A Novel Fuzzy Ant Colony System for Parameter Determination of Fuzzy Controllers

C.-W. Tao, J.-S. Taur, J.-T. Jeng, and W.-Y. Wang

Abstract

In this paper, a novel fuzzy ant colony system (FACS) with a fuzzy mechanism and a fuzzy probable mechanism is presented for parameter determinations. Based on the fuzzy rules, the transition behavior of ants is simulated. The fuzzy probable mechanism is introduced with fuzzy probable rules to implement the diverse searching. The fuzzy probable rules are proposed to have the fuzziness in the antecedent parts and the probability in the consequent parts. To indicate the effectiveness, the fuzzy ant colony system is applied to find the proper parameters of the fuzzy sliding controllers for swinging and balancing the inverted pendulum and cart system. Also, the comparisons between the proposed fuzzy ant colony system and other ant colony optimization algorithms are provided in the simulations.

Keywords: Ant Colony System, Optimization Problems, Fuzzy Control, Sliding Mode Control, Inverted Pendulum and Cart System.

1. Introduction

In recent years, many different global optimization methods based on natural laws and behaviors, such as ant colony optimization algorithms, particle swarm optimization algorithms, genetic algorithms and neural network algorithms [1-5], have attracted lets of attentions and have been applied to find the best solutions in various complex scientific and engineering systems.

The ant colony optimization algorithm is a meta-heuristic optimization approach inspired by the foraging behavior of real ants. Based on the laying and detection of pheromone concentration, an indirect communication called stigmergy [6] is carried out by ants to find the shortest path from the nest to a food source. The ant system (AS) is proposed by Dorigo to find the optimal solution of the Traveling Salesman Problem [7]. It is found in the applications of the AS algorithm that the AS algorithm is easy to cause the searching trapped in a local solution [8]. To alleviate the difficulty in the AS algorithm, the ant colony system (ACS) is presented in [8]. Also, the Max-Min Ant System (MMAS) is introduced in [9] to improve the performance of the AS algorithm.

Since the fuzzy logic techniques [10-12],[21] consist of linguistic rules based on simple concepts and experiences, fuzzy logic techniques might be a good alternative to describe and simulate the foraging behavior of ants [13]. The fuzzy ant is proposed by Rozin and Margliot to obtain the probability of choosing either left or right branch with a simple fuzzy mechanism [13]. With the probability acquired using the simple fuzzy mechanism, the stochastic and the averaged models for the ant colony are indicated to result in a more analytical and feasible description of the ant behavior [13]. The fuzzy ant-based clustering algorithm is designed in [14] to improve the data partition with the fuzzy C-means algorithm.

In general, the parameter determination of fuzzy controllers is not trivial and the trial and error techniques are quite time consuming. The ant colony optimization algorithm has been well utilized to search the best control action from the candidate actions for each rule in the fuzzy-Q learning controller [15] and the fuzzy controller [16]. Also, the rule base of the fuzzy controller is reduced and optimized with the ant colony optimization algorithm in [17].

According to the above discussion, a novel fuzzy ant colony system (FACS) algorithm is proposed to have the spirit of the foraging behavior of ants for the parameter determination. Based on the traditional ACS algorithm, new fuzzy transition probability rules are designed to determine the transition path for ants. The fuzzy transition probability rules consist of the fuzzy mechanism and the fuzzy probable mechanism for different cases. The fuzzy probable mechanism is introduced with the fuzzy probable rules to implement the diverse searching. The fuzzy probable rules are proposed to have fuzziness in the antecedent parts and fuzziness with probability in the consequent parts. For example, a fuzzy probable rule would have the following form:

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If x_1 *is* A_1, x_2 *is* A_2, …, x_n *is* A_n, *then* y *is probably B.*

Since the experts might not be very sure about all the factors (in the antecedent parts) that would affect the results (in the consequent parts) then the results would be expected to have chance to be different than the results in the experts' mind. However, the more fit to the antecedent part, the less chance would have for the unexpected results. To describe this usual situation in the real life, the fuzzy probable rules seem to be reasonable, especially for the simulation of the behavior of ants. To indicate the effectiveness, the fuzzy ant colony system is applied to find the proper parameters of the fuzzy sliding controllers for swinging-up and balancing the inverted pendulum and cart system. Also, the comparisons between the proposed fuzzy ant colony system and other ant colony optimization algorithms are provided in the simulations.

The paper is organized as follows. In Section Ⅱ, the basic concepts of ACS algorithm is reviewed, respectively. The design of the proposed FACS algorithm is presented in Section III. Section IV provides the algorithm for the proper determination of the parameters of the fuzzy sliding mode controllers with FACS. In Section V, the fuzzy hierarchical swing-up and sliding position controller is introduced. Simulation results and comparisons are included in Section VI. SectionVII states the conclusion.

2. Basic Concepts of ACS

The inspiration of the Ant Colony System (ACS) is taken from the social behavior of ants [8][7]. The artificial ant based on the biological knowledge and techniques is designed to help parameter determination in different engineering, control and computational area. By the experiments on observing the ants' behavior, M. Dorigo found that the ant will secrete the pheromone while moving on the route. The ants could communicate indirectly with others via the pheromone. In this section, the basic ACS framework was reviewed.

A. *Transition Probability Rule*

Ants prefer to moving from one node through the short edges with a high amount of pheromone to another node. According to this observation, the main idea of the ACS is based on the ant transition probability rule to select the next node. Let d_{ij} being the Euclidean distance between city *i* and city *j*. $\eta_{ij} = d_{ij}^{-1}$ is the visibility value of moving from city *i* to city *j*. As in ACS [8], the transition probability rule for the ant k at the city *i* to select the city *j* as the next city is

$$j = \begin{cases} \arg\max_{l \in N_i^k} \left\{ [\tau_{il}]^{\alpha} [\eta_{il}]^{\beta} \right\}, & if \quad q \le q_0 \\ \gamma, & if \quad q > q_0 \end{cases} \tag{1}$$

with

$$P_{ij}^k(t) = \frac{[\tau_{ij}]^{\alpha} [\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k} [\tau_{il}]^{\alpha} [\eta_{il}]^{\beta}}, \quad if \quad j \in N_i^k \tag{2}$$

where q is a random variable uniformly distributed in [0,1], q_0 is a predefined parameter ($0 \le q_0 \le 1$), and γ is a random variable selected with the roulette wheel selection method according to the probability distribution given in Eq.2. N_i^k is a set of feasible cities to be visited by the ant k at the city *i* (the set of feasible cities is updated for each ant after every movement). It can be seen that to select the city *j* as the next city with the transition probability rule is based on the amount of the pheromone concentration τ_{ij} and the visibility η_{ij}. The parameters α and β represent the relative importance of the pheromones concentration and the visibility value.

