Pre-semicompact *L*-subsets in fuzzy lattices

Shi-Zhong Bai

Abstract

A new concept of pre-semicompact L-subsets is introduced, where L is a fuzzy lattice. The pre-semicompact L-subset is semi-precompact, hence it is also semicompact and strongly compact. It is described with collection of pre-semiclosed L-subset, constant α-net and α-level filter base. It is a good L-extension; it is hereditary for pre-semiclosed L-subset; it is finitely additive; it is invariant under ζ'-continuous mapping as well as under pre-semi-irresolute mapping. Every set with finite support is pre-semicompact.

Keywords: Fuzzy lattice, Pre-semiopen set, Constant α-net, α-level filter base, Pre-semicompact L-subset.

1. Introduction

The concept of compactness is one of the most important concepts in topology. Since fuzzy topological spaces were introduced, various kinds of fuzzy compactness have been established [1, 2, 4-16]. Among those, in [7], Kudri introduced good fuzzy compact L-subsets in L-topological spaces. In [1,2,4,8-10], some definitions were introduced in L-topological spaces, respectively, which are semicompact L-subsets, and almost compact, strong compact, α-compact, S-closed, P-closed and semi-precompact L-subsets as well.

In this paper, along the lines of compactness defined by Kudri [7], we introduce a new definition of pre-semicompact L-subsets in L-topological spaces, where L is a fuzzy lattice. The pre-semicompact L-subset is semi-precompact, hence it is also semicompact and strongly compact L-subset. In general, semi-precompactness need not imply pre-semicompactness. Pre-semicompactness is a good extension. Also we obtain different characterizations and study some of its properties.

Corresponding Author: Department of Mathematics, Wuyi University, Guangdong 529020, P.R.China
E-mail: shizhongbai@yahoo.com.cn
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2. Preliminaries

Throughout this paper X and Y will be non-empty ordinary sets and $L = L(\le, \vee, \wedge, ')$ will be a fuzzy lattice, i.e. a completely distributive lattice with a smallest element 0 and largest element 1 $(0 \ne 1)$ and with an order reversing involution $a \to a'(a \in L)$. L^X will denote the lattice of all L-subsets of X, and if $A \subset X$, x_A will denote the characteristic function of A. An element p of L is called prime iff $p \ne 1$ and whenever $a, b \in L$ with $a \wedge b \le p$ then $a \le p$ or $b \le p$. The set of nonunit prime elements of L will be denoted by $pr(L)$. An element α of L is called union-irreducible or co-prime iff whenever $a, b \in L$ with $\alpha \le a \vee b$ then $\alpha \le a$ or $\alpha \le b$. The set of nonzero union-irreducible elements of L will be denoted by $M(L)$. The set of all nonzero union- irreducible elements of L^X will be denoted by $M(L^X)$. The definitions of the net, α-net ($\alpha \in M(L)$) and constant α-net can be seen in [15].

In this paper, we will denote L-topological space by L-ts. Then the interior, closure, semiinterior and semiclosure[3, 8] of an L-subset f will be denoted f°, f^-, $f_\circ$ and f_-, respectively.

Definition 2.1 [3]: Let (X, δ) be an L-ts and $f \in L^X$. Then f is called pre-semiopen iff $f \le (f^-)_\circ$, and pre-semiclosed iff $f \ge (f^\circ)_-$.

It is clear that every semi-preopen L-subset [4] is pre-semiopen, every semiopen L-subset [3] is semi-preopen and every pre-open L-subset [1, 4] is semi-preopen. That none of the converses need be true is shown by the Example 3.3 in [3].

Definition 2.2 [3]: Let (X, δ) and (Y, τ) be two L-ts's. A mapping $f : (X, \delta) \to (Y, \tau)$ is called:

(1) pre-semicontinuous iff $f^{-1}(g)$ is pre-semiopen in (X, δ) for each $g \in \tau$.

(2) pre-semiirresolute iff $f^{-1}(g)$ is pre-semiopen in (X, δ) for each pre-semiopen set g in (Y, τ).

Definition 2.3 [1]: Let $\alpha \in M(L)$ and $g \in L^X$. A collection η of L-subsets is said to form an α-level filter base in the L-subset g iff for any finite subcollection $\{f_1,\ldots,f_n\}$ of η, there exists $x\in X$ with $g(x)\geq\alpha$ such that $(\wedge_{i=1}^{n} f_i)(x)\geq\alpha$. When g is the whole space X, then η is an α-level filter base iff for any finite subcollection $\{f_1,\ldots,f_n\}$ of η, there exists $x\in X$ such that $(\wedge_{i=1}^{n} f_i)(x)\geq\alpha$.

