A New Ranking Method for Intuitionistic Fuzzy Numbers

Hassan Mishmast Nehi

Abstract

In this paper I will introduce the trapezoidal intuitionistic fuzzy numbers (IF numbers) and will prove some operations for them. Also I am going to propose a new ordering method for IF numbers in which I will consider two characteristic values of membership and non-membership functions for an IF number. These values are defined by the integral of the inverse fuzzy membership and non-membership functions multiplied by the grade with powered parameter.

Keywords: Fuzzy number, Intuitionistic fuzzy set, Ranking function, Trapezoidal fuzzy number.

1. Introduction

In real world, we frequently deal with vague or imprecise information. Information available is sometimes vague, sometimes inexact or sometimes insufficient. Out of several higher order fuzzy sets, intuitionistic fuzzy sets (IFS) [1, 2] have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Due to the some reason evaluation of non-membership values is not also always possible and consequently there remains a part indeterministic on which hesitation survives. Certainly fuzzy sets theory is not appropriate to deal with such problem, rather intuitionistic fuzzy sets(IFS) theory is more suitable. Also, Ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. It is due to the fact that fuzzy numbers are not linearly ordered. This problem is also important in the case of intuitionistic fuzzy numbers.

The paper is organized as follows. Firstly, in section 2, I will give some basic definitions and properties on fuzzy numbers and intuitionistic fuzzy sets. In section 3, I will introduce the definition of intuitionistic fuzzy numbers (IFN) and make some operations on IF numbers. In section 4, I will review some ranking methods for intuitionistic fuzzy numbers. Then the characteristic value for fuzzy number introduced by Chiao [3] is generalized in the case of intuitionistic fuzzy numbers and I will propose a ranking method for IF numbers in section 5.

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2. Basic definitions and properties

In this section I will review the basic concepts of fuzzy numbers and intuitionistic fuzzy sets.

2.1 Fuzzy numbers

The most important subfamily of all fuzzy sets are fuzzy numbers. It is not surprising since the predominant carrier of information are numbers. The notion of a fuzzy number was introduced by Dubois and Prade [4, 5].

Definition 2.1: A fuzzy set A of the real line $\mathbb{R}$ with membership function $\mu_A = \mathbb{R} \to [0,1]$ is called a fuzzy number if

a) A is normal, i.e. there exist an element x_0 such that $\mu(x_0) = 1$,

b) A is fuzzy convex, i.e. $\forall x_1, x_2 \in \mathbb{R}, \forall \lambda \in [0,1]$
$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2),$$

c) μ_A is upper semicontinuous,

d) A is bounded.

It is known that for any fuzzy number A, there exists four numbers $a_1, a_2, a_3, a_4 \in \mathbb{R}$ and two functions $f_A, g_a : \mathbb{R} \to [0,1]$ where f_A is nondecreasing and g_a is nonincreasing, such that we can describe a membership function μ_A in a following manner

$$\mu_A(x) = \begin{cases} 0 & if \quad x < a, \\ f_A(x) & if \quad a_1 \leq x \leq a_2, \\ 1 & if \quad a_2 \leq x \leq a_3, \\ g_A(x) & if \quad a_4 < x, \end{cases}$$

functions f_A and g_a are called the left side and the right side of a fuzzy number A, respectively.

A useful tool for dealing with fuzzy numbers shown in Figure 1 is their α-cuts. The α-cut of a fuzzy number A is a nonfuzzy set defined as

$$A_\alpha = \{x \in \mathbb{R} \mid \mu_A(x) \ge \alpha\}.$$

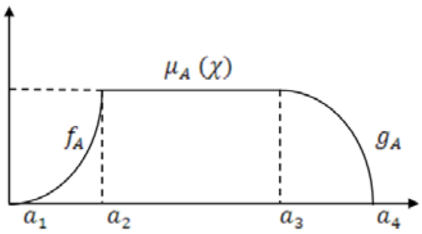

Figure 1. Fuzzy Number.

