Identification and Fuzzy Controller Design for Nonlinear Uncertain Systems with Input Time-Delay

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Abstract

This paper considers the identification and fuzzy controller design for nonlinear uncertain systems in presence of unknown input time-delay. Firstly, a time-delay Takagi-Sugeno-Kang (TSK) type fuzzy neural system (TDFN) is proposed to identify a class of nonlinear input time-delay systems. The input-output signals of nonlinear systems are used to identify the system dynamics and unknown time-delay, and then construct the system in the form TSK-fuzzy time-delay model. Each fuzzy rule has a corresponding linear system with an input time-delay as the consequent part. Based on parallel distribution compensation (PDC) approach, the Smith predictor compensation and dominate pole assignment technique are then adopted to design the fuzzy PID controller. Several simulations are shown to demonstrate the effectiveness and control performance of the proposed approach.

Keywords: Nonlinear control, time-delay systems, TSK type fuzzy model, PID controller, neural networks, Smith predictor

1. Introduction

Dynamic systems with time-delay or dead-time are usually common in chemical processes and long transmission lines [5, 9]. This phenomenon is generally caused by variety of sources, including physical transport delay (e.g., in a rolling mill or in a chemical plant), signal transmission delay (e.g., in an earth based satellite control system or in a system controlled over a network) [2, 5, 6, 9, 19, 20, 30]. A typical approach for the analysis and synthesis of a nonlinear system with time delay is local linearization [2, 30]. This only guarantees the local stability of a nonlinear system. Another approach for chemical processes was proposed recently via sliding mode controller [4]. Neural network based controller were introduced to treat a class of nonlinear time-delay systems [6, 7]

Recently, intelligent systems have attracted great attention from both academic and industrial communities. They have been successfully used in a wide variety of applications [3, 9-10, 12-18]. In particular, the neuro-fuzzy systems combining the advantages of fuzzy logic systems and neural networks have become a very active subject in many scientific and engineering areas, such as, model reference control problems, PID controller tuning, signal processing, etc. [10, 12-18]. To date, the fuzzy neural system has been used an alternative approach to conventional controller techniques for complex control problems. These systems possess the properties of a parallel computation scheme, easy to implement, fuzzy logic inference system, and parameter convergence [12-17]. The fuzzy rules can be designed and trained from linguistic information and numeric data. That is, it is then easy to design a fuzzy neural system to achieve a satisfactory level of accuracy by manipulating the network structure and learning algorithm of the fuzzy neural system.

The TSK-type fuzzy dynamic model, proposed by Takagi and Sugeno [25, 26], is described by fuzzy *IF-THEN* rules, which represents local linear input-output relations of nonlinear systems. During the past few decades, several stable fuzzy controller design approach based on the TSK-type fuzzy model have been proposed. These fuzzy models are obtained directly from the dynamic mathematical functions of the nonlinear system. However, the major advantage of fuzzy control approach is model free design, this therefore presents a contradiction. Herein, we propose the Time-Delay-Fuzzy-Neuro systems (TDFN systems) to identify and control a class of nonlinear uncertain systems with input time-delay. Using the TDFN system, the input-output states of nonlinear system are first used to develop the approximated fuzzy model. After the training process, the system is represented by several TSK fuzzy if-then rules. The resulting part of each fuzzy rule is a linear system with input time delay. Subsequently, a novel method to design the Smith predictor based fuzzy PID controller for nonlinear uncertain systems is proposed. Based on the parallel distribution compensation (PDC) approach, the Smith predictor compensation and domi-

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Manuscript received 12 Feb. 2008; revised 1 Jan. 2009; accepted 8 June 2009.

nate pole assignment technique are then adapted to design the fuzzy PID controller. Simulation results of a second order system and the continuous stirred tank reactor (CSTR) systems are shown to demonstrate the effectiveness of our approach.

The paper is organized as follows. In Section 2, nonlinear time-delay systems, TSK-type fuzzy logic systems, the so-called TDFN system, and the corresponding learning algorithm are introduced. Section 3 introduces the Smith Predictor (SP) and the SB-based fuzzy controller design via the PDC approach. Section 4 gives several simulation results for system identification and control to illustrate the effectiveness of our approach. The paper is concluded in Section 5.

2. Time-Delay Fuzzy Neural (TDFN) Systems

A. *Description of Nonlinear Systems with Input Time-delay*

In general, most physical systems are nonlinear and complex, and can be described as the following system

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^r$ are the state vector and control input vector, respectively. Without loss of generality, the equilibrium point is set to be the origin. It is well known that there is no generalization method to analyze the nonlinear system (1) [25, 26]. Therefore, the linearized system is usually adopted to approximate system (1) to possess the system's local properties and characteristics. Hence, the linearized system at the origin equilibria for (1) can be described as

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2}$$

where $A \in \Re^{n\times n}$ and $B \in \Re^{n\times r}$.

Time-delay or dead-time is frequently encountered in dynamical systems. This phenomenon is generally from the result of transmission of information between different parts of the systems [2, 8, 9, 19, 20, 27, 28]. Herein, we consider the transmission time-delay, the control input is thus replaced by $u(t-\theta)$. Therefore,

$$\dot{x}(t) = f(x(t), u(t-\theta)) \tag{3}$$

where x is assumed to be measurable and system (3) is controllable. In this paper, the nonlinear uncertain system with unknown input dead-time system (3) is considered. The system characteristics and time-delay parameter are identified using the TDFN system. Subsequently, a fuzzy PID controller approach is presented to treat the tracking control problem.

B. *TSK-type Fuzzy Model with Input Time-delay*

Many real world systems are nonlinear and a rigorous mathematical model is difficult to obtain. In a lot of literature [10, 12, 13, 17, 18, 26], the systems were represented as a TSK-type fuzzy model, which describes local linear input-output relations of nonlinear systems. Thus, in this way, the nonlinear system (3) can also be approximated by the TSK-type fuzzy model as several fuzzy rules. Each rule has a linear system with input time-delay for the consequent part. System (3) is described by the following form (ith fuzzy rule):

Rule$_i$: If z_1 is F_{i1} and … and z_g is F_{ig}

Then $\dot{x}(t) = A_i x(t) + B_i u(t-\theta_i^h)$, for i=1,2,…,l (4)

where z_i is the premise input variable, F_{ij} is a fuzzy set, l is the rule number, $x(t) \in \Re^n$ is the system state variable, $u(t-\theta_i^h) \in \Re^r$ is the control input, θ_i^h is the time-delay, h=1,2, .., r, and

$$A_i = \begin{bmatrix} a_{11}^i & \dots & a_{1n}^i \\ \vdots & & \vdots \\ a_{n1}^i & \dots & a_{nn}^i \end{bmatrix}, \quad B_i = \begin{bmatrix} b_{11}^i & \dots & b_{1r}^i \\ \vdots & & \vdots \\ b_{n1}^i & \dots & b_{nr}^i \end{bmatrix}.$$

Therefore, for a given input-output pair of ($x(t)$, $u(t)$), the final outputs of the fuzzy systems are inferred as follows:

$$\begin{aligned} Y &= \frac{\sum_{i=1}^{l} \mu_i(z(t))[A_i x(t) + B_i u(t-\theta_i^h)]}{\sum_{i=1}^{l} \mu_i(z(t))} \\ &= \sum_{i=1}^{l} h_i(z(t))[A_i x(t) + B_i u(t-\theta_i^h)] \end{aligned} \tag{5}$$

where $z(t) = \begin{bmatrix} z_1(t) & z_2(t) & \dots & z_g(t) \end{bmatrix}^T$ and

$$h_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j(t))) = \frac{\mu_i(z(t))}{\sum_{i=1}^{l} \mu_i(z(t))}. \tag{6}$$

In addition, it is assumed that $\mu_i(z(t)) \geq 0$ and $\sum_{i=1}^{l} \mu_i(z(t))$ >0, for i=1,2,…, l for all t [25, 26]. Therefore, $h_i(z(t)) \geq 0$, for i=1,2,…, l and $\sum_{i=1}^{l} h_i(z(t)) = 1$ for all t.

C. *Time Delay Fuzzy Neural (TDFN) Systems*

Herein, time-delay TSK-type fuzzy logic system (4) is implemented in a neural network structure, called Time-Delay TSK-type Fuzzy Neuro systems (TDFN systems). The TDFN system contains the concept of the neural network and the ability of learning from information. The structure diagram of the TDFN system is shown in Fig. 1. We indicate the signal propagation and the basic function of every node in each layer.