Lemma 2.4 [9]: Let $(X,\Im)$ be a topological space, f be an L-subset in the L-ts $(X,\omega(\Im))$ and $p\in pr(L)$. Then we have

(1) $(f^{-})^{-1}(\{t\in L: t\not\leq p\})\subset(f^{-1}(\{t\in L: t\not\leq p\}))^{-}$.

(2) $(f^{\circ})^{-1}(\{t\in L: t\not\leq p\})\subset(f^{-1}(\{t\in L: t\not\leq p\}))^{\circ}$.

Definition 2.5 [1]: Let (X,δ) be an L-ts and $g\in L^X$, $r\in L$.

(1) A collection $\mu=\{f_i\}_{i\in J}$ of L-subsets is called an r-level cover of g iff $(\vee_{i\in J} f_i)(x)\not\leq r$ for all $x\in X$ with $g(x)\geq r'$. If each f_i is open then μ is called an r-level open cover of g. If g is the whole space X, then μ is called an r-level cover of X iff $(\vee_{i\in J} f_i)(x)\not\leq r$ for all $x\in X$.

(2) An r-level cover $\mu=\{f_i\}_{i\in J}$ of g is said to have a finite r-level subcover if there exists a finite subset F of J such that $(V_{i\in F} f_i)(x)\not\leq r$ for all $x\in X$ with $g(x)\geq r'$.

Definition 2.6. Let (X,δ) be an L-ts and $g\in L^X$. g is said to be:

(1) Compact [7] iff for every prime $p\in L$ and every collection $\{f_i\}_{i\in J}$ of open L-subsets with $(V_{i\in J} f_i)(x)\not\leq p$ for all $x\in X$ with $g(x)\geq p'$, there exists a finite subset F of J such that $(V_{i\in F} f_i)(x)\not\leq p$ for all $x\in X$ with $g(x)\geq p'$, i.e. every p-level open cover of g has a finite p-level subcover, where $p\in pr(L)$. If g is the whole space, then the L-ts (X,δ) is called compact.

(2) Semicompact [8] iff every p-level semiopen cover of g has a finite p-level subcover, where $p\in pr(L)$. If g is the whole space, then the L-ts (X,δ) is called semicompact.

(3) Strong compact [10] iff every p-level pre-open cover of g has a finite p-level subcover, where $p\in pr(L)$. If g is the whole space, then the L-ts (X,δ) is called strong compact.

(4) semi-precompact [4] iff every p-level semi- pre-open cover of g has a finite p-level subcover, where $p\in pr(L)$. If g is the whole space, then we say that the L-ts (X,δ) is called semi-precompact.

Theorem 2.7 [7]: Let (X,δ) be an L-ts, $g\in L^X$ and β be a subbase for δ. g is compact iff every p-level cover consisting of subbasic open L-subsets has a finite p-level subcover, where $p\in pr(L)$.

Theorem 2.8 [7]: Let (X,δ) and (Y,τ) be two L-ts's and $f:(X,\delta)\to(Y,\tau)$ be a continuous mapping with $f^{-1}(y)$ is finite for every $y\in Y$. If $g\in L^X$ is compact in (X,δ), then $f(g)$ is compact in (Y,τ), where $f(g)(y)=\vee\{g(x): x\in f^{-1}(y)\}$.

Theorem 2.9 [7]: Let (X,δ) be an L-ts. If g is a compact L-subset and h is a closed L-subset, then $h\wedge g$ is compact.

3. Pre-semicompact L-subsets and its characterizations

Definition 3.1: Let (X,δ) be an L-ts and $g\in L^X$. g is called pre-semicompact iff every p-level pre-semiopen cover of g has a finite p-level subcover, where $p\in pr(L)$. If g is the whole space, then we say that the L-ts (X,δ) is pre-semicompact.

Example 3.2: If $M(L)=\varnothing$, then every L-subset is pre-semicompact. One of such De Morgan algebra is $L=\{0,a,b,c,1\}$, where a,b,c are incomparable with each other.

