Stability Analysis and Controller Design of the Nonlinear Switched Systems via T-S Discrete-Time Fuzzy Model

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Abstract

In this paper, we proposed an innovative representation modeling of the Takagi-Sugeno (T-S) fuzzy switched discrete-time system. The simulation of stability analysis methods based on Lyapunov stability theorem to study the stability and switching law design for the T-S fuzzy switched discrete-time systems. Sufficient conditions for quadratic asymptotic stability are presented and stabilizing switching laws of state-dependent form are designed. Furthermore, these methods can be applied to cases when all individual systems are unstable. The parallel distributed compensation (PDC) is employed to design fuzzy controllers from the T-S fuzzy models. The stabilization analysis is reduced to a problem of finding a common Lyapunov function for a set of linear matrix inequalities. Finally, a numerical example and an illustrative example based on the chemical process example are given to show the merits of the proposed approach, respectively.

Keywords: Switched system, T-S fuzzy model, Lyapunov function, Stability, Switching law, Linear matrix inequalities (LMIs).

1. Introduction

Switched systems have attracted considerable attention from many researchers in the field of hybrid dynamic systems. Switched systems are a class of hybrid dynamical systems consisting of several continuous-time or discrete-time subsystems, and a rule that orchestrates the switching sequences between them. The switching rule, determined by time or system state, or both, or other supervisory logic decisions, yields different switching signals and decides the categories of switched systems [1, 2]. There are many practical examples such as, air traffic management, chemical process, automated vehicles, robotics, computer disk system, batch processes etc, can be appropriately described by the switched model [3-8].

Recently, some stability conditions and stabilization approaches have been proposed for the switched discrete-time systems. Ref. [9] studied stability property for the switched systems which are composed of a continuous-time LTI subsystem and a discrete-time LTI subsystem, when the two subsystems are Hurwitz and Schur stable, respectively, and have shown that if the subsystem matrices are symmetric, then a common Lyapunov function exists for the two subsystems and that the switched system is exponentially stable under arbitrary switching. Ref. [10, 11] studied robust stability analysis and control synthesis of uncertain switched systems. Ref. [12] has shown to achieve controllability for a switched linear system, it is sufficient to use cyclic and synchronous switching paths and constant control laws. Ref. [13] studied the quadratic stabilization of discrete-time switched linear systems when a designed switching rule is imposed upon the feedback controller of subsystems, and studied quadratic stabilization of switched system with norm-bounded time-varying uncertainties. The event-driven scheduling strategy for constructing switching law to stabilize the switched system presented in [14]. There exists a switched quadratic Lyapunov function to check asymptotic stability of the switched discrete-time system in [15].

However, as yet control of nonlinear system is a challenging task because no systematic mathematical tools to help find necessary and sufficient conditions to guarantee the stability and performance. Fuzzy control is good at handing ill-defined and complex systems because it has simple properties and is easy to implement. During the past two decades, it has been widely accepted and researched in academic and industrial society [16-29]. Takagi-Sugeno (T-S) fuzzy models can provide an effective representation of complex nonlinear systems in terms of fuzzy sets and fuzzy reasoning applied to a set of linear input/output (I/O) submodels, i.e., the T-S fuzzy models can readily represent the nonlinear systems or uncertain systems by appropriate transformation [17]. In stability analysis of T-S fuzzy system, main approaches have been based on Lyapunov direct method [19-21, 24-26]. In addition, the control design is carried out based on the fuzzy model via the so-called parallel distributed compensation (PDC) scheme [27-29]. The idea

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Manuscript received 17 Oct. 2008; revised 15 April 2009; accepted 8 Dec. 2009.

is that for each local linear model, a linear feedback control is designed. By sharing the same premises as those of the fuzzy plant model, the resulting overall controller, which is nonlinear in general, is achieved "by fuzzy blending" of each individual linear controller.

In this paper, we propose an innovative representation modeling of the T-S fuzzy switched discrete-time system. We study the problems of stability and controller design for switched discrete-time systems using T-S fuzzy model. First, the T-S fuzzy model approach is extended to the stability analysis and control design for switched discrete-time systems. Then, we use the Lyapunov direct method, combined with linear matrix inequalities (LMIs) [30] to study the stability and controller design of fuzzy switched systems. Furthermore, these methods can be applied to cases when all individual systems are unstable.

The remainder content is organized as follows: following the introduction, Section 2 reviews the previous works for switched system and the T-S fuzzy discrete-time model is introduced and then we generally state the stability analysis and design of T-S fuzzy discrete-time model. In Section 3, system description and stability analysis for T-S fuzzy switched discrete-time system are presented. In Section 4, the controller design via parallel distributed compensation for T-S fuzzy switched discrete-time system is proposed. In Section 5, a numerical example and a chemical process example application are given to show the advantage of the presented method. Finally, some conclusions are drawn in Section 6.

2. System Description and Problem Statement

The nonlinear switched discrete-time systems can be described as follows:

$$x(k+1) = f_{\sigma(x)}\left(x(k), u(k)\right) \tag{1}$$

where $\{f_\sigma : \sigma \in l\}$ is a set of sufficiently regular functions from R^n to R^n that is parameterized by some index set $l, \sigma(x): R^n \to \{1, 2, \ldots, N\}$ is a piecewise constant function of state variable x(k), called a switch signal, i.e., the matrix $f_{\sigma(x)}(x)$ switches between matrices $f_1(x,u),\ f_2(x,u), \ldots,\ f_N(x,k)$ belonging to the set $f \equiv \{f_1, f_2, \ldots, f_N\}$ and $f_l,\ \ l \in \{1, 2, \ldots, N\}$.

In this section, we shall introduce some supporting result and system description. To begin with, we consider a fuzzy switched discrete-time system described by T-S fuzzy model for switched nonlinear systems (1)

Plant Rule i for individual system:

If $x_1(k)$ *is* M_{i1}, $x_2(k)$ *is* M_{i2},…, $x_n(k)$ *is* M_{in}

Then $x(k+1) = A_{\sigma(x(k))\_i}x(k) + B_{\sigma(x(k))\_i}u(k),$ (2)

where M_{ij} is the fuzzy set, $x_1(k), x_2(k), \ldots, x_n(k)$ are the premise variables, $x(k) \in R^n$ is the state vector and $i = 1, 2, \cdots, r$. r and n are the number of fuzzy rule and state variable respectively.

The T-S fuzzy switched discrete-time system (2) can be represented as follows:

Plant Rule i for individual system:

If $x_1(k)$ *is* M_{i1}, $x_2(k)$ *is* M_{i2},…, $x_n(k)$ *is* M_{in}

Then $x(k+1) = A_{l\_i}x(k) + B_{l\_i}u(k)$ (3)

By the product inference engine and center average defuzzification, the final output of the system (3) is inferred as

$$x(k+1) = \sum_{i=1}^{r} h_{l\_i}(x(k))\left\{A_{l\_i}x(k) + B_{l\_i}u(k)\right\} \tag{4}$$

where $h_{l\_i}(k) = \dfrac{\prod_{j=1}^{n} M_{l\_ij}\left(x_{l\_j(k)}\right)}{\sum_{i=1}^{r}\prod_{j=1}^{n} M_{l\_ij}\left(x_{l\_j(k)}\right)}$ and $M_{l\_ij}\left(x_{l\_j}(k)\right)$ is the firing strength of membership function M_{l_ij}.

According to the above, the following fuzzy controller is employed to deal with the stabilization problem of the T-S fuzzy switched discrete-time system (4).

Controller Rule i for individual system *l*:

If x_1 *is* M_{i1}, x_2 *is* M_{i2},…, x_n *is* M_{in}

Then $u(k) = -F_{l\_i}x(k)$ (5)

where M_{ij} is the fuzzy set and the same with the T-S fuzzy switched system, F_{l_i} is the local feedback gain vector, and $i = 1, 2, \cdots, r$. r is the number of fuzzy rule.

The final output of controller (5) is represented by

$$u(k) = -\sum_{i=1}^{r} h_{l\_i}(x(k))F_{l\_i}x(k) \tag{6}$$

For simplicity, we use the symbol h_i to denote $h_i(x(k))$.

