

Fuzzy Generalized Hybrid Aggregation Operators and its Application in Fuzzy Decision Making

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Abstract

The hybrid averaging (HA) is an aggregation operator that uses the weighted average (WA) and the ordered weighted averaging (OWA) operator in the same formulation. In this paper, we introduce several generalizations of the HA operator by using generalized and quasi-arithmetic means, fuzzy numbers and order inducing variables in the reordering step of the aggregation process. We present the fuzzy generalized hybrid averaging (FGHA) operator, the fuzzy induced generalized hybrid averaging (FIGHA) operator, the Quasi-FHA operator and the Quasi-FIHA operator. The main advantage of these operators is that they generalize a wide range of fuzzy aggregation operators that can be used in a wide range of applications such as decision making problems. For example, we could mention the fuzzy induced hybrid averaging (FIHA), the fuzzy weighted generalized mean (FWGM) and the fuzzy induced generalized OWA (FIGOWA). We end the paper with an application of the new approach in a decision making problem.

Keywords: *Aggregation operators, Fuzzy numbers, Hybrid averaging, OWA operator, Decision making.*

1. Introduction

The hybrid averaging (HA) operator [1] is an aggregation operator that uses the weighted average (WA) and the ordered weighted averaging (OWA) operator [2-38] in the same formulation. An interesting extension of the HA operator for situations where it is not possible to use exact numbers because the information is uncertain, is the fuzzy HA (FHA) operator. It represents the uncertain information to be aggregated with the HA operator by using FNs.

Another interesting aggregation operator is the generalized OWA (GOWA) operator [8, 29]. It generalizes the

OWA operator by using generalized means. Recently [17], a further generalization has been suggested by using order inducing variables in the GOWA operator. This operator is known as the induced GOWA (IGOWA) operator. The main advantage of this operator is that it generalizes a wide range of mean operators such as the induced OWA (IOWA), the induced ordered weighted quadratic averaging (IOWQA) and the generalized mean (GM). The IGOWA operator can be further generalized by using quasi-arithmetic means [17]. The result is the Quasi-IOWA operator. Other generalizations of the OWA operator by using generalized or quasi-arithmetic means can be found in [3-4,8,12,14,16-18,38].

Going a step further, in this paper we introduce the fuzzy generalized hybrid averaging (FGHA) and the fuzzy induced generalized hybrid averaging (FIGHA) operator. They are new aggregation operators that use the main characteristics of the IGOWA, the FOWA and the HA operator. Thus, these operators use generalized means in the HA operator and in uncertain situations where the available information can not be represented with exact numbers but it is possible to use FNs. By using the HA operator, the FIGHA considers the WA and the IOWA (or the OWA) in the same problem. In decision making problems, this implies that the FIGHA operator considers the subjective probability and the attitudinal character of the decision maker in the same formulation. One of the key features of the FGHA and the FIGHA operator is that they generalize a wide range of aggregation operators such as the FGM, the FWGM, the FIGOWA, the FWA, the FOWA, the fuzzy induced HA (FIHA), the fuzzy induced hybrid quadratic averaging (FIHQQA), the fuzzy induced hybrid geometric averaging (FIHGA) and the fuzzy induced hybrid harmonic averaging (FIHHA) operator. Thus, they are able to provide a more complete view of the decision problem to the decision maker, so he will be able to make the appropriate decision according to his interests.

We further generalize the FGHA and the FIGHA by using quasi-arithmetic means. The result is the Quasi-FHA and the Quasi-FIHA operator. We also develop a numerical example of the new approach in a decision making problem about selection of strategies. With this example, we will see that depending on the aggregation operator used, the results may lead to different decisions.

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In order to do so, this paper is organized as follows. In Section 2 we briefly describe some basic concepts about the FNs, the FHA and the IGOWA operator. In Section 3 we present the FGHA and the FIGHA operator. Section 4 analyses some basic families of the FIGHA operator. Section 5 introduces the Quasi-FHA and the Quasi-FIHA operators. In Section 6 we develop a numerical example of the new approach. Finally, in Section 7 we summarize the main conclusions of the paper.

2. Preliminaries

In this Section, we give a brief overview of the FNs, the FHA and the IGOWA operator.

A. Fuzzy Numbers

The FN was first introduced by [39-40]. Since then, it has been studied and applied by a lot of authors such as [41-42].

