# Method for Multiple Attribute Decision-making under Risk with Interval Numbers

Xin Zhang and Peide Liu


## Abstract

**Multi-attribute decision problems with uncertainty largely exist in the real world, it is very necessary to solve these problems. The paper proposes a grey correlation method based on maximizing deviation to solve the multiple attribute decision-making problems under risk in which the weight is unknown and attribute values are interval numbers. In the grey correlation method, according to different attributes and their natural states, the decision matrix is normalized and ideal/ negative ideal solutions are also determined respectively; then, maximizing deviation method is used to determine the weights of attributes, and grey correlation method is used to rank the alternatives by using relative closeness coefficients. The computational results from an illustrative example have shown that the proposed approach is feasible and effective for the multiple attribute decision making problems under risk.**


***Keywords: Grey correlation method; Multi-attribute decision making (MADM); Risk decision; Weight.***

## 1. Introduction

Multi-attribute decision making is widely used in the field of society, economy, management, military affairs and engineering technology, such as investment decision making, project evaluation, economic benefit evaluation and staff evaluation etc. Some methods have been proposed to solve the multi-attribute decision making problems [1-4], but in most of which the attribute values of the alternatives must be known in advance. However, in the real decision making process, the decision makers sometimes face the uncertain condition, and the attribute values take the form of random variables and they can vary with natural state. The decision makers can't know the real state in the future, but they can give various possible natural states and can quantify the randomness by setting the probability distribution. This decision making problem is called multi-attribute decision making under risk [5]. In addition, in the condition that the probability of the natural state is given, the decision makers are difficult to give the evaluation values for some attributes. In this case, the interval fuzzy numbers can be considered. Therefore, it is of great theoretical and practical significance to research the multi-attribute decision making problems under risk with interval numbers.

At present, relatively less research work on multi-attribute decision making problems under risk have been done. Rao and Xiao (2006) [6] researched the mixed multi-attribute decision making problem under risk, where the attribute weight information is unknown and the attribute values are mixed with the exact number, the interval number and linguistic fuzzy number. Firstly, according to the expected value, the risk decision making problem is transformed to the certain one, then the dynamic mixed multi-attribute decision making method under risk based on the gray matrix closeness coefficients is proposed. Luo and Liu (2004) [7] adopted the same method proposed by Rao and Xiao(2006) to solve the decision making problem under risk with the attribute weights unknown and the attribute values taking the form of interval numbers. In the method, the decision making problem under risk is transformed to certain one using the expected values, and then the grey fuzzy relationship method and two-basic-point method were proposed. Yu, Wang and Liu, etc. (2003) [5] researched the multi-attribute decision making problem under risk in the condition that the attribute weights were unknown and the attribute values were real numbers, and the multi-attribute decision making problem under risk was not transformed to certain one using the expectation values, but the weighted distances were calculated in the different attributes and natural states, and the relative mathematical model was established. Yao (2006) [8] proposed a TOPSIS method to solve the decision making problem under risk in which attribute values took the form of real number. In the method, the ideal and negative ideal solutions with different attributes and natural states were standardized and defined. Then the TOPSIS method of decision making problem under risk was realized and was verified by an application case.

According to the above statement, the method used in literatures [6, 7] can only transformed the risk decision making problem to deterministic one by the expected

Corresponding Author: Peide Liu is with the School of Information Management, Shandong Economic University.
E-mail: peide.liu@gmail.com
Xin Zhang is with the School of Information Management, Shandong Economic University, 7366 Erhuandong Rd., Jinan Shandong 250014, China.
Manuscript received 24 Dec. 2008; revised 13 Dec. 2009; accepted 13 Aug. 2010.

value. However, because the expectation values of different attributes in different states may be equal, the alternatives will not be ranked in this condition, so, these methods have some obvious weakness. Literatures [5] and [8], based on calculating the weighted distances in the different attributes and natural states, overcame the weakness in the method shown in literatures [6,7]. Especially, the method proposed in the literature [8] got the reasonable ranking. This paper, based on literature [8], proposed a grey correlation decision making method based on the attribute values taking the form of interval values. Firstly, the decision making matrix is standardized and the ideal and negative ideal solutions are defined aiming at different attributes and natural states; then the maximizing deviation method is proposed to determine the weights and the grey relevant degree method to rank the alternatives are extended to the risk decision making area.

