Decision Making with Distance Measures and Linguistic Aggregation Operators

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Abstract

We present a new decision making model with distance measures by using linguistic aggregation operators. We introduce a new aggregation operator called the linguistic ordered weighted averaging distance (LOWAD) operator. This aggregation operator provides a parameterized family of linguistic aggregation operators that includes the maximum distance, the minimum distance, the linguistic normalized Hamming distance and the linguistic weighted Hamming distance, among others. We study some of its main properties and different families of LOWAD operators such as the median-LOWAD, the Olympic-LOWAD, the S-LOWAD and the centered-LOWAD. We also develop an application of the new approach in a decision making problem concerning human resource management.

Keywords: Decision making, Hamming distance, Linguistic aggregation operators, OWA operator, Human resource management.

1. Introduction

In the literature, we find a wide range of methods for decision making [1-9]. A very useful technique for doing so are the distance measures [6-7,10-17]. The key feature of using distance measures in decision making is the possibility of comparing the results with an ideal one in order to take a decision. Thus, by doing this comparison, the alternative with the closest result to zero is the optimal choice because this implies that it is the alternative with the closest result to the ideal. One of the distance measures that can be used in the analysis is the Hamming distance [10]. Since its appearance, the Hamming distance has been studied and applied in a lot of problems, refer, e.g., to [6-7,11-17].

Often, when using the Hamming distance, it is interesting to normalize it by using the arithmetic mean or the weighted average. Therefore, we get the normalized Hamming distance (NHD) and the weighted Hamming distance (WHD), respectively. However, sometimes it is better to use another approach to normalize the Hamming distance such as the use of the ordered weighted averaging (OWA) operator [18]. Thus, with the OWA operator, the normalization process reflects a parameterized family of distance aggregation operators that range from the maximum distance to the minimum distance and including the NHD. Since its appearance, the OWA operator has been studied in a wide range of studies such as [19-36]. Note that the use of the OWA operator in distance measures have been studied in [6-7,11,13-14].

When using the OWA operator, it is assumed that the available information is given by exact numbers. However, this may not be the real situation found in the decision making problem. Sometimes, the available information is very uncertain and it cannot be analysed with exact numbers. In these cases, it is better to use another approach such as the use of linguistic information [37-49]. In order to assess the problem with the OWA operator when the available information is given in the form of linguistic variables, it has been suggested the linguistic OWA (LOWA) operator [38]. Since its appearance, it has been studied by a lot of authors such as [37-42]. Note that in the literature we may find other approaches for dealing with uncertain information such as the use of interval numbers [6], fuzzy numbers [6], rough sets [50], grey sets [51] and vague sets [52].

The objective of this paper is to suggest the use of linguistic information in decision making problems with distance measures. Thus, we will be able to provide a model that is able to assess the information in situations with high degree of uncertainty by using linguistic variables. For doing so, we will suggest a new type of linguistic aggregation operator for distance measures: the linguistic ordered weighted averaging distance (LOWAD) operator. It is a new aggregation operator that provides a parameterized family of linguistic aggregation operators such as the linguistic maximum distance, the linguistic minimum distance, the linguistic normalized Hamming distance (LNHD), the linguistic weighted Hamming distance (LWHD), the S-LOWAD, the

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olympic-LOWAD, etc. We will study some of its main properties. The main advantage of the LOWAD operator is that it gives a more complete view of the decision problem and then, it is easier to select the alternative that is more in accordance with our interests.

We will also develop an application of the new approach in a decision making problem about selection of human resources. The main advantage of this model in human resource management is that it can assess uncertain situations with linguistic information and it gives a more complete view of the problem to the decision maker because it considers a wide range of linguistic aggregation operators. Therefore, the decision maker will use the particular cases that are in accordance with its interests.

In order to do so, this paper is organized as follows. In Section 2 we briefly review the linguistic approach to be used in the paper, the LOWA operator and the Hamming distance. In Section 3 we present the LOWAD operator and in Section 4 we analyse different types of LOWAD operators. Section 5 develops a numerical example of the new approach. Finally, Section 6 summarizes the main findings of the paper.