A FN is a fuzzy subset of a universe of discourse that is both convex and normal. Note that the FN may be considered as a generalization of the interval number although it is not strictly the same because the interval numbers may have different meanings. In the literature, we find a wide range of FNs [41-45]. For example, a trapezoidal FN (TpFN) A of a universe of discourse R can be characterized by a trapezoidal membership function $A = (\underline{a}, \bar{a})$ such that

$$\begin{aligned} \underline{a}(\alpha) &= a_1 + \alpha(a_2 - a_1), \\ \bar{a}(\alpha) &= a_4 - \alpha(a_4 - a_3). \end{aligned} \quad (1)$$

where $\alpha \in [0, 1]$ and parameterized by (a_1, a_2, a_3, a_4) where $a_1 \leq a_2 \leq a_3 \leq a_4$, are real values. Note that if $a_1 = a_2 = a_3 = a_4$, then, the FN is a crisp value and if $a_2 = a_3$, the FN is represented by a triangular FN (TFN). Note that the TFN can be parameterized by (a_1, a_2, a_4) .

In the following, we are going to review some basic FN arithmetic operations as follows. Let A and B be two TFN, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. Then:

- 1) $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
- 2) $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.
- 3) $A \times k = (k \times a_1, k \times a_2, k \times a_3)$; for $k > 0$.

Note that other operations could be studied [39-45] but in this paper we will focus on these ones.

B. Fuzzy Hybrid Averaging Operator

The FHA operator is an aggregation operator that uses the weighted average (WA) and the OWA operator in the same formulation. Then, it is possible to consider in the same decision making problem, the attitudinal character of the decision maker and its subjective probability. It also deals with uncertain environments where the available information cannot be assessed with exact numbers but it is possible to find approximate results using FNs.

It can be defined as follows.

Definition 1: Let Ψ be the set of FNs. A FHA operator of dimension n is a mapping $FHA: \Psi^n \rightarrow \Psi$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$FHA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{j=1}^n w_j b_j \quad (2)$$

where b_j is the j th largest of the \hat{a}_i ($\hat{a}_i = n\omega_i \tilde{a}_i$, $i = 1, 2, \dots, n$), $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighting vector of the \tilde{a}_i , with $\omega_i \in [0, 1]$ and the sum of the weights is 1, and the \tilde{a}_i are FNs.

Note that the FHA operator can be extended by using order inducing variables, obtaining the fuzzy induced HA (FIHA) operator. It can be defined as follows.

Definition 2: Let Ψ be the set of FNs. A FIHA operator of dimension n is a mapping $FIHA: \Psi^n \times \Psi^n \rightarrow \Psi$ that has an associated weighting vector W of dimension n with the sum of the weights equal to 1 and $w_j \in [0, 1]$, such that:

$$FIHA(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \sum_{j=1}^n w_j b_j \quad (3)$$

where b_j is the \hat{a}_i value ($\hat{a}_i = n\omega_i \tilde{a}_i$, $i = 1, 2, \dots, n$) of the IOWA pair $\langle u_i, \tilde{a}_i \rangle$ having the j th largest u_i , u_i is the order inducing variable, $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighting vector of the \tilde{a}_i , with $\omega_i \in [0, 1]$ and the sum of the weights is 1, and the \tilde{a}_i are FNs.

Similar to the FOWA operator, it is possible to analyze different properties of the FHA operator. Note that in this case, we should also consider the problem of comparing FNs in the reordering process. For simplicity, we recommend to follow the policy explained in [12-13].

C. The Induced Generalized OWA Operator

The induced generalized OWA (IGOWA) operator [17] is a generalization of the OWA operator by using generalized means and order inducing variables. It includes a wide range of mean operators such as the usual OWA, the IOWA, the induced OWG operator, the ordered weighted quadratic averaging operator (OWQA), etc. It can be defined as follows.

Definition 3: An IGOWA operator of dimension n is a mapping $IGOWA: R^n \times R^n \rightarrow R$ defined by an associated weighting vector W of dimension n such that $\sum_{j=1}^n w_j = 1$ and $w_j \in [0, 1]$, a set of order-inducing variables u_i , and a parameter $\lambda \in (-\infty, \infty)$, according to the following formula:

$$IGOWA(\langle u_1, a_1 \rangle, \dots, \langle u_n, a_n \rangle) = \left(\sum_{j=1}^n w_j b_j^\lambda \right)^{1/\lambda} \quad (4)$$

where (b_1, \dots, b_n) is (a_1, a_2, \dots, a_n) reordered in decreasing order of the values of the u_i , the u_i are the order-inducing

variables, a_i are the argument variables.

For further reading on generalized OWAs and its recent developments, see for example [3-4,8,12,14,16-18,38].

If we replace b^λ with a general continuous strictly monotone function $g(b)$ [3], then, the IGOWA operator becomes the Quasi-IOWA operator [17]. It can be formulated as follows.