The remainder of the paper is organized as follows: Section 2 describes the decision making problem; Section 3 introduces the decision making method and the steps; Section 4 presents an illustrative example to verify effectiveness of this method; Finally, conclusions are given in Section 5.

## 2. The Description of the Decision Making Problem

Supposed that in the decision making problem under risk there are $m$ evaluation alternatives $A=(a_1,a_2,\cdots,a_m)$ and $n$ evaluation attributes $C=(c_1,c_2,\cdots,c_n)$. $w_j$ is the weight of the attribute $c_j$, where $0\le w_j\le 1$, $\sum_{j=1}^{n} w_j=1$. The weight vector $W=(w_1,w_2,\ldots,w_n)$ is unknown. There are $l_j$ possible states $\Theta_j=(\theta_1,\theta_2,\ldots,\theta_{l_j})$ with respect to the attribute $c_j$. $p_j^t$ is the occurring probability with respect to the state $\theta_t$ of the attribute $c_j$, where $0\le p_j^t\le 1$, $\sum_{t=1}^{l_j} p_j^t=1$. $x_{ij}^t$ is the attribute value of alternative $a_i$ under the attribute $c_j$ and the state $\theta_t$, and $x_{ij}^t$ takes the form of interval number[ $x_{ij}^{tL}$ , $x_{ij}^{tU}$ ] (The data in this decision making under risk are shown in Table 1). With these conditions, we can rank the alternatives of this multi-attribute decision making problem under risk.

Table 1.multi-attribute decision making data under risk.

| | $c_1$ | | | $\cdots$ | $c_n$ | | |
|---|---|---|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ $\cdots$ | $\theta_{l_1}$ | $\cdots$ | $\theta_1$ | $\theta_2$ $\cdots$ | $\theta_{l_n}$ |
| | $p_1^1$ | $p_1^2$ $\cdots$ | $p_1^{l_1}$ | $\cdots$ | $p_n^1$ | $p_n^2$ $\cdots$ | $p_n^{l_n}$ |
| $a_1$ | $x_{11}^1$ | $x_{11}^2$ $\cdots$ | $x_{11}^{l_1}$ | $\cdots$ | $x_{1n}^1$ | $x_{1n}^2$ $\cdots$ | $x_{1n}^{l_n}$ |
| $a_2$ | $x_{21}^1$ | $x_{21}^2$ $\cdots$ | $x_{21}^{l_1}$ | $\cdots$ | $x_{2n}^1$ | $x_{2n}^2$ $\cdots$ | $x_{2n}^{l_n}$ |
| $\cdots$ | $\cdots$ | $\cdots$ $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $a_m$ | $x_{m1}^1$ | $x_{m1}^2$ $\cdots$ | $x_{m1}^{l_1}$ | $\cdots$ | $x_{mn}^1$ | $x_{mn}^2$ $\cdots$ | $x_{mn}^{l_n}$ |

## 3. The Decision Making Method and Steps

A. *The calculation rules of interval numbers [9]*

Supposed that $a=[a^L,a^U]$ and $b=[b^L,b^U]$ are two positive interval numbers, then the following calculation rules can be obtained:

$$a+b=[a^L,a^U]+[b^L,b^U]=[a^L+b^L,a^U+b^U] \quad (1)$$

$$a-b=[a^L,a^U]-[b^L,b^U]=[a^L-b^U,a^U-b^L] \quad (2)$$

$$ab=[a^L,a^U]\times[b^L,b^U]=[a^Lb^L,a^Ub^U] \quad (3)$$

$$a/b=[a^L,a^U]/[b^L,b^U]=[a^L/b^U,a^U/b^L] \quad (4)$$

$$\lambda a=\lambda[a^L,a^U]=[\lambda a^L,\lambda a^U] \quad \lambda>0 \quad (5)$$

B. *The decision making method*

(1) The initialization of the decision making matrix

In order to avoid the influence of the different physical dimensions on the decision making result, the decision making matrix should be standardized. The most familiar index (attribute) types are benefit index ($I_1$) and cost index ($I_2$).