B. *Local Adaptive rule*

The main goal of the local adaption is to diversify the search by decreasing the pheromone concentration on the travelled edges. In the search process of ants, the pheromone concentration on the path from city *i* to city *j* is updated with the local adaptive rule,

$$\tau_{ij}(t+1) = (1-\rho_l)\tau_{ij}(t) + \rho_l \tau_0 \tag{3}$$

where $\rho_l \in (0,1]$ is the pheromone evaporation coefficient, and τ_0 is the initial value of pheromone concentration.

C. *Global Adaptive Rule*

When all ants have completed their tours, ACS algorithms allow the best ant (i.e., the ant with the shortest route) to update the pheromone. This adaptive rule is known as the global adaptive rul*e* with

$$\tau_{ij}(t+1) = (1-\rho_g)\tau_{ij}(t) + \rho_g \Delta\tau_{ij}^{best} \tag{4}$$

and $\Delta\tau_{ij}^{best}$ is:

$$\Delta\tau_{ij}^{best} = \frac{Q}{L^{best}} \tag{5}$$

where $\rho_g \in (0,1]$ is the pheromone evaporation coefficient of global adaptive rule, $\Delta\tau_{ij}^{best}$ is pheromone increment of the best ant in each iteration, Q is a constant relative to the pheromones level released by ants, and L^{best} is described as the length of the best route(shortest route).

3. Fuzzy Ant Colony System Algorithm

In this section, the design of the Fuzzy Ant Colony System (FACS) is described. As in the ACS, the pheromone is locally updated to increase the exploration capability of the FACS. Also, the pheromone is globally updated for FACS to emphasize the best paths for the next iteration. To design a fuzzy ant colony system, the new fuzzy transition probability rules are presented. Let the pheromone on the path between the city *i* and city *j* be denoted as τ_{ij} and the visibility on the path between the city *i* and city *j*, be η_{ij}. With the spirit of the transition probability rules for ACS, the fuzzy transition probability rules for FACS are proposed as

$$J = \begin{cases} J_{fm}, q \le q_0 \\ J_{fpm}, q > q_0 \end{cases} \tag{6}$$

where J_{fm} is the output of the fuzzy mechanism FM, J_{fpm} is the output of the fuzzy probable mechanism FPM. q is a random variable uniformly distributed in [0,1], q_0 is a predefined parameter ($0 \le q_0 \le 1$).

A. *Fuzzy Mechanism (FM)*

In the fuzzy mechanism (FM), an incomplete fuzzy rule base with simple fuzzy rules is constructed. Since the ants are assumed to take the pheromone concentration and the visibility on the paths into account for the decision on the next city to visit, the fuzzy rule for the ant *k* at the city *i* would select the city *j* to be the next city to visit can be expressed as

$$\begin{aligned} &Rule\ j: If\ \tau_{ij}\ is\ relative\ A_{j1}\ and\ \eta_{ij}\ is\ relative\ A_{j2}, \\ &\quad then\ the\ next\ city\ s\ is\ j. \end{aligned} \tag{7}$$

where A_{j1} and A_{j2} ($1 \le j \le n$, $j \ne i$,) are the fuzzy sets large and Large defined on the input universe of discourse *U*. The parameter *n* represents the number of cities in N_i^k. N_i^k is a set of feasible cities that remain to be visited by the ant *k* at the city *i* (the set of feasible cities is updated for each ant after every movement). Also, from *Rule j*, it is easy to see that output fuzzy sets are fuzzy singletons. In the traditional transition probability rule (see Eq.1), the parameters $\alpha \ge 1$ and $\beta \ge 1$ represent the significance degrees of the pheromone and the visibility, respectively. The larger α (β) is, the factor pheromone (visibility) is considered to be more significant. To consider the effectiveness of the parameters α and β in the fuzzy transition probability rule, the membership functions of the fuzzy set $relative\ A_{j1}$ and $relative\ A_{j2}$ can be defined as

$$\begin{aligned} \mu_{relative\ A_{j1}}(\tau_{ij}) &= [\mu_{A_{j1}}(\tau_{ij})]^{\alpha} \\ \mu_{relative\ A_{j2}}(\eta_{ij}) &= [\mu_{A_{j2}}(\eta_{ij})]^{\beta} \end{aligned} \tag{8}$$

Thus, the appropriate degree of each rule can be defined as

$$\begin{aligned} F_{ij} &= \mu_{relative\ A_{j1}}(\tau_{ij}) \cdot \mu_{relative\ A_{j2}}(\eta_{ij}) \\ &= [\mu_{A_{j1}}(\tau_{ij})]^{\alpha} \cdot [\mu_{A_{j2}}(\eta_{ij})]^{\beta}. \end{aligned} \tag{9}$$

As in the traditional transition probability rule, when $q \le q_0$, ants will select a path on which the pheromone concentration and visibility is relatively large. That is, the ants will select the city *j* if $\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta} = \max_{l \in N_i^k} \{\tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}\}$. Since the output fuzzy sets are fuzzy singletons, the next city selected with the maximum defuzzification would be $j = s^*$, such that

$$[\mu_{A_{s^*1}}(\tau_{is^*})]^{\alpha} \cdot [\mu_{A_{s^*2}}(\eta_{is^*})]^{\beta} = \sup_{s \in N_i^k} [\mu_{A_{s1}}(\tau_{is})]^{\alpha} \cdot [\mu_{A_{s2}}(\eta_{is})]^{\beta} \tag{10}$$

where N_i^k is a set of feasible cities to be visited by the ant *k* at the city *i*. It can be seen that if $\mu_{A_{j1}}(\tau_{ij}) = \tau_{ij}$ and $\mu_{A_{j2}}(\eta_{ij}) = \eta_{ij}$, the ant with the fuzzy transition probability rule would select the same next city as the ant with the traditional transition probability rule when $q \le q_0$.