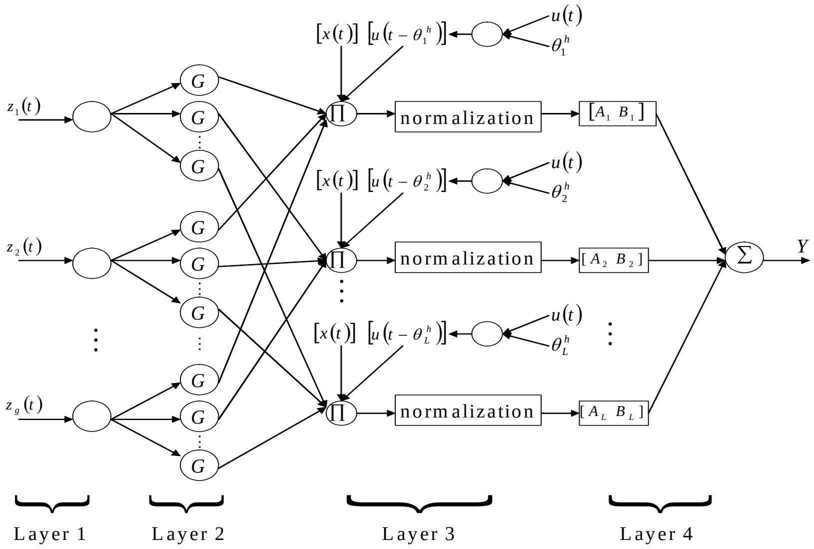


Figure 1. Diagram of time-delay TSK-type fuzzy neuro (TDFN) systems.

In the following description, $x_i^{(k)}$ denotes the *i*th input of a node in the *k*th layer; $O_i^{(k)}$ denotes the *i*th output in layer *k*; and $net_i^{(k)}$ denotes the integration function result of the *i*th node in layer *k*.

Layer 1: input layer

For the *i*th node of layer 1, the net input and output are:

$$O_i^{(1)} = x_i^{(1)}(t)\,. \tag{7}$$

Each node of the first layer transports the input to the membership layer.

Layer 2: membership layer

Each node performs a membership function, the input/output representation is

$$O_{ij}^{(2)} = e^{-\frac{(x_{ij}^{(2)}(t)-\hat{m}_{ij})^2}{(\hat{\sigma}_{ij})^2}} \tag{8}$$

where $x_{ij}^{(2)} = O_i^{(1)}$, $\hat{m}_{ij}$ and $\hat{\sigma}_{ij}$ denote the center and width of the membership function.

Layer 3: rule layer

The links in this layer are used to implement the antecedent matching

$$O_j^{(3)} = \frac{net_j^{(3)}}{\sum_{k=1}^{r} net_k^{(3)}}\,,\quad net_j^{(3)} = \prod_{i=1}^{g} w_{ij}^{(3)} u_{ij}^{(3)} \tag{9}$$

where $w_{ij}^{(3)} = 1$ and $x_{ij}^{(3)} = O_{ij}^{(2)}$. Note that the normalization is performed in this layer for convenience. Therefore, the fire strength of the *i*th rule is

$$\mu_j = \prod_{i=1}^{g} e^{\left(-\frac{(z_i(t)-\hat{m}_{ij})^2}{(\hat{\sigma}_{ij})^2}\right)} = e^{\left(\sum_{i=1}^{g}-\frac{(z_i(t)-\hat{m}_{ij})^2}{(\hat{\sigma}_{ij})^2}\right)}. \tag{10}$$

Layer 4: output layer

The output of TDFN system is a linear combination of the consequence, i.e.,

$$net_j^{(4)} = \sum_{i=1}^{l} w_{ij}^{(4)} x_{ij}^{(4)} \tag{11}$$

where $x_{ij}^{(4)} = O_{ij}^{(3)}$ and $w_{ij}^{(4)} = \hat{A}_i x(t) + \hat{B}_i u\left(t-\hat{\theta}_i^h\right)$. Thus,

$$Y = O^{(4)} = \sum_{i=1}^{l} \hat{h}_i(z(t))\left[\left(\hat{A}_i x(t) + \hat{B}_i u\left(t-\hat{\theta}_i^h\right)\right)\right]. \tag{12}$$

where

$$\hat{h}_j(z(t)) = \frac{net_j^{(3)}}{\sum_{i=1}^{l} net_i^{(3)}} = \frac{\mu_j(z(t))}{\sum_{i=1}^{l} \mu_i(z(t))}\,.$$

Note that the symbol ^ denotes the identification result by TDFN system. Obviously, the adjustable parameters of TDFN system are matrices $\hat{A}_i$ (i.e.,$[a_{pq}^i]$), $\hat{B}_i$ (i.e.,$[b_{vh}^i]$), time-delay $\hat{\theta}_i^h$, mean $\hat{m}_{ij}$ and variance $\hat{\sigma}_{ij}$.

D. *Learning Algorithm*

Given a nonlinear uncertain system with input time delay (3), the objective of learning is to model it by the TDFN system from input and output data. If the unknown system can be approximated by the learned TDFN system, then, a fuzzy PID controller designed based on the TDFN system is available for the original system. Herein, the back-propagation learning algorithm is used to train the TDFN model.

Firstly, we define the error function as

$$E = \frac{1}{2}\left\|D-Y\right\|^2 = \frac{1}{2}\sum_j \left(d_j - y_j\right)^2 \tag{13}$$

where $D=[d_1\ d_2\ \dots\ d_m]^T$ and $Y=[y_1\ y_2\ \dots\ y_m]^T$ denote the actual output and the TDFN system's output, respectively. Using the back-propagation learning algorithm, the update law of parameters W is

$$W(k+1)=W(k)+\eta\left(-\frac{\partial E(k)}{\partial W}\right) \tag{14}$$

where W denotes the adjustable parameters of TDFN system- $\hat{A}_i,\hat{B}_i,\hat{\theta}_h,\hat{m}_{ij},\hat{\sigma}_{ij}$, η denotes the corresponding learning rate. Based on the gradient method, the following terms can be obtained by the chain rule. Our remaining work is to derive the gradient of error function (13) with respect to each TDFN's parameter. Firstly, the gradient for elements of matrices $\hat{A}_i$ and $\hat{B}_i$ are

$$\begin{aligned}-\frac{\partial E}{\partial \hat{a}^i_{pq}}&=-\frac{\partial E}{\partial y_p}\frac{\partial y_p}{\partial net^{(4)}_p}\frac{\partial net^{(4)}_p}{\partial w^{(4)}_{ip}}\frac{\partial w^{(4)}_{ip}}{\partial \hat{a}^i_{pq}}\\ &=\left(d_p-f^{(4)}_p\right)\cdot O^{(3)}_i x_q,\ \ p,q=1,2,\dots,n\end{aligned} \tag{15}$$

$$\begin{aligned}-\frac{\partial E}{\partial \hat{b}^i_{vh}}&=-\frac{\partial E}{\partial y_v}\frac{\partial y_v}{\partial net^{(4)}_v}\frac{\partial net^{(4)}_v}{\partial w^{(4)}_{iv}}\frac{\partial w^{(4)}_{iv}}{\partial \hat{b}^i_{vh}},\\ &=\left(d_v-y_v\right)\cdot O^{(3)}_i u_h\left(t-\hat{\theta}^h_i\right)\\ &\qquad v=1,2,\dots,n,\ h=1,2,\dots,r.\end{aligned} \tag{16}$$

Subsequently, the derivation of the time-delay parameter cannot be obtained directly. Thus, the control input $u\left(t-\hat{\theta}^h_i\right)$ is approximated in first order representation [24]

$$u_h\left(t-\hat{\theta}^h_i\right)\cong u_h(t)-\hat{\theta}^h_i\cdot\dot{u}_h(t),\quad h=1,2,\dots,r \tag{17}$$

Thus, we choose

$$\begin{aligned}-\frac{\partial E}{\partial \hat{\theta}^h_i}&=-\frac{\partial E}{\partial y_j}\frac{\partial y_j}{\partial net^{(4)}_j}\frac{\partial net^{(4)}_j}{\partial w^{(4)}_{ij}}\frac{\partial w^{(4)}_{ij}}{\partial \hat{\theta}^h_i}\\ &=\left(d_j-y_j\right)\cdot O^{(3)}_i\cdot B_i\cdot\left(-\dot{u}_h\right),\ h=1,2,\dots,r.\end{aligned} \tag{18}$$