For the analysis and synthesis of T-S fuzzy switched discrete-time system (4), some helpful lemmas and one definition are given below.

Lemma 1: There exists a switching law for the nominal switched discrete-time system $x(k+1) = A_l x(k)$ such that the system is asymptotically stable if there exist a symmetric matrix $P > 0$, positive constants α_l $(1 \le l \le r)$ satisfying $\sum_{l=1}^{r}\alpha_l = 1$ such that

$$\sum_{l=1}^{r} \alpha_l (A_l^T P A_l) - P < 0 \tag{7}$$

Proof: If there exist a symmetric matrix $P > 0$, positive constants α_l $(1 \le l \le r)$ satisfying $\sum_{l=1}^{r}\alpha_l = 1$ such that the inequality (7) holds, and the equality (7) is equivalent to the following inequality

$$\sum_{l=1}^{r} \alpha_l (A_l^T P A_l - P) < 0,\ \ l = 1, 2, \cdots, r$$

Then, for $\forall x(k) \in \mathrm{R}^n, x(k) \ne 0$

$$x^T(k)[\sum_{l=1}^{r}\alpha_l(A_l^T PA_l - P)]x(k) < 0 \quad (8)$$

Therefore, it follows that for any k, at least there exists an $l \in \{1, 2, \ldots, r\}$ such that

$$x^T(k)[A_l^T PA_l - P]x(k) < 0 \quad (9)$$

From (9), it implies that a convex combination of the corresponding Lyapunov function (8) is negative along the trajectory and from (9) at least one must be negative. Thus, the nominal switched discrete-time system $x(k+1) = A_l x(k)$ is asymptotically stable.

Lemma 2 [31]: For any matrices $A_1, A_2, \cdots, A_N$ with the same dimensions, the following inequality folds for any positive constant ε,

$$\begin{aligned}&(\Sigma_{i=1}^N A_i)^T(\Sigma_{i=1}^N A_i)\\ &\le (1+\varepsilon)A_1^T A_1 + (1+\varepsilon^{-1})(1+\varepsilon)A_2^T A_2 + (1+\varepsilon^{-1})^2(1+\varepsilon)A_3^T A_3 + \cdots\\ &\cdots + (1+\varepsilon^{-1})^{N-2}(1+\varepsilon)A_{N-1}^T A_{N-1} + (1+\varepsilon^{-1})^{N-1}A_N^T A_N \quad (10)\end{aligned}$$

Lemma 3 [31]: Let $\alpha_l = \dfrac{1}{(1+\varepsilon^{-1})^{l-1}(1+\varepsilon)}, l = 1, 2, \ldots, r-1$ and $\alpha_r = \dfrac{1}{(1+\varepsilon^{-1})^{r-1}}$, then $\alpha_l \in [0,1]$ $(1 \le l \le r)$ and $\Sigma_{l=1}^r \alpha_l = 1$ for any positive constant ε.

Definition 1: The T-S fuzzy switched discrete-time system (3) is said to be asymptotically stable via switching if there exists a switching law $\sigma(x)$, a positive definite Lyapunov function $V(k) = x^T(k)Px(k)$ and a positive constant β such that

$$\Delta V(k) = V(k+1) - V(k) < -\beta x^T(k)x(k) \quad (11)$$

holds for all trajectories of T-S fuzzy switched discrete-time system (3).

3. Stability Analysis

By the system description and problem statement, we first analyze the stability of open-loop fuzzy switched system (3). We consider the T-S fuzzy switched discrete-time system with two individual systems and each system has r rules (i.e. $i = 1, 2, \ldots, r$). In the light of Lemma 1, the stability of the T-S fuzzy switched discrete-time systems (3) is equivalent to as following

$$x(k+1) = (\alpha\sum_{i=1}^{r} h_{1\_i}A_{1\_i} + (1-\alpha)\sum_{i=1}^{r} h_{2\_i}A_{2\_i})x(k) \quad (12)$$

To investigate the stability of system (12), we choose the Lyapunov function candidate as

$$V(x(k)) = x^T(k)Px(k) \quad (13)$$

where $x^T(k) = \begin{bmatrix} x_1^T & x_2^T \end{bmatrix}^T$ and $P = diag\{P_1, P_2\}$ are unique real symmetric positive-define matrices satisfying the Lyapunov equation (14)

$$A_{l\_i\_ll}^T P_l A_{l\_i\_ll} - P_l = -Q_l \quad (14)$$

which $l = 1, 2$ and $A_{l\_i} = \begin{bmatrix} A_{l\_i\_11} & A_{l\_i\_12} \\ A_{l\_i\_21} & A_{l\_i\_22} \end{bmatrix}$, and the difference of the Lyapunov can be written as follows:

$$\begin{aligned}\Delta V(k) &= V(x(k+1)) - V(x(k))\\ &= x^T(k)\Big[(\alpha\sum_{i=1}^{r} h_{1\_i}A_{1\_i} + (1-\alpha)\sum_{i=1}^{r} h_{2\_i}A_{2\_i})^T\\ &\quad P(\alpha\sum_{i=1}^{r} h_{1\_i}A_{1\_i} + (1-\alpha)\sum_{i=1}^{r} h_{2\_i}A_{2\_i}) - P\Big]x(k) \quad (15)\end{aligned}$$

Therefore, $\Delta V(x(k)) = V(x(k+1)) - V(x(k))$ is negative along a switching law then the T-S fuzzy switched discrete-time system is asymptotically stable.

It will be convenient throughout this chapter to use the following notations:

$$\bar{A}_{l\_jk} = \max(\|A_{l\_i\_jk}\|) \quad P_{\lambda_l} = Max[\lambda(P_l)] \; l = 1, 2$$

$$\begin{aligned}c &= -\|Q_m\| + P_{\lambda_1} + \sum_{\substack{i=1\\ i\ne k_i}}^{r} \|A_{1\_i\_11}\|^2 P_{\lambda_1} + \|\bar{A}_{1\_21}\|^2 P_{\lambda_2}\\ &\quad + \|\bar{A}_{1\_11}\| P_{\lambda_1} \|\bar{A}_{1\_12}\| + \|\bar{A}_{1\_21}\| P_{\lambda_2} \|\bar{A}_{1\_22}\|,\\ d &= \|\bar{A}_{2\_11}\|^2 P_{\lambda_1} + \|\bar{A}_{2\_21}\|^2 P_{\lambda_2} + \|\bar{A}_{2\_11}\| P_{\lambda_1} \|\bar{A}_{2\_12}\|\\ &\quad + \|\bar{A}_{2\_21}\| P_{\lambda_2} \|\bar{A}_{2\_22}\|,\\ e &= \|\bar{A}_{1\_12}\| P_{\lambda_1} \|\bar{A}_{1\_11}\| + \|\bar{A}_{1\_22}\| P_{\lambda_2} \|\bar{A}_{1\_21}\|\\ &\quad + \|\bar{A}_{1\_12}\|^2 P_{\lambda_1} + \|\bar{A}_{1\_22}\|^2 P_{\lambda_2},\\ f &= \|\bar{A}_{2\_12}\| P_{\lambda_1} \|\bar{A}_{2\_11}\| + \|\bar{A}_{2\_22}\| P_{\lambda_2} \|\bar{A}_{2\_21}\| + \|\bar{A}_{2\_12}\|^2 P_{\lambda_1}\\ &\quad - \|Q_m\| + P_{\lambda_2} + \sum_{\substack{i=1\\ i\ne k_i}}^{r} \|A_{2\_i\_22}\|^2 P_{\lambda_2}.\end{aligned}$$

Now, one sufficient condition for the stability of T-S fuzzy switched discrete-time system (3) with $N = 2$ is addressed as the following Theorem 1.

