Ecological System Control by PDC-Based Fuzzy-Sliding-Mode Controller

Chih-Min Lin, Yi-Jen Mon, Ming-Chia Li, and Daniel S. Yeung

Abstract

This paper develops a new design method of parallel-distributed-compensation (PDC) fuzzy- sliding-mode controller. This controller is then applied to control an ecological system. A new fuzzy-blending-sliding-surface based on the PDC concept is introduced. By using this sliding-surface, a PDC-based sliding-mode controller is designed for an ecological system which is a multi-input multi- output nonlinear system subject to unpredictable but bounded disturbances. The Lyapunov stability condition is achieved in this design, so that the system's stability can be guaranteed. This PDC-based fuzzy-sliding-mode controller is applied to keep biomasses of the ecological system within a small neighborhood of the unique nontrivial optimal equilibrium state. By applying this controller, the accumulative yield of harvest is better than that by using state feedback control and open loop control.

Keywords: Ecological system, Fuzzy control, Sliding-mode control.

1. Introduction

For the analysis of ecosystems which can include nonlinear phenomena such as predator switching, food limitations, saturation of predator attack capacities, etc., interaction in multispecies communities is a highly nonlinear affair [4]. The ecomodels also have to include explicitly possible effects of environmental disturbances. A great amount of effort has been devoted to the study of vulnerability and non-vulnerability of ecosystems subject to continual, unpredictable, but bounded disturbances due to changes in climatic conditions, diseases, migrating species, etc. [4, 6]. The state feedback control method has been proposed by Lee and Leitmann [6] to control the disturbed ecological system.

Fuzzy logic control has become a powerful tool in control engineering, especially for systems that are subjected to nonlinearities and unknown disturbances [2, 7]. Most of the existing works on stability analysis of multi-input multi-output (MIMO) fuzzy control systems are based on the Takagi-Sugeno (T-S) type fuzzy model [5, 12, 13, 15]. A nonlinear plant was approached by a T-S fuzzy linear model, and then a model-based fuzzy control was developed to stabilize the T-S fuzzy linear model. Some useful stability and robustness criteria for T-S type fuzzy logic control have also been developed [3, 14]. Among the aforementioned methodologies, the fuzzy-parallel-distributed-compensation (fuzzy-PDC) design approach is attractive since it is simple and natural [14]. The fuzzy-PDC control approach used in model-based systems is also shown to be an efficient and systematic method to guarantee the system to be globally stable.

Although the sliding-mode control has been addressed extensively in recent years [11], the input matrix of the MIMO system poses a challenge for sliding mode controller design. The derivation of the MIMO controller is not easy, especially for MIMO nonlinear systems, even though the final controller is very simple and has striking resemblance to its counterpart in the single-input single-output (SISO) case. Several fuzzy-sliding-mode controllers have been developed by defining a suitable sliding surface [1, 8-10]. However, these sliding surfaces are fixed so that they are not flexible enough to deal with a fuzzy PDC system which is variable based on the fuzzy inference.

By introducing a new fuzzy-blending-sliding-surface based on the PDC concept, this paper proposes an easy and effective fuzzy-sliding-mode controller for MIMO nonlinear systems. In this design, the sliding-surface is variable based on the PDC concept, so that it is more efficient for a fuzzy control system. Moreover, the system is guaranteed to be stable in the sense of Lyapunov. This controller is then applied to control an ecological system to demonstrate its effectiveness.

2. PDC-based fuzzy-sliding-mode controller design

Consider a nonlinear system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{G}(\boldsymbol{x}(t))\boldsymbol{u}(t) + \boldsymbol{d}(t) \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in R^n$ denotes the state vector, $\boldsymbol{u}(t) = [u_1(t), u_2(t), \cdots, u_n(t)]^T \in R^m$ denote the control

Corresponding Author: Chih-Min Lin. Chih-Min Lin is with the department of Electronic Engineering, Yuan Ze University, 135, Far-East Rd., Tao-Yuan, Taiwan, 320.
E-mail: cml@saturn.yau.edu.tw
Manuscript received 27 Nov. 2008; revised 25 June 2009; accepted 21 July 2009.

input, $\boldsymbol{d}(t)=[d_1(t), d_2(t),\cdots, d_n(t)]^T \in R^n$ denotes a bounded but unknown disturbance, and $\boldsymbol{f}(\boldsymbol{x}(t))\in R^n$ and $\boldsymbol{G}(\boldsymbol{x}(t))\in R^{n\times m}$ are known bounded nonlinear functions.

The system dynamics can be captured by a set of fuzzy implications which characterize local relations in the state space. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The Takagi-Sugeno fuzzy system is described by fuzzy IF-THEN rules, which locally represent the linear input-output relations of the system.

