Global exponential stability of delayed fuzzy cellular neural networks

Man-chun Tan

Abstract

New sufficient conditions are proposed for the existence, uniqueness and global exponential stability of the equilibrium point of a general class of fuzzy cellular neural networks with time-varying delays. The new criteria possess the structure of linear matrix inequalities, which can be solved efficiently by the interior-point algorithm. Theoretical analysis and numerical examples are given to show that the new criteria extend and improve some existing results in the literature.

Keywords: Delayed neural networks, Global exponential stability, Fuzzy cellular neural networks, Linear matrix inequality (LMI).

1. Introduction

System stability and performance are important concerns in the design of neural networks and fuzzy control systems (see [1-12] and references therein). To deal with the vagueness in mathematical modeling of real world problems, Yang et al. introduced the fuzzy cellular neural networks (FCNNs) in [13][14]. In a FCNN each cell contains fuzzy operating abilities, and the entire network is governed by cellular computing laws. FCNNs are proved to be important in practical applications [15][16]. It has been recognized that the time delay is an inherent feature of signal transmission between neurons. The existence of time delays may degrade system performance and cause oscillation in a network, leading to instability. Therefore, the study of time delay effects on stability and convergent dynamics of fuzzy neural networks has received much attention [17]- [28]. In [17] and [18], the authors obtained some exponential stability conditions by applying linear matrix inequality (LMI) and Lyapunov- Krasovskii functional. In [27] and [28], some results were derived by employing the properties of M -matrix.

The objective of this paper is to study a more general class of FCNNs model which includes the models in [17], [18], [27] and [28] as its special cases. Some stability criteria ensuring the exponential stability of such models are presented. Theoretical analysis and numerical examples are given to show that the results in this paper extend and improve some previous ones in [17], [18], [27] and [28].

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2. Problem Statement and Preliminaries

In the following, we consider the following fuzzy cellular neural network (FCNN) model with time-varying delays:

$$\dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_j(t))) + \sum_{j=1}^{n} c_{ij} u_j + I_i + \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j(t-\tau_j(t))) + \bigwedge_{j=1}^{n} \zeta_{ij} u_j + \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j(t-\tau_j(t))) + \bigvee_{j=1}^{n} \delta_{ij} u_j, \quad (1)$$

where $i = 1, 2, \cdots, n$, $\alpha_{ij}, \beta_{ij}, \zeta_{ij}$ and δ_{ij} are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively. $d_i > 0$ represents the passive decay rates to the state of *i*th unit at time *t*. a_{ij} and b_{ij} are elements of feedback template, and c_{ij} are elements of feed-forward template. $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operation, respectively. x_i, u_i and I_i denote state, input and bias of the ith neurons, respectively. $f_i(\cdot)$ and $g_i(\cdot)$ are the activation functions. $\tau_i(t)$ is the bounded transmission delay with $0 \le \tau_i(t) \le \tau$.

To obtain our results, we make the following assumption:

Assumption (A): The activation functions $f_i(\cdot)$ and $g_i(\cdot)$ are Lipschitz continuous, i.e., there exist constants $k_i > 0, l_i > 0$ such that for $i = 1, 2, \cdots, n$,

$$|f_i(\xi_1) - f_i(\xi_2)| \le k_i |\xi_1 - \xi_2|,$$
$$|g_i(\xi_1) - g_i(\xi_2)| \le l_i |\xi_1 - \xi_2|, \forall \xi_1, \xi_2 \in \mathbb{R}.$$

For convenience, we introduce some notations and lemmas. For real symmetric matrices X and Y,

$X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite (respectively, semi-positive definite).
$D = diag\{d_1, d_2, \cdots, d_n\}$, $K = diag\{k_1, k_2, \cdots, k_n\}$, $L = diag\{l_1, l_2, \cdots, l_n\}$,
$x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T$, $A = [a_{ij}]_{n\times n}$, $B = [b_{ij}]_{n\times n}$,
$u = [u_1, u_2, \cdots, u_n]^T$, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t))]^T$,
$g(x(t-\tau(t))) = [g_1(x_1(t-\tau_1(t))), g_2(x_2(t-\tau_2(t))), \cdots, g_n(x_n(t-\tau_n(t)))]^T$. For any vector $v = [v_1, v_2, \cdots v_n]^T \in \mathbb{R}^n$, $|v| = [|v_1|, |v_2|, \cdots, |v_n|]^T$, $\|v\| = \left(\sum_{i=1}^{n} |v_i|^2\right)^{\frac{1}{2}}$.
For any matrix $Q = (q_{ij})_{n\times n}$, $|Q| = (|q_{ij}|)_{n\times n}$, $\|Q\| = (\lambda_{\max}(Q^T Q))^{\frac{1}{2}}$, where $\lambda_{\max}(Q^T Q)$ denotes the maximum eigenvalue of $Q^T Q$.

Definition 1: The equilibrium point $x^* = [x_1^*, x_2^*, \cdots, x_n^*]^T$ of (1) is said to be globally exponentially stable, if there exist $\varepsilon > 0$ and $M \geq 1$, such that for any solution $x(t)$ of system (1) with initial function

$$x(s) = \phi(s), s \in [-\tau, 0],$$

where $\phi \in C([-\tau, 0], \mathbb{R}^n)$, one has

$$\|x(t) - x^*\| \leq M e^{-\varepsilon t} \sup_{-\tau \leq s \leq 0} \|\phi(s) - x^*\|, \forall t \geq 0.$$

Lemma 1 (Forti and Tesi, [32]): If $H : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function and satisfies the following conditions:
(i) $H(x) \neq h(y)$ for all $x \neq y$,
(ii) $\|H(x)\| \to \infty$ as $\|x\| \to \infty$,,
then, $H(x)$) is homeomorphism of $\mathbb{R}^n$.

