Fuzzy Sliding Controller Design with Adaptive Approximate Error Feedback

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Abstract

In this paper, a novel state error feedback sliding controller is proposed. In the controller, an optimal feedback gain is required and in this study it is assumed to be unknown. Usually, a rudimentary feedback gain is used. Besides, in order to approximate the state error feedback sliding controller with the optimal feedback gain, an adaptive fuzzy system is employed. Thus, the proposed control scheme consists of an adaptive fuzzy system and a state error feedback sliding controller with a rudimentary feedback gain. In the system framework, the rudimentary state error feedback sliding controller can be viewed as the approximate error estimator of the adaptive fuzzy system. Therefore, such an estimated error can be fed back to the learning of the fuzzy system through a modified adaptive law. With such an approximate error feedback, it is clearly evident from our simulation that the learning speed of the proposed learning scheme is faster than that of the original scheme. Also, with the proposed controller, the system stability not only is guaranteed, but also becomes more stable.

Keywords: adaptive sliding control, adaptive fuzzy control, feedback control.

1. Introduction

Various uncertainties exist in practical control applications, such as parameters variations, external disturbances, coupling, and system model errors. Among those uncertainties, coupling and external disturbances are usually difficult to model or estimate. If these considered uncertainties are bounded, the sliding control technique is a powerful scheme of these control problems with uncertainties [1-4]. When an appropriate sliding controller is employed, the state trajectory can approach to the sliding surface and stays on it in a finite time. Generally, the conventional sliding controller design has a large and inelastic control component to guarantee the system stability. It is often obtained based on the upper bounds of the system uncertainties. Since the robust controller can only use such bounds to ensure the stability if no other information is used, chattering phenomena seems unavoidable [18-20]. Please note that the integral sliding surface [29] or other sliding surfaces [30-31] are not considered in this study.

An idea of resolving the above problem is to incorporate certain learning capability into the system to provide information for the robust controller. In the sliding control design for uncertain systems, adaptive approximation techniques are usually employed as learning mechanisms. In this kind of approaches, numerical [27-28, 32-34] or intelligent [12-17] approximation systems are used to estimate unknown parameters in the control law [12-17, 26-28] or to adjust the boundary layer to eliminate chattering phenomena [21-26]. In this study, an adaptive fuzzy approximation system is considered. Adaptive fuzzy systems are adaptive systems with the incorporation of linguistic fuzzy information in a form of fuzzy IF-THEN rules [5-6][8-9] and are usually employed to estimate some elements in the so-call equivalent controller. Basically, these design approaches are similar to that of indirect adaptive fuzzy control systems [6][10-11]. In this paper, a novel idea for adaptive fuzzy sliding control systems is proposed. By referring to [14], it can be found that the stable (sliding) condition, which is the negative Lyapunov energy derivative, is the key idea of the system stability. Thus, it is possible to consider the traditional stable process of approaching the sliding surface only in the design process. Of course, this design direction may violate the basis of the variable structure system design [7]. But, it may exist interesting and useful design methodologies under such a deign philosophy.

As mentioned, the design idea is to use the condition of the negative Lyapunov energy derivative. First, as traditional approaches, the feedback linearization is utilized to define a state-error-feedback sliding controller which is also used to define the condition of the negative Lyapunov energy derivative. Since the optimal feedback gain is uncertain, an adaptive fuzzy system is employed to approximate the control action of the optimal state-

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error-feedback sliding controller. Then, a new adaptive law containing an approximate error feedback scheme is proposed in this paper and its stability is also proved via Lyapunov stable theorems. It is easy to see that the main difference of this approach is the use of an approximate error estimator for the fuzzy system. From our simulation, it is evident that the proposed approach indeed has excellent control performance.

This paper is organized as follows. After this introduction section, the ideas of state-error-feedback sliding controller and fuzzy system are introduced in section II. In section III, the proposed approximate error feedback adaptive law is presented. In section IV, the simulations with the use of an inverted pendulum system are illustrated to verify the correctness of our approach. Finally, conclusive remarks are given in section V.

