

Backstepping Nonlinear Control Using Nonlinear Parametric Fuzzy Systems

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Abstract

Based on nonlinear parametric fuzzy systems, an adaptive backstepping controller is proposed for a class of strict-feedback nonlinear systems. The nonlinear parametric fuzzy systems are capable of automatically learning their membership functions and tuning their weightings. Since the adjustable parameters of the membership functions nonlinearly appear in the fuzzy systems, the adaptive laws are derived by estimating the derivative of the fuzzy systems. Moreover, the stability of the closed-loop system is analyzed by means of Lyapunov theory, and some tracking performance is guaranteed. Finally, two examples are provided to demonstrate the effectiveness and applicability of the proposed scheme.

Keywords: Fuzzy control, backstepping design, nonlinear control.

1. Introduction

The strict-feedback nonlinear control systems have a kind of lower triangular form. More specifically, consider a class of strict-feedback nonlinear systems as

$$\begin{aligned}\dot{x}_1 &= {}^1f(x_1) + {}^1g(x_1)x_2 \\ \dot{x}_2 &= {}^2f(x_1, x_2) + {}^2g(x_1, x_2)x_3 \\ &\vdots \\ \dot{x}_n &= {}^nf(x_1, x_2, \dots, x_n) + {}^ng(x_1, x_2, \dots, x_n)u\end{aligned}\quad (1)$$

where kf and kg is the nonlinear system functions, $u \in R$ is the system input. Since the nonlinear system functions kf and kg in \dot{x}_k only depend on the state vector ${}^k\mathbf{x} = [x_1, x_2, \dots, x_k]^T$, which is fed back to the subsystem, such systems are called strict-feedback nonlinear systems [1].

In the past decade, backstepping technique has been widely used for strict-feedback nonlinear control systems [2]. Backstepping technique has the advantage of

avoiding the cancellation of useful nonlinearities in the design process compared with feedback linearization methods [3]. The process of the backstepping technique is that an appropriate state and a virtual control are selected for each internal subsystem, then the state equation is rewritten in terms of them, and finally Lyapunov functions are chosen for these subsystems so that the true controller integrating the individual controls of these subsystems guarantees the stability of the overall system. Because of the development of the fuzzy systems and neural networks, many researchers [4-9] have integrated the backstepping technique with the fuzzy systems or neural networks in order to control strict-feedback nonlinear systems with unknown system dynamics. Based on backstepping approach, a neural-based adaptive observer for state estimation has been presented in [7]. In [9], an adaptive fuzzy backstepping control scheme for a class of uncertain multiple-input-multiple-output nonlinear systems with block-triangular forms has been proposed.

Because fuzzy systems are universal approximators [10], adaptive control schemes incorporating them have grown rapidly [10-25]. In [10], the structure of the fuzzy systems consists of a linear combination of a set of fuzzy basis functions. Since the dynamics of most practical control plants is uncertain, nonlinear and complicated, it is difficult to determine the initial location of the membership functions. Thus, some fuzzy controllers which can automatically learn the membership functions and tune the weightings by appropriate adaptation laws [19-22] are more suitable to model the practical system behavior. Such controllers are called nonlinear parametric controllers. In general, gradient approaches which need to calculate the derivative of the fuzzy systems are used to obtain adaptive laws for the design of online controllers.

This paper uses the nonlinear parametric fuzzy systems to develop an adaptive backstepping controller for a class of strict-feedback nonlinear systems, and the adaptive laws are derived by estimating the derivative of the fuzzy systems. Prior knowledge on the location of membership functions and the calculation of the derivative of the fuzzy systems are not required. Also, the proposed method guarantees the boundedness of all the signals in the closed-loop system and some tracking performance.

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Manuscript received 4 Nov. 2008; revised 22 July 2009; accepted 8 Dec. 2009

The paper is organized as follows. In Section 2, the description of the nonlinear parametric fuzzy systems is presented. Section 3 presents the design of backstepping controller for a class of strict-feedback nonlinear systems. The design of adaptive backstepping controller using nonlinear parametric fuzzy systems is given in Section 4. Simulation results of two examples are shown to verify the performance of the proposed method in Section 5. Section 6 gives the conclusions.

2. Description of nonlinear parametric fuzzy systems

Fuzzy systems can be divided into two parts: some fuzzy IF-THEN rules and a fuzzy inference engine. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ to an output linguistic variable y . The i th fuzzy IF-THEN rule is written as

$$\mathbf{R}^i: \text{If } x_1 \text{ is } A_1^i \text{ and } \dots \text{ and } x_n \text{ is } A_n^i \text{ then } y \text{ is } B^i, \tag{2}$$

where $A_1^i, A_2^i, \dots, A_n^i$ and B^i are fuzzy sets. Suppose that the operations of produce inference, center-average, and singleton fuzzifier are used. Then, the output of the fuzzy system can be expressed as