Lemma 2 [31]: For any $\varepsilon > 0$ $x, y \in \mathbb{R}^n$, and positive definite matrix $Q \in \mathbb{R}^{n\times n}$, the following matrix inequality holds:

$$2x^T y \leq \varepsilon x^T Q x + \varepsilon^{-1} y^T Q^{-1} y.$$

Lemma 3 (Schur Complement [29]): The linear matrix inequality (LMI)

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

where $Q(x) = Q^T(x), R(x) = R^T(x)$, is equivalent to

$$R(x) > 0 \text{ and } Q(x) - S(x) R^{-1}(x) S^T(x) > 0.$$

Lemma 4: [15] Let x_i, y_i be two states of system (1), then we have

$$\left| \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(y_j) \right| \leq \sum_{j=1}^{n} |\alpha_{ij}| |g_j(x_j) - g_j(y_j)|,$$

$$\left| \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} g_j(y_j) \right| \leq \sum_{j=1}^{n} |\beta_{ij}| |g_j(x_j) - g_j(y_j)|.$$

3. Existence and uniqueness analysis

In this section, we present sufficient conditions for the existence and uniqueness of the equilibrium point for the FCNN model (1).

Theorem 1: Under the assumption (A), the FCNN model (1) has a unique equilibrium point for every input *u* if there exist three positive diagonal matrices diagonal matrices $P = diag\{p_1, p_2, \cdots, p_n\}$, $Q = diag\{q_1, q_2, \cdots, q_n\}$, $W = diag\{w_1, w_2, \cdots, w_n\}$ and number $\varepsilon \in (0,1)$, such that

$$\begin{aligned} \Omega_1 \triangleq\ & 2WD - KPK - LQL - |WAP^{-1}A^T W| \\ & -(1-\varepsilon)^{-1} |WBQ^{-1}B^T W| \\ & -\varepsilon^{-1} W(|\alpha| + |\beta|) Q^{-1} (|\alpha| + |\beta|)^T W > 0. \end{aligned} \quad (2)$$

Proof: Let us define a mapping that is associated with (1):

$$h_i(x_i) = -d_i x_i + \sum_{j=1}^{n} a_{ij} f_j(x_j) + \sum_{j=1}^{n} b_{ij} g_j(x_j) + \sum_{j=1}^{n} c_{ij} u_j + I_i$$
$$+ \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) + \bigwedge_{j=1}^{n} \zeta_{ij} u_j + \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) + \bigvee_{j=1}^{n} \delta_{ij} u_j, i = 1, 2, \cdots, n. \quad (3)$$

If $x^* = [x_1^*, x_2^*, \cdots, x_n^*]^T$ is an equilibrium point of FCNN model (1), we have $H(x^*) = 0$, where $H(x) = [h_1(x_1), h_2(x_2), \cdots, h_n(x_n)]^T$.

By the topological degree theory [32], we know that the FCNN model (1) has a unique equilibrium point for every input vector *u* if $H(x)$ given by (3) is homeomorphism of $\mathbb{R}^n$.

From (3), we get

$$h_i(x_i) - h_i(y_i) = -d_i(x_i - y_i) + \sum_{j=1}^{n} a_{ij}(f_j(x_j) - f_j(y_j))$$
$$+ \sum_{j=1}^{n} b_{ij}(g_j(x_j) - g_j(y_j)) + \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(y_j)$$
$$+ \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} g_j(y_j). \quad (4)$$

Multiplying the both sides of (4) by $2(x_i - y_i) w_i$, we obtain

$$2(x_i - y_i) w_i (h_i(x_i) - h_i(y_i)) = -2(x_i - y_i) w_i d_i (x_i - y_i)$$
$$+ 2(x_i - y_i) w_i [\sum_{j=1}^{n} a_{ij}(f_j(x_j) - f_j(y_j)) + \sum_{j=1}^{n} b_{ij}(g_j(x_j) - g_j(y_j))]$$
$$+ 2(x_i - y_i) w_i [\bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(y_j)$$

$$+\bigvee_{j=1}^{n}\beta_{ij}g_j(x_j)-\bigvee_{j=1}^{n}\beta_{ij}g_j(y_j)]\,. \tag{5}$$

From Lemma 4, we get

$$(x_i-y_i)w_i[\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j)-\bigwedge_{j=1}^{n}\alpha_{ij}g_j(y_j)]$$

$$\le|x_i-y_i|\,w_i\left|\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j)-\bigwedge_{j=1}^{n}\alpha_{ij}g_j(y_j)\right|$$

$$\le|x_i-y_i|\,w_i\sum_{j=1}^{n}|\alpha_{ij}||g_j(x_j)-g_j(y_j)|$$

and, similarly,

$$(x_i-y_i)w_i[\bigvee_{j=1}^{n}\beta_{ij}g_j(x_j)-\bigvee_{j=1}^{n}\beta_{ij}g_j(y_j)]$$

$$\le|x_i-y_i|\,w_i\sum_{j=1}^{n}|\beta_{ij}||g_j(x_j)-g_j(y_j)|.$$

From (5), we have

$$\begin{aligned}2(x-y)^TW(H(x)-H(y))&\le-2(x-y)^TWD(x-y)\\&+2(x-y)^TWA(f(x)-f(y))\\&+2(x-y)^TWB(g(x)-g(y))\\&+2|x-y|^TW(|\alpha|+|\beta|)|g(x)-g(y)|.\end{aligned} \tag{6}$$