2. Preliminaries

In this Section, we briefly review the linguistic approach to be used throughout the paper, the LOWA operator and the Hamming distance.

A. *The Linguistic Approach*

Usually, people are used to work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of the real world cannot be assessed in a quantitative form. Instead, it is possible to use a qualitative one, i.e., with vague or imprecise knowledge. In this case, a better approach may be the use of linguistic assessments instead of numerical values. The linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [49].

We have to select the appropriate linguistic descriptors for the term set and their semantics. One possibility for generating the linguistic term set consists in directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [37]. For example, a set of seven terms S could be given as follows:

$$S = \{s_1 = N, s_2 = VL, s_3 = L, s_4 = M, s_5 = H, s_6 = VH, s_7 = P\}$$

Note that N = *None*, VL = *Very low*, L = *Low*, M = *Medium*, H = *High*, VH = *Very high*, P = *Perfect*. Usually, in these cases, it is required that in the linguistic term set there exists:

- A negation operator: Neg(s_i) = s_j such that $j = g+1-i$.
- The set is ordered: $s_i \leq s_j$ if and only if $i \leq j$.
- Max operator: Max(s_i, s_j) = s_i if $s_i \geq s_j$.
- Min operator: Min(s_i, s_j) = s_i if $s_i \leq s_j$.

Different approaches have been developed for dealing with linguistic information such as [37-49,53]. In this paper, we will follow the ideas of [41]. Then, in order to preserve all the given information, we extend the discrete linguistic term set S to a continuous linguistic term set $\hat{S} = \{s_\alpha \mid s_1 < s_\alpha \leq s_t, \alpha \in [1, t]\}$, where, if $s_\alpha \in S$, we call s_α the original linguistic term, otherwise, we call s_α the virtual linguistic term.

Consider any two linguistic terms $s_\alpha, s_\beta \in \hat{S}$, and μ, μ_1, $\mu_2 \in [0, 1]$, we define some operational laws as follows [41]:

- $\mu s_\alpha = s_{\mu\alpha}$.
- $s_\alpha + s_\beta = s_\beta + s_\alpha = s_{\alpha+\beta}$.
- $s_\alpha - s_\beta = s_{\alpha-\beta}$.
- $(s_\alpha)^\mu = s_{\alpha^\mu}$.
- $s_\alpha \times s_\beta = s_\beta \times s_\alpha = s_{\alpha\times\beta}$.

B. *The Linguistic OWA Operator*

In the literature, we find a wide range of linguistic aggregation operators [37-42]. In this study, we will consider the linguistic ordered weighted averaging (LOWA) operator with its particular cases that include among others the linguistic average (LA) and the linguistic weighted average (LWA). Note that we follow the ideas developed by Xu in [41]. Then, we should point out that the LOWA operator we are going to use is also known as the extended OWA (EOWA) operator [41].

Definition 1: A LOWA operator of dimension n is a mapping $LOWA$: $S^n \rightarrow S$, which has an associated weighting vector W such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$LOWA\,(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \sum_{j=1}^{n} w_j s_{\beta_j} \tag{1}$$

where s_{β_j} is the jth largest of the s_{α_i}.

From a generalized perspective of the reordering step, it is possible to distinguish between the descending LOWA (DLOWA) and the ascending LOWA (ALOWA) operator. The weights of these operators are related by $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the DLOWA (or LOWA) operator and w^*_{n+1-j} the jth weight of the ALOWA operator. Note that the ALOWA operator is known in other studies as the inverse LOWA (I-LOWA) operator [37].

The LOWA operator provides a parameterized family of aggregation operators that includes as special cases the LA and the linguistic weighted average (LWA). The

LA is obtained when all the weights w_j are equal for all j. The LWA is obtained if the ordered position of the s_{β_j} is the same as the ordered position of the s_{α_i}.