2. State-error-feedback sliding controller and fuzzy system

Consider a class of nth-order SISO nonlinear systems described by

$$\begin{aligned} x^{(n)} &= f(\mathbf{x}) + g(\mathbf{x})u = f + gu, \\ y &= x, \end{aligned} \tag{1}$$

where $f \in \mathrm{R}^{\mathrm{n}}$ is a unknown bounded continuous function, $g \in \mathrm{R}^{\mathrm{n}}$ is a known bounded positive-definite continuous function, $u \in \mathrm{R}$ is the control input of the system, and $y \in \mathrm{R}$ is the system output. Define the state vector as $\mathbf{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T = [x_1, \cdots, x_n]^T \in \mathrm{R}^{\mathrm{n}}$. Assume that all states are measurable. The reference trajectory is $\mathbf{y}_m = [y_m, \dot{y}_m, \cdots, y_m^{(n-1)}]^T \in \mathrm{R}^{\mathrm{n}}$. The tracking error is $e = y_m - y$ and whose vector form is $\mathbf{e} = [e, \dot{e}, \cdots, e^{(n-1)}]^T = [e_1, e_2, \cdots, e_n]^T \in \mathrm{R}^{\mathrm{n}}$.

The sliding surface considered is defined as [1]

$$s(\mathbf{e}) = (\frac{d}{dt} + \lambda)^{n-1} e, \quad \lambda > 0, \tag{2}$$

where λ is a sliding constant. Then, the time derivative of s is

$$\dot{s} = (\frac{d}{dt} + \lambda)^{n-1} \dot{e} = e^{(n)} + E_S, \tag{3}$$

where

$$E_s = \sum_{i=1}^{n-1} \frac{(n-1)!\ \lambda^{n-i}}{(n-i)!(i-1)!} e^{(i)}. \tag{4}$$

Due to $e^{(n)} = y_m^{(n)} - y^{(n)} = y_m^{(n)} - f - gu - d$, then

$$\dot{s} = y_m^{(n)} - f - gu - d + E_S. \tag{5}$$

The task of u is to drive the states to the sliding surface and to stay on it in a finite time [1]-[4]. The feedback linearization method is usually used to define the reference controller (u^*) of u as

$$u^* = g^{-1}(-f + y_m^{(n)} + E_s). \tag{6}$$

By using (6), (5) can be rewritten as

$$\dot{s} = y_m^{(n)} - f - gu + E_s + gu^* - gu^* = g(u^* - u). \tag{7}$$

Define the Lyapunov function as $V = \frac{1}{2}s^2$. The time derivative of V is

$$\dot{V} = s\dot{s} = sgu^* - sgu. \tag{8}$$

Since $0 < g < \infty$ is limited and known, a sliding controller is defined as

$$u = u_s = C_g^* s, \tag{9}$$

where $C_g^* > 0$ is an optimal control gain. Referring to (2), (9) can be said to be a optimal state-error-feedback sliding controller. Substituting (9) into (8), we get

$$\dot{V} = s\dot{s} = sgu^* - C_g^* g s^2. \tag{10}$$

Since u^* is bounded, (10) indicates that a suitable large C_g^* can ensure $\dot{V} < 0$. This stability notion is similar to that in [14].

