On Lacunary Almost Statistical Convergence of Generalized Difference Sequences of Fuzzy Numbers

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Abstract

The purpose of this paper is to introduce the concepts of lacunary almost statistical convergence and strongly almost convergence of generalized difference sequences of fuzzy numbers. We obtain some results related to these concepts. It is also shown that lacunary almost $\Delta^m{}_\theta$ - statistical convergence and strongly almost $\Delta^m{}_\theta$- convergence are equivalent for Δ^m- bounded sequences of fuzzy numbers.

Keywords: Fuzzy number, statistical convergence, lacunary sequence, difference sequence.

1. Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [25] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events and fuzzy mathematical programming. Matloka [15] introduced bounded and convergent sequences of fuzzy numbers and studied their properties. Matloka [15] also has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by Nanda [19], Nuray [20], Kwon [6], Savas [21], Wu and Wang [4], Bilgin [3], Basarir and Mursaleen [2,16], Aytar [1], Fang and Huang [15], Esi [31] and many others.

The notion of statistical convergence was introduced by Fast [9] and Schoenberg [22] independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Later on it was further investigated from the sequence space point of view and linked with summability theory by Fridy [13], Šalat [12] , Tripathy [14] , Connor [10] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone- $\breve{C}$ ech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability.

2. Definitions and Preliminaries

The definitions of statistical convergence and strong *p*-Cesaro convergence of a sequence of real numbers were introduced in the literature independently of one another. A different line of development was witnessed since their first appearance. However the two definitions can be simply related to one another in general and are equivalent for bounded sequences. The idea of statistical convergence depends on the density of subsets of the set N of natural numbers. The density of a subset E of N is defined by

$$\delta(E) = \frac{1}{n}\sum_{k=1}^{n} \chi_E(k) \to 0 \text{ as } n \to \infty,$$

provided the limit exists where χ_E is the characteristic function of E. It is clear that any finite subset of N has zero natural density and $\delta(E^c) = 1 - \delta(E)$. A sequence (x_k) is said to be statistically convergent L to if for every $\varepsilon > 0$, $\delta(\{k \in N : |x_k - L| \geq \varepsilon\}) = 0$. In this case we write $S - \lim x_k = L$. Let $C(R^n) = \{A \subset R^n : A$ compact and convex$\}$. The space $C(R^n)$ has a linear structure induced by the operations

$$A + B = \{a + b : a \in A, b \in B\}$$

and

$$\lambda A = \{\lambda a : a \in A\}$$

for $A, B \in C(R^n)$ and $\lambda \in R.$ The Hausdorff distance between A and B of $C(R^n)$ is defined as

$$\delta_\infty(A, B) = \max\left\{\sup_{a\in A} \inf_{b\in B} \|a - b\|, \sup_{b\in B} \inf_{a\in A} \|a - b\|\right\}.$$

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It is well known that $\left(C\left(R^n\right), \delta_\infty\right)$ is a complete metric space.

A fuzzy number is a function X from R^n to [0, 1] which is normal, fuzzy convex, upper-semi-continuous and the closure of $\{x \in R^n : X(x) > 0\}$ is compact. These properties imply that for each $0 < \alpha \le 1$, the α – level set $[X]^\alpha = \{x \in R^n : X(x) \ge \alpha\}$ is a nonempty compact convex subset of R^n, with support $X^0 = \{x \in R^n : X(x) > 0\}$. Let $L\left(R^n\right)$ denote the set of all fuzzy numbers. The linear structure of $L\left(R^n\right)$ induces the addition $X+Y$ and the scalar multiplication $\lambda X, \lambda \in R,$ in terms of α- level sets, $|X+Y|^\alpha = |X|^\alpha + |Y|^\alpha$, and $|\lambda X|^\alpha = \lambda |X|^\alpha$ for each $0 \le \alpha \le 1$. Define, for each $1 \le q < \infty$, said to

$$d_q(X,Y) = \left(\int_0^1 \delta_\infty\left(X^\alpha, Y^\alpha\right)^q \, d\alpha \right)^{1/q}$$

and

$$d_\infty = \sup_{0 \le \alpha \le 1} \delta_\infty\left(X^\alpha, Y^\alpha\right),$$

where δ_∞ is the Hausdorff metric. Clearly $d_\infty(X,Y) = \lim_{q \to \infty} d_q(X,Y)$ with $d_q \le d_r$, if $q \le r$ [4]. Throughout the paper, d will denote d_q with $1 \le q \le \infty$.