Example 3.3: Let $X=[0,1]$ and $L=\{0\}\cup[0.1,0.9]\cup\{1\}$. For each $a\in L$ define $a'=1-a$. Let f be an L-set on X defined as $f(x)=0.9$, for all $x\in X$. Take $\delta=\{0,f,1\}$, then $\delta=\{0,f,1\}$ is a topology on X. Clearly, any L-set in (X,δ) is pre-semicompact.

Theorem 3.4: Let (X,δ) be an L-ts. Then $g\in L^X$ is pre-semicompact iff for every $\alpha\in M(L)$ and every collection $\{h_i\}_{i\in J}$ of pre-semiclosed L-subsets with $(\wedge_{i\in J} h_i)(x)\not\geq\alpha$ for all $x\in X$ with $g(x)\geq\alpha$, there is a finite subset F of J such that $(\wedge_{i\in F} h_i)(x)\not\geq\alpha$ for all $x\in X$ with $g(x)\geq\alpha$.

Proof. This follows immediately from the Definition 3.1.

Theorem 3.5: Let (X,δ) be an L-ts. Then $g\in L^X$ is pre-semicompact iff for every $p\in pr(L)$ and every collection $\{f_i\}_{i\in J}$ of pre-semiopen L-subsets with

$(\vee_{i\in F} f_i \vee g')(x) \not\leq p$ for all $x \in X$, there is a finite subset F of J such that

$$(V_{i\in F} f_i \vee g')(x) \not\leq p \text{ for all } x \in X.$$

Proof: Necessity: Let $p \in pr(L)$ and $\{f_i\}_{i\in J}$ be a collection of pre-semiopen L-subsets with $(\vee_{i\in F} f_i \vee g')(x) \not\leq p$ for all $x \in X$. Then $(V_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Since g is pre-semicompact, there is a finite subset F of J such that $(\vee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Take an arbitrary $x \in X$. If $g'(x) \leq p$ then

$$g'(x) \vee (\vee_{i\in F} f_i)(x) = (\vee_{i\in F} f_i \vee g')(x) \not\leq p$$

because $(\vee_{i\in F} f_i)(x) \not\leq p$. If $g'(x) \not\leq p$ then we have

$$g'(x) \vee (\vee_{i\in F} f_i)(x) = (\vee_{i\in F} f_i \vee g')(x) \not\leq p.$$

Thus, we have $(V_{i\in F} f_i \vee g')(x) \not\leq p$ for all $x \in X$.

Sufficiency: Let $p \in pr(L)$ and $\{f_i\}_{i\in J}$ be a p-level pre-semiopen cover of g. Then $(V_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Hence, $(\vee_{i\in F} f_i \vee g')(x) \not\leq p$ for all $x \in X$. From the hypothesis, there is a finite subset F of J such that

$$(V_{i\in F} f_i \vee g')(x) \not\leq p \text{ for all } x \in X.$$

Then $(V_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g'(x) \leq p$. Thus, g is pre-semicompact.

Definition 3.6: Let (X,δ) be an L-ts, x_α be an L-point in $M(L^X)$ and $S = (S_m)_{m\in D}$ be a net. x_α is called a pre-semicluster point of S iff for each pre-semiclosed L-subset f with $f(x) \not\geq \alpha$ and for all $n \in D$, there is $m \in D$ such that $m \geq n$ and $S_m \not\leq f$, i.e. $h(S_m) \not\leq f(Supp\ S_m)$.

Theorem 3.7: Let (X,δ) be an L-ts. Then $g \in L^X$ is pre-semicompact iff every constant α-net in g, where $\alpha \in M(L)$, has a pre-semicluster point in g with height α.