In addition, for parameters $\hat{m}_{ij}$ and $\hat{\sigma}_{ij}$, we have

$$\begin{aligned}-\frac{\partial E}{\partial \hat{m}_{ij}}&=-\frac{\partial E}{\partial net^{(3)}_i}\frac{\partial net^{(3)}_i}{\partial O^{(2)}_{ij}}\frac{\partial O^{(2)}_{ij}}{\partial net^{(2)}_{ij}}\frac{\partial net^{(2)}_{ij}}{\partial \hat{m}_{ij}}\\ &=\sum_j\frac{-\partial E}{\partial net^{(4)}_j}\frac{\partial net^{(4)}_j}{\partial O^{(3)}_i}\frac{\partial O^{(3)}_i}{\partial net^{(3)}_i}\frac{\partial net^{(3)}_i}{\partial O^{(2)}_{ij}}\frac{\partial O^{(2)}_{ij}}{\partial net^{(2)}_{ij}}\frac{\partial net^{(2)}_{ij}}{\partial \hat{m}_{ij}}\\ &=\sum_j\left(d_j-y_j\right)\sum_{k_1}^{l}net^{(3)}_{k_1}\left(w^{(4)}_{ij}-w^{(4)}_{k_1 j}\right)\left(\sum_{k_2}^{r}net^{(3)}_{k_2}\right)^{-2}\\ &\quad\cdot O^{(3)}_j\frac{2\left(z_i-\hat{m}_{ij}\right)}{\hat{\sigma}^2_{ij}}\end{aligned} \tag{19}$$

$$\begin{aligned}-\frac{\partial E}{\partial \hat{\sigma}_{ij}}&=-\frac{\partial E}{\partial net^{(3)}_i}\frac{\partial net^{(3)}_i}{\partial O^{(2)}_{ij}}\frac{\partial O^{(2)}_{ij}}{\partial net^{(2)}_{ij}}\frac{\partial net^{(2)}_{ij}}{\partial \hat{\sigma}_{ij}}\\ &=\sum_j\frac{-\partial E}{\partial net^{(4)}_j}\frac{\partial net^{(4)}_j}{\partial O^{(3)}_i}\frac{\partial O^{(3)}_i}{\partial net^{(3)}_i}\frac{\partial net^{(3)}_i}{\partial O^{(2)}_{ij}}\frac{\partial O^{(2)}_{ij}}{\partial net^{(2)}_{ij}}\frac{\partial net^{(2)}_{ij}}{\partial \hat{\sigma}_{ij}}\\ &=\sum_j\left(d_j-y_j\right)\sum_{k_1}^{l}net^{(3)}_{k_1}\left(w^{(4)}_{ij}-w^{(4)}_{k_1 j}\right)\left(\sum_{k_2}^{r}net^{(3)}_{k_2}\right)^{-2}\\ &\quad\cdot O^{(3)}_j\frac{2\left(z_i-\hat{m}_{ij}\right)^2}{\hat{\sigma}^3_{ij}}.\end{aligned} \tag{20}$$

Finally, the update laws for TDFN system are

$$\hat{a}^i_{pq}(k+1)=\hat{a}^i_{pq}(k)-\eta_{\hat{A}}\left(-\frac{\partial E}{\partial \hat{a}^i_{pq}}\right) \tag{21a}$$

$$\hat{b}^i_{vh}(k+1)=\hat{b}^i_{vh}(k)-\eta_{\hat{B}}\left(-\frac{\partial E}{\partial \hat{b}^i_{vh}}\right) \tag{21b}$$

$$\hat{\theta}^h_i(k+1)=\hat{\theta}^h_i(k)-\eta_{\hat{\theta}_h}\left(-\frac{\partial E}{\partial \hat{\theta}^h_i}\right) \tag{21c}$$

$$\hat{m}_{ij}(k+1)=\hat{m}_{ij}(k)-\eta_{\hat{m}}\left(-\frac{\partial E}{\partial \hat{m}_{ij}}\right) \tag{21d}$$

$$\hat{\sigma}_{ij}(k+1)=\hat{\sigma}_{ij}(k)-\eta_{\hat{\sigma}}\left(-\frac{\partial E}{\partial \hat{\sigma}_{ij}}\right). \tag{21e}$$

By the Lyapunov approach, the convergence of parameters is guaranteed if the learning rates are chose according the following theorem.

Theorem 1: Consider the system identification using the TDFN system, the parameters W of TDFN system will converge to optimum if the corresponding learning rate is chosen as

$$0<\eta_i(k)<\frac{2}{\left(P_{i,\max}\right)^2},\ i=1,\dots,5 \tag{22}$$

where η_i and $P_{i,\max}=\left.\frac{\partial O^{(4)}}{\partial W_i}\right|_{\max}$ are the corresponding learning rates and the maximum gradient for $\hat{A}_i,\hat{B}_i$, $\hat{\theta}^h_i$, $\hat{m}_{ij}$, and $\hat{\sigma}_{ij}$, respectively. In addition, the optimal (or adaptive) learning rate at time index k is

$$\eta^*(k)=\frac{1}{\left\|\frac{\partial O^{(4)}(k)}{\partial W}\right\|^2}. \tag{23}$$

which guarantees the fast convergence.

Proof: Please see Appendix.

Remark 1: The initialization of TDFN system can be chosen as- matrix $\hat{A}$ is set to be a zero matrix, $\hat{B}$ is set to be $[0\ 0\ \dots\ 1]^T$, delay-free $\hat{\theta}_i^h = 0$ is chose, mean and variance are chosen randomly in discussion region. Additionally, the initialization also can be chosen by the nominal plant of nonlinear system.

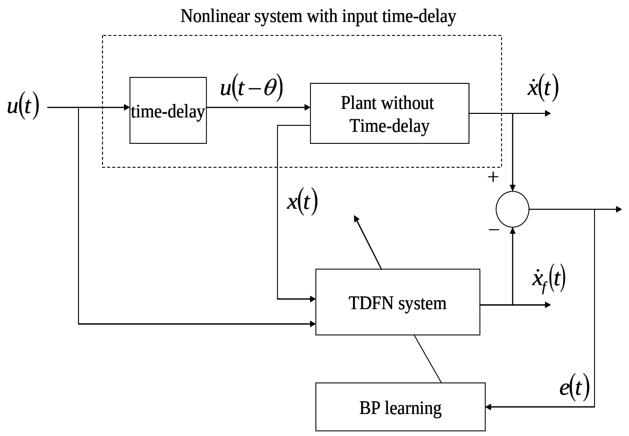


Figure 2. Identification scheme of nonlinear time-delay system using the TDFN system.

E. *System Identification Using the TDFN System*

Based on the above discussion, the TDFN system can be used to approximate any continuous function. The training scheme for system identification is shown in Fig. 2. Obviously, the objective of TDFN system is to adjust parameters through the gradient method to minimize the approximation error between TDFN output Y and system function $f(x,u(t\text{-}\theta))$. After training, the nonlinear system with time-delay (3) can be represented (or be approximation) by several TSK-type fuzzy models with input time-delay (4). Therefore, the controller design can be performed by the so-called parallel distributed compensation (PDC) approach.

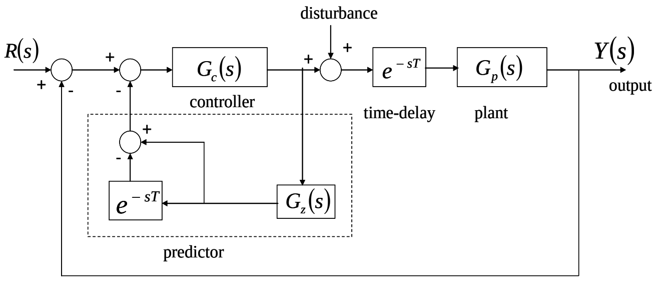


Figure 3. Smith predictor control scheme for the chemical processes systems.