The famous space $\hat{c}$ of all almost convergent sequences was introduced by Lorentz [14] and several authors such Duran [6], King [11] have studied almost convergent sequences. Maddox [17, 18] has defined x to be strongly almost convergent to number L if $\frac{1}{n}\sum_{k=1}^{n} |x_{k+m} - L| \to 0$ as $n \to \infty$, uniformly in m.

The idea of difference sequences of real numbers was first introduced by Kizmaz [9] and this concept was generalized by Et. and Basarir [7].

Let w be the set of all sequences of fuzzy numbers. The operator $\Delta^m : w \to w$ is defined by

$$\left(\Delta^0 X\right)_k = X_k,\ \left(\Delta' X\right)_k = \Delta' X_k = X_k - X_{k+1},\ (k = 0, 1, \ldots),$$

$$\Delta^m = \Delta' \circ \Delta^{m-1},\ (m \ge 2)$$

Now we will extend the notions of strongly lacunary almost convergence and lacunary almost statistical convergence of sequences of real numbers to the idea of sequences of fuzzy numbers using the generalized difference operator Δ^m and the sequence $k = (k_r)$. By a lacunary sequence $\theta = (k_r);\ r = 0, 1, 2, \ldots,$ where $k_0 = 0,$ we mean an increasing sequence of non negative integers with $h_r = (k_r - k_{r-1}) \to \infty$ as $r \to \infty$. The intervals determined by θ will denote by $I_r = (k_{r-1}, k_r]$ and $q_r = \frac{k_r}{k_{r-1}}$.

Definition1: Let $\theta = (k_r)$ be a lacunary sequence. A sequence $X = (X_k)$ be a sequence of fuzzy numbers is to be lacunary almost ${\Delta^m}_\theta$- statistically convergent to fuzzy number X_0 if for every $\varepsilon > 0$,

$$\frac{1}{h_r}\left|\left\{k \in I_r :\ d\left(\Delta^m X_{k+i}, X_0\right) \ge \varepsilon\right\}\right| \to 0 \ as\ r \to \infty,$$

uniformly in i. In this case we write $X_k \to X_0\left(\hat{s}\left({\Delta^m}_\theta\right)\right)$ or $\hat{s}\left({\Delta^m}_\theta\right) - \lim X_k = X_0$. The set of all lacunary almost ${\Delta^m}_\theta$- statiscally convergent sequences of fuzzy numbers is denoted by $\hat{s}\left({\Delta^m}_\theta\right)$. In the special case $\theta = 2^r$ for all $r \in N$, we shall write $\hat{s}\left(\Delta^m\right)$ instead of $\hat{s}\left(\Delta^m\right)$ and we said that X is lacunary almost Δ^m - statistically convergent to the fuzzy number X_0.

Definition2: Let $\theta = (k_r)$ be a lacunary sequence. A sequence $X = (X_k)$ be a sequence of fuzzy numbers and $p = (p_k)$ be a sequence of strictly positive real numbers. Then the sequence $X = (X_k)$ is said to be lacunary strongly ${\Delta^m}_\theta$- convergent if there is a fuzzy number X_0 such that

$$\frac{1}{h_r}\sum_{k \in I_r}\left[d\left(\Delta^m X_{k+i}, X_0\right)\right]^{p_k} \to 0 \text{ as } r \to \infty,$$

uniformly in i. In this case we write $X_k \to X_0\left(\left[M_\theta, p, {\Delta^m}_\theta\right]\right)$. We shall use $\left[M_\theta, p, {\Delta^m}_\theta\right]$ to denote the set of all lacunary strongly almost ${\Delta^m}_\theta$- convergent sequence of fuzzy numbers. In the special case $\theta = \left(2^r\right)$ for all $r \in N$ and $p_k = 1$ for all $k \in N$, we shall write $\left[AC, p, \Delta^m\right]$ and $\left[AC, {\Delta^m}_\theta\right]$, respectively, instead of $\left[M_\theta, p, {\Delta^m}_\theta\right]$. If $X \in \left[M_\theta, p, \Delta^m\right]$ then we say that X is strongly lacunary almost Δ^m- Cesaro summable.