B. *Fuzzy Probable Mechanism (FPM)*

To implement the searching diversity of the traditional transition probability rule, a fuzzy probable mechanism is introduced in this subsection. In the traditional transition probability rule, the next city γ for the ant *k* at city *i* is randomly selected with the roulette wheel selection approach according to the probability P_{ij}^k in Eq.2, when $q > q_0$. The purpose of random selection mechanism is to diverse the searching behavior of the ants, and to enhance the possibility of finding the optimal solution. Unlike the process for $q \le q_0$, the city *j* with $\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta} = \max_{l \in N_i^k} \{\tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}\}$ is not necessary to be selected as the next city. The city *j* with $\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta} = \max_{l \in N_i^k} \{\tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}\}$ just has the largest chance to be selected as the next city. With these observations, the fuzzy rule in Eq.7 can be revised as

$$\begin{aligned} &Rule\ j: If\ \tau_{ij}\ is\ relative\ A_{j1}\ and\ \eta_{ij}\ is\ relative\ A_{j2}, \\ &\quad then\ the\ next\ city\ s\ is\ probably\ j. \end{aligned} \tag{11}$$

Since the rule in Eq.11 combines the fuzziness in the antecedent part and the fuzziness with probability in the consequent part, it is called a fuzzy probable rule. Then the probable appropriate degree of a fuzzy probable rule can be obtained to be the product of the appropriate degree from the antecedent part and the probability from the consequent part. Let the membership functions of the fuzzy set $relative\ A_{j1}$ and $relative\ A_{j2}$ be defined as in Eq.8. With the normalized appropriate degree

$$F_{ij}{}^{k} = \frac{\mu_{relative\ A_{j1}}(\tau_{ij}) \cdot \mu_{relative\ A_{j2}}(\eta_{ij})}{\sum_{j=1}^{n} \mu_{relative\ A_{j1}}(\tau_{ij}) \cdot \mu_{relative\ A_{j2}}(\eta_{ij})} = \frac{[\mu_{A_{j1}}(\tau_{ij})]^{\alpha} \cdot [\mu_{A_{j2}}(\eta_{ij})]^{\beta}}{\sum_{j=1}^{n} [\mu_{A_{j1}}(\tau_{ij})]^{\alpha} \cdot [\mu_{A_{j2}}(\eta_{ij})]^{\beta}} \tag{12}$$

and the probability of selecting city *j* as the next city for the ant *k* at city *i*, $\tilde{P}_{ij}{}^{k}$, the probable appropriate degree of the fuzzy probable rule in Eq.11 is

$$\begin{aligned} \tilde{F}_{ij}{}^{k} &= \frac{\mu_{relative\ A_{j1}}(\tau_{ij}) \cdot \mu_{relative\ A_{j2}}(\eta_{ij})}{\sum_{j=1}^{n} \mu_{relative\ A_{j1}}(\tau_{ij}) \cdot \mu_{relative\ A_{j2}}(\eta_{ij})} \cdot \tilde{P}_{ij}{}^{k} \\ &= \frac{[\mu_{A_{j1}}(\tau_{ij})]^{\alpha} \cdot [\mu_{A_{j2}}(\eta_{ij})]^{\beta}}{\sum_{j=1}^{n} [\mu_{A_{j1}}(\tau_{ij})]^{\alpha} \cdot [\mu_{A_{j2}}(\eta_{ij})]^{\beta}} \cdot \tilde{P}_{ij}{}^{k} \end{aligned} \tag{13}$$

To obtain $\tilde{P}_{ij}{}^{k}$, one random number $0 \le r_l \le 1$ is generated for each possible city *l* in N_i^k. Then

$$\tilde{P}_{ij}^{k} = \frac{r_j}{\sum_{l=1}^{n} r_l} \tag{14}$$

The maximum defuzzification is applied again to have the next city $j = s^*$ with the fuzzy probable mechanism when $q > q_0$, and

$$\begin{aligned} &\frac{[\mu_{A_{s^*1}}(\tau_{is^*})]^{\alpha} \cdot [\mu_{A_{s^*2}}(\eta_{is^*})]^{\beta}}{\sum [\mu_{A_{s^*1}}(\tau_{is^*})]^{\alpha} \cdot [\mu_{A_{s^*2}}(\eta_{is^*})]^{\beta}} \cdot \tilde{P}_{is^*}{}^{k} \\ &= \sup_{s \in N_i^k} \frac{[\mu_{A_{s1}}(\tau_{is})]^{\alpha} \cdot [\mu_{A_{s2}}(\eta_{is})]^{\beta}}{\sum [\mu_{A_{s1}}(\tau_{is})]^{\alpha} \cdot [\mu_{A_{s2}}(\eta_{is})]^{\beta}} \cdot \tilde{P}_{is}{}^{k}. \end{aligned} \tag{15}$$

That is, $\tilde{F}_{is^*}^{k} = \sup_{s \in N_i^k} \tilde{F}_{is}^{k}$. Likewise, if $\mu_{A_{j1}}(\tau_{ij}) = \tau_{ij}$, $\mu_{A_{j2}}(\eta_{ij}) = \eta_{ij}$, the city *j* with $\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta} = \max_{l \in N_i^k} \{\tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}\}$ can be considered to have largest chance to be selected as the next city. Therefore, the ant with the fuzzy probable mechanism would have the capability of diverse searching as the ant with traditional transition probability rules when $q > q_0$.

C. *Local Update Rule*

As in the ACS, the main goal of the local update rule is to diversify the search by decreasing the pheromone concentration on the travelled edges, and the pheromone is locally updated to increase the exploration capability of the FACS. The local update rule in Eq.3 is also adopted for the FACS.

D. *Global Update Rule*

The best path is emphasized for the next iteration with the pheromone globally updated for FACS. The global update rule for FACS is defined to be the same as Eq.4.

To indicate the effectiveness of the proposed fuzzy ant colony system algorithm (FACS), the FACS is applied to find the effective parameters for the fuzzy sliding controllers to swing and balance the inverted pendulum and cart system in the next sections.

4. Parameter Determination with the FACS

With the FACS algorithm, a discrete solution space is considered because the path selections of an ant in each step are limited. The approach to generate nodes and paths in [18] are used to construct the search network for ants, and the diagram (as in [18]) of the sample nodes and paths is shown in Fig.1. Assume that the essential part of a fuzzy system is a knowledge base consisting of *m* fuzzy IF-THEN rules, and the output fuzzy sets are fuzzy singletons. Let the parameters of the consequent part of the fuzzy controller be p_1, p_2, …, p_r. For each parameter p_l, $1 \le l \le r$, there are two digits in front of the decimal point and three digits behind the decimal point. The parameters p_l, $1 \le l \le r$ are taken as the variables with five digits to be optimized by the FACS algorithm.