From Lemma 2, we obtain

$$\begin{aligned}&2(x-y)^TWA(f(x)-f(y))\le\\&\quad(f(x)-f(y))^TP(f(x)-f(y)),\\&\quad+(x-y)^TWAP^{-1}A^TW(x-y)\end{aligned} \tag{7}$$

$$\begin{aligned}&2(x-y)^TWB(g(x)-g(y))\\&\quad\le(1-\varepsilon)(g(x)-g(y))^TQ(g(x)-g(y))\\&\quad+(1-\varepsilon)^T(x-y)^TWBQ^{-1}B^TW(x-y)\end{aligned} \tag{8}$$

and

$$\begin{aligned}&2|x-y|^TW(|\alpha|+|\beta|)|g(x)-g(y)|\\&\quad\le\varepsilon|g(x)-g(y)|^TQ|g(x)-g(y)|\\&\quad+\varepsilon^{-1}|x-y|^TW(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^TW|x-y|.\end{aligned} \tag{9}$$

It follows from assumption (A) that

$$\begin{aligned}&(f(x)-f(y))^TP(f(x)-f(y))\le(x-y)^TKPK(x-y),\\&(g(x)-f(y))^TQ(g(x)-g(y))\le(x-y)^TLQL(x-y).\end{aligned} \tag{10}$$

From (6)-(10), we get

$$\begin{aligned}&2(x-y)^TW(H(x)-H(y))\le-2|x-y|^TWD|x-y|\\&+|x-y|^T(KPK+LQL+|WAP^{-1}A^TW|\\&+(1-\varepsilon)^{-1}|WBQ^{-1}B^TW|)|x-y|\\&+\varepsilon^{-1}|x-y|^TW(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^TW|x-y|\\&=-|x-y|^T\Omega_1|x-y|\le-\kappa\|x-y\|^2,\end{aligned} \tag{11}$$

where $\kappa=\lambda_{\min}(\Omega_1)>0$. Hence we obtain

$$\begin{aligned}-(x-y)^TW(H(x)-H(y))&=\|(x-y)^TW(H(x)-H(y))\|\\&\le\bar{w}\|x-y\|\|H(x)-H(y)\|,\end{aligned} \tag{12}$$

where $\bar{w}=\max_{1\le i\le n}\{w_i\}$.

From (11) and (12), we get

$$2\bar{w}\|x-y\|\|H(x)-H(y)\|\ge\kappa\|x-y\|^2. \tag{13}$$

It follows from (13) that $H(x)\ne H(y)$ when $x\ne y$ holds. Letting $y=0$ and $x\ne y$, from (13), we also get

$$\frac{\kappa}{2\bar{w}}\|x\|\le\|H(x)-H(0)\|\le\|H(x)\|+\|H(0)\|.$$

Since $\|H(0)\|$ is finite, we can conclude that $\|H(x)\|\to\infty$ as $\|x\|\to\infty$. From Lemma 1, the map $H(x)$ given by (3) is homeomorphism of $\mathbb{R}^n$. This completes the proof.

4. Exponential stability analysis

Theorem 2: Let $\tau_i'(t)\le\mu_i$ and $\mu_i\in[0,1)$, $i=1,2,\cdots,n.$ Under the assumption (A), the FCNN model (1) has a unique equilibrium point, which is globally exponentially stable if there exist positive diagonal matrices $P=diag\{p_1,p_2,\cdots,p_n\}$, $Q=diag\{q_1,q_2,\cdots,q_n\}$ and $W=diag\{w_1,w_2,\cdots,w_n\}$ and number $\varepsilon\in(0,1)$, such that

$$\begin{aligned}\Omega_2\triangleq\;&2WD-KPK-LSL-|WAP^{-1}A^TW|\\&-(1-\varepsilon)^{-1}|WBQ^{-1}B^TW|\\&-\varepsilon^{-1}W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^TW>0,\end{aligned} \tag{14}$$

where $S=diag\{\frac{q_1}{1-\mu_1},\frac{q_2}{1-\mu_2},\cdots,\frac{q_n}{1-\mu_n}\}$.

Proof: Since $\mu_i\in[0,1)$ and $S=Q\bullet diag\{\frac{1}{1-\mu_1},\frac{1}{1-\mu_2},\cdots,\frac{1}{1-\mu_n}\}$, we know that $Q\le S$. From (2) and (14) we get $\Omega_1\ge\Omega_2>0$ .By Theorem 1, the condition (14) implies the existence and uniqueness of equilibrium point of model (1).

Let $x^*=[x_1^*,x_2^*,\cdots,x_n^*]^T$ be an equilibrium point of FCNN model (1), then x^* satisfies

$$0=-d_ix_i^*+\sum_{j=1}^{n}a_{ij}f_j(x_j^*)+\sum_{j=1}^{n}b_{ij}g_j(x_j^*)+\sum_{j=1}^{n}c_{ij}u_j+I_i$$

$$+\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j^*)+\bigwedge_{j=1}^{n}\zeta_{ij}u_j+\bigvee_{j=1}^{n}\beta_{ij}g_j(x_j^*)+\bigvee_{j=1}^{n}\delta_{ij}u_j.$$

We shift the equilibrium point x^* of (1) to the origin by using the transformation $z_i(\cdot)=x_i(\cdot)-x_i^*$, $i=1,2,\cdots,n$. System (1) can be transformed to the following form:

$$\dot{z}_i(t)=-d_iz_i(t)+\sum_{j=1}^{n}a_{ij}\varphi_j(z_j(t))+\sum_{j=1}^{n}b_{ij}\psi_j(z_j(t-\tau_j(t)))$$
$$+\bigwedge_{j=1}^{n}\alpha_{ij}g_j(z_j(t-\tau_j(t))+x_j^*)-\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j^*)$$
$$+\bigvee_{j=1}^{n}\beta_{ij}g_j(z_j(t-\tau_j(t))+x_j^*)-\bigvee_{j=1}^{n}\beta_{ij}g_j(x_j^*) \quad (15)$$

where
$\varphi_i(z_i(t))=f_i(z_i(t)+x_i^*)-f_i(x_i^*),\psi_i(z_i(t))=g_i(z_i(t)+x_i^*)-g_i(x_i^*)$.