Proof: Necessity. Let $\alpha \in M(L)$ and $S = (S_m)_{m\in D}$ be a constant α-net in g without any pre-semicluster point with height α in g. Then for each $x \in X$ with $g(x) \geq \alpha$, x_α is not a pre-semicluster point of S, i.e. there are $n_x \in D$ and a pre-semiclosed L-subset f_x with $f_x(x) \not\geq \alpha$ and $S_m \leq f_x$ for each $m \geq n_x$. Let $x^1,\ldots,x^k$ be elements of X with $g(x^i) \geq \alpha$ for each $i \in \{1,\ldots,k\}$. Then there are $n_{x_1},\ldots,n_{x_k} \in D$ and pre-semiclosed L-subset f_{x_i} with $f_{x_i}(X^i) \not\geq \alpha$ and $S_m \leq f_{x_i}$ for each $m \geq n_{x_i}$ and for each $i \in \{1,\ldots,k\}$. Since D is a directed set, there is $n_o \in D$ such that $n_o \geq n_{x_i}$ for each $i \in \{1,\ldots,k\}$ and $S_m \leq f_{x_i}$ for $i \in \{1,\ldots,k\}$ and each $m \geq n_o$. Now, consider the family $\mu = \{f_x\}_{x\in X}$ with $g(x) \geq \alpha$. Then $(\wedge_{f_x\in\mu} f_x)(y) \not\geq \alpha$ for all $y \in X$ with $g(y) \geq \alpha$ because $f_y(y) \not\geq \alpha$. We also have that for any finite subfamily $\nu = \{f_{x_1},\ldots,f_{x_k}\}$ of μ, there is $y \in X$ with $g(y) \geq \alpha$ and $(\wedge_{i=1}^k f_i)(y) \geq \alpha$ since $S_m \leq \wedge_{i=1}^k f_{x_i}$ for each $m \geq n_o$ because $S_m \leq f_{x_i}$ for each $i \in \{1,\ldots,k\}$ and for each $m \geq n_o$. Hence, by the Theorem 3.4, g is not pre-semicompact.

Sufficiency: Suppose that g is not pre-semicompact. Then by the Theorem 3.4, there exist $\alpha \in M(L)$ and a collection $\mu = (f_i)_{i\in J}$ of pre-semiclosed L-subsets with $(\wedge_{i\in J} f_i)(x) \not\leq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$, but for any finite subfamily ν of μ there is $x \in X$ with $g(x) \geq \alpha$ and $(\wedge_{i\in\nu} f_i)(x) \geq \alpha$. Consider the family of all finite subsets of μ, $2^{(\mu)}$, with the order $\nu_1 \leq \nu_2$ iff $\nu_1 \subset \nu_2$. Then $2^{(\mu)}$ is a directed set. So, writing x_α as S_ν for every $\nu \in 2^{(\mu)}$, $(S_\nu)_{\nu\in 2^{(\mu)}}$ is a constant α-net in g because the height of S_ν for all $\nu \in 2^{(\mu)}$ is α and $S_\nu \leq g$ for all $\nu \in 2^{(\mu)}$, i.e. $g(x) \geq \alpha$. $(S_\nu)_{\nu\in 2^{(\mu)}}$ also satisfies the condition that for each pre-semiclosed L-subset $f_i \in \nu$ we have $x_\alpha = S_\nu \leq f_i$. Let $y \in X$ with $g(y) \geq \alpha$. Then $(\wedge_{i\in J} f_i)(y) \not\geq \alpha$, i.e. there exists $j \in J$ with $f_j(y) \not\geq \alpha$. Let $\nu_o = \{f_j\}$. So, for any $\nu \geq \nu_o$, $S_\nu \leq \wedge_{f_i\in\nu} f_i \leq \wedge_{f_i\in\nu_o} f_i = f_j$.

Thus, we get a pre-semiclosed L-subset f_j with $f_j(y) \geq \alpha$ and $\nu_o \in 2^{(\mu)}$ such that for any $\nu \geq \nu_o$, $S_\nu \leq f_j$. That means that $y_\alpha \in M(L^X)$ is not a pre-semicluster point $(S_\nu)_{\nu\in 2^{(\mu)}}$ for all $y \in X$ with $g(y) \geq \alpha$. Hence, the constant α-net $(S_\nu)_{\nu\in 2^{(\mu)}}$ has no pre-semicluster point in g with height α.

Corollary 3.8: An L-ts (X,δ) is pre-semicompact iff every constant α-net in (X,δ) has a pre-semicluster point with height α, where $\alpha \in M(L)$.

Definition 3.9: Let (X,δ) be an L-ts and η an α-level filter base, where $\alpha \in M(L)$. An L-point $x_r \in M(L^X)$ is called a pre-semicluster point of η, if $\wedge_{f\in\eta} f_{\sim}(x) \geq r$, where $f_{\sim} = \wedge\{g \in L^X :\ g$ is pre-semiclosed and $g \geq f\}$ [3].

Theorem 3.10: Let (X,δ) be an L-ts. Then $g \in L^X$ is pre-semicompact iff every α-filter base in g, where $\alpha \in M(L)$, has a pre-semicluster point x_α in g with height α.