Definition3: Let $\theta = (k_r)$ be a lacunary sequence. A sequence $X = (X_k)$ be a sequence of fuzzy numbers is said to be Δ^m- bounded if th set $\left\{\left(\Delta^m X_k\right) : k \in N\right\}$ of fuzzy numbers is bounded. By $\ell_\infty\left(\Delta^m\right)$ we shall denote the set of all Δ^m- bounded sequences of fuzzy numbers.

3. Main Results

In this section we give some inclusion relations between strongly lacunary almost $\left({\Delta^m}_\theta\right)$- convergence and lacunary almost $\left(\Delta^m{}_\theta\right)$- statistical convergence and show that they are equivalent for $\left(\Delta^m\right)$- bounded sequences of fuzzy numbers. We also study the inclusion $\hat{s}\left(\Delta^m\right) \subset \hat{s}\left(\Delta^m{}_\theta\right)$ under certain restrictions on $\theta = \left(k_r\right)$. The proof of the following results is easy, so omitted.

Theorem1: If $\left(X_k\right)$, $\left(Y_k\right) \in \hat{s}\left(\Delta^m{}_\theta\right)$ and $c \in R$ then

(i) $\hat{s}\left(\Delta^m{}_\theta\right)$-lim$c\, X_k = c\ \hat{s}\left(\Delta^m{}_\theta\right) - \lim X_k$

(ii) $\hat{s}\left(\Delta^m{}_\theta\right) - \lim\left(X_k + Y_k\right) = \ \hat{s}\left(\Delta^m{}_\theta\right) - \lim X_k$
$+\hat{s}\left(\Delta^m{}_\theta\right) - \lim Y_k$

Theorem2: Let $\theta = \left(k_r\right)$ be a lacunary sequence and $\left(p_k\right)$ be bounded sequence of positive real number then

(i) $X_k \to X_0\left(\left[M_\theta, p, \Delta^m{}_\theta\right]\right)$ implies $X_k \to X_0\left(\hat{s}\left(\Delta^m{}_\theta\right)\right)$,

(ii) $X \in \ell_\infty\left(\Delta^m\right)$ and $X_k \to X_0\left(\hat{s}\left(\Delta^m{}_\theta\right)\right)$ imply $X_k \to X_0\left(\left[M_\theta, p, \Delta^m{}_\theta\right]\right)$,

(iii) $\hat{s}\left(\Delta^m{}_\theta\right) \cap \ell_\infty\left(\Delta^m\right) = \left[M_\theta, p, \Delta^m{}_\theta\right] \cap \ell_\infty\left(\Delta^m\right)$,

where $0 < h =$ inf $p_k \le p_k \le$ sup $p_k = H$ and ℓ_∞ is the set of all bounded sequence of fuzzy numbers.

Proof :

(i) Let $\varepsilon > 0$ and $X_k \to X_0\left(\ \left[M_\theta, p, \Delta^m{}_\theta\right]\ \right)$. Then we can write

$$\frac{1}{h_r}\sum_{k\in I_r}\left[d\left(\Delta^m X_{k+i}, X_0\right)\right]^{p_k}$$

$$\ge \frac{1}{h_r}\sum_{\substack{k\in I_r\\ d\left(\Delta^m X_{k+i}, X_0\right)\ge\varepsilon}}\left[d\left(\Delta^m X_{k+i}, X_0\right)\right]^{p_k}$$

$$\ge \frac{1}{h_r}\sum_{\substack{k\in I_r\\ d\left(\Delta^m X_{k+i}, X_0\right)\ge\varepsilon}}\varepsilon^{p_k}$$

$$\ge \frac{1}{h_r}\sum_{\substack{k\in I_r\\ d\left(\Delta^m X_{k+i}, X_0\right)\ge\varepsilon}}\min\left(\varepsilon^h, \varepsilon^H\right)$$

$$\ge \frac{1}{h_r}\left|\left\{k \in I_r : d\left(\Delta^m X_{k+i}, X_0\right) \ge \varepsilon\right\}\right| \min\left[\varepsilon^h, \varepsilon^H\right],$$

uniformly in *i*. where $0 < h =$ inf $p_k \le p_k \le$ sup $p_k = H < \infty$.