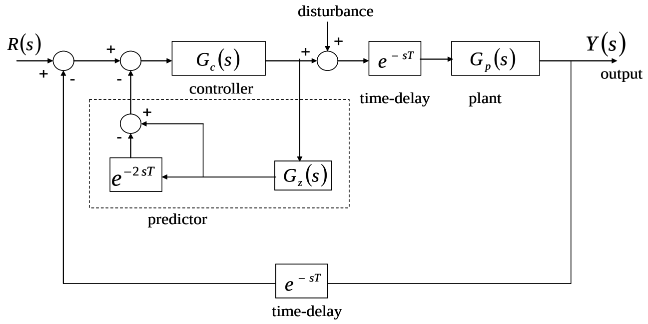


Figure 4. Smith predictor control scheme for the long transmission lines systems.

3. Fuzzy Controller Design via Smith Predictor

A. *Smith Predictor*

The Smith predictor (SP) controller approach was proposed by Smith [23, 24]. It is well known as an effective compensator for a process with time-delay [9, 23, 30]. The controller scheme in chemical processes and long transmission lines are shown in Figs. 3 and 4, respectively. $G_p(s)$ is the plant, $G_c(s)$ is the controller, and $G_z(s)$ is the dynamic model of the plant, termed the *predictor*. $R(s)$ and $Y(s)$ are the input and output of the system, respectively. In this way, the effect of the time-delay in the feedback loop can be minimized and allows us to use conventional controllers, such as PI, PD, or PID controllers, for $G_C(s)$. In the perfect matched case, i.e., $G_z(s)= G_p(s)$, the closed-loop transfer function between $R(s)$ and $C(s)$ is

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)e^{-sT}}{1+G_c(s)G_p(s)}. \qquad (24)$$

Note that the effect of the time-delay can be eliminated by the Smith predictor compensator. Herein, the predicted plant can be obtained directly from the identification result of the TDFN system.

B. *Smith Predictor Based Fuzzy Controller Design*

The controller design is carried out based on the fuzzy model via the so-called PDC approach [2, 3, 25, 26]. The concept can be described as: for each local linear model, a linear feedback control is designed. In general, the resulting overall controller is nonlinear, which is a combination of each individual linear controller by the fuzzy logic system [25, 26].

In this paper, the Smith-predictor (SP) based fuzzy PID controller is developed by the PDC scheme. Because of the limitation of the Smith predictor, the single input single output (SISO) systems are considered. Similar to the system description, the ith fuzzy controller rules can be described as

Rule$_i$:

If $z_1(t)$ is $\hat{F}_{i1}$ and … $z_g(t)$ is $\hat{F}_{ig}$

Then $u_i(t) = K_{P_i}\delta_i(t) + K_{I_i}\int \delta_i(t)dt + K_{D_i}\dot{\delta}(t),$ (25)

for $i = 1,2,\ldots,l.$

where

$$e(t) = r(t) - y(t) \quad (26)$$

$$\delta_i(t) = e(t) - y_{SP_i}(t) + y_{SP_i}(t - \hat{\theta}_i) = r(t) - y(t) - y_{SP_i}(t) + y_{SP_i}(t - \hat{\theta}_i). \quad (27)$$

Herein, $y(t)$ is the nonlinear input time-delay systems output, $r(t)$ is the reference tracking trajectory, and $y_{SP_i}(t)$ is the output of the Smith predictor for the ith fuzzy PID control rule. Therefore, the controller signal is

$$u(t) = \frac{\sum_{i=1}^{l}\hat{M}_i(z(t))\left[K_{P_i}\delta(t) + K_{I_i}\int \delta(t)dt + K_{D_i}\dot{\delta}(t)\right]}{\sum_{i=1}^{r}M_i(z(t))} = \sum_{i=1}^{l}\hat{h}_i(z(t)).\left[K_{P_i}\delta(t) + K_{I_i}\int \delta(t)dt + K_{D_i}\dot{\delta}(t)\right] \quad (28)$$

Subsequently, the parameters selection of PID controller is carried out by the dominant-pole-assignment [1, 22]. Figure 5 summaries the design of ith fuzzy PID controller rule based on Smith predictor. Obviously, the closed-loop system characteristic function for each fuzzy rule is

$$1 + G_C(s)G_P(s)e^{-\theta s} = 0\,. \quad (29)$$

Here, the SP scheme is adopted to eliminate the effect of the time-delay. As (24), the closed-loop characteristic equation is

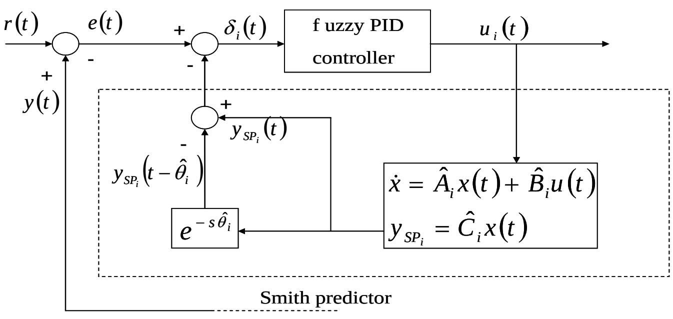


Figure 5. Smith predictor based fuzzy PID control scheme for the ith fuzzy control rule.

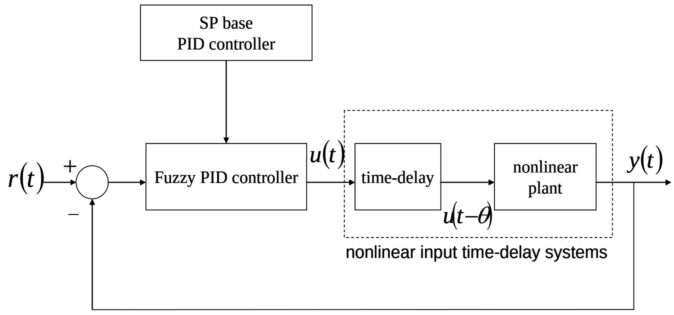


Figure 6. SP-based fuzzy PID control scheme for nonlinear input time-delay systems.

$$1 + G_{C_i}(s)G_{P_i}(s) = 0\,. \quad (30)$$

For the ith fuzzy rule

$$G_{c_i}(s) = \frac{K_{D_i}s^2 + K_{P_i}s + K_{I_i}}{s} \quad (31)$$

$$G_{p_i} = \frac{b_{i_{n-2}}s^{n-2} + b_{i_{n-3}}s^{n-3} + \cdots + b_{i_1}s + b_{i_0}}{s^n + a_{i_{n-1}}s^{n-1} + \cdots + a_{i_1}s + a_{i_0}} \quad (32)$$

In addition, we select the dominate-poles as being located on

$$P_{i1,2} = w_{i0}\left(-\xi_{i0} \pm j\sqrt{1+\xi_{i0}^{\ 2}}\right) = w_{i0}e^{j(\pi-\gamma_i)}, i = 1,2\ldots,l \quad (33)$$

$$P_{i3} = -\alpha_{i0}w_{i0}, i = 1,2,\ldots,l\,. \quad (34)$$

Thus, we define $q_i(w_{i0})$, $\kappa_i(w_{i0})$, and $\phi_i(w_{i0})$ as

$$G_{p_i}\left(w_{i0}e^{j(\pi-\gamma_i)}\right) = q_i(w_{i0})e^{j\phi_i(w_{i0})} \quad (35)$$

$$G_{p_i}(-\alpha_{i0}w_{i0}) = -\kappa_i(w_{i0}). \quad (36)$$

Then, the PID controller parameters for each fuzzy controller rule can be obtained [1, 23]

$$K_{P_i} = -\Big(\frac{\alpha_{i0}^2\kappa_i(w_{i0})\sin(\gamma_i + \phi_i) + \kappa_i(w_{i0})\sin(\gamma_i - \phi_i)}{q_i(w_{i0})\kappa_i(w_{i0})(\alpha_{i0}^2 - 2\alpha_{i0}\cos\gamma_i + 1)\sin\gamma_i} + \frac{\alpha_{i0}q_{i0}(w_{i0})\sin 2\gamma_i}{q_i(w_{i0})\kappa_i(w_{i0})(\alpha_{i0}^2 - 2\alpha_{i0}\cos\gamma_i + 1)\sin\gamma_i}\Big) \quad (37a)$$

$$K_{I_i} = -\Big(\frac{\alpha_{i0}w_{i0}q_i(w_{i0})\sin\gamma_i}{q_i(w_{i0})\kappa_i(w_{i0})(\alpha_{i0}^2 - 2\alpha_{i0}\cos\gamma_i + 1)\sin\gamma_i} + \frac{\alpha_{i0}w_{i0}\kappa_i(w_{i0})(\sin(\gamma_i - \phi_i) + \alpha_{i0}\sin\phi_i)}{q_i(w_{i0})\kappa_i(w_{i0})(\alpha_{i0}^2 - 2\alpha_{i0}\cos\gamma_i + 1)\sin\gamma_i}\Big) \quad (37b)$$

$$K_{D_i} = \Big(\frac{-\alpha_{i0}q_i(w_{i0})\sin\gamma_i}{w_{i0}q_i(w_{i0})\kappa_i(w_{i0})(\alpha_{i0}^2 - 2\alpha_{i0}\cos\gamma_i + 1)\sin\gamma_i} - \frac{\kappa_i(w_{i0})(\alpha_{i0}\sin(\gamma_i + \phi_i) - \sin\phi_i)}{w_{i0}q_i(w_{i0})\kappa_i(w_{i0})(\alpha_{i0}^2 - 2\alpha_{i0}\cos\gamma_i + 1)\sin\gamma_i}\Big). \quad (37c)$$