We introduce some notations that will be used below:
$z(t)=[z_1(t),z_2(t),\cdots,z_n(t)]^T$,

$$\Phi(x(t))=[\varphi_1(z_1(t)),\varphi_2(z_2(t)),\cdots,\varphi_n(z_n(t))]^T,$$
$$\Psi(x(t-\tau(t)))=[\psi_1(z_1(t-\tau_1(t))),\psi_2(z_2(t-\tau_2(t))),\cdots,\psi_n(z_n(t-\tau_n(t)))]^T.$$

It follows from assumption (A) that

$$|\varphi_i(z_i)|\le k_i|z_i|,\quad |\varphi_i(0)|=0,$$
$$|\psi_i(z_i)|\le l_i|z_i|,\quad |\psi_i(0)|=0, i=1,2,\cdots,n. \quad (16)$$

Obviously, the global exponential stability of equilibrium point x^* of model (1) is equivalent to the global exponential stability of the origin of (15).

We choose a Lyapunov functional as follows

$$V(t,z_t)=\sum_{i=1}^{n}w_iz_i^2(t)+\sum_{i=1}^{n}\frac{q_i}{1-\mu_i}\int_{t-\tau_i(t)}^{t}\psi_i^2(z_i(s))ds. \quad (17)$$

The derivative of $V(t,z_t)$ along the trajectory of system (15) is

$$\dot{V}(t,z_t)=2\sum_{i=1}^{n}w_iz_i(t)\dot{z}_i(t)$$
$$+\sum_{i=1}^{n}\frac{q_i}{1-\mu_i}(\psi_i^2(z_i(t))-(1-\tau_i'(t))\psi_i^2(z_i(t-\tau_i(t))))$$
$$=-2z^T(t)WDz(t)+2z^T(t)WA\Phi(z(t))$$
$$+2z^T(t)WB\Psi(z(t-\tau(t)))$$
$$+2\sum_{i=1}^{n}w_iz_i(t)(\bigwedge_{j=1}^{n}\alpha_{ij}g_j(z_j(t-\tau_j(t))+x_j^*)-\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j^*))$$
$$+2\sum_{i=1}^{n}w_iz_i(t)(\bigvee_{j=1}^{n}\beta_{ij}g_j(z_j(t-\tau_j(t))+x_j^*)-\bigvee_{j=1}^{n}\beta_{ij}g_j(x_j^*))$$
$$+\sum_{i=1}^{n}\frac{q_i}{1-\mu_i}(\psi_i^2(z_i(t))-(1-\tau_i'(t))\psi_i^2(z_i(t-\tau_i(t)))). \quad (18)$$

By Lemma 4, we have

$$2w_iz_i(t)(\bigwedge_{j=1}^{n}\alpha_{ij}g_j(z_j(t-\tau_j(t))+x_j^*)-\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j^*))$$
$$\le 2w_i|z_i(t)|\left|\bigwedge_{j=1}^{n}\alpha_{ij}g_j(z_j(t-\tau_j(t))+x_j^*)-\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j^*)\right|$$
$$\le 2w_i|z_i(t)|\sum_{j=1}^{n}|\alpha_{ij}||g_j(z_j(t-\tau_j(t))+x_j^*)-g_j(x_j^*)|$$
$$=2w_i|z_i(t)|\sum_{j=1}^{n}|\alpha_{ij}||\psi_j(z_j(t-\tau_j(t)))| \quad (19)$$

and, similarly,

$$2w_iz_i(t)(\bigvee_{j=1}^{n}\beta_{ij}g_j(z_j(t-\tau_j(t))+x_j^*)-\bigvee_{j=1}^{n}\beta_{ij}g_j(x_j^*))$$
$$\le 2w_i|z_i(t)|\sum_{j=1}^{n}|\beta_{ij}||\psi_j(z_j(t-\tau_j(t)))|. \quad (20)$$

From (18)-(20), we obtain

$$\dot{V}(t,z_t)\le -2z^T(t)WDz(t)+2z^T(t)WA\Phi(z(t))$$
$$+2z^T(t)WB\Psi(z(t-\tau(t)))$$
$$+2\sum_{i=1}^{n}w_i|z_i(t)|\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)|\psi_j(z_j(t-\tau_j(t)))|$$
$$+\sum_{i=1}^{n}(\frac{q_i}{1-\mu_i}(\psi_i^2(z_i(t))-q_i\psi_i^2(z_i(t-\tau_i(t))))$$
$$=-2z^T(t)WDz(t)+2z^T(t)WA\Phi(z(t))$$
$$+2z^T(t)WB\Psi(z(t-\tau(t)))$$
$$+2|z^T(t)|W(|\alpha|+|\beta|)|\Psi(z(t-\tau(t)))|$$
$$+\Psi^T(z(t))S\Psi(z(t))$$
$$-\Psi^T(z(t-\tau(t)))Q\Psi(z(t-\tau(t))). \quad (21)$$