According to the definition of a fuzzy number it is seen at once that every α-cut of a fuzzy number is a closed interval. Hence we have $A_\alpha = \{A_L(\alpha), A_U(\alpha)\}$, where

$$A_L(\alpha) = \inf\{x \in \mathbb{R} \mid \mu_A(x) \ge \alpha\}$$

$$A_U(\alpha) = \sup\{x \in \mathbb{R} \mid \mu_A(x) \ge \alpha\}$$

Another important notion connected with fuzzy numbers is an expected interval $EI(A)$ of a fuzzy number A, introduced independently by Dubios and Prade [4] and Heilpern [7]. It is given by

$$EI(A) = [E_*(A), E^*(A)]$$

$$= [\int_0^1 A_L(\alpha)d\alpha, \int_0^1 A_U(\alpha)d\alpha].$$

It can be shown that if A is a fuzzy number with continuous and strictly monotonic sides f_A and g_a then for

$$E_* = a_2 - \int_{a_1}^{a_2} f_A(x)dx, \quad (1)$$

$$E_* = a_3 - \int_{a_3}^{a_4} g_A(x)dx, \quad (2)$$

2.2 Intuitionistic fuzzy sets

Let X be the universal set. In the following, I will describe those aspects of intuitionistic fuzzy sets [1, 2], [8] which will be needed in our next discussion.

Definition 2.2 [1, 2]: An intuitionistic fuzzy set (IFS) A in X is given by

$$A = \{(x, \mu_A(x), v_A(x)) \mid x \in X\}$$

where the functions μ_A, v_A define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X$, $0 \le \mu(x) + v(x) \le 1$.

Obviously, every fuzzy set has the form

$$A = \{(x, \mu_A(x), \mu_{A'}(x)) \mid x \in X\}.$$

For each IFS $A = \{(x, \mu_A(x), \mu_{A'}(x)) \mid x \in X\}$. in X, we will call

$$\prod_A = 1 - \mu(x) - v(x),$$

the intuitionistic fuzzy index of X in A. It is obvious that $0 \le \prod_A \le 1, \forall x \in X$.

Atanassov [1, 2] has also defined two kinds of α-cut for intuitionistic fuzzy sets. Namely

$$A_\alpha = \{x \in \mathbb{R} \mid \mu_A(x) \ge \alpha\},$$

$$A_\alpha = \{x \in \mathbb{R} \mid v_A(x) \ge \alpha\}.$$

Corollary 2.3 [1, 2]: For every two IF sets A and B of the set X, the following properties are valid

$(i)\, A \subset B if x \in X, [\mu_A(x) \le \mu_B(x), v_A(x) \ge v_B(x)],$

$(ii)\, A = B if x \in X, [\mu_A(x) = \mu_B(x), v_A(x) = v_B(x)],$

$(iii)\, \overline{A} = \{(x, \mu_A(x), v_A(x)) \mid x \in X\},$

$(iv)\, A \cap B = \{(x, \min(\mu_A(x), \mu_B(x))),$
$(x, \max(v_A(x), v_B(x))) \mid x \in X\}$

$(iv)\, A \cup B = \{(x, \max(\mu_A(x), \mu_B(x))),$
$(x, \min(v_A(x), v_B(x))) \mid x \in X\}$

3. Intuitionistic fuzzy numbers

Here I will introduce the intuitionistic fuzzy number (IFN) and trapezoidal intuitionistic fuzzy number (TIFN) via P. Grzegrorzewski [6], then I will prove some properties for them.

Definition 3.1: An IFS $A = \{(x, \mu_A(x), v_A(x)) \mid x \in X\}$ is called IF-normal, if there exist at least two points $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1, v_A(x_1) = 1,$

It is easily seen that given intuitionistic fuzzy set A is IF-normal if there is at least one point that surely belongs to A and at least one point which does not belong to A.

Definition 3.2: An IFS $A = \{(x, \mu_A(x), v_A(x)) \mid x \in X\}$ of the real line is called IF-convex, if $\forall x_1, x_2 \in \mathbb{R}, \forall \lambda \in [0,1]$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \ge \mu_A(x_1) \wedge \mu_A(x_2)$$

$$v_A(\lambda x_1 + (1-\lambda)x_2) \ge v_A(x_1) \wedge v_A(x_2)$$

Thus A is IF-convex if its membership function is fuzzy convex and its nonmembership function is fuzzy concave.

Definition 3.3: An IFS $A = \{(x, \mu_A(x), v_A(x)) \mid x \in X\}$

of the real line is called an intuitionistic fuzzy number (IFN) if

a) A is IF-normal,
b) A is IF-convex,
c) μ_A is upper semicontinuous and v_A is lower semicontinuous,
d) $A = \{x \in X \mid v_A(x) < 1\}$ is bounded.