The SP based fuzzy PID controller scheme is summarized as Fig. 6.

To guarantee the stability of the fuzzy model, the following discussion is introduced. The SISO time-delay system for each fuzzy rule is rewritten as

$$G_{p_i} e^{-s\hat{\theta}_i} = \frac{b_{i_{n-2}} s^{n-2} + b_{i_{n-3}} s^{n-3} + \cdots + b_{i_1} s + b_{i_0}}{s^n + a_{i_{n-1}} s^{n-1} + \cdots + a_{i_1} s + a_0} e^{-s\hat{\theta}_i}, \quad i = 1, 2, \ldots, l \tag{38}$$

The PID controller (31) and Smith predictor scheme are used. Thus, the closed-loop system of the ith fuzzy rule is

$$\frac{Y_i(s)}{R(s)} = \frac{G_{c_i}(s) G_{p_i}(s) e^{-s\hat{\theta}_i}}{1 + G_{c_i}(s) G_{p_i}(s)} = \frac{Num(s)}{Den(s)} \tag{39}$$

where

$$\begin{aligned} Num(s) = {} & [b_{i_{n-2}} K_{D_j} s^n + \left(b_{i_{n-2}} K_{P_j} + b_{i_{n-3}} K_{D_j}\right) s^{n-1} \\ & + \cdots + \left(b_{i_1} K_{I_j} + b_{i_0} K_{P_j}\right) s + b_{i_0} K_{I_i}] e^{-s\hat{\theta}_i} \\ Den(s) = {} & s^{n+1} + \left(a_{i_{n-1}} + b_{i_{n-1}} K_{D_j}\right) s^n \\ & + \left(a_{i_{n-2}} + b_{i_{n-2}} K_{P_j} + b_{i_{n-3}} K_{D_j}\right) s^{n-1} + \cdots + b_{i_0} K_{I_i}. \end{aligned}$$

Transfer (39) as state representation

$$\dot{x}(t) = \sum_{i=1}^{l} \sum_{j=1}^{l} \hat{h}_i(z(t)) \hat{h}_j(z(t)) \left[Q_{ij} x(t) + G_{ij} r\left(t - \hat{\theta}_i\right) \right] \tag{40a}$$

$$\begin{aligned} y = {} & \sum_{i=1}^{l} \sum_{j=1}^{l} \hat{h}_i(z(t)) \hat{h}_j(z(t)) \\ & \times \begin{bmatrix} b_{i_0} K_{I_j} & b_{i_1} K_{I_j} + b_{i_0} K_{P_j} & \cdots & b_{i_{n-2}} K_{D_j} \end{bmatrix} x(t) \end{aligned} \tag{40b}$$

where

$$Q_{ij} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -b_{i_0} K_{I_i} & \cdots & \cdots & -a_{i_{n-2}} - b_{i_{n-2}} K_{P_j} - b_{i_{n-3}} K_{D_j} & -a_{i_{n-1}} - b_{i_{n-2}} K_{D_j} \end{bmatrix}$$

$$G_{ij} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} r\left(t - \hat{\theta}_i\right).$$

Therefore, choose a Lyapunov candidate function as

$$V(x(t)) = x^T(t) P x(t). \tag{41}$$

Hence,

$$\begin{aligned} \dot{V}(x(t)) = {} & \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) \\ = {} & \sum_{i=1}^{l} \sum_{j=1}^{l} \hat{h}_i(z(t)) \hat{h}_j(z(t)) \\ & \times \left[Q_{ij} x(t) + G_{ij} r\left(t - \hat{\theta}_i\right) \right]^T P x(t) + x^T(t) P \\ & \times \sum_{i=1}^{l} \sum_{j=1}^{l} \hat{h}_i(z(t)) \hat{h}_j(z(t)) \left[Q_{ij} x(t) + G_{ij} r\left(t - \hat{\theta}_i\right) \right] \\ = {} & \sum_{i=1}^{l} \sum_{j=1}^{l} \hat{h}_i(z(t)) \hat{h}_j(z(t)) \left[x^T(t) \left(Q_{ij}^T P + P Q_{ij}\right) \right. \\ & \left. \times x(t) + r^T\left(t - \hat{\theta}_i\right) \left(G_{ij}^T P + P G_{ij}\right) r\left(t - \hat{\theta}_i\right) \right]. \end{aligned} \tag{42}$$

According to the Lyapunov stability theorem and results of the literature [11, 25, 26], the fuzzy time-delay model system (4) is stable if there exists a positive definite matrix P satisfying

$$\left(Q_{ij}^T P + P Q_{ij}\right) < 0 \quad \text{and} \quad \left(G_{ij}^T P + P G_{ij}\right) < 0, \quad \forall i, j. \tag{43}$$

Finally, the controller design flow chart, shown in Fig. 7, summarizes our control approach using the TDFN system.

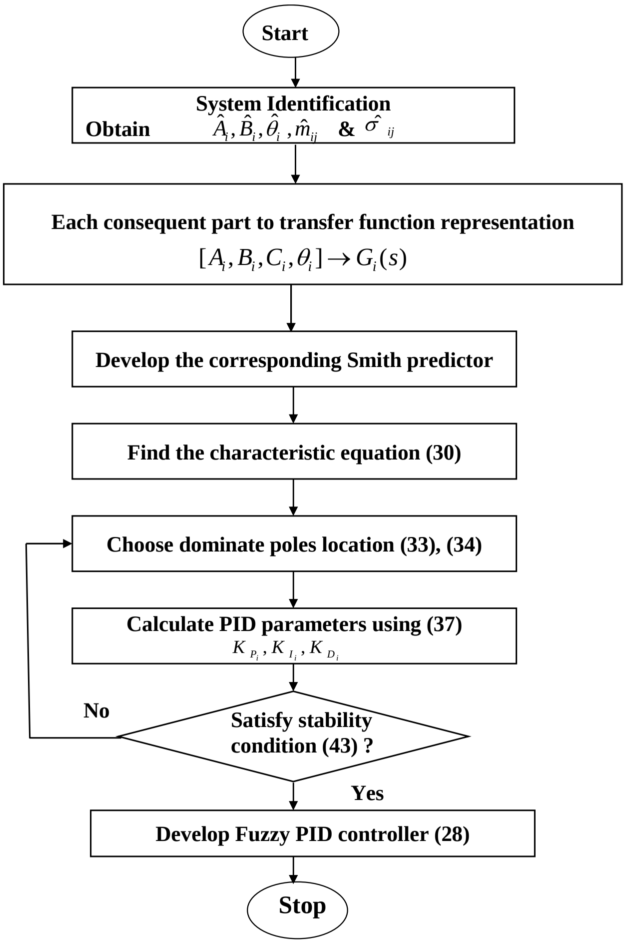


Figure 7. Summary of fuzzy PID controller design by the TDFN system.

4. Simulation Results

Example 1: Second order nonlinear time delay system

Consider the Van der Pol oscillatior with time-delay 0.5 second [12],

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= (1-x_1^2)x_2 - x_1 + u(t-0.5)\end{aligned} \tag{44}$$

The following three fuzzy models can be developed by Jacobian method (local approximation) [25, 26].