The fuzzy system is as follows: [13]

$$\begin{aligned}&\textit{Rule } i: \textit{ IF } \; x_1(t) \; \textit{is} \; M_{i1} \ldots \textit{and} \; x_n(t) \; \textit{is} \; M_{in}\\ &\textit{THEN} \quad \dot{\boldsymbol{x}}(t)=\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\boldsymbol{u}(t)\end{aligned} \tag{2}$$

where $A_i \in R^{n\times n}$ are system matrices; $\boldsymbol{B}_i \in R^{n\times m}$ are input matrices; i=1,2,...,r; r is the number of IF-THEN rules; M_{ik}; k=1,2,3,...,n are the fuzzy sets. Then the nonlinear system in (1) can be represented as the following controllable fuzzy inference system [3, 14]:

$$\begin{aligned}\dot{\mathrm{x}}(t)&=\frac{\sum_{i=1}^{r}\mu_i(\boldsymbol{x}(t))[\boldsymbol{A}_i\mathrm{x}(t)+\boldsymbol{B}_i\mathrm{u}(t)]}{\sum_{i=1}^{r}\mu_i(\boldsymbol{x}(t))}\\ &=\sum_{i=1}^{r}w_i(\mathrm{x}(t))[\mathrm{A}_i\mathrm{x}(t)+\mathrm{B}_i\mathrm{u}(t)]\end{aligned} \tag{3}$$

where

$$\mu_i(\boldsymbol{x}(t))=\prod_{\mathrm{k}=1}^{n}M_{i\mathrm{k}}(x_{\mathrm{k}}(t))\,; \tag{4}$$

$$w_i(\boldsymbol{x}(t))=\frac{\boldsymbol{\mu}_i(\boldsymbol{x}(t))}{\sum_{i=1}^{r}\boldsymbol{\mu}_i(\boldsymbol{x}(t))}\,; \tag{5}$$

and $M_{i\mathrm{k}}(x_{\mathrm{k}}(t))$ is the grade of membership of $x_{\mathrm{k}}(t)$ in $M_{i\mathrm{k}}$. In this paper, assume $\mu_i(\boldsymbol{x}(t))\geq 0$ for $i=1,2,\cdots,r$ and $\sum_{i=1}^{r}\mu_i(\boldsymbol{x}(t))>0$ for all t. Therefore, there exist $w_i(\boldsymbol{x}(t))\geq 0$ for $i=1,2,\cdots,r$ and $\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))=1$.

Equation (1) can be rewritten as

$$\begin{aligned}\dot{\boldsymbol{x}}(t)&=\boldsymbol{f}(\boldsymbol{x}(t))+\boldsymbol{G}(\boldsymbol{x}(t))\boldsymbol{u}(t)+\boldsymbol{d}(t)\\ &=\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))[\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\boldsymbol{u}(t)]\\ &\quad+[\boldsymbol{f}(\boldsymbol{x}(t))-\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))\boldsymbol{A}_i\boldsymbol{x}(t)]\\ &\quad+[\boldsymbol{G}(\boldsymbol{x}(t))-\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))\boldsymbol{B}_i\boldsymbol{u}(t)]+\boldsymbol{d}(t)\end{aligned} \tag{6}$$

Assume that the fuzzy sliding-mode controller shares the same fuzzy sets with the fuzzy system (2), the following fuzzy rules are employed to deal with the above control system:

$$\begin{aligned}&\textit{Rule } j: \textit{ IF } \; x_1(t) \; \textit{is} \; M_{j1} \ldots \textit{and} \; x_n(t) \; \textit{is} \; M_{jn}\\ &\textit{THEN } \boldsymbol{u}_j(t)=-\boldsymbol{K}\boldsymbol{x}(t)-\boldsymbol{F}\,\mathrm{sgn}(S_j(t))\end{aligned} \tag{7}$$

for $j=1,2,\cdots,r$, where $\boldsymbol{K}\in R^{m\times n}$ is a state feedback gain matrix; $\boldsymbol{F}\in R^m$ is a robust gain vector and $\mathrm{sgn}(\cdot)$ is a sign function.

The sliding surface for each linear model (2) is defined as

$$S_j(t)=\boldsymbol{C}^T[\mathrm{x}(t)-\int_0^t(\boldsymbol{A}_j-\boldsymbol{B}_j\boldsymbol{K})\boldsymbol{x}(\tau)d\tau] \tag{8}$$

where $\boldsymbol{C}\in R^n$ is a vector with positive elements and it is chosen to govern a stable sliding surface.