In this type of operator it is possible to use different measures for characterizing the weighting vector W by using the same measures that it has been used for the OWA operator [18,31-32,54-57] such as the attitudinal character or the measure of dispersion. Note that the attitudinal character is used for measuring the degree of optimism of the decision maker and the measure of dispersion for measuring the entropy of the weighting vector.

C. *The Hamming Distance*

The normalized Hamming distance [10] is a useful technique for calculating the differences between two elements, two sets, etc. In fuzzy set theory, it can be useful, for example, for the calculation of distances between fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. For two sets A and B it can be defined as follows.

Definition 2: A normalized Hamming distance of dimension n is a mapping d_H: $R^n \times R^n \rightarrow R$ such that:

$$d_H(A, B) = \left(\frac{1}{n} \sum_{i=1}^{n} | a_i - b_i | \right) \tag{2}$$

where a_i and b_i are the ith arguments of the sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, respectively.

Sometimes, when normalizing the Hamming distance we prefer to give different weights to each individual distance. Then, the distance is known as the weighted Hamming distance. It can be defined as follows.

Definition 3: A weighted Hamming distance of dimension n is a mapping d_{WH}: $R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$. Then:

$$d_{WH}(A, B) = \left(\sum_{i=1}^{n} w_i | a_i - b_i | \right) \tag{3}$$

where a_i and b_i are the ith arguments of the sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, respectively.

Note that the formulations shown above are the general expressions. For the formulation used in fuzzy set theory, see for example [2-3,5-6,15,58-60]. Basically, the main change is that we consider the arguments to be between [0, 1] while in this definition we allow the arguments to be between $[-\infty, \infty]$.

3. The Linguistic OWA Distance Operator

The LOWAD operator is a distance measure that uses the OWA operator in the normalization process of the Hamming distance. Moreover, due to the fact that the environment is very uncertain, it uses linguistic variables instead of numerical ones in order to assess the information. Therefore, this operator is very practical to assess situations with a high degree of uncertainty in the information. For two sets $X = \{s_{X_1}, s_{X_2}, \ldots, s_{X_n}\}$ and $Y = \{s_{Y_1}, s_{Y_2}, \ldots, s_{Y_n}\}$, it can be defined as follows.

Definition 4: A LOWAD operator is a mapping *LOWAD*: $S^n \times S^n \rightarrow S$ that has an associated weighting vector W such that $w_j \in [0, 1]$ and the sum of the weights is 1, then:

$$LOWAD\,(X, Y) = \sum_{j=1}^{n} w_j s_{\beta_j} \tag{4}$$

where s_{β_j} is the jth largest of the $|s_{X_i} - s_{Y_i}|$ and $|s_{X_i} - s_{Y_i}|$ is the argument variable represented in the form of individual distances.

Remark 1: Note that if $s_{X_i} = s_{Y_i}$ for all $i \in [1, n]$, $LOWAD\,(X, Y) = 0$. Note also that $LOWAD\,(X, Y) = LOWAD\,(Y, X)$.

Remark 2: From a generalized perspective of the reordering step it is possible to distinguish between descending (DLOWAD) and ascending (ALOWAD) orders. The weights of these operators are related by $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the DLOWAD (or LOWAD) operator and w^*_{n+1-j} the jth weight of the ALOWAD operator.

Remark 3: If B is a vector corresponding to the ordered arguments s_{β_j}, we shall call this the ordered argument vector and W^T is the transpose of the weighting vector, then, the LOWAD operator can be expressed as:

$$LOWAD\,(X, Y) = W^T B \tag{5}$$

Remark 4: Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, then, the LOWAD operator can be expressed as:

$$LOWAD\,(X, Y) = \frac{1}{W} \sum_{j=1}^{n} w_j s_{\beta_j} \tag{6}$$

The LOWAD operator is commutative, monotonic, bounded and idempotent. These properties can be demonstrated with the following theorems.

Theorem 1 (Commutativity): Assume f is the LOWAD operator, then

$$f(X, Y) = f(U, V) \tag{7}$$

where (U, V) is any permutation of the arguments (X, Y).