Rule_1 : If x_1 is about -1

$$\text{Then } \dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-0.5) \tag{45a}$$

Rule_2 : If x_1 is about 0

$$\text{Then } \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-0.5) \tag{45b}$$

Rule_3 : If x_1 is about 1

$$\text{Then } \dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-0.5) \tag{45c}$$

where the membership functions are described in Fig. 8. Using the TDFN system to treat this problem, we obtain the identification results as shown in Table 1 (fuzzy model parameters) and Fig. 9 (membership functions of TDFN system). Obviously, the identification results of TDFN are different from the Jacobian results (45). To show the effectiveness and the adaptive property of TDFN system, Fig. 10 shows the testing results (open loop system) with initial condition $x^T = [x_1 \quad x_2] = [0 \quad 0]$ and $u(t) = \sin(2\pi t)$. From the comparison of trajectory x_2- Fig. 10(b), we observe that the TDFN system can approximate the original system quite accurately. Obviously, the Jacobian method (45) results a divergent result of x_2 (or large approximated error) occurs in this case even x_1 approximation is well.

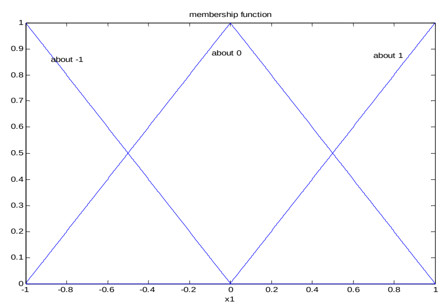


Figure 8. Membership functions of fuzzy model (45) for approximating system (44).

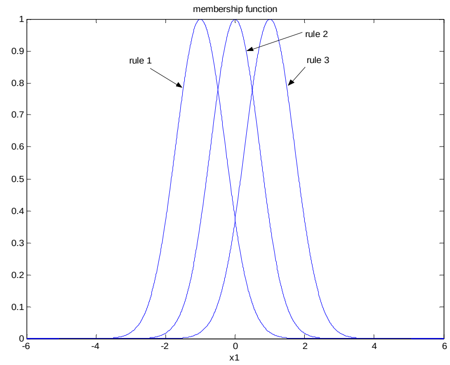
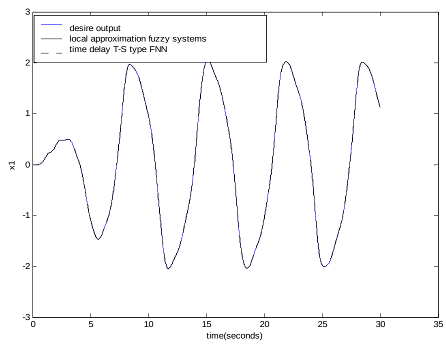


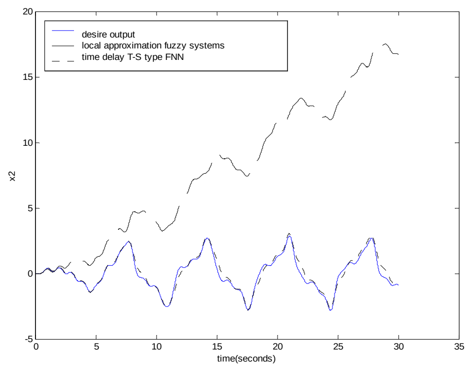
Figure 9. Membership functions of the TDFN system (after training).

Table 1. Training results of the TDFN system for Example 1.

	A_i	B_i	θ_i
Rule_1	$\begin{bmatrix} -0.0000 & 1.0000 \\ -1.0312 & -1.2462 \end{bmatrix}$	$\begin{bmatrix} 0.0000 \\ 0.9832 \end{bmatrix}$	0.5017
Rule_2	$\begin{bmatrix} -0.0000 & 1.0000 \\ -0.9594 & 2.9750 \end{bmatrix}$	$\begin{bmatrix} 0.0000 \\ 1.0432 \end{bmatrix}$	0.5027
Rule_3	$\begin{bmatrix} -0.0000 & 1.0000 \\ -1.0155 & -1.2879 \end{bmatrix}$	$\begin{bmatrix} 0.0000 \\ 0.9865 \end{bmatrix}$	0.5026

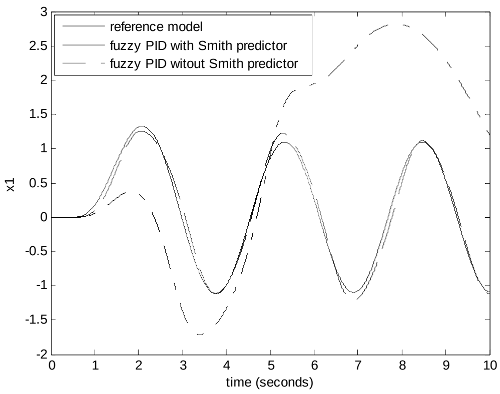


(a) state trajectories x_1

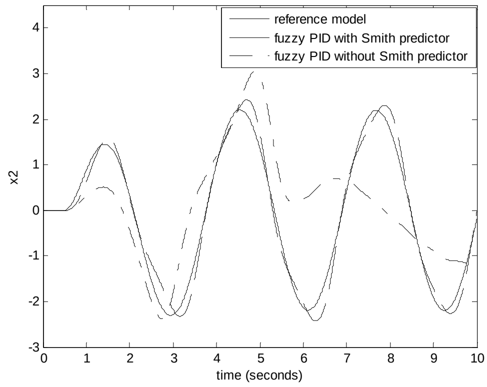


(b) state trajectories x_2

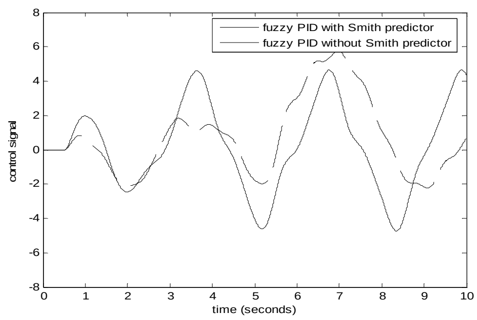
Figure 10. Testing results of system identification in Example 1.



(a) state x_1



(b) state x_2



(c) Control signal

Figure 11. Comparison results of Example 1 in control. (a) state trajectory x_1 (b) state trajectory x_2 (c) control signal.

In addition, the proposed controller scheme- SP based fuzzy PID controller is used to solve the tracking control problem. Herein, we choose

$$\begin{aligned} &\text{Rule}_1 : w_0 = 3,\ \alpha = 0.333,\ \xi = 0.833 \\ &\text{Rule}_2 : w_0 = 3,\ \alpha = 1,\ \xi = 0.833 \\ &\text{Rule}_3 : w_0 = 3,\ \alpha = 0.1,\ \xi = 1 \end{aligned} \tag{46}$$

and the corresponding PID parameters are

$$\begin{aligned} &\text{Rule}_1 : K_P = 12.406,\ K_I = 8.61,\ K_D = 4.55 \\ &\text{Rule}_2 : K_P = 24,\ K_I = 12.12,\ K_D = 11.424 \\ &\text{Rule}_3 : K_P = 9.618,\ K_I = 2.653,\ K_D = 4.92. \end{aligned} \tag{47}$$

The symmetric matrix P is

$$P = \begin{bmatrix} 0.5794 & 0.3877 & 0.0447 \\ 0.3877 & 0.7061 & 0.0468 \\ 0.0447 & 0.0468 & 0.0568 \end{bmatrix}.$$

The reference model is chosen as

$$\begin{aligned} &\dot{x}_{d1} = x_{d2} \\ &\dot{x}_{d2} = -\frac{8}{3}x_{d1} - \frac{8}{3}x_{d2} + 6\cdot\varphi(t) \\ &r(t) = x_{d1}. \end{aligned} \tag{48}$$

where $\varphi(t) = \sin(2t)$. Comparison results of state trajectories and control effort are shown in Fig. 11. Obviously, system output can follow the desired trajectories and the control performance is better than fuzzy PID without the Smith predictor. It shows the effectiveness of our approach.