Definition 4: A Quasi-IOWA operator of dimension n is a mapping *QIOWA*: $R^n \times R^n \rightarrow R$ defined by an associated weighting vector W of dimension n such that $\sum_{j=1}^n w_j = 1$ and $w_j \in [0, 1]$, and by a strictly monotonic continuous function $g(b)$, as follows:

$$QIOWA (\langle u_1, a_1 \rangle, \dots, \langle u_n, a_n \rangle) = g^{-1} \left(\sum_{j=1}^n w_j g(b_{(j)}) \right) \quad (5)$$

where the b_j are the argument values a_i of the Quasi-IOWA pairs $\langle u_i, a_i \rangle$ ordered in decreasing order of their u_i values.

3. Fuzzy Generalized Hybrid Aggregation Operators

In this Section, we are going to consider two generalizations: the fuzzy generalized hybrid averaging (FGHA) operator and the fuzzy induced generalized hybrid averaging (FIGHA) operator. As the FIGHA generalizes the FGHA, we will mostly use the FIGHA operator in this paper. The FGHA operator is an extension of the HA operator that uses generalized means and uncertain information that can be represented by using FNs. Thus, we can represent the problems considering the best and worst result and the possibility that the internal values of the FN will occur. With this generalization, we include in the same formulation a lot of aggregation operators such as the FHA, the fuzzy hybrid quadratic averaging (FHQA), the FGOWA and the FWGM. It can be defined as follows.

Definition 5: Let Ψ be the set of FNs. A FGHA operator of dimension n is a mapping *FGHA*: $\Psi^n \rightarrow \Psi$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$FGHA (\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \left(\sum_{j=1}^n w_j b_j^\lambda \right)^{1/\lambda} \quad (6)$$

where b_j is the j th largest of the \hat{a}_i ($\hat{a}_i = n\omega_i \tilde{a}_i$, $i = 1, 2, \dots, n$), $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighting vector of the \tilde{a}_i , with $\omega_i \in [0, 1]$ and the sum of the weights is 1, the \tilde{a}_i are FNs and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

The FIGHA operator is an extension of the FGHA operator for situations where we want to deal with more

complex reordering processes in the aggregation of the FNs. It uses order-inducing variables in the reordering of the FNs and it includes the FGHA as a particular case. It can be defined as follows.

Definition 6: Let Ψ be the set of FNs. A FIGHA operator of dimension n is a mapping *FIGHA*: $\Psi^n \times \Psi^n \rightarrow \Psi$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^n w_j = 1$ and $w_j \in [0, 1]$, such that:

$$FIGHA (\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \left(\sum_{j=1}^n w_j b_j^\lambda \right)^{1/\lambda} \quad (7)$$

where b_j is the \hat{a}_i value ($\hat{a}_i = n\omega_i \tilde{a}_i$, $i = 1, 2, \dots, n$) of the IOWA pair $\langle u_i, \tilde{a}_i \rangle$ having the j th largest u_i , u_i is the order inducing variable, $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighting vector of the \tilde{a}_i , with $\omega_i \in [0, 1]$ and the sum of the weights is 1, the \tilde{a}_i are FNs, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

As we can see the FGHA operator is a particular case of the FIGHA operator when the reordering of the u_i is equal to j .

Note that if $\lambda \leq 0$, we can only use positive numbers R^+ , in order to get consistent results. Note also that different types of FNs could be used in the aggregation such as Triangular FNs, Trapezoidal FNs, L-R FNs, interval-valued FNs, intuitionistic FNs [21,39], generalized FNs, and more complex structures [12-13,19,24-25,31,39-45].

In this case, the reordering of the arguments has an additional difficulty because now we are using FNs. Therefore, in some cases it is not clear which FN is higher, so we need to establish an additional criteria for ranking FNs. For simplicity, we recommend the following procedure. Select the FN with the highest value in the membership level $\alpha = 1$. Note that if the membership level $\alpha = 1$ is an interval, then, we will calculate the average of the interval. Note also that this problem is especially relevant for the FGHA because the FIGHA solves this issue with the order inducing variables that do not need to rank FNs. However, the final result is a FN so if we have different final results, then, we need to establish a ranking between them.

From a generalized perspective of the reordering step, it is possible to distinguish between the descending FIGHA (DFIGHA) and the ascending FIGHA (AFIGHA) operator. The weights of these operators are related by $w_j = w_{n-j+1}^*$, where w_j is the j th weight of the DFIGHA and w_{n-j+1}^* the j th weight of the AFIGHA operator.

If B is a vector corresponding to the ordered arguments b_j^λ , we shall call this the ordered argument vector and W^T is the transpose of the weighting vector, then, the FIGHA operator can be expressed as:

$$FIGHA(\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \left(W^T B \right)^{1/\lambda} \quad (8)$$

Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^n w_j \neq 1$, then, the FIGHA operator can be expressed as:

$$FIGHA(\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \left(\frac{1}{W} \sum_{j=1}^n w_j b_j^\lambda \right)^{1/\lambda} \quad (9)$$

The FIGHA operator is a mean or averaging operator. This is a reflection of the fact that the operator is monotonic and idempotent. These properties can be proved with the following theorems.