Theorem 1: Consider the T-S fuzzy switched discrete-time system (3) with $N = 2$, there exists a switching law such that the system (3) is asymptotically stable, if there are matrices $P_1, P_2 > 0$ and a constant $\alpha \in (\bar{\Lambda}_1 \cap \bar{\Lambda}_2)$, where

$$\bar{\Lambda}_1 = \{\alpha | \alpha(c-d) + d - P_{\lambda_1} < 0\} \quad (16a)$$

$$\bar{\Lambda}_2 = \{\alpha | \alpha(e-f) + f - P_{\lambda_2} < 0\} \quad (16b)$$

Proof: By the Lemma 2, the difference of the Lyapunov function (15) is

$$\Delta V(k) = x^T(k)((1+\varepsilon)\alpha^2 A_1^T PA_1 + (1+\varepsilon^{-1})(1-\alpha)^2 A_2^T PA_2 - P)x(k)$$

In the light of Lemma 3, let $\alpha = (1+\varepsilon)^{-1}$, then

$$\begin{aligned}\Delta V(k) \le x^T(k)\Bigg[&\alpha\left(\sum_{i=1}^{r} h_{1\_i}A_{1\_i}\right)^T P\left(\sum_{i=1}^{r} h_{1\_i}A_{1\_i}\right)\\ &+(1-\alpha)\left(\sum_{i=1}^{r} h_{2\_i}A_{2\_i}\right)^T P\left(\sum_{i=1}^{r} h_{2\_i}A_{2\_i}\right) - P\Bigg]x(k) \quad (17)\end{aligned}$$

Using the properties of matrix norm and above notations, we can change (17) into

$$\Delta V(k) \le \|x_1\|^2 \Big[\alpha(-\|Q_m\| + P_{\lambda_1} + \sum_{\substack{i=1\\ i\ne k_i}}^{r} \|A_{1\_i\_11}\|^2 P_{\lambda_1} + \|A_{1\_21}\|^2 P_{\lambda_2} + \|A_{1\_11}\| P_{\lambda_1} \|A_{1\_12}\| + \|A_{1\_21}\| P_{\lambda_2} \|A_{1\_22}\|) + (1-\alpha)(\|A_{2\_11}\|^2 P_{\lambda_1} + \|A_{2\_21}\|^2 P_{\lambda_2} + \|A_{2\_11}\| P_{\lambda_1} \|A_{2\_12}\| + \|A_{2\_21}\| P_{\lambda_2} \|A_{2\_22}\|) - P_{\lambda_1} \Big]$$
$$+ \|x_2\|^2 \Big[\alpha(\|A_{1\_12}\| P_{\lambda_1} \|A_{1\_11}\| + \|A_{1\_22}\| P_{\lambda_2} \|A_{1\_21}\| + \|A_{1\_12}\|^2 P_{\lambda_1} + \|A_{1\_22}\|^2 P_{\lambda_2}) + (1-\alpha)(\|A_{2\_12}\| P_{\lambda_1} \|A_{2\_11}\| + \|A_{2\_22}\| P_{\lambda_2} \|A_{2\_21}\| + \|A_{2\_12}\|^2 P_{\lambda_1} - \|Q_m\| + P_{\lambda_2} + \sum_{\substack{i=1\\ i\ne k_i}}^{r} \|A_{2\_i\_22}\|^2 P_{\lambda_2}) - P_{\lambda_2} \Big]$$

Therefore, we get following inequality (18)

$$\Delta V(k) \le \|x_1\|^2 \left[\alpha(c-d) + d - P_{\lambda_1} \right] + \|x_2\|^2 \left[\alpha(e-f) + f - P_{\lambda_2} \right] \tag{18}$$

If we choose $\alpha \in (\bar{\Lambda}_1 \cap \bar{\Lambda}_2)$, then $\Delta V(k) < 0$. Hence, the system (3) is asymptotically stable. Proof of Theorem 1 is completed.

Switching Law: T-S fuzzy switched discrete-time system (3) with $N = 2$ is switched to or stay at mode l at sampling step k if (19) is satisfied at k.

$$x^T(k)[(\sum_{i=1}^{r} h_{l\_i} A_{l\_i})^T P (\sum_{i=1}^{r} h_{l\_i} A_{l\_i}) - P] x(k) < 0$$
$$l \in \{1,2\},\ i = 1,2,\ldots,r \tag{19}$$

Now, the result also can be extended to arbitrary N individual subsystems. Hence, the system has been described as follows:

$$\dot{x}(k+1) = \sum_{i=1}^{r} h_{l\_i} A_{l\_i} x(k) \tag{20}$$

where

$$A_{l\_i} = \begin{bmatrix} A_{l\_i\_11} & A_{l\_i\_12} & \cdots & A_{l\_i\_1N} \\ A_{l\_i\_21} & A_{l\_i\_22} & \cdots & A_{l\_i\_2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{l\_i\_N1} & A_{l\_i\_N2} & \cdots & A_{l\_i\_NN} \end{bmatrix}, l \in \{1,2,\ldots,N\}.$$

In the light of Lemma 2, the stability of T-S fuzzy switched system (4) is equivalent to that of the following system

$$x(k+1) = \sum_{l=1}^{N} \sum_{i=1}^{r} \alpha_l h_{l\_i} A_{l\_i} x(k) \tag{21}$$

To investigate the stability of system, we choose the Lyapunov function candidate as

$$V(x(k)) = x^T(k) P x(k) \tag{22}$$

where $x^T(k) = \left[x_1^T\ x_2^T \cdots x_k^T \right]^T$ and $P = diag\{P_1, P_2, \ldots, P_K\}$ are unique real symmetric positive-definite matrices satisfying the Lyapunov equation (14) which $l = 1,2,\ldots,K$.

Without loss of generality, $A_{l\_i\_ll}$ are Hurwitz matrices.

Theorem 2: Consider the T-S fuzzy switched discrete-time system (3) with arbitrary N systems, there exist a switching law such that the system (3) is asymptotically stable if there are matrices $P_1, P_2, \ldots, P_K > 0$ and constants α_l satisfy

$$\sum_{l=1}^{N} \alpha_l = 1 \text{ and } 0 < \alpha_l < 1 \tag{23a}$$

$$(-Q_m + \sum_{\substack{i=1\\ i\ne k_i}}^{r} \|A_{m\_i\_mm}^T P_m A_{m\_i\_mm}\|) + \sum_{\substack{l=1\\ l\ne m}}^{N} \frac{\alpha_l}{\alpha_m} (\bar{A}_{l\_mm}^T P_m \bar{A}_{l\_mm}) + \sum_{l=1}^{N} \sum_{\substack{n\ne m\\ n=1}}^{K} \frac{\alpha_l}{\alpha_n} (\bar{A}_{l\_mn}^T P_m \bar{A}_{l\_mn}) < 0 \tag{23b}$$

where $\bar{A}_{l\_mm} = \max_{l\ne k_i} \|A_{l\_i\_mm}\|$, $\bar{A}_{l\_mn} = \max \|A_{l\_i\_mn}\|$ for $m,n = 1,2,\ldots,K$, $i = 1,2,\ldots,r$ and $\alpha_l = \frac{1}{(1+\varepsilon^{-1})^{l-1}(1+\varepsilon)}$ $l = 1,2,\ldots,N-1$, $\alpha_N = \frac{1}{(1+\varepsilon^{-1})^{N-1}}$.