Hence, the overall fuzzy sliding-mode controller is given by

$$\begin{aligned}\boldsymbol{u}(t)&=\sum_{j=1}^{r}w_j(\boldsymbol{x}(t))\boldsymbol{u}_j(t)=-\boldsymbol{K}\boldsymbol{x}(t)-\boldsymbol{F}\sum_{j=1}^{r}w_j(\boldsymbol{x}(t))\,\mathrm{sgn}(S_j(t))\\ &=-\boldsymbol{K}\boldsymbol{x}(t)-\boldsymbol{F}\,\mathrm{sgn}(S(t))\end{aligned} \tag{9}$$

where a new sliding-surface is defined as

$$S(t)=\boldsymbol{C}^T[\boldsymbol{x}(t)-\int_0^t\sum_{j=1}^{r}w_j(\boldsymbol{x}(\tau))\{(\boldsymbol{A}_j-\boldsymbol{B}_j\boldsymbol{K})\boldsymbol{x}(\tau)\}d\tau] \tag{10}$$

This is a new-defined sliding surface called as fuzzy-blending-sliding-surface, since it is blended from each sliding-surface defined in (8). This is a variable sliding-surface based on the fuzzy inference.

Substituting (9) into (6), (1) becomes a closed-loop nonlinear control system

$$\begin{aligned}\dot{\mathrm{x}}(t)&=\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))[\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}]\boldsymbol{x}(t)-\sum_{i=1}^{r}w_i(\mathrm{x}(t))\boldsymbol{B}_i\boldsymbol{F}\,\mathrm{sgn}(S)\\ &\quad+[\boldsymbol{f}(\boldsymbol{x}(t))-\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))\boldsymbol{A}_i\boldsymbol{x}(t)]\\ &\quad+[\boldsymbol{G}(\boldsymbol{x}(t))-\sum_{i=1}^{r}\sum_{j=1}^{r}w_i(\boldsymbol{x}(t))w_j(\boldsymbol{x}(t))\boldsymbol{B}_i\boldsymbol{u}_j(t)]+\boldsymbol{d}(t)\\ &=\sum_{i=1}^{r}w_i(\mathrm{x}(t))[\mathrm{A}_i-\mathrm{B}_i\mathrm{K}]\mathrm{x}(t)-\sum_{i=1}^{r}w_i(\mathrm{x}(t))\mathrm{B}_i\mathrm{F}\mathrm{sgn}(S)\\ &\quad+\boldsymbol{\Delta f}(\boldsymbol{x}(t))+\boldsymbol{\Delta G}(\boldsymbol{x}(t))+\boldsymbol{d}(t)\\ &=\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))[\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}]\boldsymbol{x}(t)\\ &\quad-\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))\boldsymbol{B}_i\boldsymbol{F}\,\mathrm{sgn}(S)+\boldsymbol{D}((\boldsymbol{x}(t)))\end{aligned} \tag{11}$$

where $\boldsymbol{D}((\mathrm{x}(t)))=\boldsymbol{\Delta f}(\mathrm{x}(t))+\boldsymbol{\Delta G}(\mathrm{x}(t))+\boldsymbol{d}(t)\in R^n$, and

$$\boldsymbol{\Delta f}(\boldsymbol{x}(t))=\boldsymbol{f}(\boldsymbol{x}(t))-\sum_{i=1}^{r}w_i(\boldsymbol{x}(t))\boldsymbol{A}_i\boldsymbol{x}(t) \tag{12}$$

$$\boldsymbol{\Delta G}(\boldsymbol{x}(t))=\boldsymbol{G}(\boldsymbol{x}(t))-\sum_{i=1}^{r}\sum_{j=1}^{r}w_i(\boldsymbol{x}(t))w_j(\boldsymbol{x}(t))\boldsymbol{B}_i\boldsymbol{u}_j(t) \tag{13}$$

Although $\boldsymbol{D}((\boldsymbol{x}(t))$ may be unknown, it is assumed to be bounded by a known constant vector $\boldsymbol{D}_{\mathrm{m}}$ such that $\boldsymbol{D}_{\mathrm{m}}-|\boldsymbol{D}((\boldsymbol{x}(t))|$ has positive elements where $|\boldsymbol{D}((\boldsymbol{x}(t))|$ denotes the absolute value of each element of $\boldsymbol{D}((\boldsymbol{x}(t))$.

For any realistic controllable system, it is reasonable to suppose that there exists a robust gain vector $\boldsymbol{F}\in R^m$

such that the vector ($D_m-\sum_{i=1}^{r}w_i(x(t))B_iF$) has negative elements. Then, the following theorem can be stated and proven.

Theorem1: If the nonlinear system (1) can be expressed as the fuzzy system in (2), and the fuzzy sliding-mode controller is designed as in (9) by choosing.

i) $K\in R^{m\times n}$ such that the eigenvalues of A_j-B_jK are located on the left-hand plane;

ii) $F\in R^{m}$ to satisfy the condition ($D_m-\sum_{i=1}^{r}w_i(x(t))B_iF$) has negative elements.

Then the closed-loop nonlinear system in (11) is guaranteed to be stable.