Theorem 1 (Monotonicity): Assume f is the FIGHA operator, if $\tilde{a}_i \geq e_i$, for all \tilde{a}_i , then:

$$f(\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) \geq f(\langle u_1, e_1 \rangle, \dots, \langle u_n, e_n \rangle) \quad (10)$$

Proof: Let

$$f(\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \left(\sum_{j=1}^n w_j b_j^\lambda \right)^{1/\lambda} \quad (11)$$

$$f(\langle u_1, e_1 \rangle, \dots, \langle u_n, e_n \rangle) = \left(\sum_{j=1}^n w_j d_j^\lambda \right)^{1/\lambda} \quad (12)$$

Since $\tilde{a}_i \geq e_i$, for all \tilde{a}_i , it follows that, $\tilde{a}_i \geq e_i$, and then

$$f(\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) \geq f(\langle u_1, e_1 \rangle, \dots, \langle u_n, e_n \rangle) \quad \blacksquare$$

Theorem 2 (Idempotency): Assume f is the FIGHA operator, if $\tilde{a}_i = \tilde{a}$, for all \tilde{a}_i , then:

$$f(\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \tilde{a} \quad (13)$$

Proof: Since $\tilde{a}_i = \tilde{a}$, for all \tilde{a}_i , we have

$$\begin{aligned} f(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) &= \left(\sum_{j=1}^n w_j b_j^\lambda \right)^{1/\lambda} = \\ &= \left(\sum_{j=1}^n w_j a^\lambda \right)^{1/\lambda} = \left(a^\lambda \sum_{j=1}^n w_j \right)^{1/\lambda} \end{aligned} \quad (14)$$

Since $\sum_{j=1}^n w_j = 1$, we get

$$f(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \tilde{a} \quad \blacksquare$$

Note that the FGHA and the FIGHA operator are not commutative because the weighted average (a part of the hybrid aggregation) is not commutative. They are not strictly bounded because in some particular situations [12], the aggregation may be lower than the minimum and higher than the maximum.

A further interesting issue to consider are the measures for characterizing the weighting vector W of the FGHA and the FIGHA operator such as the attitudinal character, the entropy of dispersion, the divergence of W and the balance operator. Note that these measures follow the same methodology as the original version developed for the OWA and the GOWA operator [12,26,29] but they do not affect directly the weighting vector ω . Note also that it is possible to use an hybrid version of

these measures that takes into account both weighting vectors.

4. Families of FIGHA Operators

In this Section, we analyze different families of FIGHA operators. We will distinguish between those found in the weighting vector W and those found in the parameter λ . The main advantage of using these families is that they can be very useful for the decision maker in some specific situations. However, each family is just one particular case. Therefore, they can only be used in some particular situations but they cannot be seen as a general model that can be used in different frameworks such as the general formulation explained in the previous Section. Note that these families could also be studied from the perspective of the FGHA operator.

A. Analyzing the Weighting Vector W

By using a different weighting vector in the FIGHA operator, we are able to obtain different types of fuzzy aggregation operators. For example, it is possible to obtain the fuzzy hybrid maximum, the fuzzy hybrid minimum, the fuzzy generalized mean (FGM), the fuzzy weighted generalized mean (FWGM), the FIGOWA operator and the FGHA operator.

Remark 1: The fuzzy hybrid maximum is obtained if $w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = \text{Max}\{\tilde{a}_i\}$. The fuzzy hybrid minimum is obtained if $w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = \text{Min}\{a_i\}$. More generally, if $w_k = 1$ and $w_j = 0$, for all $j \neq k$, we get for any λ , $FIGHA(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = b_k$, where b_k is the k th largest argument \tilde{a}_i . The FGM is found when $w_j = 1/n$, and $\omega_i = 1/n$, for all \tilde{a}_i . The FWGM is obtained when $w_j = 1/n$, for all \tilde{a}_i . The FIGOWA is found when $\omega_i = 1/n$, for all \tilde{a}_i . The FGHA operator is formed when the reordering of the order inducing variables u_i is equal to b_j . That is, to the reordering of the j th largest argument.

Remark 2: Following a similar methodology as it has been developed in [2-4,9,11-17,23,26-30,35], we could study other particular cases of the FIGHA operator. For example, when $w_{j^*} = 1/m$ for $k \leq j^* \leq k + m - 1$ and $w_{j^*} = 0$ for $j^* > k + m$ and $j^* < k$, we are using the window-FIGHA operator. Note that k and m must be positive integers such that $k + m - 1 \leq n$.