Proof: Let

$$f(X, Y) = \sum_{j=1}^{n} w_j s_{\beta_j} \tag{8}$$

$$f(U, V) = \sum_{j=1}^{n} w_j s_{\chi_j} \tag{9}$$

Since (U, V) is a permutation of (X, Y), we have $s_{\beta_j} = s_{\chi_j}$, for all j, and then

$$f(X, Y) = f(U, V) \quad \blacksquare$$

Theorem 2 (Monotonicity): Assume f is the LOWAD operator, if $|s_{X_i} - s_{Y_i}| \geq |s_{U_i} - s_{V_i}|$, for all i, then

$$f(X, Y) \geq f(U, V) \tag{10}$$

Proof: Let

$$f(X, Y) = \sum_{j=1}^{n} w_j s_{\beta_j} \tag{11}$$

$$f(U, V) = \sum_{j=1}^{n} w_j s_{\chi_j} \tag{12}$$

Since $|s_{X_i} - s_{Y_i}| \geq |s_{U_i} - s_{V_i}|$, for all i, it follows that, $s_{\beta_j} \geq s_{\chi_j}$, and then

$$f(X, Y) \geq f(U, V) \quad \blacksquare$$

Theorem 3 (Boundedness): Assume f is the LOWAD operator, then

$$\text{Min}\{|s_{X_i} - s_{Y_i}|\} \leq f(X, Y) \leq \text{Max}\{|s_{X_i} - s_{Y_i}|\} \tag{13}$$

Proof: Let $\max\{|s_{X_i} - s_{Y_i}|\} = c$, and $\min\{|s_{X_i} - s_{Y_i}|\} = d$, then

$$f(X, Y) = \left(\sum_{j=1}^{n} w_j s_{\beta_j}^{\lambda}\right)^{1/\lambda} \leq \left(\sum_{j=1}^{n} w_j c^{\lambda}\right)^{1/\lambda} =$$

$$= \left(c^{\lambda} \sum_{j=1}^{n} w_j\right)^{1/\lambda} \tag{14}$$

$$f(X, Y) = \left(\sum_{j=1}^{n} w_j s_{\beta_j}^{\lambda}\right)^{1/\lambda} \geq \left(\sum_{j=1}^{n} w_j d^{\lambda}\right)^{1/\lambda} =$$

$$= \left(d^{\lambda} \sum_{j=1}^{n} w_j\right)^{1/\lambda} \tag{15}$$

Since $\sum_{j=1}^{n} w_j = 1$, we get

$$f(X, Y) \leq c \tag{16}$$

$$f(X, Y) \geq d \tag{17}$$

Therefore,

$$\text{Min}\{|s_{X_i} - s_{Y_i}|\} \leq f(X, Y) \leq \text{Max}\{|s_{X_i} - s_{Y_i}|\} \quad \blacksquare$$

Theorem 4 (Idempotency): Assume f is the LOWAD operator, if $|s_{X_i} - s_{Y_i}| = s_{\alpha}$, for all i, then

$$f(X, Y) = s_{\alpha} \tag{18}$$

Proof: Since $|s_{X_i} - s_{Y_i}| = s_{\alpha}$, for all i, we have

$$f(X, Y) = \sum_{j=1}^{n} w_j s_{\beta_j} = \sum_{j=1}^{n} w_j s_{\alpha} = s_{\alpha} \sum_{j=1}^{n} w_j \tag{19}$$

Since $\sum_{j=1}^{n} w_j = 1$, we get

$$f(X, Y) = s_{\alpha} \quad \blacksquare$$

Note that the idempotency is found when analysing the individual distances between the elements of both sets. However, if all the arguments of both sets are equal, then, the aggregation process is always 0.