Supposed that after standardization the index value is $r_{ij}^t=[r_{ij}^{tL},r_{ij}^{tU}]$, the standardization method is show as follows:

$$\begin{cases} r_{ij}^{tL}=x_{ij}^{tL}\Big/\sqrt{\sum_{i=1}^{m}\left[(x_{ij}^{tL})^2+(x_{ij}^{tU})^2\right]} \\ r_{ij}^{tU}=x_{ij}^{tU}\Big/\sqrt{\sum_{i=1}^{m}\left[(x_{ij}^{tL})^2+(x_{ij}^{tU})^2\right]} \end{cases} \quad (6a)$$

For $j\in I_1 (i=1,2,\cdots,m; j=1,2,\cdots,n; t=1,2,\cdots,l_j)$

$$\begin{cases} r_{ij}^{tL}=\left(1/x_{ij}^{tU}\right)\Big/\sqrt{\sum_{i=1}^{m}\left[\left(1/x_{ij}^{tU}\right)^2+\left(1/x_{ij}^{tL}\right)^2\right]} \\ r_{ij}^{tU}=\left(1/x_{ij}^{tL}\right)\Big/\sqrt{\sum_{i=1}^{m}\left[\left(1/x_{ij}^{tU}\right)^2+\left(1/x_{ij}^{tL}\right)^2\right]} \end{cases} \quad (6b)$$

For $j\in I_2 (i=1,2,\cdots,m; j=1,2,\cdots,n; t=1,2,\cdots,l_j)$

(2) Define the distance of the interval numbers

Supposed that $a=\left[a^L,a^U\right]$ and $b=\left[b^L,b^U\right]$ are two positive interval numbers, the distance between $a$ and $b$ can be defined as follows:

$$d(a,b)=\frac{\sqrt{2}}{2}\sqrt{(a^L-b^L)^2+(a^U-b^U)^2} \tag{7}$$

(3) The calculation of the attribute weight

The uncertainty of the attribute weight can cause the uncertainty of the ranking order of the alternatives. Generally, if the difference is smaller among the attribute values of all decision making alternatives under the attribute $c_j$, then the effect of the attribute weight on the alternative is smaller. Whereas, if the attribute $c_j$ can make the attribute values of all the decision making alternatives have a bigger deviation, it can be concluded that this attribute plays a more important role in the decision making process. So in the view of ranking or selecting alternatives, the bigger the deviation of the attribute value is, the bigger the weight is. If the deviation is smaller, the weight should be smaller [10].

To the attribute $c_j$, the distance between alternative $a_i$ and alternative $a_k$ in the status $\theta_t$ can be defined as follows:

$$d_{ijk}^t(r_{ij}^t,r_{kj}^t)=\frac{\sqrt{2}}{2}\sqrt{(r_{ij}^{tL}-r_{kj}^{tL})^2+(r_{ij}^{tU}-r_{kj}^{tU})^2} \tag{8}$$

Then the deviation of attribute $c_j$ between alternative $a_i$ and alternative $a_k$ is defined as follows:

$$D_{ijk}=\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t) \tag{9}$$

To the attribute $c_j$, the weighted deviation $D_{ij}$ between alternative $a_i$ and all the other alternatives is defined as follows:

$$D_{ij}(w_j)=w_j\sum_{k=1}^{m}D_{ijk}=w_j\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t) \tag{10}$$

To the attribute $c_j$, the weighted deviation $D_j$ between all of the alternatives and the other alternatives is:

$$D_j(w_j)=\sum_{i=1}^{m}D_{ij}(w_j)=w_j\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t) \tag{11}$$

$$D(w_j)=\sum_{j=1}^{n}D_j(w_j)=\sum_{j=1}^{n}\left(w_j\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t)\right)$$

is the total deviation between all the attributes and all the alternatives.