Proof: Necessity: Assume that η is an α-level filter base in g with no pre-semicluster point in g with height α, where $\alpha \in M(L)$. Then for each $x \in X$ with $g(x) \geq \alpha$, x_α is not a pre-semicluster point of η, i.e. there is $f_x \in \eta$ with $(f_x)_{\sim}(x) \ngeq \alpha$. Hence, $((f_x)_{\sim})'(x) \nleq \alpha' = p \in pr(L)$. This means that the collection $\{((f_x)_{\sim})'\}_{x\in X}$ with $g(x) \geq \alpha$ is a p-level pre-semiopen cover of g. Since g is pre-semicompact, there are $(f_{x_1})_{\sim},...,(f_{x_n})_{\sim}$ such that $(\vee_{i=1}^{n}((f_{x_i})_{\sim})')(x) \nleq p$ for all $x \in X$ with $g(x) \geq p' = \alpha$. Hence, $(\wedge_{i=1}^{n}(f_{x_i})_{\sim})(x) \ngeq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$ which implies that $(\wedge_{i=1}^{n} f_{x_i})(x) \ngeq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$. This is a contradiction.

Sufficiency: Suppose that g is not pre-semicompact. Then there is a p-level pre-semiopen cover μ of g with no finite p-level subcover, where $p \in pr(L)$. Hence, for each finite subcollection $\{h_1,...,h_n\}$ of μ, there exists $x \in X$ with $g(x) \geq p'$ such that $(\vee_{i=1}^{n} h_{i\sim})(x) \leq p$, i.e. $(\wedge_{i=1}^{n} h_i')(x) \geq p' = \alpha \in M(L)$. Thus, $\eta = \{h' : h \in \mu\}$ forms an α-level filter base in g. By the hypothesis, μ has a pre-semicluster point $y_\alpha \in M(L^X)$ in g with height α, i.e. $g(y) \geq \alpha$ and $(\wedge_{h\in\mu}(h')_{\sim})(y) = (\wedge_{h\in\mu} h')(y) \geq \alpha$. Then $(\wedge_{h\in\mu} h')(y) \leq p$ which yields a contradiction.

Corollary 3.11: An L-ts (X,δ) is pre-semicompact iff every α-filter base has a pre-semicluster point with height α, where $\alpha \in M(L)$.

Lemma 3.12: Let $(X,\Im)$ be a topological space, f be an L-subset in the L-ts $(X,\omega(\Im))$ and $p \in pr(L)$. Then we have

(1) $(f_{-})^{-1}(\{t \in L : t \nleq p\}) \subset (f^{-1}(\{t \in L : t \nleq p\}))_{-}$.

(2) $(f_{\circ})^{-1}(\{t \in L : t \nleq p\}) \subset (f^{-1}(\{t \in L : t \nleq p\}))_{\circ}$.

Proof: This is similar to the Lemma 4.1 in [9].

Lemma 3.13: Let $(X,\Im)$ be a topological space and $A \subset X$. Then A is pre-semiopen in $(X,\Im)$ iff x_A is pre-semiopen in the L-ts $(X,\omega(\Im))$.

Proof: A is pre-semiopen in $(X,\Im)$ iff $A \subset (A^{-})_{\circ}$ iff $x_A \leq X_{(A^{-})_{\circ}} = ((x_A)^{-})_{\circ}$ (the equality is due to the *Lemma 4.2 [8]:* and the Lemma 3.6 in [9]) iff x_A is pre-semiopen in $(X,\omega(\Im))$.

We say that a topology space $(X,\Im)$ is pre-semicompact iff every pre-semiopen cover of X has a finite subcover.

Theorem 3.14: Let $(X,\Im)$ be a topological space. Then $(X,\Im)$ is pre-semicompact iff the L-ts $(X,\omega(\Im))$ is pre-semicompact.

Proof: Necessity: Let $p \in pr(L)$ and $\{f_i\}_{i\in J}$ be a p-level pre-semiopen cover of $(X,\omega(\Im))$. Then $(V_{i\in J} f_i)(x) \nleq p$ for all $x \in X$. Hence for each $x \in X$ there is $i \in J$ such that $f_i(x) \nleq p$, i.e. $x \in f_i^{-1}(\{t \in L : t \nleq p\})$.