As shown in Fig.1, XY plane is gridded to have 10*z nodes with lines, $L_1, L_2, \cdots, L_z$ perpendicular to X axis and the nodes on each line representing the number $0, 1, 2, \ldots, 9$. The number of line, $z = r * 5$ is equal to the number of parameters times the number of digits for each parameter. Ants start from the origin point and move through line L_1 to line L_z. In each step, ants only allow to choose one node on each line L_i, and one circulation is completed when all ants move to the line L_z. As indicated in Fig.1, the selected nodes on the $[5(l-1)+1]^{th}$ to the $5l^{th}$ lines represent the five digits of the parameter p_l. After a circulation, the moving path of k^{th} ant can be represented by $Path^k = \{(0, y_{j(0)}), (1, y_{j(1)}^k), (2, y_{j(2)}^k), \ldots, (i, y_{j(i)}^k), \ldots, (z, y_{j(z)}^k)\}$, where $(i, y_{j(i)}^k)$ represents the $j(i)^{th}$ node of the i^{th} line selected by the ant *k*. The parameter $y_{j(i)}^k$ indicates the value of the node $(i, y_{j(i)}^k)$. Note that when $i = 0$, the ant is at the origin $(0,\, y^k_{j(0)})$ and $y^k_{j(0)} = 0$.

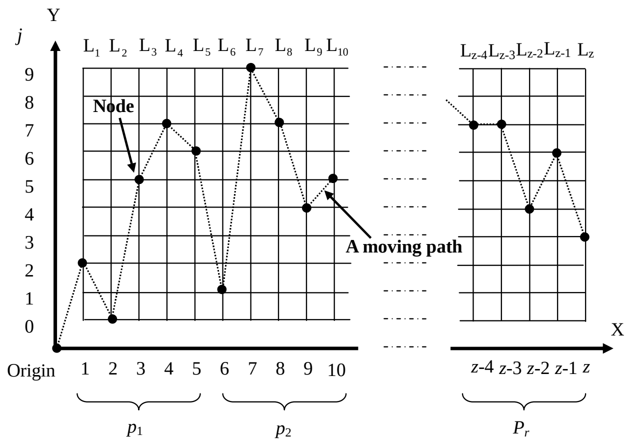

Fig.1. Diagram of a sample set of nodes and paths.

With the $Path^k$ gone through by the k^{th} ant, the parameters $p_1^k,\ p_2^k,\ \ldots,\ p_r^k$ are calculated as

$$\begin{cases} p_1^k = y^k_{j(1)}\cdot 10^1 + y^k_{j(2)}\cdot 10^0 + y^k_{j(3)}\cdot 10^{-1} + y^k_{j(4)}\cdot 10^{-2} + y^k_{j(5)}\cdot 10^{-3} \\ \quad\vdots \\ p_r^k = y^k_{j(z-4)}\cdot 10^1 + y^k_{j(z-3)}\cdot 10^0 + y^k_{j(z-2)}\cdot 10^{-1} + y^k_{j(z-1)}\cdot 10^{-2} + y^k_{j(z)}\cdot 10^{-3} \end{cases} \tag{16}$$

These parameters are taken for the consequent parts of the fuzzy controller. To evaluate the effectiveness of the fuzzy controller with the parameters obtained using the proposed FACS, the sum of the absolute error (*AE*)

$$AE = \sum |e| \tag{17}$$

is used as the performance criterion. The parameter determination process with the FACS is illustrated in Fig.2, and is summarized as follows.

Step1: Initialize the parameters for FACS.

Step2: Set the iteration number I_t to be one.

Step3: Set the line index $i = 0$.

Step4: Set the ant index $k = 1$.

Step5: Calculate the visibility of the path between the node $(i,\, y^k_{j(i)})$ on line L_i and the nodes on line L_{i+1}. The formula for the visibility in [18] is adopted,

$$\eta((i,\, y^k_{j(i)}),(i+1,\eta)) = \frac{10 - \left|\eta - y^*_{j(i+1)}\right|}{10},\ 0 \le \eta \le 9,\ \eta \in Z \tag{18}$$

where $(i+1, \eta)$ represents the η^{th} node on line L_{i+1}, $0 \le \eta \le 9$ is an integer, and the $y^*_{j(i+1)}$ is the value of the node with the best performance on line L_{i+1} in previous iterations.

Step6: For the ant *k* at the $node(i,\, y^k_{j(i)})$ on line L_i, the next node on line L_{i+1} is determined with the fuzzy mechanism when $q \le q_0$. However, when $q > q_0$, the next node is determined according to the proposed fuzzy probably mechanism to diversify the searching.

Step7: The local information is updated. The local update rule in Eq.3 can be revised as

$$\tau((i,\, y^k_{j(i)}),\,(i+1,\, y^k_{j(i+1)})) = (1-\rho_l)\tau(i+1,\, y^k_{j(i+1)}) + \rho_l \tau_0$$

Step8: If the ant *k* has completed the travel from line L_i to the line L_{i+1}, and $k \ne N_a$, then go back to Step5 with $k = k+1$. Otherwise, go to next step.

Step9: If all the ants have completed the travel to the line L_z, then go to Step10, otherwise go back to step4 with $i = i+1$.

Step10: Calculate the parameters $p_1^k,\ p_2^k,\ \ldots,\ p_r^k$, for $1 \le k \le N_a$.

Step11: Substitute the parameters $p_1^k,\ p_2^k,\ \ldots,\ p_r^k$ into the fuzzy controller, and evaluate the parameters $p_1^k,\ p_2^k,\ \ldots,\ p_r^k$, $1 \le k \le N_a$ with the performance criterion in Eq.17.

Step12: If the stop condition is satisfied, then stop the FACS. Otherwise, go to Step13.

Step13: The pheromone is globally updated for FACS to emphasize the best path for the next iteration. Let ($p_1^{k^*},\ p_2^{k^*},\ \ldots,\ p_r^{k^*}$) represent the parameters with the best performance at the iteration I_t. The pheromone on $Path^{k^*}$ is globally updated with the following rule,

$$\tau((i,\, y^{k^*}_{j(i)}),\,(i+1,\, y^{k^*}_{j(i+1)})) = (1-\rho_g)\cdot\tau(i+1,\, y^{k^*}_{j(i+1)}) + \rho_g \cdot \Delta\tau^{best} \tag{19}$$

where $\Delta\tau^{best} = \dfrac{Q}{AE^{best}_{I_t}}$, and $AE^{best}_{I_t}$ is the smallest sum of the absolute errors in the iteration I_t.

Step14: If the maximum iteration number is reached, stop the FACS. Otherwise go back to step3 for next iteration with $I_t = I_t + 1$.