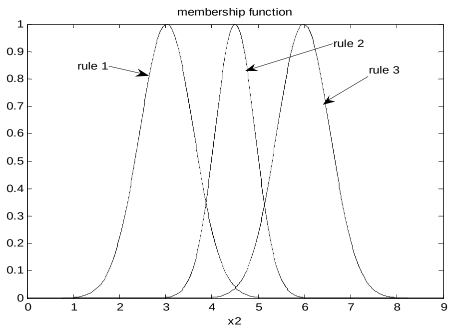


Figure 12. Membership functions of CSTR system.

Example 2: Control of continuous stirred tank reactor (CSTR) system

Consider the CSTR systems as [3, 20, 28]

$$\dot{x}_1 = -x_1 + D_a(1 - x_1)\exp\left(\frac{x_2}{1 + x_2/\varepsilon}\right) \tag{49a}$$

$$\dot{x}_2 = -x_2 + \mathrm{B}D_a(1 - x_1)\exp\left(\frac{x_2}{1 + x_2/\varepsilon}\right) - \beta_r(x_2 - x_3) \tag{49b}$$

$$\dot{x}_3 = \beta_c(x_2 - x_3) + (x_{3f} - x_3)u \tag{49c}$$

$$y = x_2. \tag{49d}$$

where x_1, x_2, and x_3 are the dimensionless reactor conversion reactor temperature and cooling jacket temperature, respectively. Our goal is to regulate reactor temperature x_2 at its desired reference model by regulating the cooling stream flow rate $u(t-\theta)$ with time delay $\theta>0$. System (49) can be represented as

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t-\theta) \quad (50a)$$
$$y(t) = C_{cs}x(t). \quad (50b)$$

where

$$f(x(t)) = \begin{bmatrix} -x_1 + D_a(1-x_1)e^{\frac{x_2}{1+x_2/\varepsilon}} \\ -x_2 + \mathrm{B}D_a(1-x_1)e^{\frac{x_2}{1+x_2/\varepsilon}} - \beta_r(x_2 - x_3) \\ \beta_c(x_2 - x_3) \end{bmatrix},$$

$$g(x(t)) = \begin{bmatrix} 0 \\ 0 \\ x_{3f} - x_3 \end{bmatrix},$$

and $C_{cs} = [0 \quad 1 \quad 0]$. Herein, the system parameters are B=11, D_a=0.14, ε=20, β_r=β_c=1.5, x_{3f}=2, and θ=0.8. Then, define the TDFN fuzzy rules as

$$\text{Rule}_i: \text{ IF } z(t) \text{ is } \hat{F}_{i1}$$
$$\text{THEN } \dot{x}(t) = \hat{A}_i x(t) + \hat{B}_i u(t - \hat{\theta}_i),\ y(t) = \hat{C}_i x(t). \quad (51)$$

where $z(t)$=$x_2(t)$ and $\hat{C}_i = [0 \quad 1 \quad 0], \forall i$. Choose three fuzzy rules. Using the TDFN system to identify the CSTR systems, we obtain the identification results as shown in Table 2 (fuzzy model parameters) and Fig. 12 (membership functions).

The dominate-poles parameters are selected as

$$\text{Rule}_1\text{: } w_o = 0.707,\ \alpha = 7.07, \xi = 0.707$$
$$\text{Rule}_2\text{: } w_o = 0.707,\ \alpha = 7.07, \xi = 0.707 \quad (52)$$
$$\text{Rule}_3\text{: } w_o = 0.707,\ \alpha = 7.07, \xi = 0.707,$$

and the corresponding PID parameters are

$$\text{Rule}_1\text{: } K_p = 8.184,\ K_I = 3.21,\ K_D = 6.81$$
$$\text{Rule}_2\text{: } K_p = 2.76,\ K_I = 1.25,\ K_D = 4.04$$
$$\text{Rule}_3\text{: } K_p = 5.686,\ K_I = 2.6,\ K_D = 4.695. \quad (53)$$

Table 2. Identification results of CSTR system.

	$\hat{A}_i$	$\hat{B}_i$	$\hat{\theta}_i$
Rule$_1$	$\begin{bmatrix} 0.0147 & 0.0246 & -0.0176 \\ 0.1318 & 0.3424 & 0.3885 \\ 0.3728 & 1.2571 & -1.0617 \end{bmatrix}$	$\begin{bmatrix} -0.0000 \\ -0.0000 \\ 2.0000 \end{bmatrix}$	0.8012
Rule$_2$	$\begin{bmatrix} 0.0002 & 0.1545 & 0.0950 \\ 0.0401 & 1.6841 & 0.9965 \\ 0.0225 & 0.6385 & 0.3762 \end{bmatrix}$	$\begin{bmatrix} -0.0000 \\ -0.0001 \\ 1.9996 \end{bmatrix}$	0.8006
Rule$_3$	$\begin{bmatrix} 0.0242 & 0.0466 & -0.0611 \\ 0.2828 & 0.5266 & -0.6879 \\ 0.3602 & 1.6563 & -2.0393 \end{bmatrix}$	$\begin{bmatrix} -0.0068 \\ -0.0636 \\ -1.5858 \end{bmatrix}$	0.8002

The symmetric matrix P is

$$P = \begin{bmatrix} 0.3002 & 0.2504 & 0.0894 \\ 0.2504 & 0.8643 & 0.2250 \\ 0.0894 & 0.2250 & 0.1642 \end{bmatrix}.$$

Figure 13 shows the comparison result of unit step response of each fuzzy rule model and actual nonlinear system with state initial values- $(x_1(0) \quad x_2(0) \quad x_3(0)) = (0 \quad 0 \quad 0)$ (solid: actual system; dash-doted: Rule$_1$; dotted: Rule$_2$; dashed: Rule$_3$). This demonstrates the effectiveness of the Smith predictor and dominate-pole assignment technique. In this way, we observe that the characteristics of CSTR system are similar to every fuzzy rule subsystem. That is, the dominate-pole assignment technique can be used to guarantee the time response characteristics and control performance by using this approach. Next, the tracking control problem is considered and the reference model is defined as [28]

$$\begin{aligned} \dot{x}_{d1}(t) &= x_{d2}(t) \\ \dot{x}_{d2}(t) &= -4x_{d1}(t) - 4x_{d2}(t) + 4\varphi(t) \\ r(t) &= x_{d1}(t) \end{aligned} \quad (54)$$

where $\varphi(t) = 4$ and $(x_{d1}(0) \quad x_{d2}(0)) = (4 \quad 4)$. The CSTR system state initial values are x(0)=[0.65 3 0.25]. The simulation result is shown in Fig. 14 (Fig 14(a): system output, Fig. 14(b): control effort). The proposed fuzzy controller is robustness even an addition disturbance $d(t)$=30 is added at time t=20. The control performance of the proposed fuzzy PID control scheme has been shown in the above simulations.

In addition, comparison results with literature [9] are presented to demonstrate our effectiveness. Note that the method of literature [9] is assumed that the time-delay should be exactly known. However, the unknown time-delay of nonlinear uncertain system (3) can be identified by the TDFN in identification procedure. Figure 15 shows the comparison results, Figs. 15(a) and 15(b) depicts the state trajectories x_1 and x_2, respectively. It is shown that the SP-based fuzzy controller has smaller tracking error. The smoothing control effort is shown in Fig. 15(c). This example shows the effectiveness of our approach.

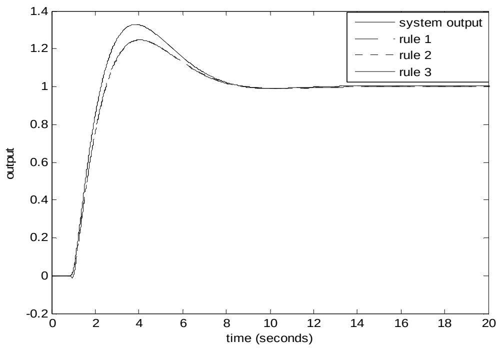
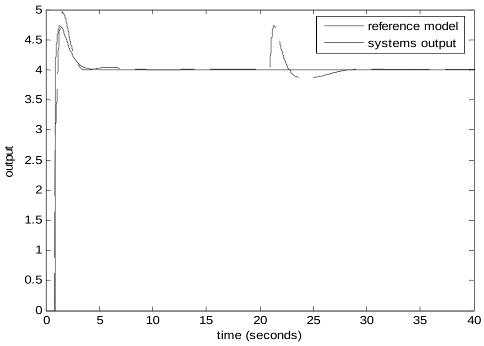
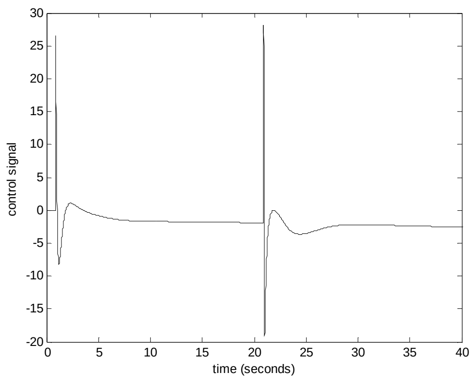


Figure 13. Comparison results of each fuzzy rule and CSTR system response.