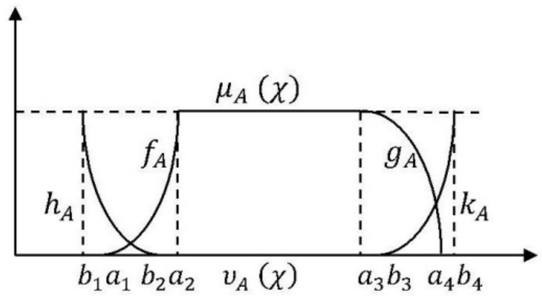

Figure 2. Intuitionistic Fuzzy Number.

From the definition given above we get at once that for any IFN A there exists eight numbers $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in \mathbb{R}$ such that $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ and four functions $f_A, g_A, h_A, k_A : \mathbb{R} \to [0,1]$, called the sides of a fuzzy number, where f_A and k_A are nondecreasing and g_A and h_A are nonincreasing, such that we can describe a membership function μ_A in form

$$\mu_A(x) = \begin{cases} 0 & if \quad x < a, \\ f_A(x) & if \quad a_1 \le x \le a_2, \\ 1 & if \quad a_2 \le x \le a_3, \\ g_A(x) & if \quad a_4 < x, \end{cases}$$

while a nonmembership function v_A has a following form

$$v_A(x) = \begin{cases} 0 & if \quad x < b, \\ h_A(x) & if \quad b_1 \le x \le b_2, \\ 1 & if \quad b_2 \le x \le b_3, \\ k_A(x) & if \quad b_3 \le x \le b_4, \\ 0 & if \quad b_4 < x. \end{cases}$$

It is worth noting that each IFN $A = \{(x, \mu_A(x), v_A(x)) \mid x \in X\}$ is a conjunction of two fuzzy numbers: A^+ with a membership function $\mu_A^{\ +}(x) = \mu_A(x)$ and A^- with a membership function $\mu_A^{\ -}(x) = 1 - v_A(x)$. It is seen that $A^+ \subseteq A^-$

In the case of intuitionistic fuzzy numbers shown in Figure 2, it is convenient to distinguish following α-cuts: $(A^+)_\alpha$ and $(A^-)_\alpha$. It is easily seen that

$$(A^+)_\alpha = \{x \in \mathbb{R} \mid \mu_A(x) \ge \alpha\} = A_\alpha,$$
$$(A^-)_\alpha = \{x \in \mathbb{R} \mid 1 - v_A(x) \ge \alpha\}$$
$$= \{x \in \mathbb{R} \mid v_A(x) \le 1 - \alpha\} = A^{1-\alpha}.$$

According to the definition it is seen at once that every α-cut, $(A^+)_\alpha$ or $(A^-)_\alpha$ is a closed interval. Hence we have $(A^+)_\alpha = [A^+{}_L(\alpha), A^+{}_U(\alpha)]$ and $(A^-)_\alpha = [A^-{}_L(\alpha), A^-{}_U(\alpha)]$, respectively, where

$$A^+{}_L(\alpha) = \inf\{x \in \mathbb{R} \mid \mu_A(x) \ge \alpha\},$$
$$A^+{}_U(\alpha) = \sup\{x \in \mathbb{R} \mid \mu_A(x) \ge \alpha\},$$
$$A^-{}_L(\alpha) = \inf\{x \in \mathbb{R} \mid v_A(x) \le 1 - \alpha\},$$
$$A^-{}_U(\alpha) = \sup\{x \in \mathbb{R} \mid v_A(x) \le 1 - \alpha\}.$$

If the sides of the fuzzy numbers A are strictly monotone then, the convention that

$$f_A^{-1}(\alpha) = A_L^+(\alpha), g_A^{-1}(\alpha) = A_U^+(\alpha), h_A^{-1}(\alpha)$$
$$= A_L^-(\alpha) and k_A^{-1}(\alpha) = A_U^-(\alpha)$$

In particular, if the decreasing functions f_A and k_A and increasing functions g_A and k_A be linear then we will have the trapezoidal intuitionistic fuzzy numbers (TIFN).