5. Fuzzy Hierarchical Swing Up and Sliding Position Controller for the Inverted Pendulum-Cart System

In order to have the FACS applied to find the proper parameters of the fuzzy hierarchical swing up and sliding position controller (FHSSC) in [12] for the inverted pendulum-cart system, the description for the design of the fuzzy hierarchical swing-up and sliding position controller is abbreviated in this section. The detail design of the fuzzy hierarchical swing up and sliding position controller for inverted pendulum and cart system is provided in our previous paper [12].

Let $x = x_1$ and $\dot{x} = \dot{x}_1 = x_3$ be the cart position and the cart velocity, $\theta = x_2$ and $\dot{\theta} = \dot{x}_2 = x_4$ be the pendulum angle and the angle velocity. $F = u$ is a control action of the system, m_c and m_p are masses of the cart and the pendulum, respective, g is the gravity acceleration, L is half distance between rotation axis of the pendulum, and $T_c = T_{cc} x_3$ is the friction force between the cart and rail.

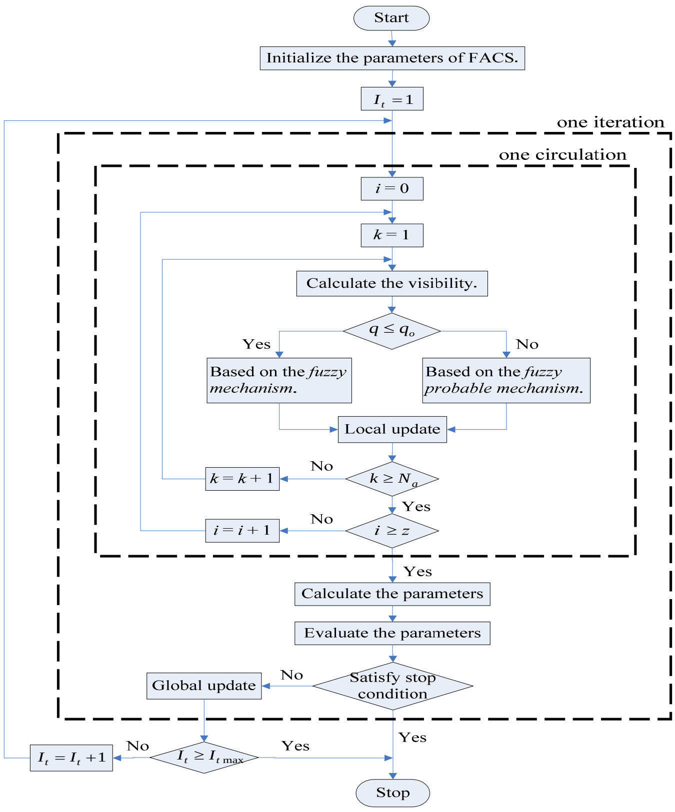

Fig.2. Parameter optimization flow diagram of the FACS.

The state equation of nonlinear inverted pendulum and cart system can be obtained as,

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = \frac{a(u - T_c - \mu x_4^2 \sin x_2) + l \cos x_2 (\mu g \sin x_2 - f_p x_4)}{J + \mu l \sin^2 x_2}$$

$$\dot{x}_4 = \frac{l \cos x_2 (u - T_c - \mu x_4^2 \sin x_2) + \mu g \sin x_2 - f_p x_4}{J + \mu l \sin^2 x_2} \tag{20}$$

and $J = m_p(4m_c + m_p)L^2 / 12(m_c + m_p)$, $l = m_p L / 2(m_c + m_p)$, $a = l^2 + \frac{J}{m_c + m_p}$, $\mu = l(m_c + m_p)$, and f_p is a constant.

The framework of the fuzzy hierarchical swing-up and sliding position controller for the inverted pendulum-cart control system with the FACS is presented in Fig.3. The fuzzy swing-up controller is designed for the swing-up control of a pendulum. Also, with the combination of sliding mode control and fuzzy control technologies, twin fuzzy sliding mode controllers are proposed to position the pendulum and cart separately. Moreover, in order to softly switch between the swing-up controller and twin fuzzy sliding mode controllers, a high level fuzzy switching controller is provided.

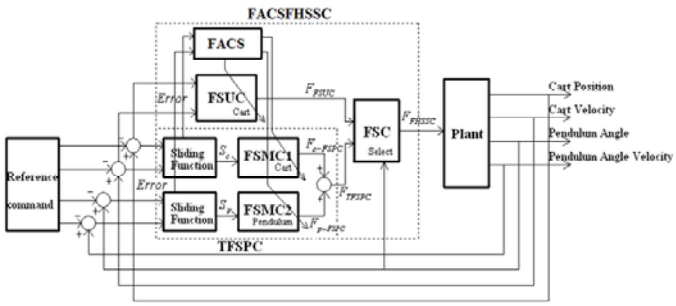

Fig.3. FHSSC with the Fuzzy Ant Colony System.

A. *Fuzzy Switching Controller (FSC)*

For the higher level fuzzy switching controller (FSC), the error of the pendulum angle e_{p2} is adopted as the input variable. The input and output membership functions are shown in Fig.4, where $\tilde{k}_{s1} = F_{FSUC}$, and $\tilde{k}_{s2} = F_{TFSPC}$. With the fuzzy sets indicated in Fig.4 the complete fuzzy rule base is presented in Table I.

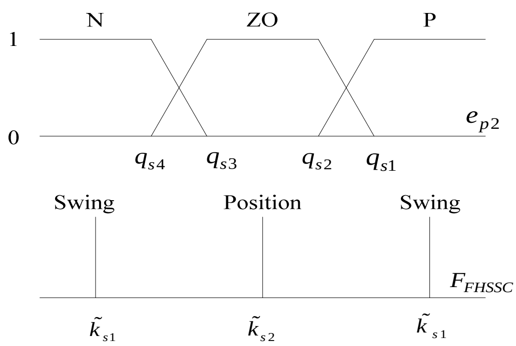

Fig.4. Membership Functions for the FSC.