Remark 3: The olympic-FIGHA is found when $w_1 = w_n = 0$, and for all others $w_{j^*} = 1/(n-2)$. A general form of the olympic-FIGHA can be used considering that $w_j = 0$ for $j = 1, 2, \dots, k, n, n-1, \dots, n-k+1$; and for all others $w_{j^*} = 1/(n-2k)$, where $k < n/2$. Note that if $k = 1$, then, this general form becomes the usual olympic aggregation. If $k = (n-1)/2$, then, this general form becomes the median-FIGHA aggregation. That is, if n is odd we assign

$w_{(n+1)/2} = 1$ and $w_{j^*} = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j^*} = 0$ for all others.

Remark 4: Another type of aggregation that could be used is the E-Z FIGHA weights. In this case, we should distinguish between two classes. In the first class, we assign $w_{j^*} = (1/q)$ for $j^* = 1$ to q and $w_{j^*} = 0$ for $j^* > q$, and in the second class, we assign $w_{j^*} = 0$ for $j^* = 1$ to $n - q$ and $w_{j^*} = (1/q)$ for $j^* = n - q + 1$ to n . If $q = 1$ for the first class, the E-Z FIGHA becomes the fuzzy hybrid maximum. And if $q = 1$ for the second class, the E-Z FIGHA becomes the fuzzy hybrid minimum.

Remark 5: For the weighted median-FIGHA, we select the argument b_k that has the k th largest argument such that the sum of the weights from 1 to k is equal or higher than 0.5 and the sum of the weights from 1 to $k - 1$ is less than 0.5.

Remark 6: A further interesting family is the S-FIGHA operator based on the S-OWA operator [27]. It can be subdivided in three classes: the “orlike”, the “andlike” and the generalized S-FIGHA operator. The generalized S-FIGHA operator is obtained when $w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, the generalized S-FIGHA operator becomes the “andlike” S-FIGHA operator and if $\beta = 0$, it becomes the “orlike” S-FIGHA operator.

Remark 7: Another family of aggregation operator that could be used is the centered-FIGHA operator. We could define a FIGHA operator as a centered aggregation operator if it is symmetric, strongly decaying and inclusive. Note that these properties have to be accomplished for the weighting vector w of the FLOWA operator but not necessarily for the weighting vector ω of the FWA. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \leq (n + 1)/2$ then $w_i < w_j$ and when $i > j \geq (n + 1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider a softening of the second condition by using $w_i \leq w_j$ instead of $w_i < w_j$. We shall refer to this as softly decaying centered-FIGHA operator. Another particular situation of the centered-FIGHA operator appears if we remove the third condition. We shall refer to it as a non-inclusive centered-FIGHA operator.

Remark 8: Other families of FIGHA operators could be studied such as the Gaussian FIGHA weights, the non-monotonic FIGHA operator, etc. For more information, see for example [2-4,9,11-17,23,26-30,35].

B. Analyzing the Parameter λ

If we analyze different values of the parameter λ , we obtain another group of particular cases such as the usual FIHA operator, the fuzzy induced hybrid geometric averaging (FIHGA) operator, the fuzzy induced hybrid

harmonic averaging (FIHHA) operator and the fuzzy induced hybrid quadratic averaging (FIHQA) operator.

Remark 9: When $\lambda = 1$, we get the FIHA operator.

$$FIGHA(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \sum_{j=1}^n w_j b_j \quad (15)$$

From a generalized perspective of the reordering step we can distinguish between the DFIHA operator and the AFIHA operator. Note that if $w_j = 1/n$, for all \tilde{a}_i , we get the FWA and if $\omega_j = 1/n$, for all \tilde{a}_i , we get the FLOWA operator. If $w_j = 1/n$, and $\omega_j = 1/n$, for all \tilde{a}_i , then, we get the fuzzy average (FA).

Remark 10: When $\lambda = 0$, the FIGHA operator becomes the FIHGA operator.

$$FIGHA(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \prod_{j=1}^n b_j^{w_j} \quad (16)$$

In this case, it is also possible to distinguish between descending (DFIHGA) and ascending (AFIHGA) orders. Note that if $w_j = 1/n$, for all \tilde{a}_i , we get the fuzzy weighted geometric average (FWGA) and if $\omega_j = 1/n$, for all \tilde{a}_i , we get the fuzzy induced OWG (FIOWG) operator. If $w_j = 1/n$, and $\omega_j = 1/n$, for all \tilde{a}_i , then, we get the fuzzy geometric mean (FGM).