$$y = \boldsymbol{\theta}^T \boldsymbol{\varphi} \tag{3}$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_h]^T$ is a weighting vector, and $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, \dots, \varphi_h]^T$ is a fuzzy basis vector, where φ_i is defined as

$$\varphi_i = \frac{\prod_{j=1}^n \mu_{A_j^i}(x_j)}{\sum_{l=1}^h \left[\prod_{j=1}^n \mu_{A_j^l}(x_j) \right]}. \tag{4}$$

where $\mu_{A_j^i}$ is the membership function value of the linguistic fuzzy variable, h is the total number of the IF-THEN rules, θ_i is the point at which $\mu_{B^i}(\theta_i) = 1$. Suppose that the membership functions and the weightings can be adjusted during identifying procedure. Then, the fuzzy system can be regarded as a nonlinear fuzzy system because the fuzzy system is nonlinear in its adjusted membership parameters.

According to the above description, define the system dynamics $f(\mathbf{x})$ as

$$f(\mathbf{x}) = \boldsymbol{\theta}^T \boldsymbol{\varphi}(\mathbf{x}, \boldsymbol{\eta}) \tag{5}$$

which is a nonlinear parametric fuzzy system, where $\boldsymbol{\eta}$ is the parameter vector of fuzzy basis functions. Assume that the membership functions are selected as Gaussian functions. Then, the parameter vector of fuzzy basis functions is $\boldsymbol{\eta} = \{\mathbf{c}, \boldsymbol{\sigma}\}$, where \mathbf{c} and $\boldsymbol{\sigma}$ are the center and spread vectors of the Gaussian functions, respectively.

3. Design of backstepping controllers for strict-feedback nonlinear systems

In this section, we describe the control problem for a class of strict-feedback nonlinear systems, and then design the backstepping controller.

First, consider a class of strict-feedback nonlinear systems as (1), where it is assumed that the nonlinear system functions ${}^k f$ are unknown, and ${}^k g = 1$. Our control objective is to develop the adaptive backstepping controller so that the state trajectory x_1 can asymptotically track a bounded command y_d .

Next, the following steps are presented for the design of the backstepping controller.

Step 1: Define a tracking error as $z_1 = x_1 - y_d$. Then, by differentiating z_1 , we have $\dot{z}_1 = \dot{x}_1 - \dot{y}_d$. Define a virtual control as

$$x_{2d} = -{}^1\hat{F} + \dot{y}_d - c_1 z_1 \tag{6}$$

where $c_1 > 0$ is a design parameter. Let $z_2 = x_2 - x_{2d}$. Then, by using the fact that $\dot{x}_1 = {}^1 f + x_2$, we obtain

$$\dot{z}_1 = {}^1 F - {}^1\hat{F} + z_2 - c_1 z_1 \tag{7}$$

where ${}^1 F = {}^1 f$.

Step 2: By differentiating z_2 , we obtain $\dot{z}_2 = \dot{x}_2 - \dot{x}_{2d}$. Similarly, define a virtual control as

$$x_{3d} = -{}^2\hat{F} - c_2 z_2 - z_1 \tag{8}$$

where $c_2 > 0$ is a design parameter. Moreover, define an error state as $z_3 = x_3 - x_{3d}$. Then, by using the fact that $\dot{x}_2 = {}^2 f + x_3$, we obtain

$$\dot{z}_2 = {}^2 F - {}^2\hat{F} + z_3 - c_2 z_2 - z_1 \tag{9}$$

where ${}^2 F = ({}^2 f - \dot{x}_{2d})$.

Step 3: Let k be a positive integer. Define an error state as $z_k = x_k - x_{kd}$. Then, by differentiating z_k , we have $\dot{z}_k = \dot{x}_k - \dot{x}_{kd}$ where $3 \leq k \leq n-1$. Define a virtual control as

$$x_{(k+1)d} = -{}^k\hat{F} - c_k z_k - z_{k-1} \tag{10}$$

where $c_k > 0$ is a design parameter. Let $z_{k+1} = x_{k+1} - x_{(k+1)d}$. Then, by using the fact that $\dot{x}_k = {}^k f + x_{k+1}$, we have

$$\dot{z}_k = {}^k F - {}^k\hat{F} + z_{k+1} - c_k z_k - z_{k-1} \tag{11}$$

where ${}^k F = ({}^k f - \dot{x}_{kd})$.

Step 4: By differentiating z_n , we have $\dot{z}_n = \dot{x}_n - \dot{x}_{nd}$. Define a control law as,

$$u = -{}^n\hat{F} - c_n z_n - z_{n-1} \tag{12}$$

where $c_n > 0$ is a design parameter. Then, by using the fact that $\dot{x}_n = {}^n f + u$, we have

$$\dot{z}_n = {}^n F - {}^n\hat{F} - c_n z_n - z_{n-1} \tag{13}$$

where ${}^n F = ({}^n f - \dot{x}_{nd})$.

