0.7 \end{bmatrix} = \bar{E}_{13}^1; \quad B_3^1 = \begin{bmatrix} 5 \\ -2 \end{bmatrix};$$

$$\bar{E}_{23}^1 = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}; \quad f_{31}^1 = \begin{bmatrix} -0.5 & 0.4 \\ 1.4 & -1 \end{bmatrix}; \quad f_{32}^1 = \begin{bmatrix} 0.5 & 0 \\ 1.5 & 3 \end{bmatrix};$$

$$A_3^2 = \begin{bmatrix} -5 & 0.7 \\ 3.9 & -13 \end{bmatrix}; \quad D_3^2 = \begin{bmatrix} -0.9 & -0.5 \\ 0.5 & -0.5 \end{bmatrix} = \bar{E}_{13}^2; \quad B_3^2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix};$$

$$\bar{E}_{23}^2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}; \quad f_{31}^2 = \begin{bmatrix} -0.8 & 0.4 \\ 1.2 & -1 \end{bmatrix}; \quad f_{32}^2 = \begin{bmatrix} 0.8 & 0 \\ 1.1 & 3 \end{bmatrix};$$

$$f_{11}^1 = f_{11}^2 = f_{22}^1 = f_{22}^2 = f_{33}^1 = f_{33}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad \Theta_1^1(t) = \Theta_1^2(t)$$

$$= \Theta_2^1(t) = \Theta_2^2(t) = \Theta_3^1(t) = \Theta_3^2(t) = \begin{bmatrix} 1 - \sin^4 t & 0 \\ 0 & 1 - \cos^4 t \end{bmatrix};$$

$$H_{11}^1(x_{11}(t)) = \frac{1}{1 + \exp(-3x_{11}(t))}; \quad H_{11}^2(x_{11}(t)) = 1 - H_{11}^1(x_{11}(t));$$

$$H_{21}^1(x_{21}(t)) = \exp(-2x_{21}^2(t)); \quad H_{21}^2(x_{21}(t)) = 1 - H_{21}^1(x_{21}(t));$$

$$H_{31}^1(x_{31}(t)) = \frac{1}{1 + \exp(-1.5x_{31}(t))}; \quad H_{31}^2(x_{31}(t)) = 1 - H_{31}^1(x_{31}(t)).$$

According to Theorem 2 and the LMIs in Remark 3 with

$$Q_1 = Q_2 = Q_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad R_1 = R_2 = R_3 = 0.5, \quad \text{we can ob-}$$

tain the positive definite matrices  $P_i$  and  $K_i^l$  for subsystem  $S_1$ ,  $S_2$ , and  $S_3$  as follows.

For subsystem  $S_1$ :

$$P_1 = \begin{bmatrix} 2.3309 & -0.5489 \\ -0.5489 & 0.6370 \end{bmatrix};$$

$$K_1^1 = [-9.1476 \ 2.8748]; \quad K_1^2 = [-9.8395 \ 3.6517];$$

$$\tilde{T}_1 = \begin{bmatrix} -9.5384 & 2.8954 & 2.4604 & -1.0351 \\ 2.8954 & -4.2891 & -1.0483 & 0.7881 \\ 2.4604 & -1.0483 & -43.8850 & 21.2144 \\ -1.0351 & 0.7881 & 21.2144 & -11.5645 \end{bmatrix}.$$

For subsystem  $S_2$ :

$$P_2 = \begin{bmatrix} 72.4048 & 18.2267 \\ 18.2267 & 5.0207 \end{bmatrix};$$

$$K_2^1 = [-97.055 \ -23.068]; \quad K_2^2 = [-135.34 \ -32.711];$$

$$\tilde{T}_2 = \begin{bmatrix} -323.8114 & -64.7081 & 310.0633 & 97.2266 \\ -64.7081 & -14.6034 & 73.9703 & 23.5882 \\ 310.0633 & 73.9703 & -9665.639 & -2306.866 \\ 97.2266 & 23.5882 & -2306.866 & -552.492 \end{bmatrix}.$$

For subsystem  $S_3$ :

$$P_3 = \begin{bmatrix} 2.6506 & 0.4360 \\ 0.4360 & 0.4408 \end{bmatrix};$$

$$K_3^1 = [-10.483 \ -0.9271]; \quad K_3^2 = [-8.275 \ -0.3615];$$

$$\tilde{T}_3 = \begin{bmatrix} -53.6903 & -4.8224 & 7.2961 & 0.6722 \\ -4.8224 & -1.0603 & 1.5202 & 0.3123 \\ 7.2961 & 1.5202 & -41.4184 & 0.2892 \\ 0.6722 & 0.3123 & 0.2892 & -1.9697 \end{bmatrix}.$$

The state responses with initial conditions  $x_1(0) = x_2(0) = x_3(0) = [-1 \ 1]^T$  are shown in Fig. 3. The controller signals are shown in Fig.4. It is obvious that they are stabilized asymptotically and a guaranteed cost of the closed-loop system is  $J_0 = 47.2571$  ( $J = 12.172 < J_0$ ).