So, $X = \bigcup_{i\in J} f_i^{-1}(\{t \in L : t \nleq p\})$. Because f_i is pre-semiopen in $(X,\omega(\Im))$, we have $f_i \leq (f_i^{-})_{\circ}$ for every $i \in J$. Hence, by the Lemmas 2.4, 3.13, we get

$$f_i^{-1}(\{t \in L : t \nleq p\}) \subset ((f_i^{-})_{\circ})^{-1}(\{t \in L : t \nleq p\}))$$
$$\subset ((f_i^{-1}(\{t \in L : t \nleq p\}))^{-}).$$

which means that $f_i^{-1}(\{t \in L : t \nleq p\})$ is pre-semiopen in $(X,\Im)$. Thus $\{f_i^{-1}(\{t \in L : t \nleq p\})\}_{i\in J}$ is a pre-semiopen cover of $(X,\Im)$. Since $(X,\Im)$ is pre-semicompact, there is a finite subset F of J such that $X = \bigcup_{i\in F} f_i^{-1}(\{t \in L : t \nleq p\})$, i.e. $(\vee_{i\in F} f_i)(x) \nleq p$ for all $x \in X$. Hence, $(X,\omega(\Im))$ is pre-semicompact.

Sufficiency: Let $\{A_i\}_{i\in J}$ be a pre-semiopen cover of $(X,\Im)$. Then by the Lemma 3.13, $\{x_{A_i}\}_{i\in J}$ is a family of pre-semiopen L-subsets in $(X,\omega(\Im))$ such that $1 = (\vee_{i\in J} x_{A_i})(x) \nleq p$ for all $x \in X$ and for all $p \in pr(L)$, i.e. $\{x_{A_i}\}_{i\in J}$ is a p-level pre-semiopen cover of $(X,\omega(\Im))$. Since $(X,\omega(\Im))$ is pre- semi-compact, there is a finite subset F of J such that $(\vee_{i\in J} x_{A_i})(x) \nleq p$ for all $x \in X$. Hence, $(\vee_{i\in F} x_{A_i})(x) = 1$ for all $x \in X$, i.e. $X = \bigcup_{i\in F} A_i$ and therefore (X,δ) is pre-semicompact.

4. Some other properties

Since every semi-preopen L-subset is pre-semiopen, every semiopen L-subset is semi-preopen and every pre-open L-subset is semi-preopen, we have

Proposition 4.1: Every pre-semicompact L-subset is semi-precompact, hence it is also semicompact and strongly compact.

In general, semi-precompactness need not imply pre-semicompactness.

Example 4.2: Let $X=\{x,y,z\}$, $L=[0,1]$, $\forall a\in L$, $a'=1-a$, and $f,g,h\in L^X$ defined as follows:

$$f(x)=0.2, f(y)=0.4, f(z)=0.5;$$
$$g(x)=0.8, g(y)=0.8, g(z)=0.6;$$
$$h(x)=0.3, h(y)=0.2, h(z)=0.4.$$

Then $\delta=\{0,f,g,1\}$ is a topology on X. By easy computations it follows that $h\le(h^-)_\circ=(f')_\circ=f'$, hence, h is a pre-semiopen set. Clearly h is not a semi-preopen set (in fact, because 0 is the only pre-open set contained in h and $0^-=0$, h is not a semi-preopen set). By the Definitions 3.1 and 2.6(4) we can obtain h is semi-precompact, but h is not pre-semicompact.

Theorem 4.3: Let (X,δ) be an L-ts and g, $h\in L^X$. If g and h are pre-semicompact then $g\vee h$ is pre-semicompact as well.

Proof: Let $p\in pr(L)$ and $\{f_i\}_{i\in J}$ be a collection of pre-semiopen L-subsets with $(V_{i\in J}f_i)(x)\not\le p$ for all $x\in X$ with $(g\vee h)(x)\ge p'$. Since p is prime, we have $(g\vee h)(x)\ge p'$ iff $g(x)\ge p'$ or $h(x)\ge p'$. So, by the pre-semicompactness of g and h, there are finite subsets E, F of J such that $(\vee_{i\in E}f_i)(x)\not\le p$ for all $x\in X$ with $g(x)\ge p'$ and $(\vee_{i\in F}f_i)(x)\not\le p$ for all $x\in X$ with $h(x)\ge p'$. Then, $(\vee_{i\in E\cup F}f_i)(x)\not\le p$ for all $x\in X$ with $g(x)\ge p'$ or $h(x)\ge p'$, i.e. $(\vee_{i\in E\cup F}f_i)(x)\not\le p$ for all $x\in X$ with $(g\vee h)(x)\ge p'$. Thus, $g\vee h$ is pre-semicompact.