Step 5: Suppose that ${}^k \hat{F} = {}^k F$ for $k=1,2,\dots,n$. Then, consider Lyapunov function as follows

$$V = \frac{1}{2} \sum_{i=1}^n z_i^2 \quad (14)$$

By differentiating (14) and using (7), (9), (11) and (13), we have

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n z_i \dot{z}_i \\ &= z_1(z_2 - c_1 z_1) + \sum_{i=2}^{n-1} z_i(z_{i+1} - c_i z_i - z_{i-1}) + z_n(-c_n z_n - z_{n-1}) \\ &= -\sum_{i=1}^n c_i z_i^2 \\ &\leq -c_1 z_1^2 \end{aligned} \quad (15)$$

From (14) and (15), we can conclude that $\lim_{t \rightarrow \infty} z_1 = 0$.

That is, the state trajectory x_1 can asymptotically track the bounded command y_d . This completes the proof.

4. Design of adaptive backstepping controller using nonlinear parametric fuzzy systems

Since the function ${}^k F$ is unknown, the backstepping controller (12) in the aforementioned discussion can not be obtained. Thus, the nonlinear parametric fuzzy system as mentioned previously is integrated with the backstepping controller and utilized to approximate the unknown system functions.

A. The derivative estimation of nonlinear parametric fuzzy systems

The estimation of the derivative of nonlinear parametric fuzzy systems is used to replace the derivative operation of the fuzzy systems. First, based on the universal approximation theorem, there exists an optimal function ${}^k F^*$ for the function ${}^k F$ such that

$${}^k F = {}^k F^* + {}^k o = {}^k \theta^{*T} {}^k \varphi(\mathbf{x}, {}^k \boldsymbol{\eta}^*) + {}^k o \quad (16)$$

where ${}^k o$ is the approximation error, and it is bounded. ${}^k \theta^*$ and ${}^k \boldsymbol{\eta}^*$ are the optimal parameter vectors of ${}^k \theta$ and ${}^k \boldsymbol{\eta}$, respectively. The output of the nonlinear parametric fuzzy system (5) can be expressed as

$${}^k \hat{F} = {}^k \hat{\boldsymbol{\theta}}^T {}^k \varphi(\mathbf{x}, {}^k \hat{\boldsymbol{\eta}}) \quad (17)$$

where ${}^k \hat{\boldsymbol{\theta}}$ and ${}^k \hat{\boldsymbol{\eta}}$ are the estimates of the optimal vectors ${}^k \theta^*$ and ${}^k \boldsymbol{\eta}^*$, respectively. By means of mean value theorem [23], there exists a point ${}^k \mathbf{z}$ between $({}^k \boldsymbol{\eta}^*, {}^k \hat{\boldsymbol{\eta}})$ such that the following equality is satisfied.

$$\begin{aligned} {}^k \varphi(\mathbf{x}, {}^k \boldsymbol{\eta}^*) - {}^k \varphi(\mathbf{x}, {}^k \hat{\boldsymbol{\eta}}) &= \frac{\partial({}^k \varphi)}{\partial({}^k \boldsymbol{\eta})} \Big|_{{}^k \boldsymbol{\eta}={}^k \mathbf{z}} ({}^k \boldsymbol{\eta}^* - {}^k \hat{\boldsymbol{\eta}}) \\ &= {}^k \mathbf{Q} ({}^k \mathbf{c}^* - {}^k \hat{\mathbf{c}}) + {}^k \mathbf{R} ({}^k \boldsymbol{\sigma}^* - {}^k \hat{\boldsymbol{\sigma}}) \end{aligned} \quad (18)$$

where ${}^k \mathbf{Q} = \frac{\partial({}^k \varphi)}{\partial({}^k \mathbf{c})} \Big|_{{}^k \mathbf{c}={}^k \mathbf{z}_c}$ and ${}^k \mathbf{R} = \frac{\partial({}^k \varphi)}{\partial({}^k \boldsymbol{\sigma})} \Big|_{{}^k \boldsymbol{\sigma}={}^k \mathbf{z}_\sigma}$. Let

${}^k \tilde{\mathbf{Q}} = {}^k \mathbf{Q}^* - {}^k \hat{\mathbf{Q}}$, ${}^k \tilde{\mathbf{R}} = {}^k \mathbf{R}^* - {}^k \hat{\mathbf{R}}$, ${}^k \tilde{\boldsymbol{\eta}} = {}^k \boldsymbol{\eta}^* - {}^k \hat{\boldsymbol{\eta}}$, and ${}^k \tilde{\varphi} = {}^k \varphi(\mathbf{x}, {}^k \boldsymbol{\eta}^*) - {}^k \varphi(\mathbf{x}, {}^k \hat{\boldsymbol{\eta}})$, where ${}^k \hat{\mathbf{Q}}$ and ${}^k \hat{\mathbf{R}}$ are the estimates of the optimal matrices ${}^k \mathbf{Q}^*$ and ${}^k \mathbf{R}^*$, respectively. Then, equation (18) can be rewritten as