Proof: Choose the Lyapunov function as (22) and the difference of Lyapunov function as

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k))$$
$$= \left[(\sum_{l=1}^{N} \sum_{i=1}^{r} \alpha_l h_{l\_i} A_{l\_i} x(k))^T P (\sum_{l=1}^{N} \sum_{i=1}^{r} \alpha_l h_{l\_i} A_{l\_i} x(k)) \right] - x(k)^T P x(k) \tag{24}$$

Then we can get

$$\Delta V(x(k)) \le \sum_{m=1}^{K} \{ \sum_{l=1}^{N} \alpha_l [\sum_{i=1}^{r} h_{l\_i} (A_{l\_i\_mm} x_m(k) + \sum_{\substack{n\ne m\\ n=1}}^{K} A_{l\_i\_mn} x_n(k))^T P_m \times \sum_{i=1}^{r} h_{l\_i} (A_{l\_i\_mm} x_m(k) + \sum_{\substack{n\ne m\\ n=1}}^{K} A_{l\_i\_mn} x_n(k))] - x_m^T(k) P_m x_m(k) \} \tag{25}$$

$$\Delta V(x(k)) \le \sum_{m=1}^{K} \{ \sum_{l=1}^{N} \alpha_l [\frac{1}{\alpha_m} (\sum_{i=1}^{r} h_{l\_i} A_{l\_i\_mm} x_m(k))^T \times P_m (\sum_{i=1}^{r} h_{l\_i} A_{l\_i\_mm} x_m(k)) + \sum_{\substack{n\ne m\\ n=1}}^{K} \frac{1}{\alpha_n} (\sum_{i=1}^{r} h_{l\_i} A_{l\_i\_mn} x_n(k))^T P_m (\sum_{i=1}^{r} h_{l\_i} A_{l\_i\_mn} x_n(k))] - x_m^T(k) P_m x_m(k) \} \tag{26}$$

From Lemma 2, Lemma 3 and (14), (26) become to following

$$\Delta V(x(k)) \le \sum_{m=1}^{K} \Big\{ x_m^T [(-Q_m + \sum_{\substack{i=1\\ i\ne k_i}}^{r} \|A_{m\_i\_mm}^T P_m A_{m\_i\_mm}\|)$$

$$+\sum_{\substack{l=1\\l\neq m}}^{N}\frac{\alpha_l}{\alpha_m}(\bar{A}_{l\_mm}^T P_m \bar{A}_{l\_mm})+\sum_{l=1}^{N}\sum_{\substack{n\neq m\\n=1}}^{K}\frac{\alpha_l}{\alpha_n}(\bar{A}_{l\_mn}^T P_m \bar{A}_{l\_mn})]\Bigg\}x_m \quad (27)$$

If (23) is satisfied, then $\Delta V(k)\le 0$. Hence, the T-S fuzzy switched system (3) is asymptotically stable. Proof of the Theorem 2 is completed.

Switching Law: T-S fuzzy switched discrete-time system (3) with arbitrary N systems is switched to or stay at mode l at sampling step k if (28) is satisfied at k.

$$x^T(k)[(\sum_{i=1}^{r}h_{l\_i}A_{l\_i})^T P(\sum_{i=1}^{r}h_{l\_i}A_{l\_i})-P]x(k)<0 \quad (28)$$

$l\in\{1,2,\ldots,N\}$ $i=1,2,\ldots,r$

The T-S fuzzy switched discrete-time system is satisfying the condition, and then the system is asymptotically stable under the switching law.

4. Controller Design

In this section, we will utilize the concept of parallel contributed compensation (PDC) to design fuzzy controller to stabilize T-S fuzzy switched discrete-time system. The design of state feedback fuzzy controller is to determine the local feedback gains F_{l_i} in the sequent parts such that the overall closed-loop T-S fuzzy switched system is stable.

By substituting (6) into (4), and using Lemma 1, the fact that the closed-loop fuzzy switched discrete-time system is stable implies that the following system is stable:

$$x(k+1)=\sum_{l=1}^{N}\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_l h_{l\_i}h_{l\_j}\left\{A_{l\_i}+B_{l\_i}F_{l\_j}\right\}x(k) \quad (29)$$

$l\in\{1,2,\ldots,N\}$, $i=1,2,\ldots,r$.

According to (29), first, the switched system with two individual systems will be discussed in this section. Hence, it is obvious that the stabilization of the T-S fuzzy switched discrete-time system (29) is equivalent to (30)

$$x(k+1)=\Bigg\{\alpha_1\sum_{i=1}^{r}\sum_{j=1}^{r}h_{1\_i}h_{1\_j}(A_{1\_i}-B_{1\_i}F_{1\_j})+\alpha_2\sum_{i=1}^{r}\sum_{j=1}^{r}h_{2\_i}h_{2\_j}(A_{2\_i}-B_{2\_i}F_{2\_j})\Bigg\}x(k) \quad (30)$$

where $\alpha_1+\alpha_2=1$

To investigate the stability of system (29), we choose the Lyapunov function candidate as

$$V(x(k))=x^T(k)Px(k) \quad (31)$$

where P is unique real symmetric positive definite matrices.

Theorem 3: The T-S fuzzy switched discrete-time system (3) with $N=2$ is asymptotically stable via the fuzzy controller (6) if there exist symmetric matrices Q and matrices K_{l_i} and $\alpha_1,\alpha_2\in[0,1]$ with $\alpha_1+\alpha_2=1$, such that the following *LMIs* are satisfied

(i) $$\left(\alpha_1A_{1\_i}Q-\alpha_1B_{1\_i}K_{1\_i}+\alpha_2A_{2\_i}Q-\alpha_2B_{2\_i}K_{2\_i}\right)^TQ^{-1}\times\left(\alpha_1A_{1\_i}Q-\alpha_1B_{1\_i}K_{1\_i}+\alpha_2A_{2\_i}Q-\alpha_2B_{2\_i}K_{2\_i}\right)-Q<0,$$
$$i=1,2,\ldots,r \quad (32a)$$

(ii) $$\left\{\frac{1}{2}\Big[\alpha_1(A_{1\_i}Q-B_{1\_i}K_{1\_j})+\alpha_1(A_{1\_j}Q-B_{1\_j}K_{1\_i})+\alpha_2(A_{2\_i}Q-B_{2\_i}K_{2\_j})+\alpha_2(A_{2\_j}Q-B_{2\_j}K_{2\_i})\Big]\right\}^TQ^{-1}$$
$$\times\left\{\frac{1}{2}\Big[\alpha_1(A_{1\_i}Q-B_{1\_i}K_{1\_j})+\alpha_1(A_{1\_j}Q-B_{1\_j}K_{1\_i})+\alpha_2(A_{2\_i}Q-B_{2\_i}K_{2\_j})+\alpha_2(A_{2\_j}Q-B_{2\_j}K_{2\_i})\Big]\right\}-Q<0,$$
$$i=1,2,\ldots,r.\quad i<j\le r. \quad (32b)$$

and the feedback gains are obtained from $K_{l\_i}=F_{l\_i}Q$ and $P=Q^{-1}$

Proof: According to (4.36). Hence,

$$x(k+1)=\sum_{l=1}^{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_l h_{l\_i}h_{l\_j}\left\{A_{l\_i}-B_{l\_i}F_{l\_j}\right\}x(k)$$
$$=\Bigg\{\alpha_1\sum_{i=1}^{r}\sum_{j=1}^{r}h_{1\_i}h_{1\_j}\left(A_{1\_i}-B_{1\_i}F_{1\_j}\right)+\alpha_2\sum_{i=1}^{r}\sum_{j=1}^{r}h_{2\_i}h_{2\_j}\left(A_{2\_i}-B_{2\_i}F_{2\_j}\right)\Bigg\}x(k) \quad (33)$$

We choose the Lyapunov function candidate as: $V(x(k))=x^T(k)Px(k)$, where $P>0$. Then

$$\Delta V(x(k))=x(k)^T\Bigg\{\Bigg[\alpha_1\sum_{i=1}^{r}h_{1\_i}^2(A_{1\_i}-B_{1\_i}F_{1\_i})+\alpha_2\sum_{i=1}^{r}h_{2\_i}^2(A_{2\_i}-B_{2\_i}F_{2\_i})\Bigg]^TP$$
$$\times\Bigg[\alpha_1\sum_{i=1}^{r}h_{1\_i}^2(A_{1\_i}-B_{1\_i}F_{1\_i})+\alpha_2\sum_{i=1}^{r}h_{2\_i}^2(A_{2\_i}-B_{2\_i}F_{2\_i})\Bigg]-P\Bigg\}x(k)$$
$$+x(k)^T\Bigg\{\Bigg[2\Bigg(\alpha_1\sum_{i<j}^{r}h_{1\_i}h_{1\_j}\bigg(\frac{(A_{1\_i}-B_{1\_i}F_{1\_j})+(A_{1\_j}-B_{1\_j}F_{1\_i})}{2}\bigg)\Bigg)+\alpha_2\sum_{i<j}^{r}h_{2\_i}h_{2\_j}\bigg(\frac{(A_{2\_i}-B_{2\_i}F_{2\_j})+(A_{2\_j}-B_{2\_j}F_{2\_i})}{2}\bigg)\Bigg]^T$$
$$\times P\Bigg[2\Bigg(\alpha_1\sum_{i<j}^{r}h_{1\_i}h_{1\_j}\bigg(\frac{(A_{1\_i}-B_{1\_i}F_{1\_j})+(A_{1\_j}-B_{1\_j}F_{1\_i})}{2}\bigg)\Bigg)+\alpha_2\sum_{i<j}^{r}h_{2\_i}h_{2\_j}\bigg(\frac{(A_{2\_i}-B_{2\_i}F_{2\_j})+(A_{2\_j}-B_{2\_j}F_{2\_i})}{2}\bigg)\Bigg]$$
$$-P\Bigg\}x(k) \quad (34)$$