The model of maximizing deviation is constructed as follows:

$$\begin{cases}\max D(w_j)=\sum_{j=1}^{n}\left(w_j\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t)\right)\\ s.t\sum_{j=1}^{n}w_j^2=1, w_j\geq 0,\ j=1,2\ldots\ldots n\end{cases} \tag{12}$$

Construct the Lagrange multiplier function:

$$L(w_j,\lambda)=\sum_{j=1}^{n}\left(w_j\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t)\right)+\lambda(\sum_{j=1}^{n}w_j^2-1)$$

Let

$$\begin{cases}\dfrac{\partial L(w_j,\lambda)}{\partial w_j}=\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t)+2\lambda w_j=0\\ \dfrac{\partial L(w_j,\lambda)}{\partial \lambda}=\sum_{j=1}^{n}w_j^2-1=0\end{cases}$$

Solve this model, it can be obtained:

$$w_j=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t)}{\sqrt{\sum_{j=1}^{n}(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t))^2}} \tag{13}$$

After normalization, the following result can be obtained:

$$w_j=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t)}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{l_j}p_j^t d_{ijk}^t(r_{ij}^t,r_{kj}^t)} \tag{14}$$

C. *The decision making method based on the grey correlation relative closeness degree*

(1) Define the ideal solution and the negative ideal solution:

Let $v_j^{t+}=[v_j^{tL+},v_j^{tU+}]$ ($j=1,2,\cdots,n, t=1,2,\cdots,l_j$) be the ideal solution of the $j^{th}$ attribute in the $t^{th}$ status, , and $v_j^{t-}=[v_j^{tL-},v_j^{tU-}]$ ($j=1,2,\cdots,n$, $t=1,2,\cdots,l_j$) be the negative ideal solution, they are defined as follows:

$$\begin{cases}v_j^{tL+}=\max_i(r_{ij}^{tL}), v_j^{tU+}=\max_i(r_{ij}^{tU})\\ v_j^{tL-}=\min_i(r_{ij}^{tL}), v_j^{tU-}=\min_i(r_{ij}^{tU})\end{cases} \tag{15}$$

(2) Calculate the grey correlation coefficient between the alternative $a_i$ and the ideal solution about the attribute $c_j$ [11].

$$\xi_{ij}^+=\frac{N+\rho M}{d_{ij}^+ + \rho M},\quad \rho\in(0,1) \tag{16}$$

where

$$d_{ij}^{+}=\sum_{t=1}^{l_j}\left[(p_j^t\times(\frac{\sqrt{2}}{2}\sqrt{(r_{ij}^{tL}-v_j^{tL+})^2+(r_{ij}^{tU}-v_j^{tU+})^2})\right],$$

$N=\underbrace{\min}_{i}\underbrace{\min}_{j}d_{ij}^{+}$, $M=\underbrace{\max}_{i}\underbrace{\max}_{j}d_{ij}^{+}$, $\rho$ is the resolution coefficient, the value is always equal to 0.5.

Then the grey correlation coefficient matrix between each alternative and the ideal solution is:

$$\xi^{+}=\begin{bmatrix}\xi_{11}^{+} & \xi_{12}^{+} & \cdots & \xi_{1n}^{+}\\ \xi_{21}^{+} & \xi_{22}^{+} & \cdots & \xi_{2n}^{+}\\ \vdots & \vdots & \vdots & \vdots\\ \xi_{m1}^{+} & \xi_{m2}^{+} & \cdots & \xi_{mn}^{+}\end{bmatrix}$$

The grey correlation degree between the alternative $a_i$ and the ideal solution is:

$$R_i^{+}=\sum_{j=1}^{n}w_j\xi_{ij}^{+},(i=1,2,\cdots,m) \quad (17)$$

(3) Calculate the grey correlation coefficient between the alternative $a_i$ and the negative ideal solution about the attribute $c_j$:

$$\xi_{ij}^{-}=\frac{N+\rho M}{d_{ij}^{-}+\rho M},\ \ \rho\in(0,1) \quad (18)$$

where

$$d_{ij}^{-}=\sum_{t=1}^{l_j}\left[(p_j^t\times(\frac{\sqrt{2}}{2}\sqrt{(r_{ij}^{tL}-v_j^{tL-})^2+(r_{ij}^{tU}-v_j^{tU-})^2})\right],$$

$N=\underbrace{\min}_{i}\underbrace{\min}_{j}d_{ij}^{-}$, $M=\underbrace{\max}_{i}\underbrace{\max}_{j}d_{ij}^{-}$, $\rho$ is the resolution coefficient, the value is always equal to 0.5.