$${}^k \tilde{\varphi} = {}^k \hat{\mathbf{Q}} {}^k \tilde{\mathbf{c}} + {}^k \tilde{\mathbf{Q}} {}^k \tilde{\mathbf{c}} + {}^k \hat{\mathbf{R}} {}^k \tilde{\boldsymbol{\sigma}} + {}^k \tilde{\mathbf{R}} {}^k \tilde{\boldsymbol{\sigma}} + {}^k v \quad (19)$$

where ${}^k v = ({}^k \mathbf{Q} - {}^k \hat{\mathbf{Q}}) {}^k \tilde{\mathbf{c}} + ({}^k \mathbf{R} - {}^k \hat{\mathbf{R}}) {}^k \tilde{\boldsymbol{\sigma}}$. Define the estimation error ${}^k \tilde{F} = {}^k F - {}^k \hat{F}$. By using (19) and (16) and after some manipulations, we have

$$\begin{aligned} {}^k \tilde{F} &= {}^k \tilde{\boldsymbol{\theta}}^T (-{}^k \hat{\mathbf{Q}} {}^k \hat{\mathbf{c}} - {}^k \hat{\mathbf{R}} {}^k \hat{\boldsymbol{\sigma}}) + {}^k \tilde{\mathbf{c}}^T {}^k \hat{\mathbf{Q}}^T {}^k \hat{\boldsymbol{\theta}} + {}^k \tilde{\boldsymbol{\sigma}}^T {}^k \hat{\mathbf{R}}^T {}^k \hat{\boldsymbol{\theta}} \\ &\quad - \sum_{j=1}^l \sum_{i=1}^h {}^k \tilde{a}_{ij} {}^k \hat{\theta}_i {}^k \hat{c}_j - \sum_{j=1}^l \sum_{i=1}^h {}^k \tilde{b}_{ij} {}^k \hat{\theta}_i {}^k \hat{\sigma}_j + {}^k \varepsilon \end{aligned} \quad (20)$$

where ${}^k \varepsilon = {}^k \tilde{\boldsymbol{\theta}}^T ({}^k \hat{\mathbf{Q}} ({}^k \mathbf{c}^* - {}^k \tilde{\mathbf{c}}) + {}^k \hat{\mathbf{R}} ({}^k \boldsymbol{\sigma}^* - {}^k \tilde{\boldsymbol{\sigma}}) + {}^k \varphi^*) + {}^k \tilde{\boldsymbol{\theta}}^T ({}^k \tilde{\mathbf{Q}} {}^k \mathbf{c}^* + {}^k \tilde{\mathbf{R}} {}^k \boldsymbol{\sigma}^*) + {}^k \tilde{\boldsymbol{\theta}}^T {}^k v + {}^k o$, ${}^k \varphi^* = {}^k \varphi(\mathbf{x}, {}^k \boldsymbol{\eta}^*)$, ${}^k \tilde{a}_{ij} = {}^k a_{ij}^* - {}^k \hat{a}_{ij}$ and ${}^k \tilde{b}_{ij} = {}^k b_{ij}^* - {}^k \hat{b}_{ij}$. ${}^k \tilde{a}_{ij}$ and ${}^k \tilde{b}_{ij}$ are the elements in the i th row and j th column of the matrices ${}^k \tilde{\mathbf{Q}}$ and ${}^k \tilde{\mathbf{R}}$, respectively. Note that for the nonlinear parametric fuzzy system, there are two estimate vectors ${}^k \hat{\boldsymbol{\theta}}_f$ and ${}^k \hat{\boldsymbol{\eta}}$, and two estimate matrices ${}^k \hat{\mathbf{Q}}$ and ${}^k \hat{\mathbf{R}}$.

B. Design of adaptive backstepping controllers

Let the control law be as follows.

$$u = u_{fb} \quad (21)$$

where u_{fb} is a fuzzy backstepping control. Based on the aforementioned discussion, the fuzzy backstepping control with the virtual controls (6), (8) and (10) can be expressed as

$$u_{fb} = -{}^n \hat{F} - c_n z_n - z_{n-1} \quad (22)$$

where ${}^k \hat{F}$ denotes the output of the k -th nonlinear parametric fuzzy system and the estimation of the system dynamics ${}^k F$. From (21) and (22), the n th order nonlinear systems can be rewritten as

$$\dot{z}_n = {}^n \tilde{F} - c_n z_n - z_{n-1} \quad (23)$$

where ${}^n \tilde{F} = {}^n F - {}^n \hat{F}$. By using the estimation error (20), equation (23) can be expressed as

$$\begin{aligned} \dot{z}_n &= {}^n \tilde{\boldsymbol{\theta}}^T (-{}^n \hat{\mathbf{Q}} {}^n \hat{\mathbf{c}} - {}^n \hat{\mathbf{R}} {}^n \hat{\boldsymbol{\sigma}}) + {}^n \tilde{\mathbf{c}}^T {}^n \hat{\mathbf{Q}}^T {}^n \hat{\boldsymbol{\theta}} + {}^n \tilde{\boldsymbol{\sigma}}^T {}^n \hat{\mathbf{R}}^T {}^n \hat{\boldsymbol{\theta}} \\ &\quad - \sum_{j=1}^l \sum_{i=1}^p {}^n \tilde{a}_{ij} {}^n \hat{\theta}_i {}^n \hat{c}_j - \sum_{j=1}^l \sum_{i=1}^p {}^n \tilde{b}_{ij} {}^n \hat{\theta}_i {}^n \hat{\sigma}_j + {}^n \varepsilon - c_n z_n - z_{n-1} \end{aligned} \quad (24)$$