The simulation result shows that subsystem  $S_2$  with larger control gain (Fig.4) and most cost. Subsystem  $S_1$  and  $S_3$  are with similar small control gains and lower cost. To get a lower cost, we reduce  $R_2$  of (3) to suppress the cost with respect to  $K_2^1$  and  $K_2^2$ . Fig. 5 shows the controller signals of  $S_2$  in different  $R_2$ . The comparison of different  $R_2$  is shown in Table 1. It shows that reducing  $R_2$  can obtain smaller controller and cost. The simulation results match the guaranteed cost control design law.

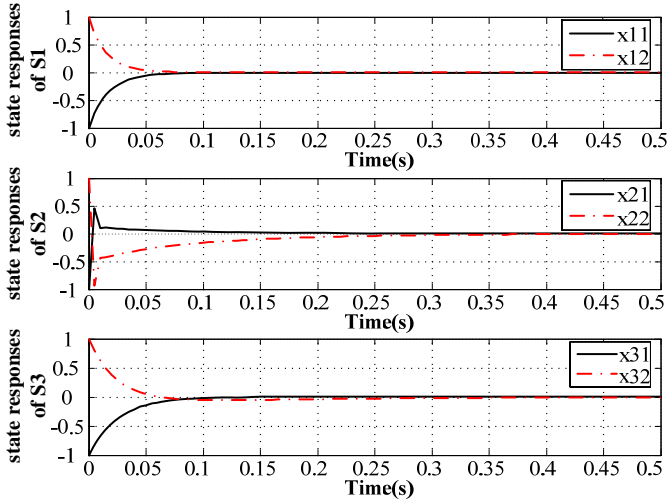


Fig. 3. The state responses of Example 2.

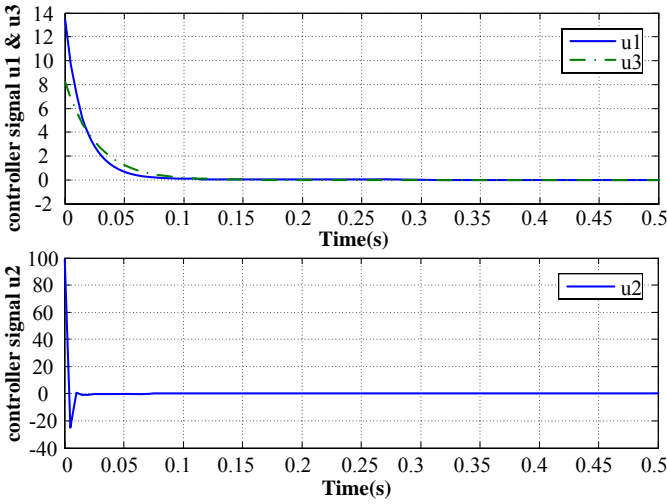


Fig. 4. The controller signals of Example 2.

The simulation result (Fig. 3) shows that subsystem  $S_1$  and  $S_3$  are with similar responses. Subsystem  $S_2$  is with worse state responses than other subsystems. While  $R_2 = 0.1$ , the response of  $x_{22}$  is much worse than  $x_{21}$ . Now, we try to improve the state responses of  $S_2$  by choosing a suitable  $Q_2$  of (3). The state responses of  $S_2$  in different  $Q_2$  are shown in Fig. 6. The comparison of choosing different  $Q_2$  is presented in Table 2. It shows that increasing the weight of  $Q_2(2, 2)$  will lead to larger control gains. And the larger control gains cause a shorter settling time. However, it should be noted that the interconnections of a large-scale system imply that every subsystems will affect each other. Hence, when we try to choose a proper cost function  $J$ , (i.e.,  $Q$  and  $R$ ) the interconnections should be concerned, too. Finally, referring to Table 1 and Table 2, we

choose  $Q_1 = Q_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ ;  $Q_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1.5 \end{bmatrix}$ ;  $R_1 = R_3 = 0.5$  and  $R_2 = 0.1$  for design this example. The simulation result is shown in Fig. 7.