(ii) Suppose that $X \in \ell_\infty\left(\Delta^m\right)$ and $X_k \to X_0\left(\hat{s}\left(\Delta^m{}_\theta\right)\right)$, since $X \in \ell_\infty$, there is a constant $T > 0$ such that $d\left(\Delta^m X_{k+i}, X_0\right) \le T$. Given $\varepsilon > 0$, we have

$$\frac{1}{h_r}\sum_{k\in I_r}\left[d\left(\Delta^m X_{k+i}, X_0\right)\right]^{p_k}$$

$$= \frac{1}{h_r}\sum_{\substack{k\in I_r\\ d\left(\Delta^m X_{k+i}, X_0\right)\ge\varepsilon}}\left[d\left(\Delta^m X_{k+i}, X_0\right)\right]^{p_k}$$

$$+\frac{1}{h_r}\sum_{\substack{k\in I_r\\ d\left(\Delta^m X_{k+i}, X_0\right)<\varepsilon}}\left[d\left(\Delta^m X_{k+i}, X_0\right)\right]^{p_k}$$

$$\le \max\left(T^h, T^H\right)\frac{1}{h_r}\left|\left\{k \in I_r : d\left(\Delta^m X_{k+i}, X_0\right) \ge \varepsilon\right\}\right| + \max\left(\varepsilon^h, \varepsilon^H\right).$$

Hence $X_0 \in \left[M_\theta, p, \Delta^m{}_\theta\right]$.

(iii) Follows from (i) and (ii). This completes the proof.

Theorem3: Let $\theta = \left(k_r\right)$ be a lacunary sequence. If a sequence $X = \left(X_k\right)$ is almost Δ^m - statistically convergent to the fuzzy number X_0 and $\lim \inf_{(r)}\left(\frac{k_r}{r}\right) > 0,$ then it is almost $\Delta^m{}_\theta$ - statistically convergent to X_0.

Proof: Given $\varepsilon > 0$ we have

$$\left|\left\{k \le r : d\left(\Delta^m X_{k+i}, X_0\right) \ge \varepsilon\right\}\right| \supset \left|\left\{k \in I_r : d\left(\Delta^m X_{k+i}, X_0\right) \ge \varepsilon\right\}\right|$$

Therefore

$$\frac{1}{r}\left|\left\{k \le r : d\left(\Delta^m X_{k+i}, X_0\right) \ge \varepsilon\right\}\right|$$

$$\ge \frac{1}{r}\left|\left\{k \in I_r : d\left(\Delta^m X_{k+i}, X_0\right) \ge \varepsilon\right\}\right|$$

$$\ge \frac{h_r}{r}\cdot\frac{1}{h_r}\left|\left\{k \in I_r : d\left(\Delta^m X_{k+i}, X_0\right) \ge \varepsilon\right\}\right|$$

Taking limit as $r \to \infty$ and using $\lim \inf_{(r)}\left(\frac{k_r}{r}\right) > 0$, we get X is $\Delta^m{}_\theta -$ statistically convergent to X_0. This completes the proof.

Theorem4: Let $0 < p_k \le q_k$ and $\left(q_k \big/ p_k\right)$ be bounded. Then $\left[M_\theta, q, \Delta^m{}_\theta\right] \subset \left[M_\theta, p, \Delta^m{}_\theta\right]$

Proof:

Let $X \in \left[M_\theta, q, \Delta^m{}_\theta\right]$. Write $w_{k,i} = \left[d\left(\Delta^m X_{k+i}, X_0\right)\right]^{q_k}$ and $\mu_k = \frac{p_k}{q_k}$, so that $0 < \mu < \mu_k \le 1$ for each k. We define the Sequences $\left(u_{k,i}\right)$ and $\left(v_{k,i}\right)$ as follows: Let $u_{k,i} = w_{k,i}$ and $v_{k,i} = 0$ if $w_{k,i} \ge 1$, and let $u_{k,i} = 0$ and $v_{k,i} = w_{k,i}$ if $w_{k,i} < 1$. Then it is clear that for all $k \in N,$ we have $w_{k,i} = u_{k,i} + v_{k,i}$, $w_{k,i}{}^{\mu_k} = u_{k,i}{}^{\mu_k} + v_{k,i}{}^{\mu_k}$. Now it follows that $u_{k,i}{}^{\mu_k} \le u_{k,i} \le w_{k,i}$ and $v_{k,i}{}^{\mu_k} \le v_{k,i}{}^{\mu}$. Therefore