Remark 11: When $\lambda = -1$, we get the FIHHA operator.

$$FIGHA(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \frac{1}{\sum_{j=1}^n \frac{w_j}{b_j}} \quad (17)$$

Note that we can distinguish between the descending FIHHA (DFIHHA) operator and the ascending FIHHA (AFIHHA) operator. Note that if $w_j = 1/n$, for all \tilde{a}_i , we get the fuzzy weighted harmonic mean (FWHM) and if $\omega_j = 1/n$, for all \tilde{a}_i , we get the fuzzy induced ordered weighted harmonic averaging (FIOWHA) operator. If $w_j = 1/n$, and $\omega_j = 1/n$, for all \tilde{a}_i , then, we get the fuzzy harmonic mean (FHM).

Remark 12: When $\lambda = 2$, we get the FIHQA operator.

$$FIGHA(\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = \left(\sum_{j=1}^n w_j b_j^2 \right)^{1/2} \quad (18)$$

In this case, we also get the descending FIHQA (DFIHQA) operator and the ascending FIHQA (AFIHQA) operator. If $w_j = 1/n$, for all \tilde{a}_i , we get the fuzzy weighted quadratic average (FWQA) and if $\omega_j = 1/n$, for all \tilde{a}_i , we get the fuzzy induced OWQA (FIOWQA) operator. If $w_j = 1/n$, and $\omega_j = 1/n$, for all \tilde{a}_i , then, we get the fuzzy quadratic average (FQA).

Remark 13: Note that other families could be obtained by using different values in the parameter λ . Note also that it is possible to study these families individually. Then, we could develop for each case, a similar analysis as it has been developed in Section 3 and 4.1, where we study different properties and families of the fuzzy hy-

brid aggregation operators.

5. The Quasi-FHA and the Quasi-FIHA Operators

As it is explained in [3], it is possible to further generalize the GOWA operator by using quasi-arithmetic means. Following the same methodology as in [3,6,17], we can suggest a similar generalization to the FGHA and the FIGHA operators by using quasi-arithmetic means. We will call these generalizations the Quasi-FHA and the Quasi-FIHA operator. The Quasi-FHA is defined as follows.

Definition 7: Let Ψ be the set of FN. A Quasi-FHA operator of dimension n is a mapping $QFHA: \Psi^n \rightarrow \Psi$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$Quasi-FHA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = g^{-1} \left(\sum_{j=1}^n w_j g(b_{(j)}) \right) \quad (19)$$

where b_j is the j th largest of the \hat{a}_i ($\hat{a}_i = n\omega_i\tilde{a}_i$, $i = 1, 2, \dots, n$), $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighting vector of the \tilde{a}_i , with $\omega_i \in [0, 1]$ and the sum of the weights is 1, the \tilde{a}_i are FNs and $g(b)$ is a strictly continuous monotonic function.

The Quasi-FIHA operator extends the Quasi-FHA operator by using order inducing variables. Thus, we get a more complete fuzzy generalized aggregation operator. It can be defined as follows.

Definition 8: Let Ψ be the set of FN. A Quasi-FIHA operator of dimension n is a mapping $QFIHA: \Psi^n \times \Psi^n \rightarrow \Psi$ that has an associated weighting vector W of dimension n such that $\sum_{j=1}^n w_j = 1$ and $w_j \in [0, 1]$, then:

$$Quasi-FIHA(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = g^{-1} \left(\sum_{j=1}^n w_j g(b_{(j)}) \right) \quad (20)$$

where b_j is the \hat{a}_i value ($\hat{a}_i = n\omega_i\tilde{a}_i$, $i = 1, 2, \dots, n$) of the IOWA pair $\langle u_i, \tilde{a}_i \rangle$ having the j th largest u_i , u_i is the order inducing variable, $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighting vector of the \tilde{a}_i , with $\omega_i \in [0, 1]$ and the sum of the weights is 1, the \tilde{a}_i are FNs, and $g(b)$ is a strictly continuous monotonic function.

As we can see, we replace b^λ of the FIGHA with a general continuous strictly monotone function $g(b)$. In this case, the weights of the ascending and descending versions are also related by $w_j = w_{n-j+1}^*$, where w_j is the j th weight of the Quasi-DFIHA and w_{n-j+1}^* the j th weight of the Quasi-AFIHA operator.

Note that if the weighting vector is not normalized,

i.e., $W = \sum_{j=1}^n w_j \neq 1$, then, the Quasi-FIHA operator can be expressed as:

$$Quasi-FIHA(\langle u_1, \tilde{a}_1 \rangle, \dots, \langle u_n, \tilde{a}_n \rangle) = g^{-1} \left(\frac{1}{W} \sum_{j=1}^n w_j g(b_{(j)}) \right) \quad (21)$$

Note that all the properties and particular cases commented in the FIGHA operator, are also included in these generalizations. Thus, for example, we could mention the problem of reordering the arguments when they are FNs. In order to solve this problem, we recommend the method explained in [12-13].