Remark 5: A further interesting issue to consider is the measures for characterizing the weighting vector W such as the attitudinal character, the entropy of dispersion, the balance operator and the divergence of W [18,31-32]. The attitudinal character can be defined as follows:

$$\alpha(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1}\right) \tag{20}$$

Note that it is possible to develop different types of measures of entropy but the most common one is based on the Shannon entropy and for the LOWAD operator is defined as follows:

$$H(W) = -\sum_{j=1}^{n} w_j \ln(w_j) \tag{21}$$

The balance operator can be defined as:

$$BAL\ (W) = \sum_{j=1}^{n} \left(\frac{n+1-2j}{n-1}\right) w_j \tag{22}$$

And the divergence of W:

$$DIV(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} - \alpha(W)\right)^2 \tag{23}$$

Note that in this case, it is also possible to distinguish between descending and ascending orders.

4. Families of LOWAD Operators

By using a different manifestation of the weighting vector in the LOWAD operator, we are able to obtain different types of aggregation operators.

Remark 6: For example, we can obtain the linguistic maximum distance, the linguistic minimum distance, the linguistic normalized Hamming distance (LNHD) and the linguistic weighted Hamming distance (LWHD).

- The linguistic maximum distance is obtained if $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$.
- The linguistic minimum distance if $w_n = 1$ and $w_j = 0$, for all $j \neq n$.
- The LNHD is found when $w_j = 1/n$, for all i.
- The LWHD is obtained when the ordered position of i is the same than j.

Remark 7: In the median-LOWAD we distinguish between two cases. If n is odd we assign $w_{(n+1)/2} = 1$ and $w_{j*} = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all others.

Remark 8: For the weighted median-LOWAD, we select the argument s_{β_k} that has the kth largest argument such that the sum of the weights from 1 to k is equal or higher than 0.5 and the sum of the weights from 1 to $k - 1$ is less than 0.5.

Remark 9: The olympic-LOWAD is found when $w_1 = w_n = 0$, and for all others $w_{j*} = 1/(n - 2)$. Thus, we are able to exclude the extreme weights from the analysis in order to get more stable results in the aggregation process. Note that if n = 3 or n = 4, the olympic-LOWAD becomes the median-LOWAD and if $m = n - 2$ and k = 2, the window-LOWAD is transformed in the olympic-LOWAD.

Remark 10: Following [23], it is possible to develop a general form of the olympic-LOWAD operator considering that $w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j^*} = 1/(n-2k)$, where $k < n/2$. Note that if $k = 1$, then, this general form becomes the usual olympic-LOWAD. If $k = (n-1)/2$, then, this general form becomes the median-LOWAD aggregation.

Remark 11: Note that it is also possible to develop the contrary case of the general olympic-LOWAD operator. In this case, $w_j = (1/2k)$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and $w_j = 0$, for all others, where $k < n/2$. Note that if $k = 1$, then, we get the contrary case of the median-LOWAD.

Remark 12: The window-LOWAD is found when $w_{j^*} = 1/m$ for $k \leq j^* \leq k+m-1$ and $w_{j^*} = 0$ for $j^* > k+m$ and $j^* < k$. Note that k and m must be positive integers such that $k+m-1 \leq n$. Also note that if $m = k = 1$, the window-LOWAD becomes the linguistic maximum distance. If $m = 1$, $k = n$, the window-LOWAD becomes the linguistic minimum distance. And if $m = n$ and $k = 1$, the window-LOWAD is transformed in the LNHD.

Remark 13: A further type of LOWAD operator is the S-LOWAD operator that it is based on the S-OWA operator [30]. It can be subdivided in three classes, the “orlike”, the “andlike” and the generalized S-LOWAD operator. The generalized S-LOWAD operator is obtained when $w_1 = (1/n)(1-(\alpha+\beta)) + \alpha$, $w_n = (1/n)(1-(\alpha+\beta)) + \beta$, and $w_j = (1/n)(1-(\alpha+\beta))$ for j = 2 to $n-1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, the generalized S-LOWAD operator becomes the “andlike” S-LOWAD operator and if $\beta = 0$, it becomes the “orlike” S-LOWAD operator. Also note that if $\alpha + \beta = 1$, we get the linguistic Hurwicz distance criteria.