In the adaptive fuzzy control system design, a fuzzy system ($\hat{u}_s$) will be used to approximate the controller (9). The fuzzy system is constructed by a set of fuzzy IF-THEN rules as

$$\begin{aligned} &R^l: \text{IF } x_1 \text{ is } A_1^l, \text{ and } \cdots, \text{ and } x_n \text{ is } A_n^l, \\ &\text{THEN } \hat{u}_s \text{ is } \theta_s^l, \qquad l = 1, 2, \cdots, M \end{aligned} \tag{11}$$

where $(x_1, x_2, \cdots, x_n)$ and $\hat{u}_s$ are the input and output of the fuzzy system, respectively, $(A_1^l, A_2^l, \cdots, A_n^l)$ and θ_s^l are the corresponding fuzzy sets, l is the rule index, and M is the number of rules. In this study, for easy computation, θ_s^l is a fuzzy singleton. The overall output of the fuzzy systems with the center-average defuzzification and product inference can be obtained as

$$\hat{u}_s(\mathbf{x}) = \frac{\sum_{l=1}^{M} \theta_s^l (\prod_{j=1}^{n} \mu_{A_j^l}(x_j))}{\sum_{l=1}^{M} (\prod_{j=1}^{n} \mu_{A_j^l}(x_j))}, \tag{12}$$

where $\mu_{A_j^l}(\cdot)$ is the fuzzy membership function of A_j^l. It should be noted that (12) is a universal function approximator [8][9]. (12) can also be rewritten as

$$\hat{u}_s(\mathbf{x}|\boldsymbol{\theta}) = \boldsymbol{\theta}_s^T \boldsymbol{\omega}_s, \tag{13}$$

where $\boldsymbol{\theta}_s = [\theta_s^1, \theta_s^2, \cdots, \theta_s^M]$ is the fuzzy singleton vector, and $\boldsymbol{\omega}_s = [\omega_s^1, \omega_s^2, \cdots, \omega_s^M]$ is the fuzzy degree vector. The elements of $\boldsymbol{\omega}_s$ are

$$\omega_s^l = \frac{\prod_{j=1}^{n} \mu_{A_j^l}(x_j)}{\sum_{l=1}^{M} (\prod_{j=1}^{n} \mu_{A_j^l}(x_j))}, \ l = 1, \cdots, M. \tag{14}$$

As mentioned earlier, (13) is employed to approximate the controller in (9) directly. In order to cope with the approximate error of the limited fuzzy system constructed from the limited rules, usually a additional robust controller (u_r) is utilized to ensure $\lim_{t\to\infty} u = u^*$. In this paper, we further proposed to employ another compensative controller (u_c) to directly cope with errors so as to improve the learning performance. Therefore, the proposed controller is

$$u = \hat{u}_s + u_c + u_r = \boldsymbol{\theta}_s^T \boldsymbol{\omega}_s + u_c + u_r \,, \tag{15}$$

The designs of the adaptive law of $\hat{u}_s$, u_c, and u_r are provided in next section.

3. Approximate Error Feedback Adaptive Law

Define the optimal fuzzy approximator as follows.

$$\hat{u}_s^*(\mathbf{x}|\boldsymbol{\theta}_s^*) = \hat{u}_s^* = \boldsymbol{\theta}_s^{*T}\boldsymbol{\omega}_s \,, \tag{16}$$

where $\boldsymbol{\theta}_s^*$ is the optimal parameter vector defined as

$$\boldsymbol{\theta}_s^* \equiv \arg\min_{\boldsymbol{\theta}_s \in \Omega_{\theta_s}} \{\sup_{\mathbf{x}\in\Omega_{\mathbf{x}}} |u_s - \hat{u}_s^*(\mathbf{x}|\boldsymbol{\theta}_s)|\} \,, \tag{17}$$

where Ω_{θ_s} and Ω_x are constant sets of the suitable bounds of $\boldsymbol{\theta}_s$ and **x**, respectively. Substituting (15) into (7), we obtain

$$\dot{s} = g(u^* - \boldsymbol{\theta}_s^T\boldsymbol{\omega}_s - u_c - u_r) = g(\tilde{u} - \boldsymbol{\theta}_s^T\boldsymbol{\omega}_s - u_r) \,, \tag{18}$$

where $\tilde{u} = u^* - u_c$ is the control error. Because u_c is a compensative controller designed for compensating the difference between u_s and $\hat{u}_s$, u_c can be selected as

$$u_c = C_g s \,, \tag{19}$$

where $0 < C_g < \infty$ is a selected gain constant. In our study, the design targets are defined as follow.