To show the stability analysis of the closed-loop sys-

tem, consider Lyapunov function as

$$V = \frac{1}{2} \sum_{k=1}^n [z_k^2 + \frac{{}^k \tilde{\theta}^T {}^k \tilde{\theta}}{{}^k \gamma_1} + \frac{{}^k \tilde{c}^T {}^k \tilde{c}}{{}^k \gamma_2} + \frac{{}^k \tilde{\sigma}^T {}^k \tilde{\sigma}}{{}^k \gamma_3} + \frac{1}{{}^k \gamma_4} \sum_{i=0}^h \sum_{j=0}^l {}^k \tilde{a}_{ij}^2 + \frac{1}{{}^k \gamma_5} \sum_{i=0}^h \sum_{j=0}^l {}^k \tilde{b}_{ij}^2] \tag{25}$$

where ${}^k \gamma_i$ is positive. By differentiating (25) and using the aforementioned discussion, we have

$$\dot{V} = \sum_{k=1}^n [z_k \dot{z}_k + \frac{{}^k \tilde{\theta}^T \dot{{}^k \tilde{\theta}}}{k \gamma_1} + \frac{{}^k \tilde{c}^T \dot{{}^k \tilde{c}}}{k \gamma_2} + \frac{{}^k \tilde{\sigma}^T \dot{{}^k \tilde{\sigma}}}{k \gamma_3} + \frac{1}{{}^k \gamma_4} \sum_{i=0}^h \sum_{j=0}^l \dot{{}^k \tilde{a}}_{ij} + \frac{1}{{}^k \gamma_5} \sum_{i=0}^h \sum_{j=0}^l \dot{{}^k \tilde{b}}_{ij}] \tag{26}$$

Suppose that the adaptive laws are given as

$$\begin{aligned} \dot{{}^k \hat{\theta}}_f &= -{}^k \gamma_1 z_k ({}^k \hat{Q} {}^k \hat{c} + {}^k \hat{R} {}^k \hat{\sigma}) \\ \dot{{}^k \hat{c}} &= {}^k \gamma_2 z_k {}^k \hat{Q}^T {}^k \hat{\theta} \\ \dot{{}^k \hat{\sigma}} &= {}^k \gamma_3 z_k {}^k \hat{R}^T {}^k \hat{\theta} \\ \dot{{}^k \hat{a}}_{ij} &= -{}^k \gamma_4 z_k {}^k \hat{\theta}_i {}^k \hat{c}_j \\ \dot{{}^k \hat{b}}_{ij} &= -{}^k \gamma_5 z_k {}^k \hat{\theta}_i {}^k \hat{c}_j \end{aligned} \tag{27}$$

Substituting for \dot{z}_k , $\dot{{}^k \hat{\theta}}_f$, $\dot{{}^k \hat{c}}$, $\dot{{}^k \hat{\sigma}}$, $\dot{{}^k \hat{a}}_{ij}$ and $\dot{{}^k \hat{b}}_{ij}$ in (26), we obtain

$$\dot{V} = -\sum_{i=1}^n c_i z_i^2 + \sum_{k=1}^n z_k {}^k \varepsilon \tag{28}$$

Then, equation (28) can be rewritten as

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n (-c_i z_i^2 + z_i {}^i \varepsilon) \\ &= \sum_{i=1}^n \left[-\left(c_i - \frac{1}{\zeta^2} \right) z_i^2 - \left(\frac{z_i}{\zeta} - \frac{\zeta {}^i \varepsilon}{2} \right)^2 + \frac{\zeta^2 {}^i \varepsilon^2}{4} \right] \\ &= \sum_{i=1}^n \left[-\left(c_i - \frac{1}{\zeta^2} \right) z_i^2 - \left(\frac{z_i}{\zeta} - \frac{\zeta {}^i \varepsilon}{2} \right)^2 \right] + \zeta^2 \sum_{i=1}^n \left(\frac{{}^i \varepsilon^2}{4} \right) \\ &= \sum_{i=1}^n \left[-\left(c_i - \frac{1}{\zeta^2} \right) z_i^2 - \left(\frac{z_i}{\zeta} - \frac{\zeta {}^i \varepsilon}{2} \right)^2 \right] + \zeta^2 \varepsilon_s \\ &\leq \sum_{i=1}^n \left[-\left(c_i - \frac{1}{\zeta^2} \right) z_i^2 \right] + \zeta^2 \varepsilon_s \end{aligned} \tag{29}$$

where $\varepsilon_s = \sum_{i=1}^n \frac{{}^i \varepsilon^2}{4}$. Let $c_i = \bar{c}_i + \underline{c}_i$, where $\underline{c}_i > \frac{1}{\zeta^2}$ and $\bar{c}_i > 0$. Then, by integrating (29) and using the fact that $V(T) \geq 0$, the following tracking performance can be obtained.