Definition 3.4: A is a trapezoidal intuitionistic fuzzy number with parameters

$$b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$$

and denoted by

$$A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4).$$

In this case we will give

$$\mu_A = \begin{cases} 0 & if \quad x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1} & if \quad a_1 \le x \le a_2, \\ 1 & if \quad a_2 \le x \le a_3, \\ \dfrac{x - a_4}{a_3 - a_4} & if \quad a_3 \le x \le a_4, \\ 0 & if \quad a_4 \le x, \end{cases}$$

and

$$\nu_A = \begin{cases} 0 & if \quad x < b_1, \\ \frac{x-b_1}{b_2-b_1} & if \quad b_1 \le x \le b_2, \\ 1 & if \quad b_2 \le x \le b_3, \\ \frac{x-b_4}{b_3-b_4} & if \quad b_3 \le x \le b_4, \\ 0 & if \quad b_4 \le x, \end{cases}$$

If in a TIFN A, we let $b_2 = b_3$ (and hence $a_2 = a_3$), we will give a triangular intuitionistic fuzzy number (TrIFN) with parameters

$$b_1 \le a_1 \le b_2 \left(a_2 = a_3 = b_3\right) \le a_4 \le b_4,$$

and denoted by $A = \left(b_1, a_1, b_2, a_4, b_4\right)$.

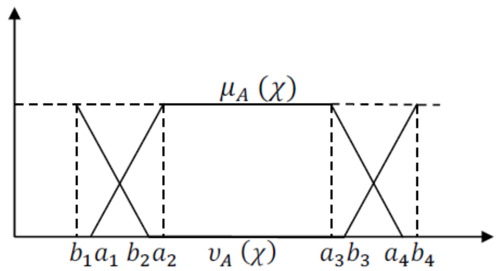

Figure 3: Trapezoidal Intuitionistic Fuzzy Number

We can prove the following properties for trapezoidal (triangular) intuitionistic fuzzy numbers shown in Figure 3.

Lemma 3.5: If $A = \left(b_1, a_1, b_2, a_2, b_3, a_4, b_4\right)$ and $B = \left(b'_1, a'_1, b'_2, a'_2, b'_3, a'_4, b'_4\right)$ be two trapezoidal intuitionistic fuzzy numbers, then $A+B$ also is a trapezoidal intuitionistic fuzzy number and

$$A+B = (b_1+b'_1, a_1+a'_1, b_2+b'_2, a_2+a'_2,$$
$$a_3+a'_3, a_4+a'_4, b_4+b'_4).$$

Proof: By Extension principle and Nguyen theorem, it is sufficient to prove that for all $\alpha \in [0,1]$

$$A_\alpha^+ + B_\alpha^+ = \left(A+B\right)_\alpha^+ \quad \text{and} \quad A_\alpha^- + B_\alpha^- = \left(A+B\right)_\alpha^-$$

For any fixed $\alpha \in [0,1]$, from the membership and non-membership function of TIFNs A and B we will have

$$A_\alpha^+ = [a_1 + \alpha\left(a_2 - a_1\right), a_4 + \alpha\left(a_4 - a_3\right)],$$
$$B_\alpha^+ = [a'_1 + \alpha\left(a'_2 - a'_1\right), a'_4 + \alpha\left(a'_4 - a'_3\right)],$$
$$A_\alpha^- = [b_1 + \alpha\left(b_2 - b_1\right), b_4 + \alpha\left(b_4 - b_3\right)],$$
$$B_\alpha^- = [b'_1 + \alpha\left(b'_2 - b'_1\right), b'_4 + \alpha\left(b'_4 - b'_3\right)],$$

Hence

$$A_\alpha^+ + B_\alpha^+ = [\left(a_1+a'_1\right) + \alpha\left\{\left(a_2+a'_2\right) - \left(a_1+a'_1\right)\right\},$$
$$\left(a_4+a'_4\right) + \alpha\left\{\left(a_4+a'_4\right) - \left(a_3+a'_3\right)\right\}]$$

which is equal to $\left(A+B\right)_\alpha^+$. Similarly, we can prove that $A_\alpha^- + B_\alpha^- = \left(A+B\right)_\alpha^-$.