Table I. Fuzzy Rule Base of FSC.

e_{p2}	N	ZO	P
F_{FHSSC}	Swing	Position	Swing

Based on the centroid defuzzification and sum-product fuzzy inference [19], the output of the fuzzy switching controller, F_{FHSSC}, is defined as

$$F_{FHSSC} = \frac{\sum_{l=1}^{3} M_l(e_{p2}) g_l}{\sum_{l=1}^{3} M_l(e_{p2})}, \; g_l \in \left\{ \tilde{k}_{si} \mid i = 1 \sim 2 \right\} \tag{21}$$

B. *Fuzzy Swing-Up Controller (FSUC)*

It is found that to consider the control of the pendulum and cart separately would be easier for the design of a controller with the rule base mechanism. However, there is only one control action allowed for the inverted pendulum-cart system. Therefore, the control action F_p for the pendulum subsystem and the control action $\tilde{F}_c$ for the cart subsystem need to be combined into one control action F for the inverted pendulum-cart system. The instinctive knowledge indicates that the control actions to move the cart and pendulum to the same direction have opposite sign. Since the main purpose for the position control of the inverted pendulum-cart system is to balance the pendulum at the straight upward direction, the combination of F_p and $\tilde{F}_c$ is defined as $F = F_p - \tilde{F}_c$. In our approach for swinging up the pendulum, only the control of moving the cart back and forth is considered. Thus, the output of fuzzy swing-up controller is defined as $F_{FSUC} = -\tilde{F}_c$ with $F_p = 0$. For the fuzzy swing-up controller, the error $E_c = X_c - R_c = [e_{c1} \quad e_{c3}]^T$ is taken as the input vector, where $R_c = [r_1 \quad r_3]^T$ is a vector of reference command, and $X_c = [x_1 \quad x_3]^T$ is a state vector with x_1 and x_3 being cart position and cart velocity, respectively. It is easy to see that the output variable is the control action F_{FSUC} of *FSUC*. Without loss of generality, the reference command is devised as $R_c = 0$. The membership functions for the input fuzzy sets of the fuzzy swing-up controller are shown in Fig.5(a) and Fig.5(b).

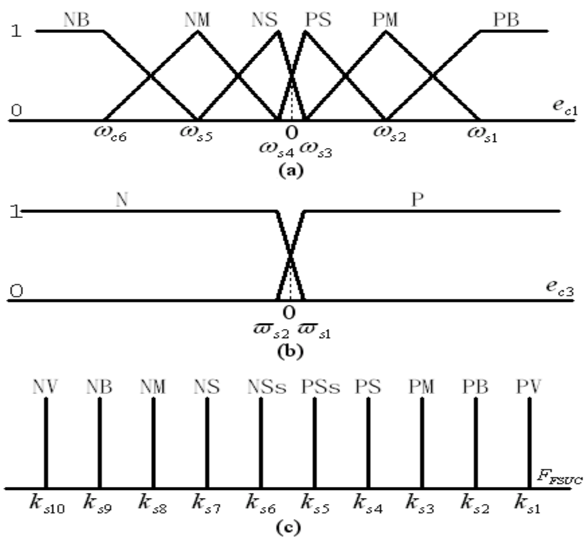

Fig.5. Membership functions for the FSUC.

With the complete fuzzy rules are indicated in Table II, and the control action of the fuzzy swing-up controller can be obtained as

$$F_{FSUC} = \frac{\sum_{s=1}^{12} M_{s1}(e_{c1}) M_{s3}(e_{c3}) g_s}{\sum_{s=1}^{12} M_{s1}(e_{c1}) M_{s3}(e_{c3})},\ g_s \in \{k_{si} \mid i = 1 \sim 10\} \quad (22)$$

Table II. Fuzzy rule base for FSUC.

e_{c3} \ e_{c1}	NB	NM	NS	PS	PM	PB
N	NV	NB	PSs	PM	PS	PB
P	NB	NS	NM	NSs	PB	PV

C. *Pseudo-Decomposed Inverted Pendulum-Car System*

To simplify the position control of an inverted pendulum-cart system, the inverted pendulum-cart system is pseudo-decomposed into a pendulum subsystem and a cart subsystem. Without loss for generality, the desired positions of pendulum and cart are assumed to be zero, $X_{dp} = 0$, $X_{dc} = 0$. Then the errors between the current positions and the desired position of pendulum and cart are

$$E_p = \begin{bmatrix} e_{p2} \\ e_{p4} \end{bmatrix} = X_p - X_{dp} = X_p = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}$$
$$E_c = \begin{bmatrix} e_{c1} \\ e_{c3} \end{bmatrix} = X_c - X_{dc} = X_c = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} \quad (23)$$

With the change of the state variables and the inclusion of the control action, the state equations of the inverted pendulum-cart system become

$$\dot{e}_{c1} = f_1 = e_{c3}$$
$$\dot{e}_{p2} = f_2 = e_{p4}$$
$$\dot{e}_{c3} = f_3 = \frac{a(F_{TFSPC} - T_c - \mu e_{p4}^2 \sin e_{p2}) + l \cos e_{p2}(\mu g \sin e_{p2} - f_p e_{p4})}{J + \mu l \sin^2 e_{p2}}$$
$$\dot{e}_{p4} = f_4 = \frac{l \cos e_{p2}(F_{TFSPC} - T_c - \mu e_{p4}^2 \sin e_{p2}) + \mu g \sin e_{p2} - f_p e_{p4}}{J + \mu l \sin^2 e_{p2}} \quad (24)$$

The inverted pendulum-cart system with state equations in Eq.20 is now pseudo-decomposed into a pendulum subsystem and a cart subsystem with state equations,

$$\dot{E}_p = A_p E_p + \begin{bmatrix} 0 \\ \Delta\phi_p \end{bmatrix} E_p + \beta_p F_{p-FSPC}$$
$$\dot{E}_c = A_c E_c + \begin{bmatrix} 0 \\ \Delta\phi_c \end{bmatrix} E_c + \beta_c F_{c-FSPC} \quad (25)$$

where

$$A_p = \begin{bmatrix} 0 & 1 \\ \frac{\mu g}{J} & \frac{-f_p}{J} \end{bmatrix},\ A_c = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-aT_{cc}}{J} \end{bmatrix} \quad (26)$$

$$\beta_p(E_p)=\begin{bmatrix}0\\ \beta_4\end{bmatrix}=\begin{bmatrix}0\\ \dfrac{l\cos e_{p2}}{J+\mu l\sin^2 e_{p2}}\end{bmatrix}, \quad (27)$$

$$\beta_c(E_c,E_p)=\begin{bmatrix}0\\ \beta_3\end{bmatrix}=\begin{bmatrix}0\\ \dfrac{-a}{J+\mu l\sin^2 e_{p2}}\end{bmatrix}$$

$$\Delta\phi_p E_p=\frac{Jl\cos e_{p2}(F_{c-FSPC}-T_c)+J\mu g(\sin e_{p2}-e_{p2})-\mu^2 g l e_{p2}\sin^2 e_{p2}}{J(J+\mu l\sin^2 e_{p2})}+\frac{\mu l f_p e_{p4}\sin^2 e_{p2}-J\mu l e_{p4}^2\cos e_{p2}\sin e_{p2}}{J(J+\mu l\sin^2 e_{p2})}$$

$$\Delta\phi_c E_c=\frac{Ja(-F_{p-FSPC}+T_{cc}e_{c3}-T_c)+a\mu l T_{cc}e_{c3}\sin^2 e_{p2}+J\mu g l\cos e_{p2}\sin e_{p2}}{J(J+\mu l\sin^2 e_{p2})}+\frac{-Ja\mu e_{p4}^2\sin e_{p2}-Jlf_p e_{p4}\cos e_{p2}}{J(J+\mu l\sin^2 e_{p2})} \quad (28)$$