$$\int_0^T z_1^2(\tau) d\tau \leq \frac{V(0)}{\bar{c}_1} + \frac{\zeta^2}{2\bar{c}_1} \int_0^T \varepsilon_s^2(\tau) d\tau \tag{30}$$

where

$$V(0) = \frac{1}{2} \sum_{k=1}^n [z_k^2(0) + \frac{{}^k \tilde{\theta}^T(0) {}^k \tilde{\theta}(0)}{{}^k \gamma_1} + \frac{{}^k \tilde{c}^T(0) {}^k \tilde{c}(0)}{{}^k \gamma_2} + \frac{{}^k \tilde{\sigma}^T(0) {}^k \tilde{\sigma}(0)}{{}^k \gamma_3} + \frac{1}{{}^k \gamma_4} \sum_{i=0}^h \sum_{j=0}^l {}^k \tilde{a}_{ij}^2(0) + \frac{1}{{}^k \gamma_5} \sum_{i=0}^h \sum_{j=0}^l {}^k \tilde{b}_{ij}^2(0)]$$

On the basis of the aforementioned description, the adaptive backstepping controller for the n th order strict-feedback nonlinear systems can be given as follows.

Theorem: Consider the n th order strict-feedback nonlinear systems represented by (1), where the nonlinear system functions ${}^k f$ are unknown, and ${}^k g = 1$. Suppose that the control law is designed as (22). Let the adaptive laws be given as (27). Then, the overall control scheme guarantees the following tracking performance

$$\begin{aligned} \int_0^T z_1^2(\tau) d\tau &\leq \frac{1}{2\bar{c}_1} \sum_{k=1}^n [z_k^2(0) + \frac{{}^k \tilde{\theta}^T(0) {}^k \tilde{\theta}(0)}{{}^k \gamma_1} + \frac{{}^k \tilde{c}^T(0) {}^k \tilde{c}(0)}{{}^k \gamma_2} + \frac{{}^k \tilde{\sigma}^T(0) {}^k \tilde{\sigma}(0)}{{}^k \gamma_3} + \frac{1}{{}^k \gamma_4} \sum_{i=0}^h \sum_{j=0}^l {}^k \tilde{a}_{ij}^2(0) \\ &\quad + \frac{1}{{}^k \gamma_5} \sum_{i=0}^h \sum_{j=0}^l {}^k \tilde{b}_{ij}^2(0)] + \rho \int_0^T \varepsilon_s^2(\tau) d\tau \end{aligned} \tag{31}$$

where $T \in [0, \infty]$, and $\rho = \frac{\zeta^2}{2\bar{c}_1}$.

5. Simulation results

In this section, the proposed fuzzy backstepping control scheme is applied to control two nonlinear systems to demonstrate its effectiveness and applicability.

Example 1: Consider the nonlinear system described as

$$\begin{aligned} \dot{x}_1 &= 0.5x_1 + x_2 \\ \dot{x}_2 &= -3x_2 - 6x_1^3 + 9 \sin(x_1) + u \end{aligned}$$

The control objective is to design a control input such that the state x_1 asymptotically tracks the bounded command y_d . The bounded command is set as $y_d(t) = 2.5 \sin(t)$. The membership functions are given as ${}^k \varphi(x_i, {}^k \hat{c}_j, {}^k \hat{\sigma}_j) = \exp(-(x_i - {}^k \hat{c}_j)^2 / {}^k \hat{\sigma}_j^2)$. The design parameters are selected as $c_1 = 20, c_2 = 20, {}^k \gamma_1 = {}^k \gamma_2 = {}^k \gamma_3 = 0.01, {}^k \gamma_4 = {}^k \gamma_5 = 0.1$. The initial state condition is set as $x(0) = [1, 1.05]$. ${}^k \hat{\theta}(0), {}^k \hat{c}(0)$, and ${}^k \hat{\sigma}(0)$ are randomly given as the interval between $[0, 0.01], [-5, 5]$, and $[0, 5]$, respectively. The simulation results are shown in Figs. 1-4. From Figs. 1 and 2, it is shown that the state $x_1(t)$ tracks the bounded command $y_d(t)$ well. Figs. 3 and 4 are the control input $u(t)$ and the tracking error $z_1(t)$, respectively.