From Lemma 2, we know

$$2z^T(t)WA\Phi(z(t))\le\Phi^T(z(t))P\Phi(z(t))$$
$$+z^T(t)WAP^{-1}A^TWz(t) \quad (22)$$

and

$$2z^T(t)WB\Psi(z(t-\tau(t)))\le(1-\varepsilon)\Psi^T(z(t-\tau(t)))Q\Psi(z(t-\tau(t)))$$
$$+(1-\varepsilon)^{-1}z^T(t)WBQ^{-1}B^TWz(t). \quad (23)$$

Since

$$2|z^T(t)|W(|\alpha|+|\beta|)|\Psi(z(t-\tau(t)))|-\varepsilon\Psi^T(z(t-\tau(t)))Q\Psi(z(t-\tau(t)))$$
$$=2|\Psi^T(z(t-\tau(t)))|(|\alpha|+|\beta|)^TW|z(t)|$$
$$-\varepsilon\Psi^T(z(t-\tau(t)))Q\Psi(z(t-\tau(t)))$$
$$=-(\varepsilon^{\frac{1}{2}}Q^{\frac{1}{2}}|\Psi(z(t-\tau(t)))|-\varepsilon^{-\frac{1}{2}}Q^{-\frac{1}{2}}(|\alpha|+|\beta|)^TW|z(t)|)^T$$
$$\times(\varepsilon^{\frac{1}{2}}Q^{\frac{1}{2}}|\Psi(z(t-\tau(t)))|-\varepsilon^{-\frac{1}{2}}Q^{-\frac{1}{2}}(|\alpha|+|\beta|)^TW|z(t)|)$$
$$+\varepsilon^{-1}|z^T(t)|W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^TW|z(t)|$$

$$\le \varepsilon^{-1}\left|z^T(t)\right|W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W\left|z(t)\right|. \quad (24)$$

From (21)-(24), we have

$$\dot V(t,z_t)\le -2z^T(t)WDz(t)+z^T(t)P\Phi(z(t))$$
$$+z^T(t)WAP^{-1}A^TWz(t)+\Psi^T(z(t))S\Psi(z(t))$$
$$+(1-\varepsilon)^{-1}z^T(t)WBQ^{-1}B^TWz(t)$$
$$+\varepsilon^{-1}\left|z^T(t)\right|W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W\left|z(t)\right|$$
$$\le\left|z^T(t)\right|\left[-2WD+KPK+LSL+\left|WAP^{-1}A^TW\right|\right.$$
$$+(1-\varepsilon)^{-1}\left|WBQ^{-1}B^TW\right|$$
$$\left.+\varepsilon^{-1}W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^TW\right]\left|z(t)\right|$$
$$=-\left|z^T(t)\right|\Omega_2\left|z(t)\right|\le -\vartheta\left\|z(t)\right\|^2, \quad (25)$$

where $\vartheta=\lambda_{\min}\{\Omega_2\}>0$.

Letting $\gamma>0$, from (17) we get

$$(e^{\gamma t}V(t,z_t))'=\gamma e^{\gamma t}V(t,z_t)+e^{\gamma t}\dot V(t,z_t)$$
$$\le \gamma e^{\gamma t}\Big(\sum_{i=1}^n w_i z_i^2(t)+\sum_{i=1}^n \frac{q_i}{1-\mu_i}\int_{t-\tau_i(t)}^t \psi_i^2(z_i(s))ds\Big)$$
$$-\vartheta e^{\gamma t}\left\|z(t)\right\|^2. \quad (26)$$

Letting $t=\eta$ and integrating the both sides of (26) on $[0,t]$, we obtain

$$e^{\gamma t}V(t,z_t)-V(0,z_t)\le\int_0^t \gamma e^{\gamma\eta}\Big(\sum_{i=1}^n w_i z_i^2(\eta)$$
$$+\sum_{i=1}^n\frac{q_i}{1-\mu_i}\int_{\eta-\tau_i(\eta)}^{\eta}\psi_i^2(z_i(s))ds\Big)d\eta$$
$$-\int_0^t \vartheta e^{\gamma\eta}\left\|z(\eta)\right\|^2 d\eta. \quad (27)$$

Taking $\bar w=\max_{1\le i\le n} w_i$, $\bar l=\max_{1\le i\le n} l_i$, $\rho=\max_{1\le i\le n}\{\frac{q_i}{1-\mu_i}\}$, $\left\|\phi\right\|=\max_{s\in[-\tau,0]}\left\|z(s)\right\|$, we have

$$\sum_{i=1}^n\frac{q_i}{1-\mu_i}\int_{-\tau_i(0)}^0\psi_i^2(z_i(s))ds\le\sum_{i=1}^n\rho\bar l^2\int_{-\tau}^0\left|z_i(s)\right|^2 ds$$
$$=\rho\bar l^2\int_{-\tau}^0\left\|z(s)\right\|^2 ds\le\rho\bar l^2\tau\left\|\phi\right\|^2,$$

and, consequently,

$$V(0,z_t)=\sum_{i=1}^n w_i z_i^2(0)+\sum_{i=1}^n\frac{q_i}{1-\mu_i}\int_{-\tau_i(0)}^0\psi_i^2(z_i(s))ds$$
$$\le(\bar w+\rho\bar l^2\tau)\left\|\phi\right\|^2. \quad (28)$$

Thus we obtain

$$\int_0^t\gamma e^{\gamma\eta}\Big(\sum_{i=1}^n w_i z_i^2(\eta)\Big)d\eta\le\gamma\bar w\int_0^t\gamma e^{\gamma\eta}\left\|z(\eta)\right\|^2 d\eta. \quad (29)$$