Then the grey correlation coefficient matrix of each alternative and the negative ideal solution is:

$$\xi^{-}=\begin{bmatrix}\xi_{11}^{-} & \xi_{12}^{-} & \cdots & \xi_{1n}^{-}\\ \xi_{21}^{-} & \xi_{22}^{-} & \cdots & \xi_{2n}^{-}\\ \vdots & \vdots & \vdots & \vdots\\ \xi_{m1}^{-} & \xi_{m2}^{-} & \cdots & \xi_{mn}^{-}\end{bmatrix}$$

The grey correlation degree of the alternative $a_i$ and the negative ideal solution is:

$$R_i^{-}=\sum_{j=1}^{n}w_j\xi_{ij}^{-},(i=1,2,\cdots,m) \quad (19)$$

(4) Calculate the relative closeness degree of each alternative:

$$C_i=R_i^{+}\Big/\left(R_i^{+}+R_i^{-}\right),(i=1,2,\cdots,m) \quad (20)$$

From the grey relational axiom [11], it can be concluded that 0< $R_i^{+}$ ≤1, 0< $R_i^{-}$ ≤1. So the relative closeness degree of the grey correlation is 0< $C_i$ <1.

(5) Ranking the alternatives:

According to the relative closeness degree of the grey correlation, the alternatives are ranked. The bigger the relative closeness is, the better the alternative is.

## 4. Illustrative Example

An enterprise plans to build a new factory, and there are four alternatives for the product sale forested in market: very good ($\theta_1$), good ($\theta_2$), medium ($\theta_3$) and bad ($\theta_4$). Now four projects are designed, where three attributes are considered: direct benefit $c_1$, indirect benefit $c_2$ and pollution loss $c_3$. The data in this decision making under risk is shown in Table 2, the problem is to select the best alternative [7].

Table 2 the decision making matrix under risk.

| | $c_1$ | | | | $c_2$ | |
|---|---|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_1$ | $\theta_2$ |
| | 0.1 | 0.2 | 0.3 | 0.4 | 0.1 | 0.2 |
| $a_1$ | [28,32] | [28,32] | [28,32] | [28,32] | [95,105] | [95,105] |
| $a_2$ | [25,31] | [30,36] | [30,36] | [30,36] | [90,116] | [97,113] |
| $a_3$ | [22,30] | [27,35] | [32,40] | [32,40] | [90,112] | [97,109] |
| $a_4$ | [20,28] | [25,33] | [30,38] | [35,43] | [90,100] | [96,106] |

Table 2 the decision making matrix under risk (cont').

| | $c_2$ | | $c_3$ | | | |
|---|---|---|---|---|---|---|
| | $\theta_3$ | $\theta_4$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
| | 0.4 | 0.3 | 0.1 | 0.2 | 0.3 | 0.4 |
| $a_1$ | [ 95,105] | [ 95,105] | [190,210] | [195,215] | [200,220] | [205,225] |
| $a_2$ | [ 97,113] | [ 97,113] | [210,230] | [215,235] | [220,240] | [225,245] |
| $a_3$ | [104,116] | [104,116] | [230,250] | [235,255] | [240,260] | [245,265] |
| $a_4$ | [101,111] | [115,125] | [250,270] | [255,275] | [260,280] | [265,285] |

The decision making steps are shown as follows:

(1) Standardize the decision making matrix (shown in table 3).