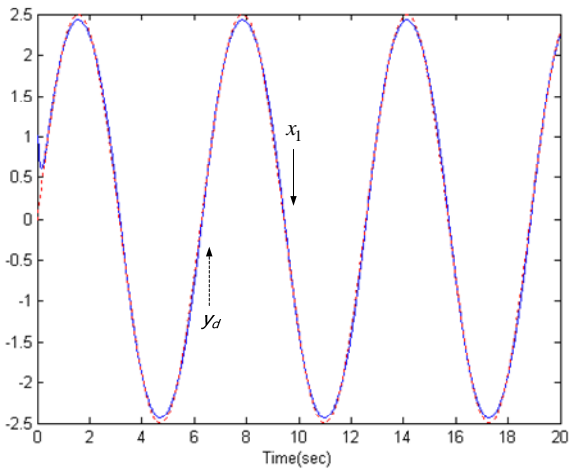


Figure 1. The state $x_1(t)$ and bounded command $y_d(t)$.

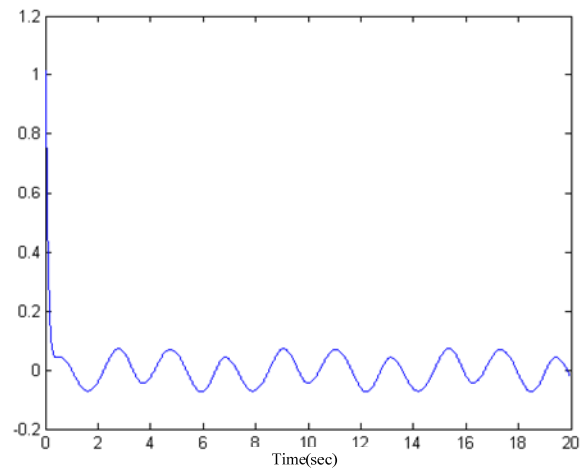


Figure 4. The tracking error $z_1(t)$.

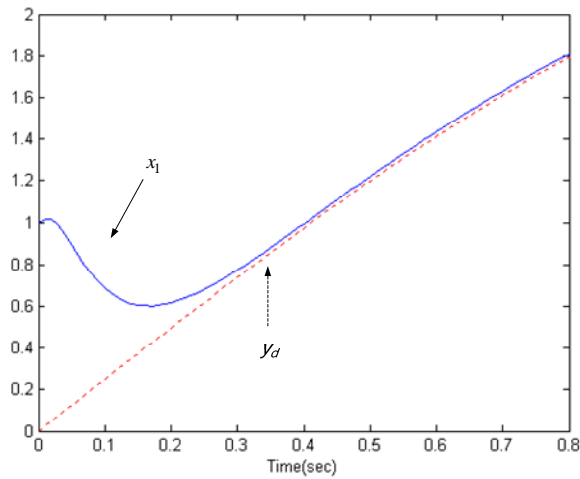


Figure 2. The state $x_1(t)$ and bounded command $y_d(t)$ (0-0.8sec).

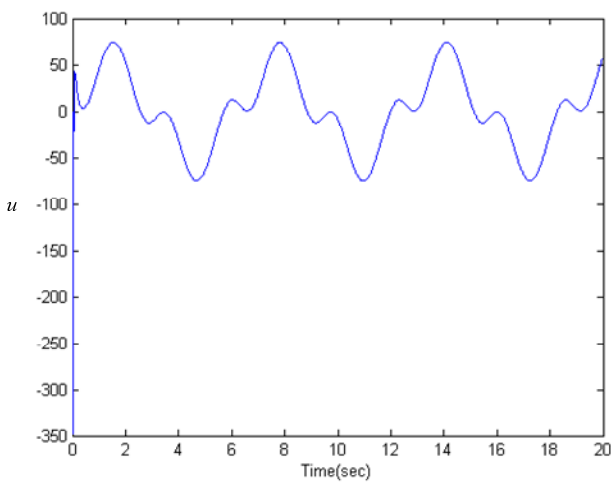


Figure 3. The control input $u(t)$.

Example 2: Consider the nonlinear system described as

$$\dot{x}_1 = 0.2x_1 + x_2$$

$$\dot{x}_2 = \frac{1 - e^{-x_1}}{1 + e^{-x_1}} - 0.5x_2 + x_1 \sin(x_2) + 0.8\sin(x_1) + u$$

The bounded command is set as $y_d(t) = \sin(0.5t) + \cos(t)$.

The membership functions are given as

$${}^k\varphi(x_i, {}^k\hat{c}_j, {}^k\hat{\sigma}_j) = \exp(-(x_i - {}^k\hat{c}_j)^2 / {}^k\hat{\sigma}_j^2)$$

The design parameters are selected as $c_1 = 10, c_2 = 15$,

${}^k\gamma_1 = {}^k\gamma_2 = {}^k\gamma_3 = 0.1, {}^k\gamma_4 = {}^k\gamma_5 = 0.5$.

The initial state condition is set as $x_1(0) = x_2(0) = 0$.

${}^k\hat{\theta}(0), {}^k\hat{c}(0)$, and ${}^k\hat{\sigma}(0)$ are randomly given as the interval between

$[0, 0.01], [-5, 5]$, and $[0, 5]$, respectively.

The simulation results are shown in Figs. 5-8.