Remark 14: Another interesting family that could be used is the centered-LOWAD operator, that it is based on the OWA version [33]. We can define a LOWAD operator as a centered aggregation operator if it is symmetric, strongly decaying and inclusive. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \leq (n+1)/2$ then $w_i < w_j$ and when $i > j \geq (n+1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider a softening of the second condition by using $w_i \leq w_j$ instead of $w_i < w_j$. We shall refer to this as softly decaying centered-LOWAD operator. Note that the LNHD is an example of this particular situation. Another particular case of the centered-LOWAD operator appears if we remove the third condition. We shall refer to it as a non-inclusive centered-LOWAD operator. For this situation, we find the median-LOWAD as a particular case.

Remark 15: Using a similar methodology, we could develop a lot of other families of LOWAD weights in a similar way as it has been developed in a lot of studies for the OWA operator such as [19-33].

Remark 16: Note that it is easy to apply these methods to the LOWAD operator because the weights are not affected by the linguistic information. Obviously, it is possible to consider more complex analysis where the weights are also linguistic variables but in this paper we will not enter in this problem.

5. Numerical Example

In the following, we are going to develop a numerical example of the new approach. We will consider a decision making problem about human resource management.

Assume that an enterprise wants to acquire a person for a new position in the company. After an application period, the company has evaluated the applications received. After careful analysis of the information, the group of experts of the enterprise considers five possible human resources.

- A_1 = Candidate 1.
- A_2 = Candidate 2.
- A_3 = Candidate 3.
- A_4 = Candidate 4.
- A_5 = Candidate 5.

When analyzing the candidates, the experts have considered the following general characteristics:

- C_1 = Experience in similar jobs.
- C_2 = Intelligence.
- C_3 = Knowledge about the job.
- C_4 = Motivation.
- C_5 = Skills of the worker.
- C_6 = Other aspects.

Due to the fact that the general characteristics are very imprecise because they contain a lot of particular aspects, the experts cannot use numerical values in the analysis. Instead, they use linguistic variables to evaluate the general results obtained for each candidate depending on the characteristic considered. In order to do so, they establish the following linguistic scale.

S = {s_1 = *Extremely low*, s_2 = *Very low*, s_3 = *Low*, s_4 = *Medium*, s_5 = *High*, s_6 = *Very high*, s_7 = *Extremely high*}.

After careful analysis of these characteristics, the experts have given the following information shown in Table 1.

Table 1. Available information about the candidates.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	S_6	S_3	S_7	S_2	S_1	S_4
A_2	S_4	S_5	S_2	S_3	S_4	S_4
A_3	S_1	S_6	S_2	S_2	S_7	S_4
A_4	S_5	S_4	S_5	S_2	S_4	S_3
A_5	S_6	S_3	S_7	S_1	S_3	S_3

According to their objectives, the enterprise establishes the following ideal candidate shown in Table 2.

Table 2. Ideal worker.

	C_1	C_2	C_3	C_4	C_5	C_6
Ideal	S_6	S_7	S_7	S_6	S_7	S_6

With this information, it is possible to develop different methods for selecting a candidate according to the interests of the company. In this example, we will consider the linguistic maximum distance, the linguistic minimum distance, the LNHD, the LWHD, the Hurwicz-LOWAD (α = 0.6), the LOWAD, the ALOWAD, the median-LOWAD, the orlike S-LOWAD (α = 0.7), the andlike S-LOWAD (β = 0.8), the step-LOWAD (k = 2) and the olympic-LOWAD. In order to aggregate the information, the group of experts calculates the attitudinal character of the enterprise. They calculate the following weighting vector W = (0.1, 0.1, 0.1, 0.2, 0.2, 0.3). With this information, it is possible to aggregate the available information in order to take a decision.

Table 3. Aggregated results 1.