(2) Calculate the weight of the index. According to formula (14), we can get

$$W=\left(\ 0.369,\ 0.202,\ 0.429\right)$$

(3) Calculate the ideal and negative ideal solutions (shown in table 4)

Table 3 the standardization of the decision making matrix.

| | $c_1$ | | | |
|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
| | 0.1 | 0.2 | 0.3 | 0.4 |
| $a_1$ | [0.363,0.414] | [0.320,0.365] | [0.296,0.338] | [0.284,0.325] |
| $a_2$ | [0.324,0.401] | [0.343,0.411] | [0.317,0.380] | [0.305,0.365] |
| $a_3$ | [0.285,0.389] | [0.308,0.400] | [0.338,0.422] | [0.325,0.406] |
| $a_4$ | [0.259,0.363] | [0.285,0.377] | [0.317,0.401] | [0.355,0.437] |

Table 3 the standardization of the decision making matrix (cont' 1).

| | $c_2$ | | | |
|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
| | 0.1 | 0.2 | 0.4 | 0.3 |
| $a_1$ | [0.335,0.370] | [0.328,0.362] | [0.318,0.352] | [0.308,0.340] |
| $a_2$ | [0.318,0.409] | [0.335,0.390] | [0.325,0.379] | [0.314,0.366] |
| $a_3$ | [0.318,0.395] | [0.335,0.376] | [0.349,0.389] | [0.337,0.376] |
| $a_4$ | [0.318,0.353] | [0.331,0.366] | [0.339,0.372] | [0.372,0.405] |

Table 3 the standardization of the decision making matrix (cont' 2).

| | $c_3$ | | | |
|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
| | 0.1 | 0.2 | 0.3 | 0.4 |
| $a_1$ | [0.381,0.421] | [0.380,0.419] | [0.380,0.418] | [0.379,0.416] |
| $a_2$ | [0.347,0.381] | [0.348,0.380] | [0.348,0.380] | [0.348,0.379] |
| $a_3$ | [0.320,0.347] | [0.320,0.348] | [0.321,0.348] | [0.322,0.348] |
| $a_4$ | [0.296,0.320] | [0.297,0.320] | [0.298,0.321] | [0.299,0.322] |

Table 4 the ideal and negative ideal solution under different attributes and status.

| | $c_1$ | | | |
|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
| | 0.1 | 0.2 | 0.3 | 0.4 |
| IS* | [0.363,0.414] | [0.343,0.411] | [0.338,0.422] | [0.355,0.437] |
| NIS+ | [0.259,0.363] | [0.285,0.365] | [0.296,0.338] | [0.284,0.325] |

* IS are the ideal solutions

+ NIS are the negative ideal solutions

Table 4 the ideal and negative ideal solution under different attributes and status (cont' 1).

| | $c_2$ | | | |
|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
| | 0.1 | 0.2 | 0.4 | 0.3 |
| IS* | [0.335,0.409] | [0.335,0.390] | [0.349,0.389] | [0.372,0.405] |
| NIS+ | [0.318,0.353] | [0.328,0.362] | [0.318,0.352] | [0.308,0.340] |

Table 4 the ideal and negative ideal solution under different attributes and status (cont' 2).

| | $c_3$ | | | |
|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
| | 0.1 | 0.2 | 0.3 | 0.4 |
| IS* | [0.381,0.421] | [0.380.0.419] | [0.380,0.418] | [0.379,0.416] |
| NIS+ | [0.296,0.320] | [0.297.0.320] | [0.298,0.321] | [0.299,0.322] |

(5) Calculate the grey correlation degree between each alternative and the ideal solution/the negative ideal solution:

$$R^+ = (0.687,\ 0.573,\ 0.574,\ 0.539)$$

$$R^- = (0.623,\ 0.528,\ 0.565,\ 0.721)$$

(6) Calculate the relative closeness degree:

$$C = (0.524,\ 0.521,\ 0.504,\ 0.428)$$

(7) Ranking:

According to the relative closeness degree, the order of the alternatives is: $a_1 \succ a_2 \succ a_3 \succ a_4$.