$$\begin{cases} \lim_{t\to\infty}(\hat{u}_s + u_c + u_r) = u^* \\ \lim_{t\to\infty}(\hat{u}_s + u_c) = u_s \end{cases} . \tag{20}$$

Now, rewrite (18) as

$$\begin{aligned} \dot{s} &= g(\tilde{u} - \boldsymbol{\theta}_s^{*T}\boldsymbol{\omega}_s + (\boldsymbol{\theta}_s^* - \boldsymbol{\theta}_s)^T \boldsymbol{\omega}_s - u_r) \\ &= g(\varepsilon_u + \tilde{\boldsymbol{\theta}}_s^T\boldsymbol{\omega}_s - u_r) \,, \end{aligned} \tag{21}$$

where $\varepsilon_u = \tilde{u} - \boldsymbol{\theta}_s^{*T}\boldsymbol{\omega}_s$ and $\tilde{\boldsymbol{\theta}}_s = \boldsymbol{\theta}_s^* - \boldsymbol{\theta}_s$. Consider a Lyapunov function as

$$V_s = \frac{1}{2}s^2 + \frac{1}{2\alpha}\tilde{\boldsymbol{\theta}}_s^T\tilde{\boldsymbol{\theta}}_s \,, \tag{22}$$

where $\alpha > 0$ is an assignable parameter. Substituting (21) into the time derivative of (22), we have

$$\begin{aligned} \dot{V}_s &= s\dot{s} - (1/\alpha)\tilde{\boldsymbol{\theta}}_s^T\dot{\boldsymbol{\theta}}_s \\ &= (1/\alpha)\tilde{\boldsymbol{\theta}}_s^T(\alpha s g \boldsymbol{\omega}_s - \dot{\boldsymbol{\theta}}_s) + sg\varepsilon_u - sgu_r \,. \end{aligned} \tag{23}$$

The adaptive law for the fuzzy system is obtained as

$$\dot{\boldsymbol{\theta}}_s = \alpha s g \boldsymbol{\omega}_s \,, \tag{24}$$

which is also obtained in various adaptive fuzzy control approaches. In this paper, we will further propose a modified adaptive law to improve the learning performance of the fuzzy adaptive control. The idea is to consider the approximate error compensation directly. In this study, we propose to add a simple approximate error feedback into the adaptive law (24). In other words, the prototype of the novel adaptive law is

$$\dot{\boldsymbol{\theta}}_s = \alpha s g \boldsymbol{\omega}_s + \beta\,\tilde{\boldsymbol{\theta}}_s \,, \tag{25}$$

where $\beta > 0$ is a constant. With (25), (23) becomes

$$\dot{V}_s = -(\beta/\alpha)\tilde{\boldsymbol{\theta}}_s^T\tilde{\boldsymbol{\theta}}_s + sg\varepsilon_u - sgu_r \,. \tag{26}$$

Suppose that u_c has a suitable C_g so that $\varepsilon_u = u^* - u_c - \boldsymbol{\theta}_s^{*T}\boldsymbol{\omega}_s \in L_\infty$. It is known that $\tilde{\boldsymbol{\theta}}_s^T\tilde{\boldsymbol{\theta}}_s \in L_\infty$ based on the universal approximator theorem for fuzzy systems [8][9]. Besides, $-(\beta/\alpha)\,\tilde{\boldsymbol{\theta}}_s^T\tilde{\boldsymbol{\theta}}_s < 0$. Thus, the robust controller (u_r) can be designed as

$$u_r = k_r \operatorname{sgn}(s) \,, \tag{27}$$

where $\operatorname{sgn}(s)$ denotes the signal function as

$$\operatorname{sgn}(s) = \begin{cases} 1 & ,\text{if } s > 0 \\ -1 & ,\text{if } s < 0 \\ 0 & ,\text{if } s = 0 \end{cases} , \tag{28}$$

and $k_r > \max(|\varepsilon_u|)$ is a constant. Substituting (25) and (27) into (26), we have

$$\dot{V}_s = \dot{V}_{s1} = -(\beta/\alpha)\tilde{\boldsymbol{\theta}}_s^T\tilde{\boldsymbol{\theta}}_s - sg\varepsilon_u - |s|g(\max(|\varepsilon_u|)) < 0 \,. \tag{29}$$