Proof:

Define a Lyapunov function $V=\frac{1}{2}S^2$. According to (10) and (11), and the fact that C has positive elements and ($\mathbf{D}_m-\sum_{i=1}^{r}w_i(\mathbf{x}(t))\mathbf{B}_i\mathbf{F}$) has negative elements, the Lyapunov stability condition can be satisfied by the following derivation:

$$\begin{aligned}\dot V &= S(t)\dot S(t)=S(t)C^T\dot x(t)\\ &-S(t)C^T\sum_{i=1}^{r}w_i(x(t))\{(A_i-B_iK)x(t)\}\\ &=S(t)C^T\{\sum_{i=1}^{r}w_i(x(t))[(A_i-B_iK)x(t)]\\ &-\sum_{i=1}^{r}w_i(x(t))B_iF\,\mathrm{sgn}(S(t))+D\}\\ &-S(t)C^T\sum_{i=1}^{r}w_i(x(t))\{(A_i-B_iK)x(t)\}\\ &=S(t)C^T\{D-\sum_{i=1}^{r}w_i(x(t))B_iF\,\mathrm{sgn}(S(t))\}\\ &=S(t)C^TD-|S(t)|C^T\sum_{i=1}^{r}w_i(x(t))B_iF\\ &\le|S(t)|C^T\{D_m-\sum_{i=1}^{r}w_i(x(t))B_iF\}\le 0\end{aligned}\tag{14}$$

Thus, the stability of the fuzzy system in (11) is guaranteed, and the proof is completed.

The design procedure of the PDC-based fuzzy-sliding-mode controller is summarized in the following steps:

Step 1: Obtain the fuzzy system of a nonlinear system by means of the method in (2).

Step 2: Choose a state feedback gain $K\in R^{m\times n}$ such that the eigenvalues of A_j-B_jK are located on the left-hand plane.

Step 3: From (10), calculate the fuzzy-blending-sliding-Surface $S(t)$.

Step 4: Choose a vector $F\in R^{m}$ such that ($\mathbf{D}_m-\sum_{i=1}^{r}w_i(\mathbf{x}(t))\mathbf{B}_i\mathbf{F}$) has negative elements.

Step 5: Finally, a fuzzy sliding-mode controller is established by (9).

3. Model of an ecological system

Consider an exploited ecosystem model [6]

$$\begin{aligned}\dot x(t)&=g(x(t))-Hx(t)+\Delta g(x(t),v(t))+u(t)\\ x(t_0)&=x^0\end{aligned}\tag{15}$$

where $H=diag(h_1,\cdots,h_n)$ is a constant harvest matrix; $x(t)=[x_1(t),\cdots,x_n(t)]^T$, $x_i>0,\ i=1,\cdots,n$ is an n- dimensional biomass vector, its ith component representing the biomass of the ith specie at time t; $g(\cdot)$ is a continuous nonlinear function; $v(t)=[v_1(t)..v_n(t)]^T$ is the uncertainty; $\Delta g(\cdot)$ denotes a continuous unknown disturbance due to diseases, migrating species and changes in climatic conditions and $u(t)=[u_1(t),\cdots,u_n(t)]^T$ is the control input. Beside the constant harvest Hx, the control input $u(t)$ may be interpreted as the additional harvest rate of the exploited ecosystem. Only $x_i>0, i=1,\cdots,n$ are considered since they represent the biomasses. Comparing (15) with (1), it is found that $f(x(t))=g(\mathrm{x}(t))-Hx(t)$, $G(x(t))=I$ and $d(t)=\Delta g(x(t),v(t))$. A constant harvest effort vector $h=[h_1,\cdots h_n]^T$ is assumed to be unique of the corresponding non-trivial solution of

$$g(x(t))-Hx=0.\tag{16}$$

Let h^* be the admissible constant harvest effort that maximizes the quantity Hx subject to (16), and let x^* be the corresponding equilibrium state of (16).

Consider an ecological system with two competing species, a simplified model can be expressed as

$$\begin{aligned}\dot x_1(t)&=x_1(t)[1-x_1(t)-0.8x_2(t)-h_1]+v_1(t)x_1(t)+u_1(t)\\ \dot x_2(t)&=x_2(t)[1-0.25x_1(t)-x_2(t)-h_2]+v_2(t)x_2(t)+u_2(t)\\ x_1(0)&=x_1^0\\ x_2(0)&=x_2^0\end{aligned}\tag{17}$$

where h_1 and h_2 are constant harvest, v_1 and v_2 are the disturbances and u_1 and u_2 are the additional harvest.

Refer to [6], it can be found that the equilibrium states are at $x_1^*=x_2^*=0.328$ for the optimal harvest efforts $h_1=h_1^*=0.41$ and $h_2=h_2^*=0.59$.

In the system described in (17), assume the disturbances are given by

$$v_1(t)=-0.2\cos(t),\qquad v_2(t)=-0.15\cos(t).\tag{18}$$

The design of the controller is to keep both species in steady equilibrium states; meanwhile, the accumulative yield ($\hat Y_{ac}$) is maximized as much as possible for the system subject to disturbances, where $\hat{Y}_{ac}$ is defined as

$$\hat{Y}_{ac}(\boldsymbol{\tau}) = \int_0^{\tau}[h_1^* x_1(t) + h_2^* x_2(t) + u_1(t) + u_2(t)]dt \tag{19}$$

Without considering the disturbance and control, the biomasses of the ecosystem (17) with initial condition $(x_1^0, x_2^0) = (0.8915,\ \ 0.8915)$ is shown in Fig. 1. When the disturbances in (18) are included in the system (17), the biomasses without control is shown in Fig. 2.