Theorem 4.4: Let (X,δ) be an L-ts and $g,h\in L^X$. If g is pre-semicompact and h is pre-semiclosed, then $g\wedge h$ is pre-semicompact.

Proof: Let $p\in pr(L)$ and $\{f_i\}_{i\in J}$ be a collection of pre-semiopen L-subsets with $(V_{i\in J}f_i)(x)\not\le p$ for all $x\in X$ with $(g\wedge h)(x)\ge p'$. Thus, $\mu=(f_i)_{i\in J}\cup\{h'\}$ is a family of pre-semiopen L-subsets with $(\vee_{k\in\mu}k)(x)\not\le p$ for all $x\in X$ with $g(x)\ge p'$. In fact, for each $x\in X$ with $g(x)\ge p'$, if $h(x)\ge p'$ then $(g\wedge h)(x)\ge p'$ which implies that $(\vee_{i\in J}f_i)(x)\not\le p$, thus $(\vee_{k\in\mu}k)(x)\not\le p$. If $h(x)\not\ge p'$ then $h'(x)\not\le p$ which implies $(\vee_{k\in\mu}k)(x)\not\le p$. From the pre-semicompactness of g, there is a finite subfamily ν of μ, say $\nu=\{f_1,\ldots,f_n,h'\}$ with $(\vee_{k\in\nu}k)(x)\not\le p$ for all $x\in X$ with $g(x)\ge p'$. Then $(\vee_{i=1}^n f_i)(x)\not\le p$ for all $x\in X$ with $(g\wedge h)(x)\ge p'$. Hence, $g\wedge h$ is pre- semicompact.

Corollary 4.5: Let (X,δ) be a pre-semicompact space and g be a pre-semiclosed L-subset. Then g is pre-semicompact.

Theorem 4.6: Let (X,δ) be an L-ts where X is a finite set. Then (X,δ) is pre-semicompact.

Proof: Let $\{f_i\}_{i\in J}$ be a p-level pre-semiopen cover of (X,δ), where $p\in pr(L)$. Then $(V_{i\in J}f_i)(x)\not\le p$ for all $x\in X$. Hence, for each $x\in X$ there is $i\in J$ such that $x\in f_i^{-1}(\{t\in L:t\not\le p\})$; i.e. $X=\bigcup_{i\in J}f_i^{-1}(\{t\in L:t\not\le p\})$. Since X is finite, there is a finite subset F of J such that $X=\bigcup_{i\in F}f_i^{-1}(\{t\in L:t\not\le p\})$, i.e. $(\vee_{i\in F}f_i)(x)\not\le p$ for each $x\in X$. So, (X,δ) is pre-semicompact.

Corollary 4.7: Let (X,δ) be an L-ts and $g\in L^X$. If g with finite support, then g is pre-semicompact.

Let (X,δ) be an L-ts. The following δ_ζ will denote the L-topology on X which has the set of all pre-semiopen L-subsets of (X,δ) as a subbase.

Theorem 4.8: Let (X,δ) be an L-ts and $g\in L^X$. Then g is pre-semicompact in (X,δ) iff g is compact in (X,δ_ζ).

Proof: Necessity: Let $p\in pr(L)$ and $\{f_i\}_{i\in J}$ be a collection of subbasic δ_ζ-open L-subsets with $(V_{i\in J}f_i)(x)\not\le p$ for all $x\in X$ with $g(x)\ge p'$. Then each f_i is pre-semiopen in (X,δ) and so $\{f_i\}_{i\in J}$ is a p-level pre-semiopen cover of g. Since g is pre-semicompact in (X,δ), there is a finite subset F of J such that $(V_{i\in F}f_i)(x)\not\le p$ for all $x\in X$ with $g(x)\ge p'$. Hence, by the Theorem 2.7, g is compact in (X,δ_ζ).

Sufficiency: Let $p \in pr(L)$ and $\{f_i\}_{i\in J}$ be a collection of pre-semiopen L-subsets in (X,δ) with $(\vee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Since every pre-semiopen L-subsets in (X,δ) is δ_ζ-open, by the compactness of g in (X,δ_ζ), there is a finite subset F of J such that $(\vee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Hence, g is pre-semicompact in (X,δ).

Corollary 4.9: An L-ts (X,δ) is pre-semicompact iff L-ts (X,δ_ζ) is compact.

Theorem 4.10: Let (X,δ) be an L-ts. If g is a pre-semicompact L-subset in (X,δ), then for each closed L-subset h in (X,δ_ζ), $h \wedge g$ is pre-semicompact in (X,δ).