From Figs. 5 and 6, it is shown that the state $x_1(t)$ tracks the bounded command well.

Figs. 7 and 8 are the control input $u(t)$ and the tracking error $z_1(t)$, respectively.

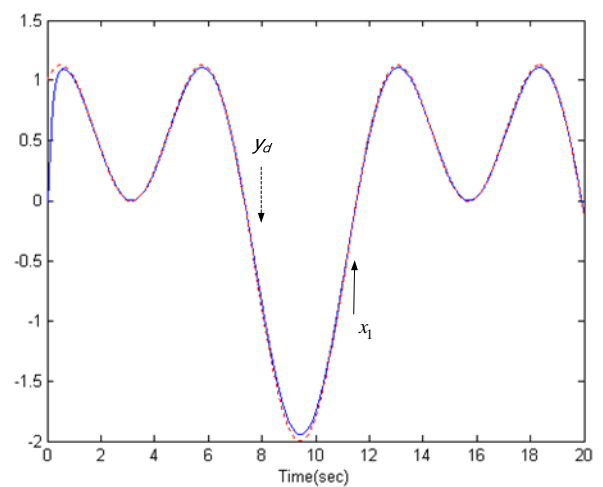


Figure 5. The state $x_1(t)$ and bounded command $y_d(t)$.

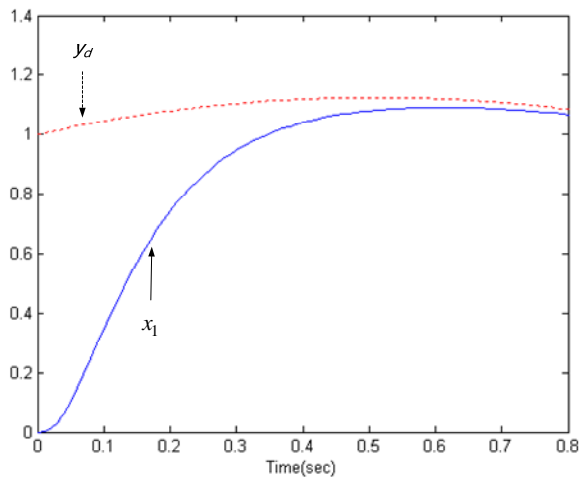


Figure 6. The state $x_1(t)$ and bounded command $y_d(t)$ (0-0.8sec).

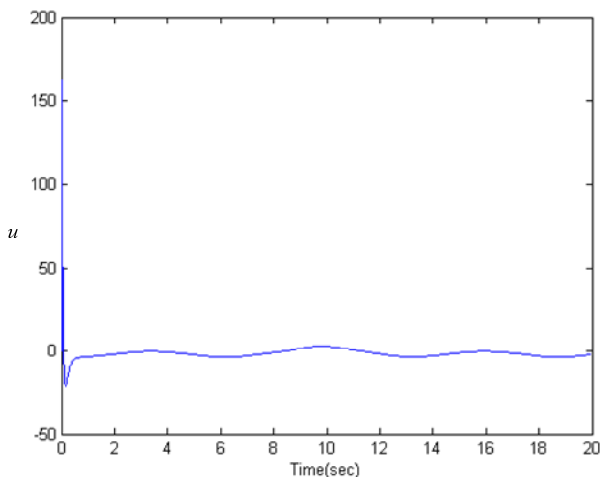


Figure 7. The control input $u(t)$.

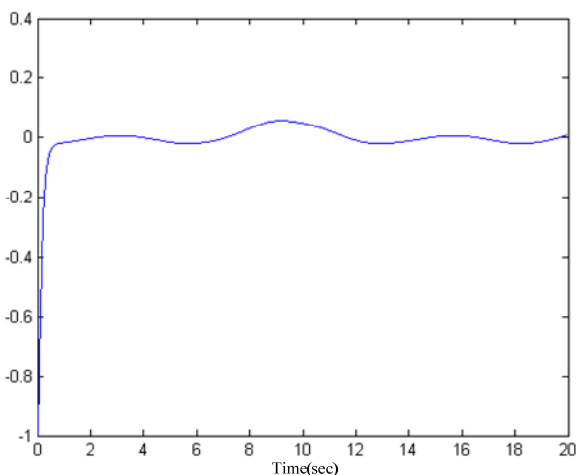


Figure 8. The tracking error $z_1(t)$.

6. Conclusions

This paper has proposed the nonlinear parametric fuzzy backstepping controller for a class of strict-feedback nonlinear systems. The estimation of the derivative of nonlinear parametric fuzzy systems is used to replace the derivative operation of the fuzzy systems. In addition, based on Lyapunov stability analysis, all signals involved into the closed-loop system have been proved to be bounded, and the tracking performance has been derived. Finally, simulation results of two examples have shown that the proposed control scheme can force the system output to track the desired trajectory well.

Acknowledgment

This work was supported by the National Science Council of Taiwan, R.O.C., under Grants NSC 96-2221-E-003-012-MY3.

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