Lemma 3.6: If $A = \left(b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4\right)$ be a trapezoidal intuitionistic fuzzy number and r be a real number then rA is a trapezoidal intuitionistic fuzzy number and

$$rA = \begin{cases} \left(rb_1, ra_1, rb_2, ra_2, ra_3, rb_3, ra_4, rb_4\right) & if \quad r > 0 \\ \left(rb_4, ra_4, rb_3, ra_3, ra_2, rb_2, ra_1, rb_1\right) & if \quad r < 0 \end{cases}$$

Proof: By Extension principle and Nguyen theorem, it is sufficient to prove that for all $\alpha \in [0,1]$ and $\forall r \in \mathbb{R}$

$$rA_\alpha^+ = \left(rA\right)_\alpha^+ \quad \text{and} \quad rA_\alpha^- = \left(rA\right)_\alpha^-.$$

For any fixed $\alpha \in [0,1]$ and $r > 0$, from the membership and non-membership function of TIFNs A and B we will have

$$rA_\alpha^+ = [ra_1 + \alpha\left(ra_2 - ra_1\right), ra_4 + \alpha\left(ra_4 - ra_3\right)],$$
$$rA_\alpha^- = [ra_1 + \alpha\left(ra_2 - ra_1\right), ra_4 + \alpha\left(ra_4 - ra_3\right)].$$

Hence

$$rA_\alpha^+ = [r\left(a_1 - ra'_1\right) - \alpha\{r\left(a_2 - a'_2\right) - r(a_1 + a'_1)\},$$
$$r\left(a_4 - ra'_4\right) + \alpha\{r\left(a_4 - a'_4\right) + r\left(a_3 - a'_3\right)].$$

which is equal to $\left(rA\right)_\alpha^+$. Similarly, we can prove that $rA_\alpha^- = \left(rA\right)_\alpha^-$.

In the case $r < 0$, we will have

$$rA_\alpha^+ = [ra_1 + \alpha\left(ra_2 - ra_1\right), ra_4 + \alpha\left(ra_4 - ra_3\right)],$$
$$rA_\alpha^- = [ra_3 + \alpha\left(ra_4 - ra_3\right), ra_1 + \alpha\left(ra_2 - ra_1\right)].$$

Hence

$$rA_\alpha^+ = [r\left(a_4 - ra'_4\right) - \alpha\{r\left(a_4 - a'_4\right) - r(a_3 + a'_3)\}$$
$$, r\left(a_1 - ra'_1\right) + \alpha\{r\left(a_2 - a'_2\right) + r\left(a_1 - a'_1\right)],$$

which is equal to $\left(rA_\alpha^+\right)$. Similarly, we can prove that $rA_\alpha^- = \left(rA\right)_\alpha^-$.

4. Some ranking methods for IFNs

It is known that there is no unique linear ordering in a family of fuzzy numbers. Thus ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetics. The same is true in the case of intuitionistic fuzzy numbers. H. B. Mitchell [9] and V. L. G. Nayagam and et. al. [10] introduced some methods for ranking intuitionistic fuzzy numbers. Below I will review the method of ranking IF numbers which suggested by Grzegorzewski in [6].

Suppose A is a subfamily of all IF numbers.

Definition 4.1: An IF number $L(A)$ is called the lower horizon of a given subfamily A if $\sup(\sup pL(A)) \le \inf(\sup pA)$ for any $A \in \mathbf{A}$. Similarly, an IF number $U(A)$ is called the upper horizon of a given subfamily A if $\sup(\sup pL(A)) \ge \inf(\sup pA)$ for any $A \in \mathbf{A}$.

It is obvious that A may have one or more horizons. Two following orders proposed by Grzegorzewski in [4]:

Definition 4.2: Let $A, B \in \mathbf{A}$. Moreover, let $H = L(A)$ and let d be a metric in the family of IF numbers. The relation $\succ_L$ in $\mathbf{A} * A$ given by $A \succ_L B \Leftrightarrow d(A,H) \ge b(B,H)$ is called the order respect to the lower horizon H.

Definition 4.3: Let $A, B \in \mathrm{A}$. Moreover, let $H = L(A)$ and let d be a metric in the family of IF numbers. The relation $\succ_U$ in $\mathbf{A} * A$ given by $A \succ_L B \Leftrightarrow d(A,H) \ge b(B,H)$ is called the order respect to the upper horizon H.

It should be noticed that both relations $\succ_L$ and $\succ_U$ are not antisymmetric and hence they are only quasi-ordering relations, not ordering relations.