Since $\int_s^{\tau+s}e^{\gamma\eta}d\eta\le\int_s^{\tau+s}e^{\gamma(s+\tau)}d\eta\le\tau e^{\gamma(s+\tau)}$, $\int_{-\tau}^0 e^{\gamma s}\left\|z(s)\right\|^2 ds\le\int_{-\tau}^0\left\|z(s)\right\|^2 ds\le\tau\left\|\phi\right\|^2$, we have

$$\int_0^t\gamma e^{\gamma\eta}\Big(\sum_{i=1}^n\frac{q_i}{1-\mu_i}\int_{\eta-\tau_i(\eta)}^{\eta}\psi_i^2(z_i(s))ds\Big)d\eta$$
$$\le\gamma\rho\int_0^t e^{\gamma\eta}\Big(\sum_{i=1}^n\int_{\eta-\tau}^{\eta}\psi_i^2(z_i(s))ds\Big)d\eta$$
$$\le\gamma\rho\bar l^2\int_0^t e^{\gamma\eta}\int_{\eta-\tau}^{\eta}\left\|z(s)\right\|^2 ds d\eta$$
$$=\gamma\rho\bar l^2\int_{-\tau}^t\Big(\int_{\max[s,0]}^{\min[s+\tau,t]}e^{\gamma\eta}d\eta\Big)\left\|z(s)\right\|^2 ds$$
$$\le\gamma\rho\bar l^2\int_{-\tau}^t\Big(\int_s^{s+\tau}e^{\gamma\eta}d\eta\Big)\left\|z(s)\right\|^2 ds$$
$$\le\gamma\rho\bar l^2\int_{-\tau}^t\tau e^{\gamma(s+\tau)}\left\|z(s)\right\|^2 ds=\gamma\rho\bar l^2\tau e^{\gamma\tau}\int_{-\tau}^t e^{\gamma s}\left\|z(s)\right\|^2 ds$$
$$=\gamma\rho\bar l^2\tau e^{\gamma\tau}\Big(\int_{-\tau}^0 e^{\gamma s}\left\|z(s)\right\|^2 ds+\int_0^t e^{\gamma s}\left\|z(s)\right\|^2 ds\Big)$$
$$\le\gamma\rho\bar l^2\tau e^{\gamma\tau}\Big(\tau\left\|\phi\right\|^2+\int_0^t e^{\gamma s}\left\|z(s)\right\|^2 ds\Big). \quad (30)$$

From (27)-(30), we obtain

$$e^{\gamma t}V(t,z_t)\le(\bar w+\rho\bar l^2\tau)\left\|\phi\right\|^2+\gamma\bar w\int_0^t e^{\gamma\eta}\left\|z(\eta)\right\|^2 d\eta$$
$$+\gamma\rho\bar l^2\tau e^{\gamma\tau}\Big(\tau\left\|\phi\right\|^2+\int_0^t e^{\gamma s}\left\|z(s)\right\|^2 ds\Big)-\int_0^t\vartheta e^{\gamma\eta}\left\|z(\eta)\right\|^2 d\eta$$

Choosing sufficiently small $\gamma>0$ such that $\gamma\bar w+\gamma\rho\bar l^2\tau e^{\gamma\tau}-\vartheta\le 0$, we have

$$e^{\gamma t}V(t,z_t)\le(\bar w+\rho\bar l^2\tau+\gamma\rho\bar l^2\tau^2e^{\gamma\tau})\left\|\phi\right\|^2\triangleq M\left\|\phi\right\|^2$$

From (17), we obtain

$$w\left\|z(t)\right\|^2\le\sum_{i=1}^n w_i z_i^2(t)\le V(t,z_t)\le M\left\|\phi\right\|^2e^{-\gamma t},$$

where $w=\min_{1\le i\le n}\{w_i\}$. That gives

$$\left\|z(t)\right\|\le\sqrt{\frac{M}{w}}\left\|\phi\right\|e^{-\frac{\gamma}{2}t}.$$

Hence the origin of (15) is globally exponentially stable. This completes the proof.

Remark 1: Suppose that inequality (22) is replaced with the following inequality

$$2z^T(t)WA\Phi(z(t))\le 2\left|z^T(t)\right|W\left|A\right|\left|\Phi(z(t))\right|$$
$$\le\left|\Phi^T(z(t))\right|P\left|\Phi(z(t))\right|+\left|z^T(t)\right|W\left|A\right|P^{-1}\left|A\right|^TW\left|z(t)\right|. \quad (31)$$

Consequently, the sufficient condition (14) can be replaced with the following stronger condition

$$\bar\Omega_2\triangleq 2WD-KPK-LSL-W\left|A\right|P^{-1}\left|A^T\right|W$$
$$-(1-\varepsilon)^{-1}W\left|B\right|Q^{-1}\left|B^T\right|W$$
$$-\varepsilon^{-1}W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^TW>0 \quad (32)$$

or, equivalently,

$$\begin{bmatrix} 2WD-KPK-LSL & W|A| & (1-\varepsilon)^{-\frac{1}{2}}W|B| & \varepsilon^{-\frac{1}{2}}W(|\alpha|+|\beta|) \\ |A|^T W & P & 0 & 0 \\ (1-\varepsilon)^{-\frac{1}{2}}|B|^T W & 0 & Q & 0 \\ \varepsilon^{-\frac{1}{2}}(|\alpha|+|\beta|)^T W & 0 & 0 & Q \end{bmatrix} > 0 \quad (33)$$

Theorem 3: Let $b_{ij}(t) \equiv 0$, $\tau_i'(t) \le \mu_i$ and $\mu_i \in [0,1)$, $i, j = 1, 2, \cdots, n.$ Under the assumption (A), FCNN model (1) has a unique equilibrium point, which is globally exponentially stable if there exist positive diagonal matrices $P = diag\{p_1, p_2, \cdots, p_n\}$, $Q = diag\{q_1, q_2, \cdots, q_n\}$ and $W = diag\{w_1, w_2, \cdots, w_n\}$, such that

$$\Omega_3 \triangleq 2WD - LSL - W|A|K - K|A^T|W - W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W > 0 \quad (34)$$

or, equivalently,

$$\begin{bmatrix} 2WD-LSL-W|A|K-K|A^T|W & W(|\alpha|+|\beta|) \\ (|\alpha|+|\beta|)^T W & Q \end{bmatrix} > 0, \quad (35)$$

where $S = diag\{\frac{q_1}{1-\mu_1}, \frac{q_2}{1-\mu_2}, \cdots, \frac{q_n}{1-\mu_n}\}$.