The sliding condition $\dot{V}_s < 0$ is satisfied. It means when the trajectory crosses a closed ball around the origin, it can never come out again [1]. Thus, the control performance is expectable [14]. Suppose $\varepsilon_u \in L_2$ and then it can be verified that $\lim_{t\to\infty} s = 0$ and $\lim_{t\to\infty} V_s(t) = 0$. In other words, the asymptotical stable is ensured [1].

Another problem is how to obtain $\tilde{\boldsymbol{\theta}}_s$ in (25). In the ideal case as (20), we know

$$u_c = u_s - \hat{u}_s = \tilde{\boldsymbol{\theta}}_s^T\boldsymbol{\omega}_s \,. \tag{30}$$

Then, $\tilde{\boldsymbol{\theta}}_s$ can be defined as

$$\tilde{\boldsymbol{\theta}}_s = (u_c\boldsymbol{\omega}_s^{-1})^T \,. \tag{31}$$

Therefore, the adaptive law is

$$\dot{\boldsymbol{\theta}}_s = \alpha s g \boldsymbol{\omega}_s + \beta(u_c\boldsymbol{\omega}_s^{-1})^T \,. \tag{32}$$

For comparison, consider the normal adaptive law as (24). The time derivative of the Lyapunov function is

$$\dot{V}_s = \dot{V}_{s2} = -sg\varepsilon_u - |s|g(\max(|\varepsilon_u|)) < 0 \,. \tag{33}$$

It can be found that $\dot{V}_s < 0$ still holds. Nevertheless, it is easy to see that $\dot{V}_{s1} < \dot{V}_{s2} < 0$. Thus, the convergent speed of the state error by using (32) is better than that of using (24). This effect is illustrated in the simulation shown in the next section.

In order to avoid the chattering phenomenon, similar to that in [1] and [18-19], the following boundary layer (L_b) is added into (27).

$$L_b = \begin{cases} 1 & \text{, if } s > \Phi \\ -1 & \text{, if } s < \Phi \\ s/\Phi & \text{, else} \end{cases}, \tag{34}$$

where $\Phi \geq 0$ is a boundary layer constant. The robust controller (27) is rewritten as

$$u_r = k_r L_b \operatorname{sgn}(s). \tag{35}$$

Other elimination methods for chattering can be found in [21-26].

Summarily, the proposed approach is

$$\begin{cases} u = \boldsymbol{\theta}_s^T \boldsymbol{\omega}_s + u_c + u_r \\ u_c = C_g s \\ u_r = k_r L_b \operatorname{sgn}(s),\ k_r > \max(|\varepsilon_u|) \\ \dot{\boldsymbol{\theta}}_s = \alpha s g \boldsymbol{\omega}_s + \beta (u_c \boldsymbol{\omega}_s^{-1})^T \end{cases}. \tag{36}$$