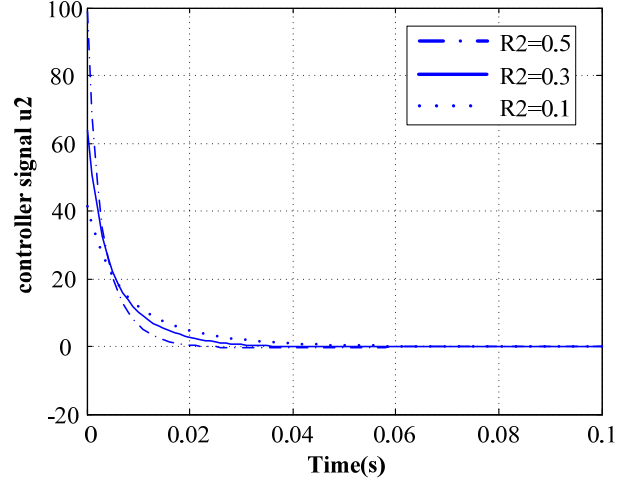


Fig. 5. The controller signals of  $S_2$  in different  $R_2$  (Example 2).

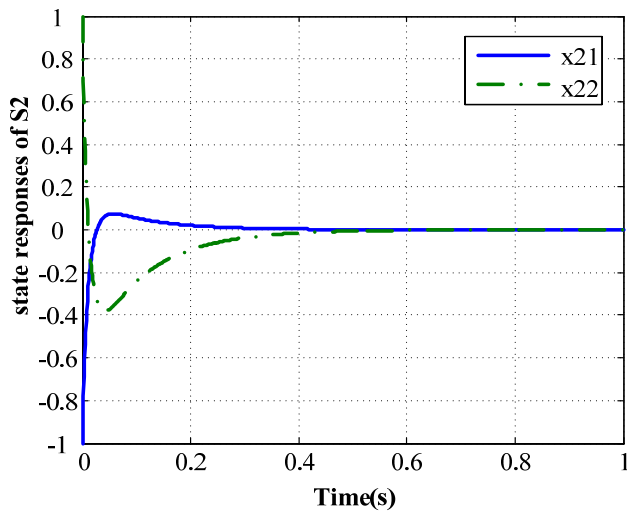
Table 1. Comparison of different  $R_2$  (Example 2).

$R_2$	$K_2^1$ & $K_2^2$	$J_0$	$J$
0.5	$K_2^1 = [-97.055 \quad -23.068]$ $K_2^2 = [-135.34 \quad -32.711]$	47.2571	12.172
0.3	$K_2^1 = [-60.466 \quad -14.608]$ $K_2^2 = [-88.844 \quad -22.29]$	25.7116	5.9114
0.1	$K_2^1 = [-39.801 \quad -9.0835]$ $K_2^2 = [-57.204 \quad -14.081]$	17.3552	3.0083
0.05	$K_2^1 = [-37.756 \quad -8.6172]$ $K_2^2 = [-55.772 \quad -13.892]$	15.8845	2.6911

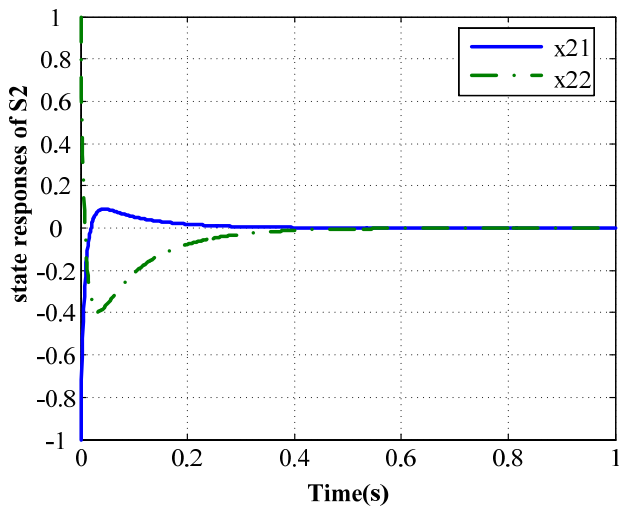
### 5. Conclusions

This paper has considered the guaranteed cost control design problems for the large-scale T-S fuzzy systems. The PDC fuzzy controllers are designed under some sufficient conditions in Theorem 1 and Theorem 2 (with uncertainties) such that the whole closed-loop systems are asymptotically stable and the upper bound of cost function is satisfied. The interactive relations among the fuzzy subsystems are considered and therefore the relaxed criteria are provided. The positive definite matrices  $P_i$  and PDC gains  $K_i^l$  can be found by LMIs tools.