	Max	Min	LNHD	LWHD	LOWAD	ALOWAD
A_1	S_6	S_0	$S_{2.66}$	S_3	$S_{1.8}$	$S_{3.6}$
A_2	S_5	S_2	$S_{2.83}$	$S_{2.7}$	$S_{2.5}$	$S_{3.3}$
A_3	S_5	S_0	$S_{2.83}$	$S_{2.5}$	S_2	$S_{3.6}$
A_4	S_4	S_1	$S_{2.66}$	$S_{2.9}$	$S_{2.3}$	S_3
A_5	S_5	S_0	$S_{2.66}$	$S_{3.1}$	$S_{1.9}$	$S_{3.4}$

Table 4. Aggregated results 2.

	Or	And	median	step	Olym.	Hurwicz
A_1	S_5	$S_{0.53}$	S_3	S_4	$S_{2.5}$	$S_{3.6}$
A_2	$S_{4.35}$	$S_{2.16}$	$S_{2.5}$	S_3	$S_{2.5}$	$S_{3.8}$
A_3	$S_{4.35}$	$S_{0.56}$	S_3	S_5	S_3	S_3
A_4	$S_{3.6}$	$S_{1.33}$	S_3	S_3	$S_{2.75}$	$S_{2.8}$
A_5	$S_{3.55}$	$S_{0.53}$	$S_{3.5}$	S_4	$S_{2.75}$	S_3

Note that in these cases, the result indicates the distance between the linguistic variables of the candidate and the ideal one. Then, the results may range from 0 to some value not higher than the maximum.

As we can see, depending on the linguistic distance aggregation operator used, the optimal choice is different. Note that the lowest value in each method is the optimal result because we are using distances.

If we establish an ordering of the candidates, a typical situation if we want to consider more than one alternative, we will get the following orders shown in Table 5.

Table 5. Ordering of the candidates.

	Ordering
Maximum dist.	$A_4 \succ A_2=A_3=A_5 \succ A_1$
Minimum dist.	$A_1=A_3=A_5 \succ A_4 \succ A_2$
LNHD	$A_1=A_4=A_5 \succ A_2=A_3$
LWHD	$A_3 \succ A_2 \succ A_4 \succ A_1 \succ A_5$
LOWAD	$A_1 \succ A_5 \succ A_3 \succ A_4 \succ A_2$
ALOWAD	$A_4 \succ A_2 \succ A_5 \succ A_1 \succ A_3$
Or-S-LOWAD	$A_5 \succ A_4 \succ A_2=A_3 \succ A_1$
And-S-LOWAD	$A_1=A_5 \succ A_3 \succ A_4 \succ A_2$
Median-LOWAD	$A_2 \succ A_1=A_3=A_4 \succ A_5$
Step-LOWAD	$A_2=A_4 \succ A_1=A_5 \succ A_3$
Olympic-LOWAD	$A_1=A_2 \succ A_4=A_5 \succ A_3$
Hurwicz-LOWAD	$A_4 \succ A_3=A_5 \succ A_1 \succ A_2$

As we can see, depending on the particular type of LOWAD operator used, the results are different. Then, depending on the method used in the LOWAD, the decision maker may select a candidate.

6. Conclusions

We have analysed the use of linguistic information in decision making with distance measures. For doing so, we have developed a new distance measure, the linguistic ordered weighted averaging distance (LOWAD) operator. It is a new aggregation operator that provides a parameterized family of linguistic aggregation operators such as the linguistic maximum distance, the linguistic minimum distance, the LNHD and the LWHD. We have studied some of its main properties. The main advantage of the LOWAD is that it is able to assess uncertain problems where the available information can not be represented with numerical values but it is possible to use linguistic ones.

The LOWAD operator can be applied in a lot of situations already considered with the Hamming distance such as in statistics, economics and engineering. In this paper, we have focused on an application in decision making regarding human resource management. The main advantage of the LOWAD in this type of problems is that it gives a parameterized family of linguistic distance aggregation operators. Then, depending on the particular case used, the results and decisions may be different.

In future research, we expect to develop further extensions of the LOWAD operator by adding new characteristics in the problem such as the use of order-inducing variables, the Euclidean distance or the Minkowski distance, and applying it to other problems.

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