4. Simulations

In our study, the inverted pendulum car system is used to illustrate the advantages of (36) and is described as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f + g(u + d),$$

$$f = \frac{g_r \sin x_1 - \dfrac{m_t L x_2^2 \sin x_1 \cos x_1}{m_c + m_t}}{L\left(\dfrac{4}{3} - \dfrac{m \cos^2 x_1}{m_c + m_t}\right)},$$

$$g = \frac{\dfrac{\cos x_1}{m_c + m_t}}{L\left(\dfrac{4}{3} - \dfrac{m \cos^2 x_1}{m_c + m_t}\right)}. \tag{37}$$

where $x_1 (rad)$ is the angle of the pole, x_2 (rad/sec.) is the angular velocity of the pole. The system parameters: g_r is the acceleration due to gravity ($9.8 m/s^2$), m_c is the mass of the car $(1.0 kg)$, m_t is the mass of the pole $(0.1 kg)$, and L=1(m) is the length of the pole. The function g of (37) is a continuous positive function [6]. The fuzzy system is built using 25 fuzzy rules. The used fuzzy sets and membership functions are shown in Fig. 1, and the initial fuzzy singleton vector is $\boldsymbol{\theta}_s(0) = 0$. The following parameters are used; $\alpha = 50$, $\Phi = 10$, $k_r = 1$, $\beta = 100$, $\lambda = 20$, and $C_g = 20$. The initial states are $x_1(0) = 15(\text{deg.})$ and $x_2(0) = 0$. $y_m = \sin(t)$ is the tracking target and the simulation time is 50 seconds.

The MATLAB function "ode45" is used to emulate the system dynamics with the fixed time step size 0.01 (sec.). Figs. 2-4 are the tracking control performance. From these results, it is evident that the convergence of the initial tracking error is fast and the final tracking error is very small. Figs. 5-7 show the total control action. The chatting phenomenon does not exist in the initial or the final stages. Fig. 8 is the compensative control action or the estimation of the approximate error to the adaptive fuzzy controller. It can be found that the approximate error is small. It implies that (32) is effective. Fig. 9 is the control action of the adaptive fuzzy controller (13). After about 15 seconds, the control action is stable. Fig. 10 is the control action of the robust controller (35). Please note that a small k_r is acceptable. It implies $\hat{u}_s + u_c \cong u^*$. Fig. 11 shows the learning results ($\boldsymbol{\theta}_s$) of the fuzzy system. After about 15 seconds the learning is convergent and the parameter drifting problem is not noticeable. Figs. 12-15 are the simulation results while (24) is used in (36) (i.e., (32) is not used). From Fig. 14, it can be found that the needed convergent time is above 40 seconds. On the other hand, the convergent time of using the approximate error feedback adaptive law (32) is about 15 seconds. It is easy to see that the convergent speed of using (24) is slower than that of using (32).

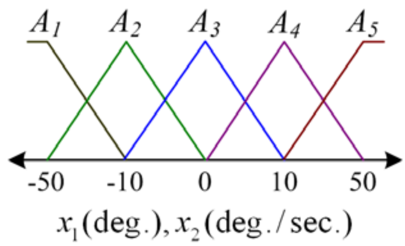

Fig. 1. The used membership functions and fuzzy sets.

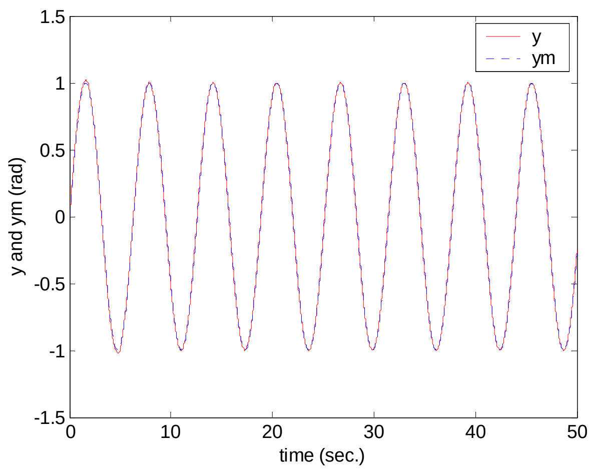

Fig. 2. The tracking control performance of (36).