Also, Grzegorzewski propose an ordering method for IF numbers by using the expected interval of an IF number. The expected interval of an IF number $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ is a crisp interval $\widetilde{EI}(A)$ given by (see [4])

$$\widetilde{EI}(A) = \left[\widetilde{E_*}(A), \widetilde{E^*}(A)\right],,$$

where

$$\widetilde{E_*}(A) = \frac{b_1 + a_2}{2} + \frac{1}{2}\int_{b_1}^{b_2} h_A(x)\,dx - \frac{1}{2}\int_{a_1}^{a_2} f_A(x)\,dx,$$

$$\widetilde{E^*}(A) = \frac{b_4 + a_3}{2} + \frac{1}{2}\int_{a_3}^{a_4} g_A(x)\,dx - \frac{1}{2}\int_{b_3}^{b_4} k_A(x)\,dx.$$

As in the case of the classical fuzzy sets, the expected value of an IF number define as follows:

Definition 4.4: The expected value $\widetilde{EV}(A)$ of an IF number $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in \mathbb{R}\}$ is the center of the expected interval of that IF number, i.e.

$$\widetilde{EV}(A) = \frac{\widetilde{E_*}(A) + \widetilde{E^*}(A)}{2}..$$

The following theorem holds:

Theorem 4.5 [6]: Let $\succ_L$ and $\succ_U$ denote the quasi-order with respect to the lower and upper horizon, respectively, based on the metric d_1 (i.e. d_p for $p = 1$). Then for any two IF numbers A and B we get

$$A \succ_L B \Leftrightarrow \widetilde{EV}(A) \ge \widetilde{EV}(B),$$

and

$$A \succ_L B \Leftrightarrow \widetilde{EV}(A) \ge \widetilde{EV}(B).$$

5. A New ranking method for IFNs

Based on the characteristic value for a fuzzy number introduced by Chiao in [3], we proposed an ordering method for intuitionistic fuzzy numbers.

Definition 5.1: Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an IF number. Let $s(r;k) = \frac{((k+1)r^k)}{2}$ be a regular reducing function with positive parameter k Then the characteristic values of membership and non-membership for IF number A with parameter k denoted by $C_\mu^k(A), C_\nu^k(A)$ respectively, are defined by

$$C_\mu^k(A) = \int_0^1 s(r;k)\left[f_A^{-1}(r) + g_A^{-1}(r)\right]dr,$$

$$C_\nu^k(A) = \int_0^1 s(r;k)\left[h_A^{-1}(r) + k_A^{-1}(r)\right]dr.$$

Simple calculation implies that

$$C_\mu^k(A) = \frac{k+1}{2}\int_0^1 r^k\left[f_A^{-1}(r) + g_A^{-1}(r)\right]dr,$$

$$C_\nu^k(A) = \frac{k+1}{2}\int_0^1 r^k\left[h_A^{-1}(r) + k_A^{-1}(r)\right]dr,$$

for $k \in [0, \infty)$..

Various approach to the definition of the characteristic values of membership and non-membership of a IF number is dependent on the parameter k. It has an interesting interpretation. Let $k = 0$, then

$$C_\mu^0(A) = \frac{1}{2}\int_0^1\left[f_A^{-1}(r) + g_A^{-1}(r)\right]dr,$$

$$C_\nu^0(A)=\frac{1}{2}\int_0^1\left[h_A^{-1}(r)+k_A^{-1}(r)\right]dr.$$

Let $k=1$, then

$$C_\mu^1(A)=\frac{1}{2}\int_0^1 r\left[f_A^{-1}(r)+g_A^{-1}(r)\right]dr,$$

$$C_\nu^1(A)=\frac{1}{2}\int_0^1 r\left[h_A^{-1}(r)+k_A^{-1}(r)\right]dr.$$

Letting k approach to ∞, it is easily shown that

$$\lim C_\mu^k(A)=\frac{f_A^{-1}+g_A^{-1}}{2}=\frac{a_2+a_3}{2},$$

$$\lim C_\mu^k(A)=\frac{h_A^{-1}+k_A^{-1}}{2}=\frac{b_2+b_3}{2},$$

which are the mean values of the intervals $[a_2,a_3]$ and $[b_2,b_3]$ respectively. Note that the larger the value k is , the less influence of the left and right functions of membership and non-membership, is on the characteristic values of the IF number.