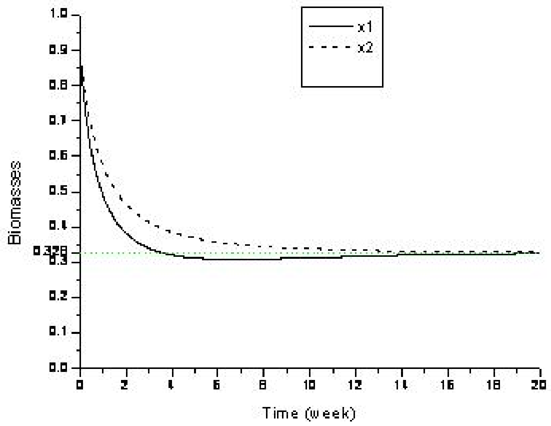

Figure 1. Biomasses of ecological system (without disturbance and without control).

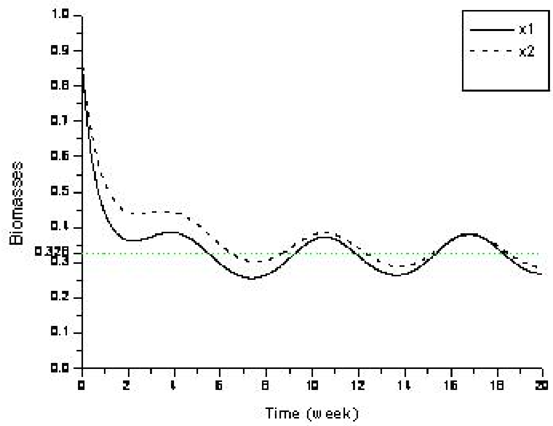

Figure 2. Biomasses of ecological system (with disturbance and without control).

4. PDC-based fuzzy-sliding-mode control of ecological system

Substituting $h_1 = h_1^* = 0.41$ and $h_2 = h_2^* = 0.59$ into (17), the disturbed system can be expressed as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{G}(\boldsymbol{x}(t))\boldsymbol{u}(t) + \boldsymbol{d}(t) \tag{20}$$

where

$$\boldsymbol{f}(\boldsymbol{x}(t)) = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}; \boldsymbol{G}(\boldsymbol{x}(t)) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$
$$\boldsymbol{d}(t) = \begin{bmatrix} v_1(t)x_1(t) \\ v_2(t)x_2(t) \end{bmatrix}; u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}; \tag{21a}$$

where

$$L_{11} = x_1(t)(l - x_1(t) - 0.8x_2(t) - 0.4l);\ L_{12} = 0;$$
$$L_{21} = 0;\ L_{22} = x_2(t)(l - 0.25x_1(t) - x_2(t) - 0.59); \tag{21b}$$

Define the state error of the ith species as

$$\tilde{x}_i = x_i - x_i^* \qquad i = 1,2\,, \tag{22}$$

where x_i^* is the equilibrium state of the ith species.

To reduce the design effort and complexity, membership functions and fuzzy rules are used as few as possible. For each fuzzy input variable $\tilde{x}_i$, two membership functions are utilized to generate four linearized models. The initial states and the equilibrium states are at $x_i^0 = 0.891$ and $x_i^* = 0.328$, respectively. Hence the variation range of $\tilde{x}_i$ will be 0.663. However, the design goal is to let $\tilde{x}_i \to 0$, so that the fuzzy models are chosen at $\tilde{x}_i = 0$ and $\tilde{x}_i = 0.3$. The block diagram of the ecological control system is shown in Fig. 3. The fuzzy sliding-mode controller is designed by following the design procedure below:

Step 1:

Because the fuzzy models are chosen at $\tilde{x}_i = 0$ and $\tilde{x}_i = 0.3$, the nonlinear system (20) can be separated as four fuzzy systems in this range. By utilizing MATLAB "linmod" command the linear state metric $\mathbf{A}_i$, *i*=1, 2, 3, 4 around the nominal point of nonlinear systems can be found easily. Four fuzzy rules are expressed as follows:

$$\begin{aligned} &Rule\ 1:\ IF\quad \tilde{x}_1 = 0\ \ and\quad \tilde{x}_2 = 0 \\ &\qquad THEN\quad \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_1\tilde{\boldsymbol{x}} + \boldsymbol{B}_1\mathrm{u} \end{aligned} \tag{23a}$$