It is straightforward to know that the uncertainties in Eq.28 satisfy the matching conditions [20] and $\Delta\phi_p=\beta_4\xi_p$, $\Delta\phi_c=\beta_3\xi_c$ can be assumed. Since the twin fuzzy sliding position controllers are devised to position the pendulum and cart for a small range of pendulum angle, $-0.4\,rad \leftrightarrow 0.4\,rad$, $\beta_4\neq 0$ and $\beta_3\neq 0$ are ensured.

D. *Design of the Twin-Fuzzy-Sliding-Position Controller (TFSPC)*

In this subsection, the design of a twin-fuzzy-sliding-position controller (TFSPC) is presented. The twin-fuzzy-sliding-position controller includes two similar position controllers, one (p-FSPC) for the position control of the pendulum and another one (c-FSPC) for position control of the cart system. In order to have the sliding mode guaranteed, the hitting condition $S^T\dot{S}<0,\quad S\neq 0$ must be satisfied. Let the sliding functions for the pendulum and cart system be

$$S_p=C_pE_p=\begin{bmatrix}c_2 & c_4\end{bmatrix}\begin{bmatrix}e_{p2}\\ e_{p4}\end{bmatrix}=c_2e_{p2}+c_4e_{p4}$$
$$S_c=C_cE_c=\begin{bmatrix}c_1 & c_3\end{bmatrix}\begin{bmatrix}e_{c1}\\ e_{c3}\end{bmatrix}=c_1e_{c1}+c_3e_{c3} \quad (29)$$

where c_1, c_2, c_3, c_4 are positive constants. From the definitions of sliding functions in Eq. 29, it is obvious that S_p and S_c are one dimension variables. To satisfy the hitting condition, two fuzzy sliding position controllers, p-FSPC and c-FSPC, are designed to have the outputs, F_{p-FSPC}, F_{c-FSPC},

$$F_{p-FSPC}=F_{peq}+(c_4\beta_4)^{-1}F_{pfs}$$
$$F_{c-FSPC}=F_{ceq}+(c_3\beta_3)^{-1}F_{cfs} \quad (30)$$

where F_{peq}, F_{ceq} are the equivalent control actions and F_{pfs}, F_{cfs} are the outputs of the fuzzy sliding mechanisms for the pendulum and cart systems, respectively.

Because the uncertain pendulum and cart systems satisfy the matching conditions, the uncertain pendulum and cart systems are invariant on the sliding surface [20]. Therefore, the equivalent control actions for the uncertain systems in the sliding mode are

$$F_{peq}=-(c_4\beta_4)^{-1}[c_2(a_{p11}e_{p2}+a_{p12}e_{p4})+c_4(a_{p21}e_{p2}+a_{p22}e_{p4})]$$
$$F_{ceq}=-(c_3\beta_3)^{-1}[c_1(a_{c11}e_{c1}+a_{c12}e_{c3})+c_3(a_{c21}e_{c1}+a_{c22}e_{c3})] \quad (31)$$

With the input and output spaces fuzzily partitioned into nine fuzzy sets, the complete fuzzy rules used in the fuzzy mechanisms of the twin-fuzzy-sliding-position controller are provided in Table III.

Table III. Fuzzy rule base for (a) pendulum control (b) cart control.

Input(S_p)	NV	NB	NM	NS	ZO	PS	PM	PB	PV
Output(F_{pfs})	PV	PB	PM	PS	ZO	NS	NM	NB	NV

(a)

Input(S_c)	NV	NB	NM	NS	ZO	PS	PM	PB	PV
Output(F_{cfs})	PV	PB	PM	PS	ZO	NS	NM	NB	NV

(b)

With the sum-product fuzzy inference and centroid defuzzification, the control actions of the provided fuzzy mechanisms are

$$F_{pfs}=-\sum_{i=1}^{9}k_{pi}\,sign(S_p)M_{pi}(S_p)$$
$$F_{cfs}=-\sum_{i=1}^{9}k_{ci}\,sign(S_c)M_{ci}(S_c) \quad (32)$$

where $k_{pi}\geq 0$, $k_{ci}\geq 0$, $\forall\, i=1\sim 9$. Substitute F_{peq}, F_{ceq}, F_{pfs}, F_{cfs} into Eq. 30, the output of the fuzzy sliding position controllers, *p-FSPC and c-FSPC*, are

$$\begin{aligned}F_{p-FSPC}&=[F_{peq}+(c_4\beta_4)^{-1}F_{pfs}]\\&=-(c_4\beta_4)^{-1}[c_2(a_{p11}e_{p2}+a_{p12}e_{p4})+c_4(a_{p21}e_{p2}+a_{p22}e_{p4})]\\&\quad-(c_4\beta_4)^{-1}[\sum_{i=1}^{9}k_{pi}sign(S_p)M_{pi}(S_p)]\\F_{c-FSPC}&=[F_{ceq}+(c_3\beta_3)^{-1}F_{cfs}]\\&=-(c_3\beta_3)^{-1}[c_1(a_{c11}e_{c1}+a_{c12}e_{c3})+c_3(a_{c21}e_{c1}+a_{c22}e_{c3})]\\&\quad-(c_3\beta_3)^{-1}[\sum_{i=1}^{9}k_{ci}sign(S_c)M_{ci}(S_c)]\end{aligned} \quad (33)$$

Then, the control action of the twin-fuzzy-sliding-position controller $F_{TFSPC}=F_{p-FSPC}+F_{c-FSPC}$ is obtained, and the sliding modes of *p-FSPC* and *c-FSPC* are guaranteed as in [12].

6. Simulation Results

The main parameters of the inverted pendulum-cart system are provided as in [12]. Also, the main parameters of the FACS are in Table IV.

Table IV. The main parameters of the FACS algorithm.