In particular, let $A=(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ be a trapezoidal IF number with membership and non-membership functions,

$$\mu_A=\begin{cases}0 & if \quad x<a_1,\\ \dfrac{x-a_1}{a_2-a_1} & if \quad a_1\le x\le a_2,\\ 1 & if \quad a_2\le x\le a_3,\\ \dfrac{x-a_4}{a_3-a_4} & if \quad a_3\le x\le a_4,\\ 0 & if \quad a_4\le x,\end{cases}$$

and

$$\nu_A=\begin{cases}0 & if \quad x<b_1,\\ \dfrac{x-b_1}{b_2-b_1} & if \quad b_1\le x\le b_2,\\ 1 & if \quad b_2\le x\le b_3,\\ \dfrac{x-b_4}{b_3-b_4} & if \quad b_3\le x\le b_4,\\ 0 & if \quad b_4\le x,\end{cases}$$

In this case we have $f_A(x)=\dfrac{x-a_1}{a_2-a_1}$, $g_A(x)=\dfrac{x-a_4}{a_3-a_4}$, $h_A(x)=\dfrac{x-b_1}{b_2-b_1}$ and $k_A(x)=\dfrac{x-b_4}{b_3-b_4}$. The inverses for these shape functions for any $r\in[0,1]$ are

$$f_A^{-1}=a_1+(a_2-a_1)r,$$
$$g_A^{-1}=a_4+(a_3-a_4)r,$$
$$h_A^{-1}=b_1+(b_2-b_1)(1-r),$$
$$k_A^{-1}=b_4+(b_3-b_4)(1-r),$$

Thus,

$$C_\mu^k(A)=\frac{k+1}{2}\int_0^1 r^k\left[f_A^{-1}(r)+g_A^{-1}(r)\right]dr,$$

$$=\frac{k+1}{2}\int_0^1 r^k\left[a_1+(a_2-a_1)r+a_4+(a_3-a_4)r\right]dr$$

$$=\frac{a_2+a_3}{2}+\frac{(a_1-a_2)+(a_4-a_3)}{2(k+2)},$$

and

$$C_\nu^k(A)=\frac{k+1}{2}\int_0^1 r^k\left[h_A^{-1}(r)+k_A^{-1}(r)\right]dr,$$

$$=\frac{k+1}{2}\int_0^1 r^k\left[b_1+(b_2-b_1)r+b_4+(b_3-b_4)r\right]dr$$

$$=\frac{b_1+b_4}{2}+\frac{(b_2-b_1)+(b_3-b_4)}{2(k+2)}.$$

Note that if A be a symmetrical trapezoidal IF number, then $C_\mu^k(A)=\dfrac{a_2+a_3}{2}, C_\nu^k(A)=\dfrac{b_1+b_4}{2}$

Now, an ordering could be given on IF numbers as is shown in the following algorithm:

Algorithm 5.2: As a ranking method, we compare two IF numbers A and B using the following steps:

Step 1) For a given k, compare $C_\mu^k(A)$ and $C_\mu^k(B)$. If they are equal, then go to the step 2. Otherwise rank A and B according to the relative position of $C_\mu^k(A)$ and $C_\mu^k(B)$.

Step 2) Compare $C_\mu^k(A)$ and $C_\mu^k(B)$. If they are equal, then conclude that A and B are equal. Otherwise rank A and B according to the relative position of $-C_\nu^k(A)$ and $-C_\nu^k(B)$.

6. Conclusion

In this paper I have introduced the definition of intuitionistic fuzzy numbers. I have also proposed the concept of Trapezoidal (triangular) intuitionistic fuzzy numbers and I have proved some operation for them. I have also proposed a method of ranking intuitionistic fuzzy numbers based on the characteristic values of membership and non-membership functions of an IF number. It is worth noting that these results are direct generalization of the results obtained for the classical fuzzy numbers.

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Corrections to "Reducing the Semantic Gap of the MRI Image Retrieval Systems Using a Fuzzy Rule Based Technique"

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In the paper "Reducing the Semantic Gap of the MRI Image Retrieval Systems Using a Fuzzy Rule Based Technique" by Abolfazl Lakdashti, M. Shahram Moin, and Kambiz Badie, the second author, M. Shahram Moin, is currently with IT Faculty, Iran Telecom Research Center, Tehran, Iran.

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[1] Abolfazl Lakdashti, M. Shahram Moin, and Kambiz Badie, "Reducing the Semantic Gap of the MRI Image Retrieval Systems Using a Fuzzy Rule Based Technique," *International Journal of Fuzzy Systems*, vol. 11, no. 4, pp. 232–249, Dec. 2009.