(8) Analysis:

The literature [7] provided two results: $a_3 \succ a_2 \succ a_1 \succ a_4$ and $a_3 \succ a_2 \succ a_4 \succ a_1$, which are very different from those obtained in this paper. The reason is that the weights adopted by the literature [7] have some errors, which are great different from those adopted in this paper. By using the weights in this paper and the method proposed by literature [7], the ranking result can be obtained: $a_1 \succ a_2 \succ a_3 \succ a_4$; whereas by using the weights in literature[7] and the method proposed by this paper, the ranking result is: $a_3 \succ a_2 \succ a_1 \succ a_4$. Therefore, it can be concluded that the method proposed by this paper is efficient. In addition, from the weight data, the weights in this paper are $W = (0.369, 0.202, 0.429)$ and the weights in the literature [7] are: $W = (0.41, 0.32, 0.27)$. The weights used in this paper aggravate the attribute "pollution loss". For the pollution loss, that for the project $a_1$ is the least, so it is normal to obtain the conclusion that $a_1$ is the optimum solution, whereas, the weights used in the literature [7] aggravate the attribute "direct benefit", so it is also very reasonable to get the conclusion that $a_3$ is the optimum solution.

## 5. Conclusions

With respect to multiple attribute decision making (MADM) problems under risk with the attribute weights unknown and the attribute values taking the form of interval numbers, a grey correlation method is proposed. Firstly, maximizing deviation method is used to determine the attribute weights, then according to different attributes and their natural states, the decision matrix is normalized and ideal/ negative ideal solutions are also determined respectively, and the grey correlation method is used to rank the alternatives by using the relative closeness coefficients. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness. The method in this paper is easy to understand and to use, and overcomes the weakness of the method proposed in literature [7], so it has good adaptability. In the future, we can study the risk decision-making problems with the continuous random variables in finite interval, and to expand the application range in MADM.

## Acknowledgment

This paper is supported by the Humanities and Social Sciences Research Project of Ministry of Education of China (No.09YJA630088), and the Natural Science Foundation of Shandong Province (No. ZR2009HL022). The Social Science Planning Project Fund of Shandong Province (No.09BSHJ03), the Soft science project Fund of Shandong Province (No.2009RKA376), and Dr. Foundation of Shandong Economic University. The authors also would like to express appreciation to the anonymous reviewers for their very helpful comments on improving the paper.

## References

[1] C. Y. Yue, Decision making theory and method .Beijing: science press, 2003.
[2] X. F. Wang, "Fuzzy Number Intuitionistic Fuzzy Arithmetic Aggregation Operators," *International Journal of Fuzzy Systems*, vol. 10, no. 2, pp. 104-111, 2008.
[3] T. Y. Chen, "Remarks on the Subtraction and Division Operations over Intuitionistic Fuzzy Sets and Interval-Valued Fuzzy Sets," *International Journal of Fuzzy Systems*, vol. 9, no. 3, pp. 169-172, 2007.
[4] S. H. Lee, W. Pedrycz, and G. Sohn, "Design of Similarity and Dissimilarity Measures for Fuzzy Sets on the Basis of Distance Measure," *International Journal of Fuzzy Systems,* vol. 11, no. 2, pp. 67-72, 2009.
[5] Y. B. Yu, B. D. Wang, P. Liu, "Risky Multiobjective Decision-Making Theory and Its Application," *Chinese Journal of Management Science*, vol. 11, no. 6, pp. 9-13, 2003.
[6] C. J. Rao, X. P. Xiao, "Method of grey matrix relative degree for dynamic hybrid multi-attribute decision making under risk," *Systems Engineering and Electronics,* vol. 28, no. 9, pp. 1353-1357, 2006.
[7] D. Luo, S. F. Liu, "Research on grey multi-criteria risk decision-making method," *Systems Engineering and Electronics*, vol. 26, no. 8, pp. 1057-1060, 2004.
[8] S. B. Yao, "Research on Risk Multi-attribute Decision Making: Theory, Methods and Applications," PhD Thesis of Huazhong University of Science and Technology, May 2006.
[9] T. H. You and Z. P. Fan, "TOPSIS Method for Multiple Attribute Decision Making with Intervals," *Journal of Northeastern University (Natural Science)*, vol. 23, no. 9, pp. 840-843, 2002.
[10] H. A Zhou and S. Y. Liu, "Projection method of fuzzy multi-attribute decision-making based on the maximal deviation model," *Systems Engineering and Electronics,* vol. 29, no. 5, pp. 741-744, 2007.
[11] S. F. Liu, T. B. Guo, and Y. G. Dang, Grey system theory and application, Beijing: science press, 1999.