Pre-multiply and post-multiply both sides of the first part of (34) by $Q=P^{-1}$, yields

$$Q\left(\alpha_1(A_{1\_i}-B_{1\_i}F_{1\_i})+\alpha_2(A_{2\_i}-B_{2\_i}F_{2\_i})\right)^TQ^{-1}$$

$$\left(\alpha_1(A_{1\_i}-B_{1\_i}F_{1\_i})+\alpha_2(A_{2\_i}-B_{2\_i}F_{2\_i})\right)Q-Q$$

In this case, the fuzzy feedback gains are $F_{l\_i}=K_{l\_i}Q^{-1}$, where $l\in\{1,2\}$, $i=1,2,\ldots,r$. Hence,

$$\left(\alpha_1A_{1\_i}Q-\alpha_1B_{1\_i}K_{1\_i}+\alpha_2A_{2\_i}Q-\alpha_2B_{2\_i}K_{2\_i}\right)^TQ^{-1}$$
$$\times\left(\alpha_1A_{1\_i}Q-\alpha_1B_{1\_i}K_{1\_i}+\alpha_2A_{2\_i}Q-\alpha_2B_{2\_i}K_{2\_i}\right)-Q<0 \quad (35)$$

Inequality (35) is the first *LMI* in Theorem 3, and then pre-multiply and post-multiply both sides of the second part of (34) by $Q=P^{-1}$ and define $F_{l\_i}=K_{l\_i}Q^{-1}$, yields

$$\left\{\frac{1}{2}\left[\alpha_1(A_{1\_i}Q-B_{1\_i}K_{1\_j})+\alpha_1(A_{1\_j}Q-B_{1\_j}K_{1\_i})\right.\right.$$
$$\left.\left.+\alpha_2(A_{2\_i}Q-B_{2\_i}K_{2\_j})+\alpha_2(A_{2\_j}Q-B_{2\_j}K_{2\_i})\right]\right\}^TQ^{-1}$$
$$\times\left\{\frac{1}{2}\left[\alpha_1(A_{1\_i}Q-B_{1\_i}K_{1\_j})+(A_{1\_j}Q-B_{1\_j}K_{1\_i})\right.\right.$$
$$\left.\left.+\alpha_2(A_{2\_i}Q-B_{2\_i}K_{2\_j})+(A_{2\_j}Q-B_{2\_j}K_{2\_i})\right]\right\}-Q<0 \quad (36)$$

The second *LMI* is yielded. If (4.38a) and (4.38b) hold, we can obtained $\dot{V}<0$. This completes the proof of theorem 3.

Switching Law: T-S fuzzy switched discrete-time system (29) with N=2 is switched to or stay at mode l at time k if (37) is satisfied at time k

$$x^T(k)\left\{\sum_{i=1}^r\sum_{j=1}^r h_{l\_i}(A_{l\_i}-B_{l\_i}F_{l\_j})^T\right.$$
$$\left.P\sum_{i=1}^r\sum_{j=1}^r h_{l\_i}(A_{l\_i}-B_{l\_i}F_{l\_j})-P\right\}x(k)<0,\quad l\in\{1,2\}. \quad (37)$$

After we have proposed one result for individual system is true, we assume the case is extended to N individual systems. Hence, according to Lemma 1, the stable of T-S fuzzy switched discrete-time system implies that of the following system

$$x(k+1)=\sum_{l=1}^N\sum_{i=1}^r\sum_{j=1}^r\alpha_lh_{l\_i}h_{l\_j}\left\{A_{l\_i}-B_{l\_i}F_{l\_j}\right\}x(k)$$
$$l\in\{1,2,\ldots,N\} \quad (38)$$

Theorem 4: Suppose the T-S fuzzy switched discrete-time system (4) with arbitrary N individual modes, then the T-S fuzzy switched discrete-time system (4) is asymptotically stabilizable via the fuzzy controller (6) if there exist symmetric matrix Q and matrices K_{l_i} and $\alpha_l\in[0,1]$ with $\sum_{l=1}^N\alpha_l=1$, such that the following *LMIs* are satisfied,

(i). $$\left(\sum_{l=1}^N\alpha_l(A_{l\_i}Q-B_{l\_i}K_{l\_i})\right)^TQ^{-1}$$
$$\left(\sum_{l=1}^N\alpha_l(A_{l\_i}Q-B_{l\_i}K_{l\_i})\right)-Q<0 \quad (39a)$$

(ii). $$\left\{\frac{1}{2}\left[\sum_{l=1}^N\alpha_l(A_{l\_i}Q-B_{l\_i}K_{l\_j}+A_{l\_j}Q-B_{l\_j}K_{l\_i})\right]\right\}^TQ^{-1}$$
$$\times\left\{\frac{1}{2}\left[\sum_{l=1}^N\alpha_l(A_{l\_i}Q-B_{l\_i}K_{l\_j}+A_{l\_j}Q-B_{l\_j}K_{l\_i})\right]\right\}-Q<0,$$
$$l\in\{1,2,\ldots,N\},\ i=1,2,\ldots,r. \quad (39b)$$

And the feedback gains are obtained from

$$K_{l\_i}=F_{l\_i}Q \quad\text{and}\quad P=Q^{-1}$$

Proof: According to (29), we consider the system with arbitrary N individual systems.

$$x(k+1)=\left\{\sum_{l=1}^N\alpha_l\left(\sum_{i=1}^r h_{l\_i}^2(A_{l\_i}-B_{l\_i}F_{l\_i})\right.\right.$$
$$\left.\left.+2\sum_{i<j}^r h_{l\_i}h_{l\_j}\left(\frac{(A_{l\_i}-B_{l\_i}F_{l\_j})+(A_{l\_j}-B_{l\_j}F_{l\_i})}{2}\right)\right)\right\}x(k) \quad (40)$$

We choose the Lyapunov function candidate as: $V(x(t))=x^T(t)Px(t)$, and the difference of Lyapunov function as:

$$\Delta V(x(k))=x^T(k+1)Px(k+1)-x(k)^TPx(k)$$
$$=x(k)^T\left\{\sum_{l=1}^N\alpha_l\left[\sum_{i=1}^r h_{l\_i}^2(A_{l\_i}-B_{l\_i}F_{l\_i})\right]^TP\right.$$
$$\left.\sum_{l=1}^N\alpha_l\left[\sum_{i=1}^r h_{l\_i}^2(A_{l\_i}-B_{l\_i}F_{l\_i})\right]-P\right\}x(k)$$
$$+x(k)^T\left\{\left[\left[2\sum_{l=1}^N\alpha_l\left(\sum_{i<j}^r h_{l\_i}h_{l\_j}\right.\right.\right.\right.$$
$$\left.\left.\left.\left(\frac{(A_{l\_i}-B_{l\_i}F_{l\_j})+(A_{l\_j}-B_{l\_j}F_{l\_i})}{2}\right)\right)\right]\right]^T$$
$$\times P\left[2\sum_{l=1}^N\alpha_l\left(\sum_{i<j}^r h_{l\_i}h_{l\_j}\right.\right.$$
$$\left.\left.\left.\left(\frac{(A_{l\_i}-B_{l\_i}F_{l\_j})+(A_{l\_j}-B_{l\_j}F_{l\_i})}{2}\right)\right)\right]-P\right\}x(k) \quad (41)$$

Pre-multiply and post-multiply both sides of the first part of (41) by $Q=P^{-1}$, yields

$$Q\left(\sum_{l=1}^N\alpha_l(A_{l\_i}-B_{l\_i}F_{l\_i})\right)^TQ^{-1}$$
$$\left(\sum_{l=1}^N\alpha_l(A_{l\_i}-B_{l\_i}F_{l\_i})\right)Q-Q$$