Proof: Since $b_{ij}(t) \equiv 0$ holds, similarly to the proof of Theorem2, we obtain

$$\begin{aligned} \dot{V}(t,z_t) &\le -2z^T(t)WDz(t) + 2z^T(t)WA\Phi(z(t)) \\ &+2|z^T(t)|W(|\alpha|+|\beta|)|\Psi(z(t-\tau(t)))| \\ &+\Psi^T(z(t))S\Psi(z(t)) - \Psi^T(z(t-\tau(t)))Q\Psi(z(t-\tau(t))), \end{aligned} \quad (36)$$

where $V(t, z_t)$ is defined as (17).

From (24) with $\varepsilon = 1$, using the fact that

$$\begin{aligned} 2z^T(t)WA\Phi(z(t)) &= 2\sum_{i=1}^n \sum_{j=1}^n z_i(t) w_i a_{ij} \varphi_j(z_j(t)) \\ &\le 2\sum_{i=1}^n \sum_{j=1}^n |z_i(t) w_i a_{ij} \varphi_j(z_j(t))| \\ &\le 2\sum_{i=1}^n \sum_{j=1}^n |z_i(t)| w_i |a_{ij}| k_j |z_j(t)| \\ &= |z^T(t)|(W|A|K + K|A|^T W)|z(t)|. \end{aligned}$$

we have

$$\begin{aligned} \dot{V}(t,z_t) &\le -2z^T(t)WDz(t) \\ &+|z^T(t)|(W|A|K + K|A|^T W)|z(t)| \\ &+\Psi^T(z(t))S\Psi(z(t)) \\ &+|z^T(t)|W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W|z(t)| \\ &\le |z^T(t)|(-2WD + LSL + W|A|K + K|A|^T W \\ &+W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W)|z(t)|. \end{aligned}$$

The following proof is similar to that of Theorem 2 and is omitted.

Theorem 4: Let $a_{ij}(t) \equiv 0$, $\tau_i'(t) \le \mu_i$ and $\mu_i \in [0,1)$, $i, j = 1, 2, \cdots, n.$ Under the assumption (A), FCNN model (1) has a unique equilibrium point, which is globally exponentially stable if there exist positive diagonal matrices $Q = diag\{q_1, q_2, \cdots, q_n\}$ and $W = diag\{w_1, w_2, \cdots, w_n\}$, and number $\varepsilon \in (0,1)$, such that

$$\begin{aligned} \Omega_4 &\triangleq 2WD - LSL - (1-\varepsilon)^{-1}|WBQ^{-1}B^T W| \\ &-\varepsilon^{-1}W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W > 0. \end{aligned} \quad (37)$$

where $S = diag\{\frac{q_1}{1-\mu_1}, \frac{q_2}{1-\mu_2}, \cdots, \frac{q_n}{1-\mu_n}\}$.

Proof: Since $a_{ij}(t) \equiv 0$ holds, similarly to the proof of Theorem 2, we obtain

$$\begin{aligned} \dot{V}(t,z_t) &\le -2z^T(t)WDz(t) + 2z^T(t)WB\Psi(z(t-\tau(t))) \\ &+2z^T(t)W(|\alpha|+|\beta|)|\Psi(z(t-\tau(t)))| \\ &+\Psi^T(z(t))S\Psi(z(t)) - \Psi^T(z(t-\tau(t)))Q\Psi(z(t-\tau(t))). \end{aligned} \quad (38)$$

From (23), (24) and (38), we have

$$\begin{aligned} \dot{V}(t,z_t) &\le -2z^T(t)WDz(t) + \Psi^T(z(t))S\Psi(z(t)) \\ &+(1-\varepsilon)^{-1}z^T(t)WBQ^{-1}B^T Wz(t) \\ &+\varepsilon^{-1}|z^T(t)|W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W|z(t)| \\ &\le |z^T(t)|[-2WD + LSL + (1-\varepsilon)^{-1}|WBQ^{-1}B^T W| \\ &+\varepsilon^{-1}W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W]|z(t)|. \end{aligned}$$

The following proof is also similar to that of Theorem 2 and is omitted.

5. Comparison and Examples

The FCNNs models in [17], [18], [27] and [28] are special cases of model (1) with $A = 0, B = 0$ or $f_j(x_j) \equiv g_j(x_j)$. To make precise comparison, we list some of their results as follows.