where

$$\boldsymbol{A}_1 = \begin{bmatrix} 0.80 & 0 \\ 0 & 0.85 \end{bmatrix}; \boldsymbol{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$\begin{aligned} &Rule\ 2:\ IF\quad \tilde{x}_1 = 0.3\ \ and\quad \tilde{x}_2 = 0 \\ &\qquad THEN\quad \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_2\tilde{\boldsymbol{x}} + \boldsymbol{B}_2\boldsymbol{u} \end{aligned} \tag{23b}$$

where

$$\boldsymbol{A}_2 = \begin{bmatrix} 0.78 & 0 \\ 0 & 0.85 \end{bmatrix}; \boldsymbol{B}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$\begin{aligned} &Rule\ 3:\ IF\quad \tilde{x}_1 = 0\ \ and\quad \tilde{x}_2 = 0.3 \\ &\qquad THEN\quad \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_3\tilde{\boldsymbol{x}} + \boldsymbol{B}_3\boldsymbol{u} \end{aligned} \tag{23c}$$

where

$$\boldsymbol{A}_3 = \begin{bmatrix} 0.79 & 0 \\ 0 & 0.83 \end{bmatrix}; \boldsymbol{B}_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$\begin{aligned} &Rule\ 4:\ IF\quad \tilde{x}_1 = 0.3\ \ and\quad \tilde{x}_2 = 0.3 \\ &\qquad THEN\quad \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_4\tilde{\boldsymbol{x}} + \boldsymbol{B}_4\boldsymbol{u} \end{aligned} \tag{23d}$$

where

$$\mathbf{A}_4 = \begin{bmatrix} 0.77 & 0 \\ 0 & 0.83 \end{bmatrix}; \mathbf{B}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

In these fuzzy rules, membership functions at $\tilde{x}_1$ and $\tilde{x}_2$ are shown in Fig. 4, which denote the degrees of $\tilde{x}_1$ and $\tilde{x}_2$ to be 0 or 0.3.

Step 2:

The state feedback gain matrix is chosen as

$$\boldsymbol{K} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (24)$$

It is obvious that the closed-loop eigenvalues of $\boldsymbol{A}_i - \boldsymbol{B}_i \boldsymbol{K}$, $i = 1,2,3,4$ are at $[-0.2 \quad -0.15]$, $[-0.22 \quad -0.15]$, $[-0.17 \quad -0.20]$ and $[-0.23 \quad -0.17]$, respectively.

Step 3:

From (10), the fuzzy-blending-sliding-surface S(t) can be established as

$$S(t) = \boldsymbol{C}^T [\tilde{\boldsymbol{x}}(t) - \int_0^t \sum_{j=1}^{4} w_i(\boldsymbol{x}(\tau)) \{ (\boldsymbol{A}_j - \boldsymbol{B}_j \boldsymbol{K}) \tilde{\boldsymbol{x}}(\tau) \} d\tau] \quad (25)$$

where C is chosen as

$$C^T = [1 \quad 1] \quad (26)$$

to govern the negative slope of the sliding surface; and

$$w_i(\boldsymbol{x}(t)) = \frac{\mu_i(\boldsymbol{x}(t))}{\sum_{i=1}^{4} \mu_i(\boldsymbol{x}(t))} \quad (27)$$

where

$$\mu_i(\boldsymbol{x}(t)) = \prod_{k=1}^{2} M_{ik}(\tilde{x}_k(t)); \quad (28)$$

$M_{ik}(\tilde{x}_k(t))$ is the grade of membership of $\tilde{x}_k(t)$ in M_{ik}.

Step 4:

Choose $\boldsymbol{F}$ to overcome the disturbances and uncertainties. Referring to the disturbances v_1 and v_2, and the uncertainties of the ith subsystem caused by the approximation, $\boldsymbol{F}$ is chosen as

$$\boldsymbol{F} = [1 \quad 1]^T. \quad (29)$$

Step 5:

From (9), the control law of system (17) is designed as

$$\boldsymbol{u}(t) = -\boldsymbol{K}\tilde{\boldsymbol{x}}(t) - \boldsymbol{F}\,\mathrm{sgn}(S(t)). \quad (30)$$

In the simulations, the sign function of (30) can be replaced by a saturation function to avoid the chattering effect.

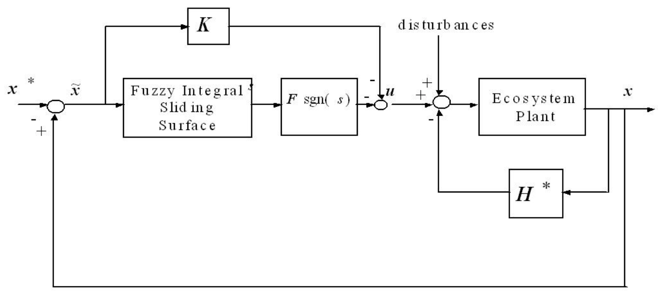

Figure 3. Fuzzy-sliding-mode control for ecological system.