Define $F_{l\_i}=K_{l\_i}Q^{-1}$, where $l\in\{1,2,\ldots,N\}$, $i=1,2,\ldots,r$. Hence

$$\left(\sum_{l=1}^N\alpha_l(A_{l\_i}Q-B_{l\_i}K_{l\_i})\right)^TQ^{-1}$$
$$\left(\sum_{l=1}^N\alpha_l(A_{l\_i}Q-B_{l\_i}K_{l\_i})\right)-Q<0 \quad (42)$$

Inequality (42) is the first *LMI* in theorem 4, and then pre-multiply and post-multiply both sides of the second part of (41) by $Q=P^{-1}$ and define $F_{l\_i}=K_{l\_i}Q^{-1}$, yields

$$\left\{\frac{1}{2}\left[\sum_{l=1}^N\alpha_l(A_{l\_i}Q-B_{l\_i}K_{l\_j}+A_{l\_j}Q-B_{l\_j}K_{l\_i})\right]\right\}^TQ^{-1}$$

$$\times\left\{\frac{1}{2}\left[\sum_{l=1}^{N}\alpha_l(A_{l\_i}Q-B_{l\_i}K_{l\_j}+A_{l\_j}Q-B_{l\_j}K_{l\_i})\right]\right\}-Q<0 \quad (43)$$

The system is satisfying the condition, and then the T-S fuzzy switched discrete-time system is asymptotically stable under the switching law. This completes the proof of theorem 4.

Switching Law: T-S fuzzy switched discrete-time system (29) with arbitrary N individual systems is switched to or stay at mode l at time k if (44) is satisfied at time k

$$x^T(k)\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_{l\_i}(A_{l\_i}-B_{l\_i}F_{l\_j})^T P\sum_{i=1}^{r}\sum_{j=1}^{r}h_{l\_i}(A_{l\_i}-B_{l\_i}F_{l\_j})-P\right\}x(k)<0,$$
$$l\in\{1,2,\ldots,N\} \quad (44)$$

5. Numerical Examples

Example 1: We consider the two individual subsystems of T-S fuzzy switched discrete-time system without input, given as follows:

Individual system 1:

Plant Rule 1: If $x_1(t)$ is $M_{11}(x_1)$ Then $\dot{x}(t)=A_{1\_1}x(t)$

Plant Rule 2: If $x_1(t)$ is $M_{21}(x_1)$ Then $\dot{x}(t)=A_{1\_2}x(t)$

Plant Rule 3: If $x_1(t)$ is $M_{31}(x_1)$ Then $\dot{x}(t)=A_{1\_3}x(t)$

Individual system 2:

Plant Rule 1: If $x_1(t)$ is $M_{11}(x_1)$ Then $\dot{x}(t)=A_{2\_1}x(t)$

Plant Rule 2: If $x_1(t)$ is $M_{21}(x_1)$ Then $\dot{x}(t)=A_{2\_2}x(t)$

Plant Rule 3: If $x_1(t)$ is $M_{31}(x_1)$ Then $\dot{x}(t)=A_{2\_3}x(t)$

where

$$A_{1\_1}=\begin{bmatrix}0.4868 & 0 & 0.0345\\ 0 & 0.1004 & 0.0118\\ 0 & 0.1254 & 1.1124\end{bmatrix},$$

$$A_{1\_2}=\begin{bmatrix}0.0343 & 0.0512 & 0.0079\\ 0 & 0.1721 & 0.0326\\ 0 & 0.1733 & 1.1008\end{bmatrix},$$

$$A_{1\_3}=\begin{bmatrix}0.0139 & 0.0664 & 0.0103\\ 0 & 0.3551 & 0.0445\\ 0 & 0.1444 & 1.0538\end{bmatrix},$$

$$A_{2\_1}=\begin{bmatrix}1.2032 & 0.0327 & 0.0496\\ 0.0325 & 0.3048 & 0.0013\\ -0.0192 & 0 & 0.2096\end{bmatrix},$$

$$A_{2\_2}=\begin{bmatrix}1.2252 & 0.0120 & 0.0172\\ -0.0193 & 0.4594 & 0.0005\\ 0.0257 & 0 & 0.4673\end{bmatrix},$$

$$A_{2\_3}=\begin{bmatrix}1.1131 & 0.0412 & 0.0255\\ 0.0055 & 0.2388 & 0.0001\\ 0.0333 & 0.0001 & 0.0945\end{bmatrix}$$

That is

$$A_{1\_1\_11}=[0.4868];\ A_{1\_1\_22}=\begin{bmatrix}0.1004 & 0.0118\\ 0.1254 & 1.1124\end{bmatrix};$$
$$A_{1\_1\_12}=[0 \quad 0.0345];\ A_{2\_2\_11}=[1.2252];$$
$$A_{2\_2\_21}=\begin{bmatrix}-0.0193\\ 0.0257\end{bmatrix};\ A_{2\_2\_22}=\begin{bmatrix}0.4594 & 0.0005\\ 0 & 0.4673\end{bmatrix}.$$

The membership function for Rules 1, 2 and 3 for each individual system are represented by the following membership functions respectively:

$$M_{11}(x_1)=M_{21}(x_1)=\begin{cases}1 & x_1\le -1\\ -x_1 & -1<x_1<0\\ 0 & 0\le x_1\end{cases}$$

$$M_{12}(x_1)=M_{22}(x_1)=\begin{cases}0 & x_1\le -1\\ x_1+1 & -1<x_1\le 0\\ -x_1+1 & 0<x_1\le 1\\ 0 & 1<x_1\end{cases}$$

$$M_{13}(x_1)=M_{23}(x_1)=\begin{cases}0 & x_1\le 0\\ x_1 & 0<x_1\le 1\\ 1 & 1<x_1\end{cases}$$

where $x(k)=[x_1(k) \quad x_2(k)]^T$.

Thus, according to Lyapunov equation (13) and let $Q_m=I$, P_m $(m=1,2)$ can be obtained as following:

$$P_1=[1.3106] \text{ and } P_2=\begin{bmatrix}1.2675 & 0.0004\\ 0.0004 & 1.2794\end{bmatrix}$$

In view of the stability conditions of Theorem 1, the inequalities (16) can be written as follows:

$$-1.7307\alpha+0.7766<0 \text{ and } 1.1438\alpha-0.7704<0$$

Thus, we can get the stable region is $0.4487<\alpha<0.6735$. The T-S fuzzy switched discrete-time system can be stabilized by switching law and it is chosen based on Lemma 1 which is given as follows:

Switching Law: T-S fuzzy switched discrete-time system is switched to or stay at mode l at sampling step k if (45) is satisfied at k.

$$x^T(k)[(\sum_{i=1}^{3}h_{l\_i}A_{l\_i})^T P(\sum_{i=1}^{3}h_{l\_i}A_{l\_i})-P]x(k)<0,\quad l\in\{1,2\}. \quad (45)$$

Example 1 is exploited to illustrate the proposed schemes, stability conditions that guarantee the T-S fuzzy switched system is asymptotically stable for construction of stabilizing switching law. The main analysis is based on the fact that the existence of a linear convex combination implies quadratic stabilizability of the switched system. For the switched systems, convex combination technique is the most suitable for application. It is still possible to find a stable convex combination for a class of switched system. Therefore, the stability analysis for the switched system, the convex combination of the whole switched system can be easily to simplify analysis. Especially, the particular method can be applied to cases when all individual subsystems are unstable. And we constructively design a switching rule which can guarantee the stability of the switched system. The trajectory of the T-S fuzzy switched discrete-time system and the switching during $k = 1, 2, \ldots, 40$ is shown in Figures 1(a)–(c) with initial value $x(0) = [100 \text{ -150 -50}]$. Via state-driven switching method (switching law), every subsystem must orchestrate the switching between them in accordance with the states. The switching signal is shown in Figure 1(d).