6. Numerical Example

In the following, we are going to develop a numerical example in order to illustrate the new approach. We will consider a decision making problem where a company is analyzing the optimal strategy for the next period. We will use different types of FIGHA (or Quasi-FIHA) operators such as the FA, the FWA, the FOWA, the FHA and the FIHA operator.

Assume that a company that it is operating in Europe and North America is analyzing its general policy for the next year and they consider 5 possible strategies to follow.

- A_1 : Expand to the Asian market.
- A_2 : Expand to the South American market.
- A_3 : Expand to the African market.
- A_4 : Do not develop any expansion.
- A_5 : Expand to the 3 continents.

In order to evaluate these strategies, the group of experts of the company considers that the key factor for the next year is the economic situation. Then, depending on the situation, the expected benefits for the company will be different. The experts have considered 5 possible situations for the next year: S_1 = Very bad, S_2 = Bad, S_3 = Normal, S_4 = Good, S_5 = Very good. The results are shown in Table 1. Note that the results are TFNs.

Table 1. Fuzzy payoff matrix.

	S_1	S_2	S_3	S_4	S_5
A_1	(20,30,40)	(40,50,60)	(30,40,50)	(50,60,70)	(60,70,80)
A_2	(60,70,80)	(30,40,50)	(50,60,70)	(50,60,70)	(20,30,40)
A_3	(40,50,60)	(60,70,80)	(20,30,40)	(50,60,70)	(30,40,50)
A_4	(60,70,80)	(60,70,80)	(50,60,70)	(30,40,50)	(10,20,30)
A_5	(30,40,50)	(60,70,80)	(60,70,80)	(30,40,50)	(30,40,50)

In this example, we assume that the experts use the following weighting vector for all the cases of the WA and the OWA operators: $W = (0.1, 0.2, 0.2, 0.2, 0.3)$. Note that they assume the following order inducing variables: $U = (14, 20, 40, 33, 28)$.

With this information, it is possible to aggregate it in

order to take a decision. The results are presented in Table 2.

Table 2. Fuzzy aggregated results.

	FA	FWA	FOWA	FHA	FIHA
A ₁	(40,50,60)	(44,54,64)	(36,46,56)	(36,45,54)	(42,52,62)
A ₂	(42,52,62)	(38,48,58)	(38,48,58)	(36,45.5,55)	(36,46,56)
A ₃	(40,50,60)	(39,49,59)	(36,46,56)	(35,44.5,54)	(39,49,59)
A ₄	(42,52,62)	(37,47,57)	(37,47,57)	(32.5,43,53)	(35,45,55)
A ₅	(42,52,62)	(42,52,62)	(39,49,59)	(37.5,47,56.5)	(37.5,47.5,57.5)

Note that these results can also be expressed using the α -cut representation that represents the membership function of the FN as it has been explained in Section 2.1. Recall that for the TFN $A = (\underline{a}, \bar{a})$, we use the following equations:

$$\begin{aligned} \underline{a}(\alpha) &= a_1 + \alpha(a_2 - a_1), \\ \bar{a}(\alpha) &= a_3 - \alpha(a_3 - a_2). \end{aligned} \tag{22}$$

where $\alpha \in [0, 1]$ and parameterized by (a_1, a_2, a_3) where $a_1 \leq a_2 \leq a_3$, are real values.

In order to present the results in a more complete way, we calculate the intervals obtained in the α -cut representation for each decimal between $[0, 1]$. Therefore, we are able to analyze the results obtained for different degrees of membership. These results are presented in Tables 3, 4 and 5. Note that we focus on the results obtained in the FWA, the FOWA and the FIHA operator.

As we can see in Tables 2, 3, 4 and 5, depending on the aggregation operator used, the optimal strategy may be different.

A further interesting issue is to establish an ordering of the strategies. This is very useful when we want to consider more than one alternative. Note that if the ordering is not clear due to the FNs, we will have to establish a criterion for comparing FNs. As it has been explained in Section 3, we recommend following the method explained in [12-13] where we calculate an average or a weighted average of the FNs. The results are shown in Table 6.

Table 3. Fuzzy aggregated results using the FWA operator.