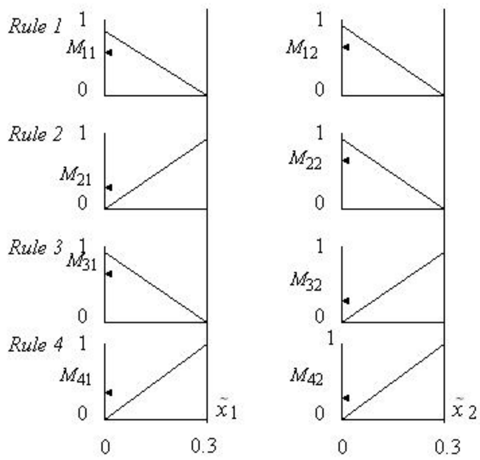

Figure 4. Example of a figure caption (8 to 10 point type).

5. Simulation

A comparison between the proposed fuzzy-sliding-mode control and the state feedback control [6] for the ecological system is made. The same parameters are used for these simulations. Considering the initial condition $(x_1^0, x_2^0) = (0.8915, \quad 0.8915)$, the simulation results for these design methods are shown in Fig. 5 and Fig. 6 for the state trajectories and control inputs. These simulation results demonstrate that the proposed fuzzy-sliding-mode controller can achieve fast response and stable steady state of the trajectories than by using the state feedback control. The comparison of accumulative yield for fuzzy-sliding-mode control, state feedback control and open-loop control is shown in Fig. 7. It is revealed that the proposed fuzzy-sliding-mode controller can obtain the best accumulative yield.

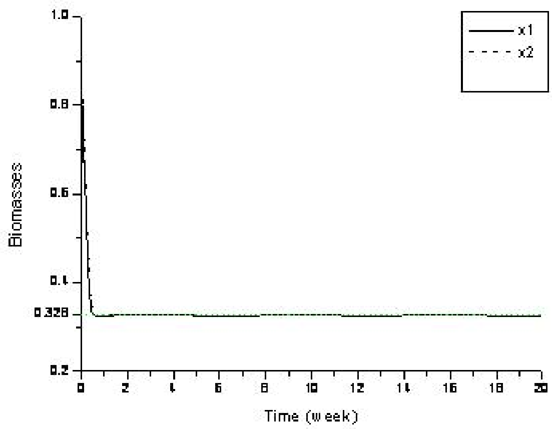

Figure 5(a). Biomasses of ecological system (with disturbance and using fuzzy-sliding-mode control).

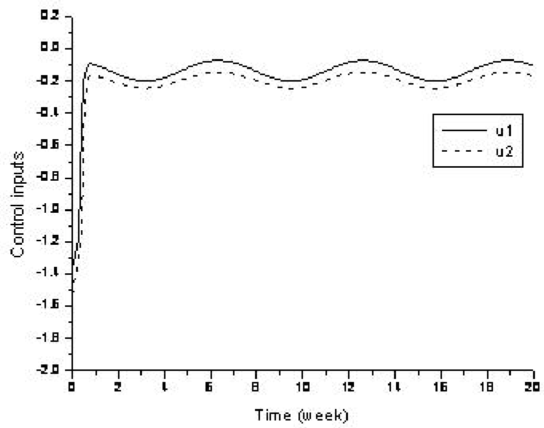

Figure 5(b). Control inputs of ecological system (with disturbance and using fuzzy-sliding-mode control).

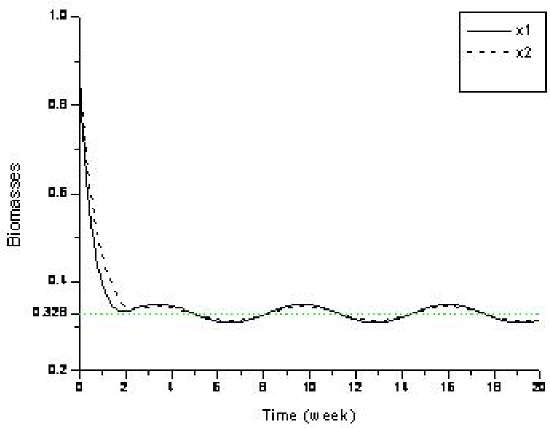

Figure 6(a). Biomasses of ecological system (with disturbance and using state feedback control).

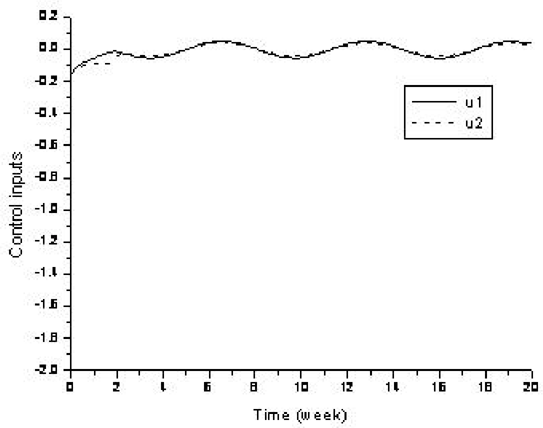

Figure 6(b). Control inputs of ecological system (with disturbance and using state feedback control).