Parameter	N_a	$I_{t\max}$	q_0	Q	α	β	ρ_l	ρ_g
Value	10	50	0.7	5	1	2	0.6	0.4

With the input parameters determined as in [12], the proposed FACS algorithm is used to find the proper output parameters k_s, k_c, k_p of the fuzzy swing up controller and the twin fuzzy sliding position controller, respectively. Note that as in section V.A. there is no necessity to find the proper output parameter for the fuzzy switching controller. Let the initial values of the output parameters, k_s, k_c, k_p be simply determined to be equally spaced as follows

$$\begin{cases} k_s(0)=[5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5] \\ k_c(0)=[-5 \quad -3.75 \quad -2.5 \quad -1.25 \quad 0 \quad 1.25 \quad 2.5 \quad 3.75 \quad 5] \\ k_p(0)=[-5 \quad -3.75 \quad -2.5 \quad -1.25 \quad 0 \quad 1.25 \quad 2.5 \quad 3.75 \quad 5] \end{cases} \tag{34}$$

The proper values k_s^*, k_c^*, k_p^* for the output parameters are obtained with the proposed FACS to be

$$\begin{cases} k_s^*=[-13 \quad -13 \quad -6.751 \quad -3.771 \quad -2.1207 \quad 2.1207 \quad 3.771 \quad 6.751 \quad 13 \quad 13] \\ k_c^*=[4.2678 \quad 4.2433 \quad 4.2415 \quad 4.2415 \quad 0 \quad 4.2415 \quad 4.2415 \quad 4.2433 \quad 4.2678] \\ k_p^*=[8.5169 \quad 4.1028 \quad 4.1016 \quad 4.0004 \quad 0 \quad 4.0004 \quad 4.1016 \quad 4.1028 \quad 8.5169] \end{cases} \tag{35}$$

To indicate the effectiveness of the FACS, the simulation results with the parameters found using the ACS and MMAS algorithms are provided for comparisons. With the same parameter values as in FACS, the values k_s^*, k_c^*, k_p^* for the output parameters obtained using ACS are in Eq.36.

$$\begin{cases} k_s^*=[-13 \quad -10.161 \quad -9.444 \quad -3.34 \quad -2.1282 \quad 2.1282 \quad 3.34 \quad 9.444 \quad 10.161 \quad 13] \\ k_c^*=[6.4188 \quad 5.8843 \quad 4.1644 \quad 3.4585 \quad 0 \quad 3.4585 \quad 4.1644 \quad 5.8843 \quad 6.4188] \\ k_p^*=[5.5994 \quad 5.3134 \quad 3.1571 \quad 2.4241 \quad 0 \quad 2.4241 \quad 3.1571 \quad 5.3134 \quad 5.5994] \end{cases} \tag{36}$$

Also, using the MMAS parameters in Table V, the k_s^*, k_c^*, k_p^* obtained with MMAS are as follows.

$$\begin{cases} k_s^*=[-13 \quad -13 \quad -6.681 \quad -6.212 \quad -1.0833 \quad 1.0833 \quad 6.212 \quad 6.681 \quad 13 \quad 13] \\ k_c^*=[8 \quad 5.425 \quad 4.978 \quad 3.3714 \quad 0 \quad 3.3714 \quad 4.978 \quad 5.425 \quad 8] \\ k_p^*=[5.6552 \quad 4.1801 \quad 2.7572 \quad 2.2786 \quad 0 \quad 2.2786 \quad 2.7572 \quad 4.1801 \quad 5.6552] \end{cases} \tag{37}$$

Table V. The main parameters of the MMAS algorithm.

Parameter	N_a	$I_{t\max}$	Q	α	β	ρ
Value	10	50	5	1	2	0.02

Let the initial condition of the inverted pendulum-cart system be $[0 \quad \pi \quad 0 \quad 0]$. With the parameters of FHSSC obtained by using the FACS, ACS, and MMAS algorithms, the simulation results for the swing-up and position control of the inverted pendulum-cart system are provided in Fig.6 and Fig.7. From Fig.6 and Fig.7, it can be seen that the FHSSC with the proper parameters found by the FACS can swing-up and balance the inverted pendulum and cart quicker. That is, the FACS is effective for the parameter determination and the FHSSC with FACS has the better performance on the swing-up and balance of the inverted pendulum-cart system.

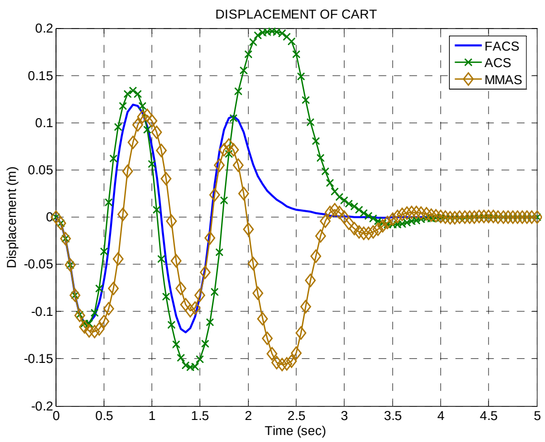

Fig.6. The displacements of the cart with the parameters of FHSSC obtained by using three different ant systems.

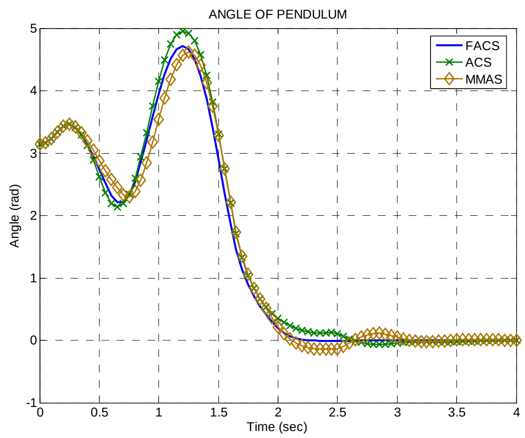

Fig.7. The angles of the pendulum with the parameters of FHSSC obtained by using three different ant systems.

7. Conclusion

In this paper, we propose a novel fuzzy ant colony system (FACS) algorithm to simulate the foraging behavior of ants. Based on the traditional ACS algorithm, new fuzzy transition probability rules are presented to select the next city. The new fuzzy transition probability rules included the fuzzy mechanism and the fuzzy probable mechanism. The proposed fuzzy probable rules reasonably diversify the searching process. To indicate the effectiveness of the fuzzy ant colony system, FACS is applied to find the proper parameters for the fuzzy controllers to swing up and balance the inverted pendulum and cart system. Also, the comparisons between the proposed fuzzy ant colony system and other ant colony optimization algorithms are provided in the simulations.

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