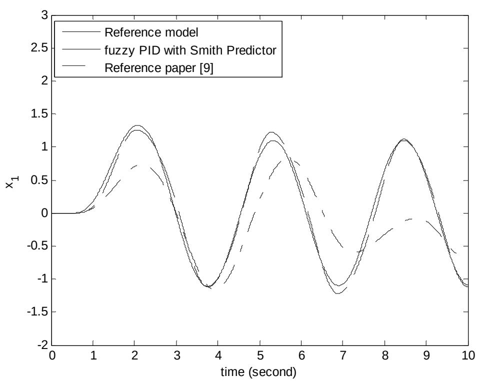


(a) system output

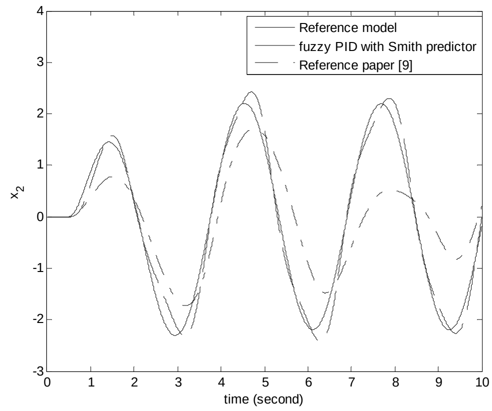


(b) control signal

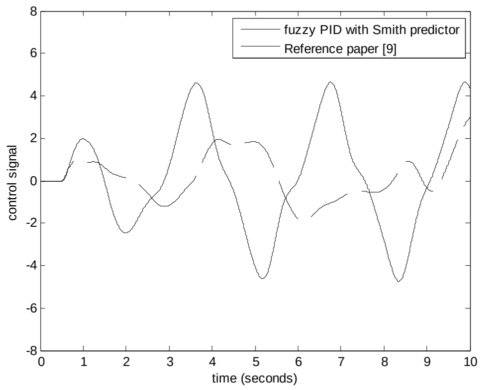
Figure 14. The simulation results of the CSTR system using the TDFN approach.



(a)



(b)



(c)

Figure 15. Comparison results of the CSTR system.

5. Conclusions

This paper has proposed the TDFN system for identifying and control a class of nonlinear uncertain systems with unknown input time-delay. The uncertain unknown system was accurately identified by training the TDFN system. The nonlinear uncertain system can be represented by several TSK-type fuzzy if-then rules and the unknown time delay can be estimated. Using the identification results, a novel method to design the SP-based fuzzy PID controller for nonlinear uncertain systems was proposed. Based on the PDC approach, the Smith predictor compensation and dominant-pole assignment technique are then adopted to design the control performance and characteristics of the closed-loop system. Finally, the stability of the closed-loop system is guaranteed by the Lyapunov approach. Several simulations are shown to demonstrate the effectiveness of the proposed TDFN approach.

Appendix

Proof of Theorem 1

First, we define Lyapunov function candidate for single output case, i.e.,

$$V(k)=E(k)=\frac{1}{2}e^2(k)\,. \tag{55}$$

The difference of cost function is

$$\begin{aligned}\Delta V(k)&=\frac{1}{2}\cdot\left[e^2(k+1)-e^2(k)\right]\\&=\frac{1}{2}\cdot\Delta e(k)\cdot\left[2\cdot e(k)+\Delta e(k)\right].\end{aligned} \tag{56}$$

In general usage, the difference of error can be represented as

$$\begin{aligned}e(k+1)&=e(k)+\Delta e(k)\\&=e(k)+\left[\frac{\partial e(k)}{\partial W}\right]^T\Delta W+H.O.T.\end{aligned} \tag{57}$$

We have

$$\begin{aligned}\Delta e(k)&\approx\frac{\partial e(k)}{\partial W}\cdot\Delta W=\frac{\partial\left[d(k)-O^{(4)}(k)\right]}{\partial W}\cdot\left(-\eta\cdot\frac{\partial E(k)}{\partial W}\right)\\&=\eta\frac{\partial O^{(4)}}{\partial W}\cdot\frac{\partial E(k)}{\partial W}\end{aligned} \tag{58}$$

and

$$\begin{aligned}\frac{\partial E(k)}{\partial W}&=\frac{\partial}{\partial W}\left(\frac{1}{2}\cdot e^2(k)\right)=e(k)\cdot\frac{\partial e(k)}{\partial W}\\&=-e(k)\cdot\frac{\partial O^{(4)}}{\partial W}\end{aligned}. \tag{59}$$

Therefore, (57) can be rewritten as

$$\Delta e(k)=-\eta\cdot\left(\frac{\partial O^{(5)}}{\partial W}\right)^2\cdot e(k)\,. \tag{60}$$

Thus,

$$\begin{aligned}\Delta V(k)&=\frac{1}{2}\left[-\eta\cdot\left(\frac{\partial O^{(4)}}{\partial W}\right)^2\cdot e(k)\right]\times[2\cdot e(k)\\&\quad-\eta\cdot\left(\frac{\partial O^{(4)}}{\partial W}\right)^2\cdot e(k)]\\&=\frac{1}{2}\cdot\eta\cdot e^2(k)\cdot\left(\frac{\partial O^{(4)}}{\partial W}\right)^2\cdot\left[-2+\eta\cdot\left(\frac{\partial O^{(4)}}{\partial W}\right)^2\right].\end{aligned} \tag{61}$$

If $\Delta V(k)<0$, the condition as (62) should be held.

$$0<\eta(k)<2\Big/\left\|\frac{\partial O^{(4)}}{\partial W}\right\|^2,\qquad\forall k \tag{62}$$

Herein, we denote

$$\begin{aligned}P_{\max}&\equiv\left[P_{1,\max}\quad P_{2,\max}\quad P_{3,\max}\quad P_{4,\max}\quad P_{5,\max}\right]^T\\&=[\max_{ij}\left|\frac{\partial O^{(4)}}{\partial\hat{m}_{ij}}\right|\;\max_{ij}\left|\frac{\partial O^{(4)}}{\partial\hat{m}_{ij}}\right|\;\max_{ij}\left|\frac{\partial O^{(4)}}{\partial a_{ij}}\right|\;\max_{ij}\left|\frac{\partial O^{(4)}}{\partial b_{ij}}\right|\\&\quad\max_i\left|\frac{\partial O^{(4)}}{\partial\hat{\theta}_i}\right|]^T.\end{aligned} \tag{63}$$

Therefore, condition (62) is rewritten as

$$0<\eta_i(k)<2\Big/\left\|P_{i,\max}\right\|^2,\qquad\forall i,k\,.$$

It is well known that the identified error $e(k)$ will tend to zero by the Lyapunov stability theorem. For obtaining fast convergence, the adaptive optimal learning rate is derived as follows. (56) gives $\Delta V(k)=-\frac{\lambda}{2}e^2(k)$ and implies

$$e^2(k+1)=(1-\lambda)e^2(k)\,. \tag{64}$$

By (64), the adaptive learning rate $\eta^*(k)=\dfrac{1}{\left\|\dfrac{\partial O^{(4)}(k)}{\partial W}\right\|^2}$ is obtained if

$$\lambda=\eta\left\|\frac{\partial O^{(4)}}{\partial W}\right\|^2\left(2-\eta\left\|\frac{\partial O^{(4)}}{\partial W}\right\|^2\right)=1\,. \tag{65}$$

This completes the proof.

Acknowledgment

The authors would like to thank the Associate Editor and anonymous reviewers for their insightful comments and valuable suggestions. This work was partially supported by the National Science Council, Taiwan, under Contracts NSC-95-2221-E-155-068-MY2.

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