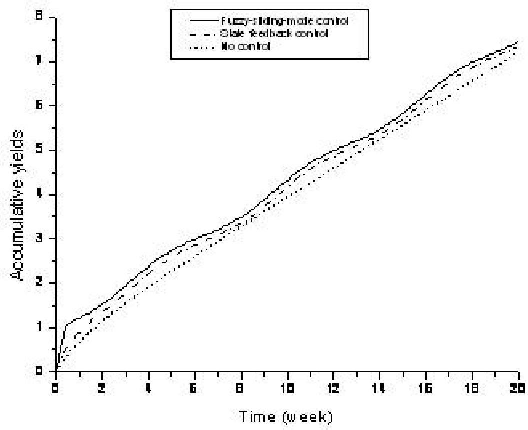

Figure 7. Accumulative yields of ecological system.

6. Conclusion

This paper considers the control of ecological systems subject to continual, unpredictable, but bounded disturbance due to changes in climatic conditions, diseases, migrating species, etc. A new fuzzy- sliding-mode controller is developed by defining a novel fuzzy-blending-sliding-surface and using a PDC design concept. This controller is then applied to control the biomasses within a small neighborhood of the unique nontrivial optimal equilibrium state of the undisturbed exploited ecosystem. Under the disturbed system, the proposed fuzzy-sliding-mode controller can achieve fast response and stable steady state of trajectories than by using the state feedback control; moreover, the accumulative yield with the fuzzy-sliding-mode controller is better than that with the state feedback control and open-loop control.

Acknowledgment

The authors appreciate valuable comments from editor and reviewers. This work was supported partially by the National Science Council of the Republic of China under Grant NSC95-2221-E-155-014-MY2.

References

[1] A. Boubakir, F. Boudjema, C. Boubakir and S. Labiod, "A fuzzy sliding mode controller using nonlinear sliding surface applied to the coupled tanks system," *Int. J. of Fuzzy Syst.,* vol. 10, no. 2, pp. 112-116, 2008

[2] S. G. Cao, N. W. Rees and G. Feng, "Stability analysis of fuzzy control systems," *IEEE Trans. Syst. Man, and Cyber*., vol. 26, pp. 201-204, 1996.

[3] B. S. Chen, C. S. Tseng, and H. J. Uang, "Robust-

ness design of nonlinear dynamic systems via fuzzy linear control," *IEEE Trans. Fuzzy Syst.*, vol. 7, pp. 571-585, 1999.

[4] B. S. Goh, *Management and analysis of biological populations*, Elsevier Science, Amsterdam, 1980.

[5] N. Golea, A. Golea, and I. Abdelmalek, "Takagi-Sugeno fuzzy systems based nonlinear adaptive control," *Int. J. Fuzzy Syst.*, vol. 8, no. 2, pp. 106-112, 2006.

[6] C. S. Lee and G. Leitmann, "On optimal long-term management of some ecological systems subject to uncertain disturbances," *Int. J. System Sci.*, vol. 14, pp. 979-994, 1983.

[7] H. X. Li and H. B. Gatland, "A new methodology for designing a fuzzy logic controller," *IEEE Trans. Syst., Man, and Cyber*., vol. 25, pp. 505-512, 1995.

[8] C. M. Lin and C. F. Hsu, "Self-learning fuzzy sliding-mode control for antilock braking systems," *IEEE Trans. Contr. Syst. Technol.*, vol. 11, no. 2, pp. 273-278, 2003.

[9] C. M. Lin and Y. J. Mon, "Decoupling control by hierarchical fuzzy sliding-mode controller," *IEEE Trans. Contr. Syst. Technol.*, vol. 13, no. 4, pp. 593-598, 2005.

[10] K. K. Shyu and H. J. Shieh, "A new switching surface sliding-mode speed controller for induction motor drive systems," *IEEE Trans. Power Elect*., vol. 11, pp. 660-667, 1996.

[11] J-J. E. Slotine and W. Li, *Applied Nonlinear Control*, Englewood Cliffs, NJ: Prentice-Hall, 1991.

[12] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern*., vol. 15, pp. 116-132, 1985.

[13] K. Tanaka and M. Sugeno, "Stability analysis and design of fuzzy control systems," *Fuzzy Sets and Syst*., vol. 45, pp. 135-156, 1992.

[14] H. O. Wang, K. Tanaka and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: stability and design issues," *IEEE Trans. Fuzzy Syst*., vol. 4, pp. 14-23, 1996.

[15] W. Y. Wang, Y. H. Chien and I. H. Li, " An on-line robust and adaptive T-S fuzzy-neural controller for more general unknown systems," *Int. J. of Fuzzy Syst.,* vol. 10, no. 1, pp. 33-43, 2008.