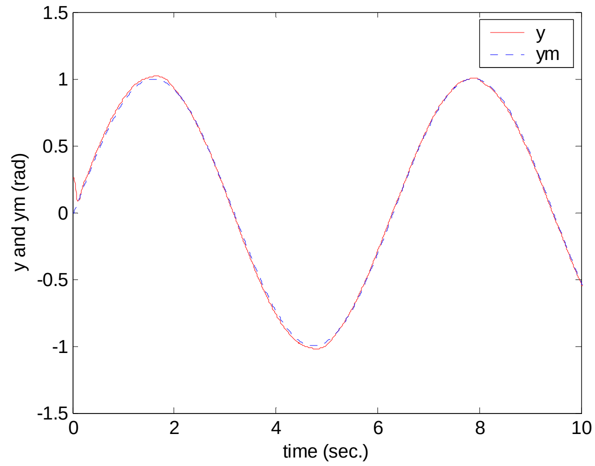

Fig. 3. The initial stage of Fig. 2.

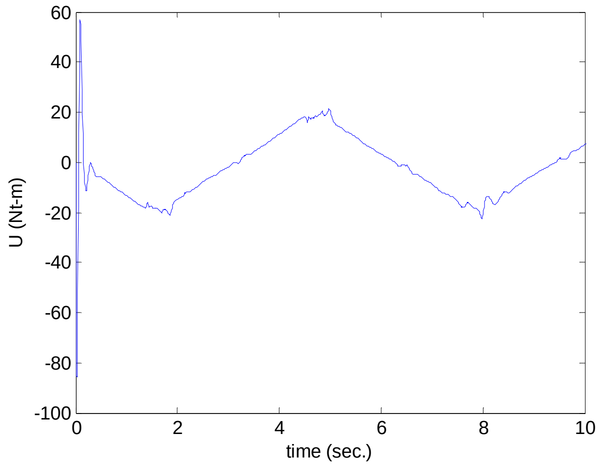

Fig. 6. The initial stage of Fig. 5.

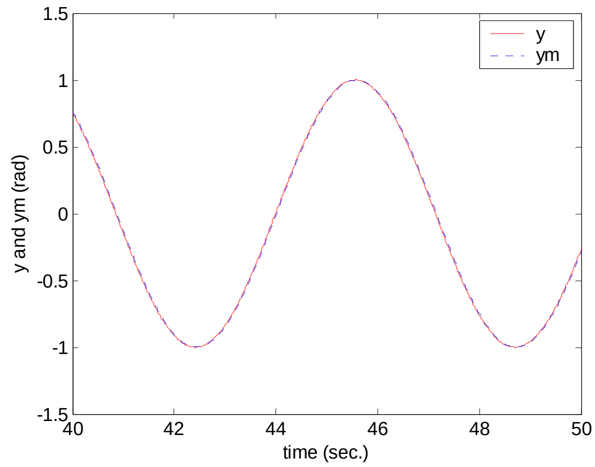

Fig. 4. The final stage of Fig. 2.

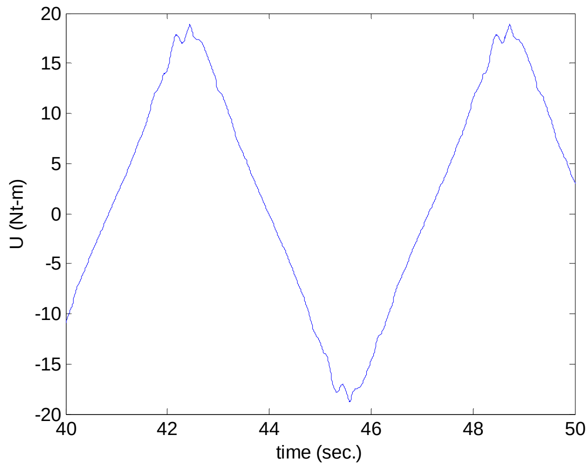

Fig. 7. The final stage of Fig. 5.