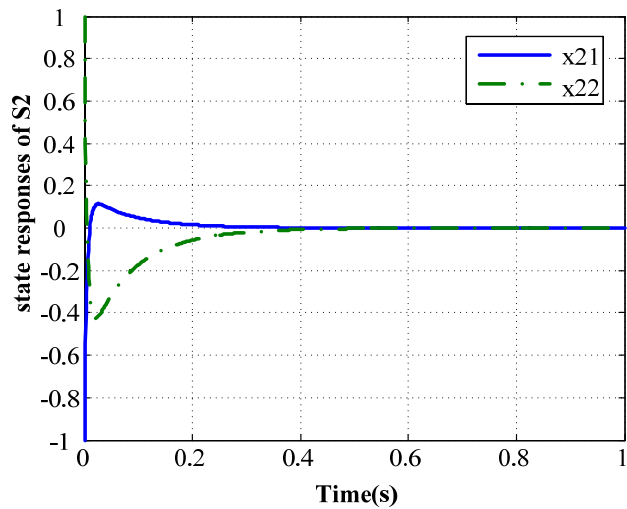
Finally, illustration examples have been given to show the effectiveness of the proposed methods.



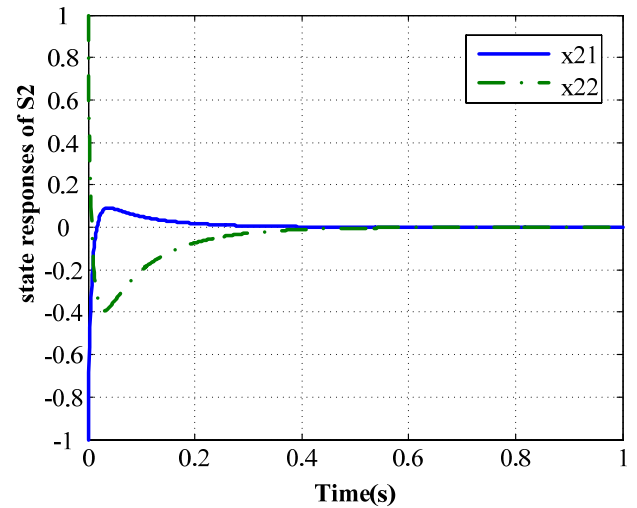
(a)  $Q_2 = \text{diag}(0.5, 0.3)$



(b)  $Q_2 = \text{diag}(0.5, 1.5)$



(c)  $Q_2 = \text{diag}(0.5, 3)$



(d)  $Q_2 = \text{diag}(0.5, 5)$

Fig. 6. The state responses of  $S_2$  in different  $Q_2$  (by Theorem 2).

Table 2. Comparison of different  $Q_2$ .

$Q_2$	$K_2^1$ & $K_2^2$	$t_s$ of $x_{21}$ (2%) $t_s$ of $x_{22}$ (2%)	$J_0$
$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}$	$K_2^1 = [-37.741 \ -8.5023]$ $K_2^2 = [-54.543 \ -13.31]$	0.2041 s 0.3700 s	16.8679
$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$	$K_2^1 = [-39.801 \ -9.0835]$ $K_2^2 = [-57.204 \ -14.081]$	0.202 s 0.364 s	17.3552
$\begin{bmatrix} 0.5 & 0 \\ 0 & 1.5 \end{bmatrix}$	$K_2^1 = [-50.756 \ -12.379]$ $K_2^2 = [-72.504 \ -18.757]$	0.194 s 0.3385 s	20.0098
$\begin{bmatrix} 0.5 & 0 \\ 0 & 3 \end{bmatrix}$	$K_2^1 = [-90.879 \ -24.89]$ $K_2^2 = [-130.12 \ -36.802]$	0.1785 s 0.2978 s	29.6715
$\begin{bmatrix} 0.5 & 0 \\ 0 & 5 \end{bmatrix}$	$K_2^1 = [-242.9 \ -73.655]$ $K_2^2 = [-348.81 \ -107.02]$	0.1643 s 0.265 s	66.4679

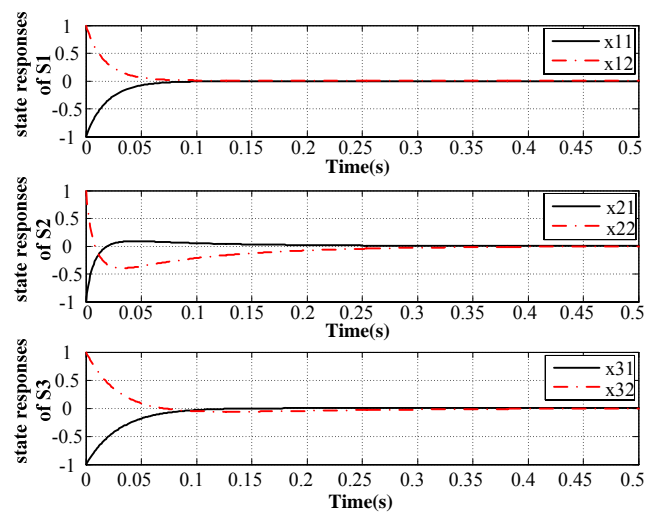


Fig. 7. The state responses of Example 2.

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