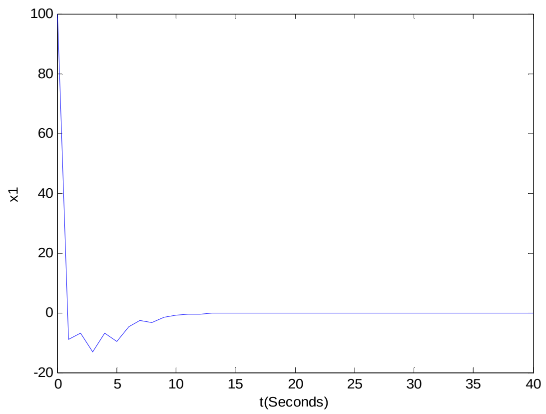

Figure 1(a). State trajectory for $x_1(k)$.

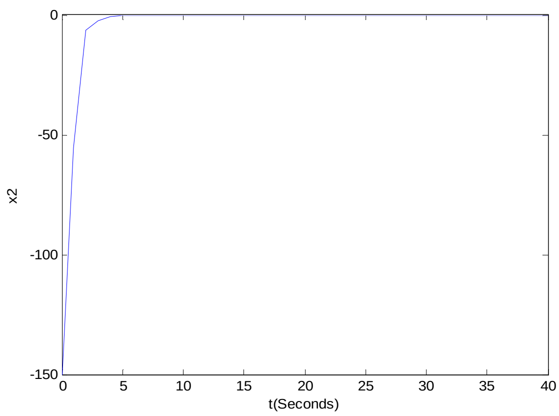

Figure 1(b). State trajectory for $x_2(k)$.

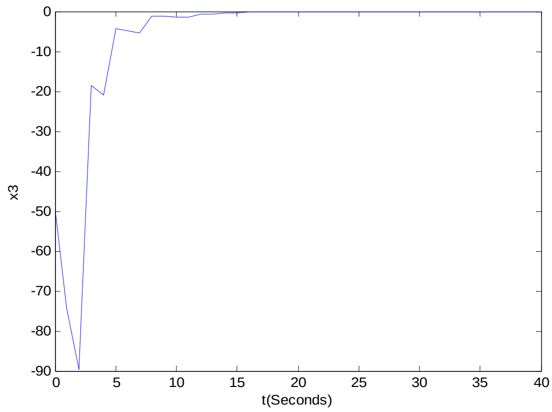

Figure 1(c). State trajectory for $x_3(k)$.

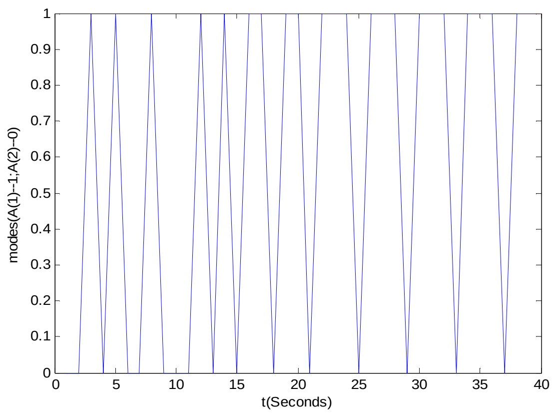

Figure 1(d). State trajectory for the switching law.

Example 2: To illustrate the proposed results, we apply the above analysis technique to a chemical process example [32]. For each model of operation, the mathematical model for the process takes the form

$$\dot{C}_A = \frac{F_\sigma}{V}\left(C_{A_\sigma} - C_A\right) - k_0 e^{-E/RT_R} C_A$$

$$\dot{T}_R = \frac{F_\sigma}{V}\left(T_{A_\sigma} - T_R\right) + \frac{(-\Delta H)}{\rho c_p} k_0 e^{-E/RT_R} C_A + \frac{Q_\sigma}{\rho c_p V} \quad (46)$$

where C_A denotes the concentration of the species A, T_R denotes the temperature of the reactor, Q_σ is the heat removed from the reactor, V is the volume of the reactor, k_0, E, ΔH are the pre-exponential constant, the activation energy and enthalpy of the reaction, c_p and ρ are the heat capacity and fluid density in the reactor, and $\sigma(t) \in \{1, 2\}$ is the discrete variable. The values of all process parameters can be found in Table 1.

The system described by (46) is a classical nonlinear system as follows:

Individual system 1 ($\sigma = 1$):

$$\dot{C}_A = -0.0334 C_A - 1.2 \times 10^9 e^{-10000 T_R} C_A + 0.026386$$

$$\dot{T}_R = -0.0334T_R + 2.4\times10^{11}e^{-10000T_R}C_A + 11.77684 + \frac{Q_\sigma}{23.9}$$

Individual system 2 ($\sigma = 2$):

$$\dot{C}_A = -0.0167C_A - 1.2\times10^{9}e^{-10000T_R}C_A + 0.0167$$

$$\dot{T}_R = -0.0167T_R + 2.4\times10^{11}e^{-10000T_R}C_A + 5.177 + \frac{Q_\sigma}{23.9}$$

It is easy to find the two steady-states when $Q_\sigma = 0$ are $(C_A, T_R)^1 = (0.57, 395.3)$ and $(C_A, T_R)^2 = (0.738, 509.12)$

In this section, we will consider designing a simple fuzzy control law to stabilize the system. As in [17], we present the following fuzzy control law for any expected operating point (x_d, u_d), which is a stationary point of the nonlinear system, and the nonlinear plant of (46) can be represented by the fuzzy rules and utilize nonsynchronous sampling; the continuous state of system can be transformed to discrete state. Hence, each individual system described as follows:

Table 1.

V	=	0.1	m^3
R	=	8.314	$kJ/kmol\cdot K$
C_{A1_s}	=	0.79	$kmol/m^3$
C_{A2_s}	=	1.0	$kmol/m^3$
T_{A1}	=	352.6	K
T_{A2}	=	310.0	K
Q_{1_s}	=	0.0	KJ/hr
Q_{2_s}	=	0.0	KJ/hr
ΔH	=	-4.78×10^4	$KJ/kmol$
k_0	=	1.2×10^9	s^{-1}
E	=	8.314×10^4	$kJ/kmol$
c_p	=	0.239	$kJ/kg\cdot K$
ρ	=	1000.0	kg/m^3
F_1	=	3.34×10^{-3}	m^3/s
F_2	=	1.67×10^{-3}	m^3/s
T_{R_s}	=	395.33	K
C_{A_s}	=	0.57	$kmol/m^3$

Individual system 1 (node 1):

Rule 1:

If the concentration of the species A is $M_{11}(x_1)$ (i.e., $x_1(k)$ is 0.57)

Then $\delta x(k+1) = A_{1\_1}\delta x(k) + B_{1\_1}\delta u(k)$

$\delta u(k) = -F_{1\_1}\delta x(k)$

Rule 2:

If the concentration of the species A is $M_{12}(x_1)$ (i.e., $x_1(k)$ is 0.738)

Then $\delta x(k+1) = A_{1\_2}\delta x(k) + B_{1\_2}\delta u(k)$

$\delta u(k) = -F_{1\_2}\delta x(k)$

Individual system 2 (node 2):

Rule 1:

If the concentration of the species A is $M_{21}(x_1)$ (i.e., $x_1(k)$ is 0.57)

Then $\delta x(k+1) = A_{2\_1}\delta x(k) + B_{2\_1}\delta u(k)$

$\delta u(k) = -F_{2\_1}\delta x(k)$

Rule 2:

If the concentration of the species A is $M_{22}(x_1)$ (i.e., $x_1(k)$ is 0.738)

Then $\delta x(k+1) = A_{2\_2}\delta x(k) + B_{2\_2}\delta u(k)$

$\delta u(k) = -F_{2\_2}\delta x(k)$

where $x(k) = [x_1(k) \quad x_2(k)]^T = [C_A \quad T_R]^T$, $\delta x(k) = x(k) - x_d$, $\delta u(k) = u(k) - u_d$, F_{1_1}, F_{1_2}, F_{2_1} and F_{2_2} are to be designed, where

$$A_{1\_1} = \begin{bmatrix} 0.9954 & 0 \\ 0.2475 & 0.9996 \end{bmatrix},\ A_{1\_2} = \begin{bmatrix} 0.6997 & 0 \\ 59.4951 & 0.9991 \end{bmatrix},$$

$$A_{2\_1} = \begin{bmatrix} 0.9971 & 0 \\ 0.2477 & 0.9996 \end{bmatrix},\ A_{2\_2} = \begin{bmatrix} 0.7003 & 0 \\ 59.3717 & 0.9991 \end{bmatrix}$$

and

$$B_{1\_1} = B_{1\_2} = B_{2\_1} = B_{2\_2} = \begin{bmatrix} 0 \\ 0.0042 \end{bmatrix}.$$