$$\frac{1}{h_r}\sum_{k\in I_r} w_{k,i}{}^{\mu_k} = \frac{1}{h_r}\sum_{k\in I_r}\left(u_{k,i}{}^{\mu_k} + v_{k,i}{}^{\mu_k}\right)$$

$$\le \frac{1}{h_r}\sum_{k\in I_r} w_{k,i} + \frac{1}{h_r}\sum_{k\in I_r} v_{k,i}{}^{\mu}.$$

Since $\mu < 1$, for each m we have

$$\frac{1}{h_r}\sum_{k\in I_r} v_{k,i}{}^{\mu} = \sum_{k\in I_r}\left(\frac{1}{h_r}\ v_{k,i}\right)^{\mu}\left(\frac{1}{h_r}\right)^{1-\mu}$$

$$\leq \left(\sum_{k\in I_r}\left[\left(\frac{1}{h_r}\ v_{k,i}\right)^{\mu}\right]^{1/\mu}\right)^{\mu}\left(\sum_{k\in I_r}\left[\left(\frac{1}{h_r}\right)^{1-\mu}\right]^{1/1-\mu}\right)^{1-\mu}$$

$$= \left(\frac{1}{h_r}\sum_{k\in I_r} v_{k,i}\right)^{\mu}$$

By Hölder's inequality and thus

$$\frac{1}{h_r}\sum_{k\in I_r} w_{k,i}{}^{\mu_k} \leq \frac{1}{h_r}\sum_{k\in I_r} w_{k,i} + \left(\frac{1}{h_r}\sum_{k\in I_r} v_{k,i}\right)^{\mu}$$

Hence $X \in \left[M_\theta, p, \Delta^m{}_\theta\right]$. This completes the proof.

Theorem 5: $\left[M_\theta, \Delta^m{}_\theta\right]_\infty = \ell_\infty\left(\Delta^m\right)$, where

$$\left[M_\theta, \Delta^m{}_\theta\right]_\infty = \left\{X = \left(X_k\right) : \frac{1}{h_r}\sum_{k\in I_r}\left[d\left(\Delta^m X_{k+i}, 0\right)\right] < \infty\right\}.$$

Proof: Let $X \in \left[M_\theta, \Delta^m{}_\theta\right]_\infty$. Then there exists a constant $k_1 > 0$ such that

$$\frac{1}{h_1} d\left[\Delta^m X_{1+i}, \overline{0}\right] \leq \frac{1}{h_r}\sum_{k\in I_r} d\left[\Delta^m X_{k+i}, \overline{0}\right] \leq k_1$$

for all i and so we have $X \in \ell_\infty\left(\Delta^m\right)$. Conversely, let $X \in \ell_\infty\left(\Delta^m\right)$. Then there exists a constant $k_2 > 0$ such that $d\left(\Delta^m X_j, 0\right) \leq k_2$ for all j, and so $\frac{1}{h_r}\sum_{k\in I_r} d\left(\Delta^m X_{k+i}, \overline{0}\right) \leq \frac{k_2}{h_r}\sum_{k\in I_r} 1 \leq k_2$ for all k and i. Thus $X \in \left[M_\theta, \Delta^m{}_\theta\right]_\infty$. This completes the proof.

4. Conclusions

We introduce and examine the concepts of lacunary almost statistical convergence and strongly almost convergence of generalized difference sequences of fuzzy numbers. We give a brief information about fuzzy numbers, the operator Δ^m, the sequence $\theta = \left(k_r\right)$, statistical convergence and using the generalized difference operator Δ^m and the sequence $\theta = \left(k_r\right)$. We define the concepts of lacunary almost statistical convergence and strongly lacunary almost convergence of sequences of fuzzy numbers. We establish some relations between strongly lacunary almost $\Delta^m{}_\theta$ - convergence and lacunary almost $\Delta^m{}_\theta$ - statistical convergence.

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