Theorem 5 (Yuan et al. 2006, [17]): Assume that $f_j(\cdot) \equiv g_j(\cdot)$, $b_{ij}(t) \equiv 0$ and $\tau_i'(t) \le \mu < 1$. Under the assumption (A), the equilibrium point of model (1) is exponentially stable if there exist positive diagonal matrices W and S, and a positive constant k such that

$$\Omega_5 \triangleq 2WD - 2kW - W|A|L - L|A^T|W - LSL$$
$$-(1-\mu)^{-1}e^{2k\tau}W(|\alpha|+|\beta|)S^{-1}(|\alpha|+|\beta|)^T W > 0 \quad (39)$$

Remark 2: Letting $f_j(\cdot) = g_j(\cdot)$ and $\mu_i = \mu \in [0,1)$, from Theorem 3, we know that $S = \frac{1}{1-\mu}Q$. Therefore, (34) can be rewritten as

$$\Omega_3 = 2WD - W|A|L - L|A^T|W - LSL$$
$$-(1-\mu)^{-1}W(|\alpha|+|\beta|)^{-1}S(|\alpha|+|\beta|)^T W > 0.$$

Since $\mu \in [0,1)$ and $k > 0$ hold, from (34) and (39) we know that $\Omega_3 > \Omega_5$, i.e., inequality (39) implies (34). Hence, to some extent, Theorem 3 improves Theorem 5.

Theorem 6 (Zhang et al. 2008, [28]): Assume that assumption (A) holds and $a_{ij}(t) \equiv 0$. The equilibrium point of model (1) is globally asymptotically stable if $D - |B|L - (|\alpha|+|\beta|)L$ is nonsingular M-matrix and $\tau_i'(t) \le 0$.

Remark 3: In [27], assuming that $b_{ij}(t) \equiv 0$, the authors proved that the model (1) is globally exponentially stable if $D - |A|L - (|\alpha|+|\beta|)L$ is a nonsingular M-matrix. Obviously, the signs of elements of weight matrices *A* and *B* of the non-fuzzy terms are ignored in [27] and [28], while they are considered in the Theorems 1, 2 and 4 of this paper. In [18], the activation functions f_j are assumed to be bounded, while this condition is removed in this paper. Examples given below will show the criteria in this paper are less conservative than the results in [18], [27] and [28]. In addition, the condition $\tau_i'(t) \le 0$ in Theorem 6 is not required in Theorems 1 through 4 in this paper.

Remark 4: The conditions (33) and (35) in this paper are expressed by LMI, which can be solved numerically using the effective interior-point algorithm [29] [30].

Now let us consider two examples.

Example 1: Suppose that a FCNN (1) has the following parameters:

$$A = B = \frac{1}{4}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad \alpha = \beta = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix},$$
$$D = \begin{bmatrix} 2.2 & 0 \\ 0 & 2.2 \end{bmatrix},$$
$$f_j(x_j) = 0.5(x_j - \sin x_j), \quad g_j(x_j) = 0.5(x_j + \cos x_j).$$

Taking $\mu_i = 0.5, \varepsilon = 0.8,\ W = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$, $P = Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $K = L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, we have

$$\Omega_2 = 2WD - KPK - LSL - |WAP^{-1}A^TW| - (1-\varepsilon)^{-1}|WBQ^{-1}B^TW|$$
$$-\varepsilon^{-1}W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W > 0$$
$$= \begin{bmatrix} 1.0125 & -0.9 \\ -0.9 & 1.0125 \end{bmatrix} > 0.$$

From Theorem 2, this model has a unique equilibrium point, which is globally exponentially stable. Since $A \ne 0$ and $B \ne 0$, the results in [17], [27] and [28] fail to this example. Since f_j and g_j are unbounded activation functions, the results in [18] are not applied for this example, either.

Example 2: Assume that the parameters of FCNN model (1) are given as follows:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 \end{bmatrix},$$
$$\alpha = \beta = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix},$$
$$\tau_i(t) \equiv \tau,\ D = 3E,\ K = L = E,$$

where $\tau > 0$ is a constant and E denotes the identity matrix.

Letting $\mu_i = 0,\quad W = 0.4E,\quad Q = E,\quad \varepsilon = 0.3,$ from (37) we have

$$\Omega_4 = 2WD - LSL - (1-\varepsilon)^{-1}|WBQ^{-1}B^TW|$$
$$-\varepsilon^{-1}W(|\alpha|+|\beta|)Q^{-1}(|\alpha|+|\beta|)^T W$$
$$= \begin{bmatrix} 0.4004 & -0.0853 & -0.0853 & -0.0853 \\ -0.0853 & 0.4004 & -0.0853 & -0.0853 \\ -0.0853 & -0.0853 & 0.4004 & -0.0853 \\ -0.0853 & -0.0853 & -0.0853 & 0.4004 \end{bmatrix}.$$

The eigen-values of Ω_4 are 0.1444 and triple 0.4857. Hence $\Omega_4 > 0$ and by Theorem 4 the equilibrium point of this FCNN model is globally exponentially stable.

Since

$$\Pi \triangleq D - |B|L - (|\alpha|+|\beta|)L$$

$$= \begin{bmatrix} 1.8 & -1.2 & -1.2 & -1.2 \\ -1.2 & 1.8 & -1.2 & -1.2 \\ -1.2 & -1.2 & 1.8 & -1.2 \\ -1.2 & -1.2 & -1.2 & 1.8 \end{bmatrix},$$

whose eigen-values are -1.8 and triple 3. Hence Π is not nonsingular M-matrix, i.e., Theorem 6 is not applicable to this example.

6. Conclusion

In this paper, the stability problem for a broad class of fuzzy cellular neural networks (FCNNs) with time-varying delays is studied. By the topological degree theory, the existence and uniqueness of the equilibrium point is analyzed. Constructing suitable Lyapunov functional, we derive some new sufficient conditions for global exponential stability of such FCNNs. These criteria in terms of linear matrix inequalities can be solved numerically using the interior-point algorithm. Theoretical analysis shows the improvement of the obtained results over some existing ones. Two examples are presented to show our results are less conservative than some previous stability criteria.

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