The membership functions for Rule 1 and Rule 2 for each individual system are

$$M_{11}(x_1) = M_{21}(x_1) = \begin{cases} 1 & x_1 \le 0.57 \\ \dfrac{0.738 - x_1}{0.168} & 0.57 < x \le 0.78 \\ 1 & x_1 > 0.738 \end{cases}$$

$$M_{12}(x_1) = M_{22}(x_1) = \begin{cases} 0 & x_1 \le 0.57 \\ \dfrac{x_1 - 0.57}{0.168} & 0.57 < x_1 \le 0.738 \\ 1 & x_1 \ge 0.738 \end{cases}$$

Using the MATLAB LMI toolbox to solve (32), we can obtain the following positive definite matrices P and feedback gain $F_{l\_i} = K_{l\_i}Q^{-1}$.

$$P = Q^{-1} = \begin{bmatrix} 0.0240 & 0.5697 \\ 0.5697 & 34.0079 \end{bmatrix},\ K_{1\_1} = \begin{bmatrix} -0.0004 & 0.0040 \end{bmatrix},$$
$$K_{1\_2} = \begin{bmatrix} 0.0218 & 0.0171 \end{bmatrix},\ K_{2\_1} = \begin{bmatrix} -0.0008 & 0.0040 \end{bmatrix},$$
$$K_{2\_2} = \begin{bmatrix} 0.0262 & 0.0170 \end{bmatrix}.$$

Then feedback gains are as follows:

$$F_{1\_1} = \begin{bmatrix} 0.0015 & 0.1156 \end{bmatrix},\ F_{1\_2} = \begin{bmatrix} 0.0070 & 0.5059 \end{bmatrix},$$
$$F_{2\_1} = \begin{bmatrix} 0.0015 & 0.1158 \end{bmatrix},\ F_{2\_2} = \begin{bmatrix} 0.0070 & 0.5056 \end{bmatrix}.$$

Hence, according to Theorem 3, the nonlinear switched system (46) is stable.

Simulation result of the trajectory of the nonlinear switched system (46) is shown in Figures 2(a)-(c) with initial value $x(0) = \begin{bmatrix} 0.14 & 404.9 \end{bmatrix}$. From these figures, we can see the system (46) is indeed stable via the controller (6). Consequently, marks by the"$*$" are the closed-loop system state at the switching time in Figure 2(c). The nonlinear switched system (46) can be stabilized by switching law and it is chosen based on the Lemma 1 which is given as follows:

Switching Law: Switched nonlinear system (46) is switched to or stay at mode l at sampling step k if (47) is satisfied at k.

$$x^T(k)\left\{ \sum_{i=1}^{2}\sum_{j=1}^{2} h_{l\_i}(A_{l\_i} - B_{l\_i}F_{l\_j})^T P \sum_{i=1}^{2}\sum_{j=1}^{2} h_{l\_i}(A_{l\_i} - B_{l\_i}F_{l\_j}) - P \right\} x(k) < 0,\ l \in \{1,2\} \qquad (47)$$

By switching law, every subsystem must orchestrate the switching between node 1 and node 2 in accordance with the states. The switching signal is shown in Figure 2(d).

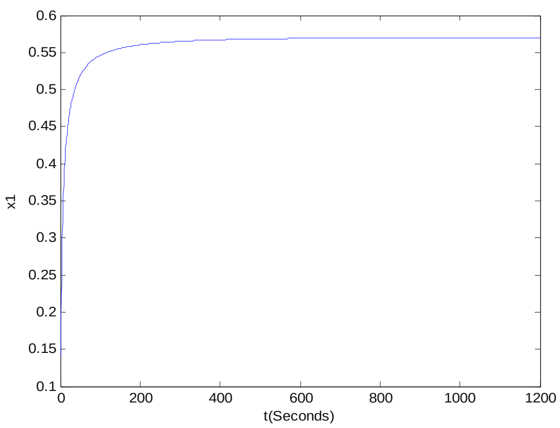

Figure 2(a). State trajectory for $x_1(k)$.

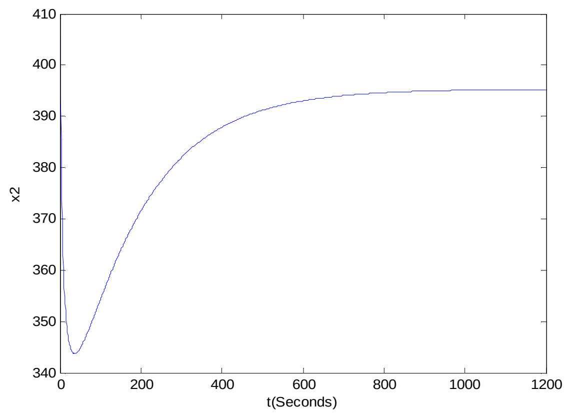

Figure 2(b). State trajectory for $x_2(t)$.

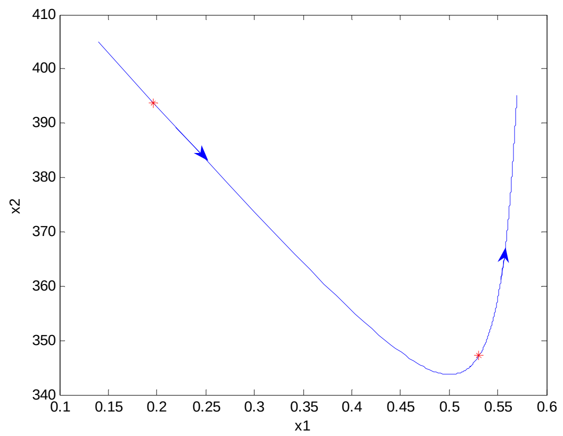

Figure 2(c). State trajectory for $x_1(t)$ vs. $x_2(t)$.

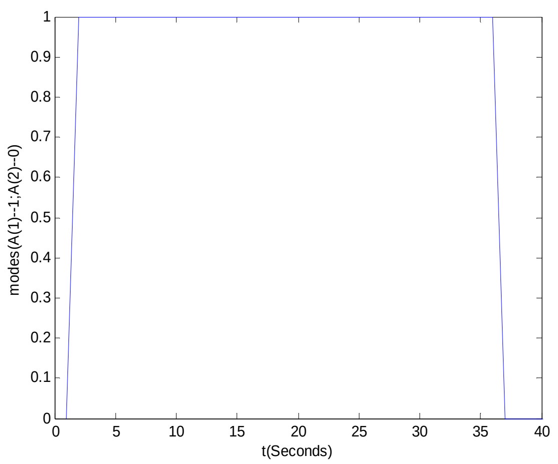

Figure 2(d). State trajectory of the switching law.

6. Conclusions

In this paper, we adopted Lyapunov stability theorem to study the stabilization analysis of a class of T-S fuzzy switched discrete-time system. The main advantages of our approach are that it can be applied to all individual switched systems which are unstable, quantify the region of stability, then we can extend to the case of arbitrary systems of T-S fuzzy switched discrete-time system and develop the simple switching rule to stabilize the T-S fuzzy switched system. The concept of parallel distributed compensation has been employed to design fuzzy controller from the T-S fuzzy models. LMI-based design procedures for fuzzy controller have been constructed using the PDC. Therefore, they can be solved very efficiently in practice by convex programming techniques for LMI's. Finally, two applicable examples illustrate the validity and effectiveness of the stability analysis and design for T-S fuzzy switched discrete-time system described in this paper.

Acknowledgment

This work is supported by the National Science Council, Taiwan, R.O.C., under grand number NSC 96-2221-E-218-040.

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