α	A ₁	A ₂	A ₃	A ₄	A ₅
0	(44, 64)	(38, 58)	(39, 59)	(37, 57)	(42, 62)
0.1	(45, 63)	(39, 57)	(40, 58)	(38, 56)	(43, 61)
0.2	(46, 62)	(40, 56)	(41, 57)	(39, 55)	(44, 60)
0.3	(47, 61)	(41, 55)	(42, 56)	(40, 54)	(45, 59)
0.4	(48, 60)	(42, 54)	(43, 55)	(41, 53)	(46, 58)
0.5	(49, 59)	(43, 53)	(44, 54)	(42, 52)	(47, 57)
0.6	(50, 58)	(44, 52)	(45, 53)	(43, 51)	(48, 56)
0.7	(51, 57)	(45, 51)	(46, 52)	(44, 50)	(49, 55)
0.8	(52, 56)	(46, 50)	(47, 51)	(45, 49)	(50, 54)
0.9	(53, 55)	(47, 49)	(48, 50)	(46, 48)	(51, 53)
1	54	48	49	47	52

Table 4. Fuzzy aggregated results using the FOWA operator.

α	A ₁	A ₂	A ₃	A ₄	A ₅
0	(36, 56)	(38, 58)	(36, 56)	(37, 57)	(39, 59)
0.1	(37, 55)	(39, 57)	(37, 55)	(38, 56)	(40, 58)
0.2	(38, 54)	(40, 56)	(38, 54)	(39, 55)	(41, 57)
0.3	(39, 53)	(41, 55)	(39, 53)	(40, 54)	(42, 56)
0.4	(40, 52)	(42, 54)	(40, 52)	(41, 53)	(43, 55)
0.5	(41, 51)	(43, 53)	(41, 51)	(42, 52)	(44, 54)
0.6	(42, 50)	(44, 52)	(42, 50)	(43, 51)	(45, 53)
0.7	(43, 49)	(45, 51)	(43, 49)	(44, 50)	(46, 52)
0.8	(44, 48)	(46, 50)	(44, 48)	(45, 49)	(47, 51)
0.9	(45, 47)	(47, 49)	(45, 47)	(46, 48)	(48, 50)
1	46	48	46	47	49

Table 5. Fuzzy aggregated results using the FIHA operator.

α	A ₁	A ₂	A ₃	A ₄	A ₅
0	(42, 62)	(36, 56)	(39, 59)	(35, 55)	(37.5, 57.5)
0.1	(43, 61)	(37, 55)	(40, 58)	(36, 54)	(38.5, 56.5)
0.2	(44, 60)	(38, 54)	(41, 57)	(37, 53)	(39.5, 55.5)
0.3	(45, 59)	(39, 53)	(42, 56)	(38, 52)	(40.5, 54.5)
0.4	(46, 58)	(40, 52)	(43, 55)	(39, 51)	(41.5, 53.5)
0.5	(47, 57)	(41, 51)	(44, 54)	(40, 50)	(42.5, 52.5)
0.6	(48, 56)	(42, 50)	(45, 53)	(41, 49)	(43.5, 51.5)
0.7	(49, 55)	(43, 49)	(46, 52)	(42, 48)	(44.5, 50.5)
0.8	(50, 54)	(44, 48)	(47, 51)	(43, 47)	(45.5, 49.5)
0.9	(51, 53)	(45, 47)	(48, 50)	(44, 46)	(46.5, 48.5)
1	52	46	49	45	47.5

Table 6. Ordering of the strategies.

	Ordering
FA	$A_2=A_4=A_5 \succ A_1=A_3$
FWA	$A_1 \succ A_5 \succ A_3 \succ A_2 \succ A_4$
FOWA	$A_3 \succ A_2 \succ A_4 \succ A_1=A_5$
FHA	$A_5 \succ A_2 \succ A_1 \succ A_3 \succ A_4$
FIHA	$A_1 \succ A_3 \succ A_5 \succ A_2 \succ A_4$

As we can see, we get different orderings depending on the aggregation operator used. The main advantage of this analysis is that the decision maker gets a more complete view of the different scenarios that could happen in the future depending on the method used. Although he will select the alternative that it is in accordance with his interests, he will be concerned on other potential results that could happen in the uncertain environment.

7. Conclusions

We have presented different generalizations of the hybrid aggregation operators such as the FGHA and the FIGHA operators. We have seen that they are extensions of the HA operator that use generalized means and uncertain information assessed with FNs. Moreover, we have seen that they include the WA and the OWA operator (and the IOWA) in the same formulation in a similar way than the HA operator. Particularly, these operators include the FWGM and the FIGOWA operator by using the hybrid formulation. It also includes a lot of other types of aggregation operators such as the FHA, the FHGA, the FHQA, the FGOWA and the FGM.

We have further generalized this approach by using

quasi-arithmetic means. As a result, we have obtained the Quasi-FHA and the Quasi-FIHA operators. We have also developed an application of these generalized aggregation operators in a decision making problem concerning the selection of strategies. Note that a lot of other applications could be developed with the FGHA and the FIGHA such as in statistics, operational research, engineering and economics.

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