Proof: Let g be a pre-semicompact L-subset in (X,δ). Then by the Theorem 4.8, g is compact in (X,δ_ζ). Since h is closed in (X,δ_ζ), by the Theorem 2.9, $h \wedge g$ is compact in (X,δ_ζ). Hence, again by the Theorem 4.8, $h \wedge g$ is pre-semicompact in (X,δ).

Definition 4.11: Let (X,δ) and (Y,τ) be two L-ts's. A mapping $f:(X,\delta)\to(Y,\tau)$ is called:

(1) ζ-continuous iff $f:(X,\delta_\zeta)\to(Y,\tau)$ is continuous.

(2) ζ'-continuous iff $f:(X,\delta_\zeta)\to(Y,\tau_\zeta)$ is continuous.

Corollary 4.12: If $f:(X,\delta)\to(Y,\tau)$ is a pre-semicontinuous (pre-semiirresolute) mapping, then f is ζ-continuous (ζ'-continuous).

Proof. This follows immediately from the definitions.

Theorem 4.13: Let $f:(X,\delta)\to(Y,\tau)$ be a ζ-continuous mapping with $f^{-1}(y)$ is finite for every $y \in Y$. If $g \in L^X$ is pre-semicompact in (X,δ), then $f(g)$ is compact in (Y,τ).

Proof: Let g is pre-semicompact in (X,δ). Then by the Theorem 4.8, g is compact in (X,δ_ζ). Since $f:(X,\delta_\zeta)\to(Y,\tau)$ is continuous, by the Theorem 2.8, $f(g)$ is compact in (Y,τ).

Corollary 4.14: Let $f:(X,\delta)\to(Y,\tau)$ be a pre-semi-continuous mapping with $f^{-1}(y)$ is finite for every $y \in Y$. If $g \in L^X$ is pre-semicompact in (X,δ), then $f(g)$ is compact in (Y,τ).

Proof: This follows immediately from the Corollary 4.12 and the Theorem 4.13.

Corollary 4.15: Let $f:(X,\delta)\to(Y,\tau)$ be a pre-semi- continuous surjection with $f^{-1}(y)$ is finite for every $y \in Y$. If (X,δ) is pre-semicompact, then (Y,τ) is compact.

Theorem 4.16: Let $f:(X,\delta)\to(Y,\tau)$ be a ζ'-continuous mapping with $f^{-1}(y)$ is finite for every $y \in Y$. If $g \in L^X$ is pre-semicompact in (X,δ), then $f(g)$ is pre-semicompact in (Y,τ).

Proof: Let g is pre-semicompact in (X,δ). Then by the Theorem 4.8, g is compact in (X,δ_ζ). Since $f:(X,\delta)\to(Y,\tau)$ is ζ'-continuous, $f:(X,\delta_\zeta)\to(Y,\tau_\zeta)$ is continuous. Hence, by the Theorem 2.8, $f(g)$ is compact in (Y,τ_ζ). So, again by the Theorem 4.8, $f(g)$ is pre-semicompact in (Y,τ).

Corollary 4.17: Let $f:(X,\delta)\to(Y,\tau)$ be a pre-semi- irresolute mapping with $f^{-1}(y)$ is finite for every $y \in Y$. If $g \in L^X$ is pre-semicompact in (X,δ), then $f(g)$ is pre-semicompact in (Y,τ).

Proof. This follows immediately from the Corollary 4.12 and the Theorem 4.16.

Corollary 4.18: Let $f:(X,\delta)\to(Y,\tau)$ be a pre-semi-irresolute surjection with $f^{-1}(y)$ is finite for every $y \in Y$. If (X,δ) is pre-semicompact, then (Y,τ) is pre-semicompact.

5. Conclusion

In recent years, research in fuzzy topology (especially the combination of fuzzy topology, fuzzy complete lattice and Domain theory) have greatly promoted the rapid development of Locale theory, Domain theory, fuzzy logic and computer science. Just due to the requirement of these related subjects, also due to the requirement of the theory of fuzzy topology itself, some specialized theory of fuzzy topology, such as semi- topological properties, weak compactness, strong compactness and closeness theory and etc., have caught much attention of many researchers. The new definition of pre-semicompact L-subsets in L-topological spaces we introduced and its different characterizations and properties we studied in this paper are bound to be applied in Domain theory and computer science.

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