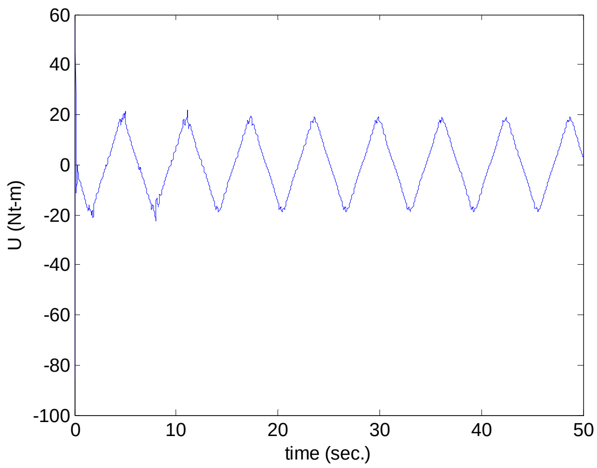

Fig. 5. The total control action (u).

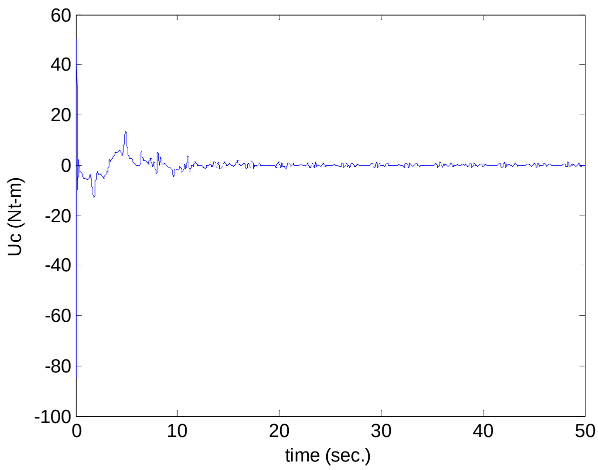

Fig. 8. The compensative control action or the approximate error estimated from (19).

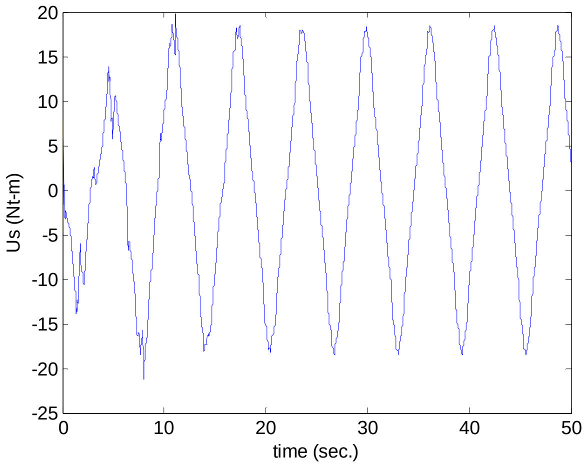

Fig. 9. The action of the adaptive fuzzy controller (13).

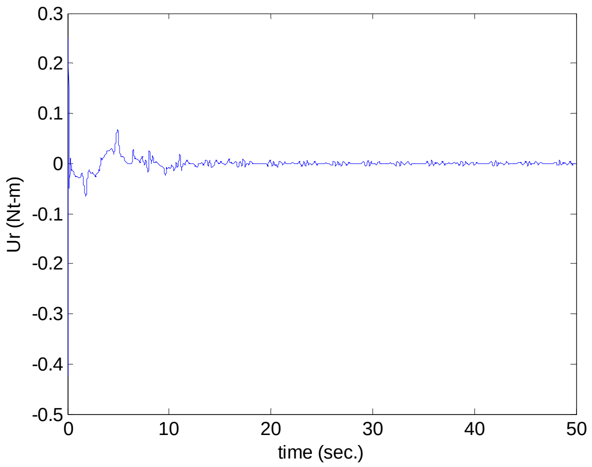

Fig. 10. The robust control action of (35).

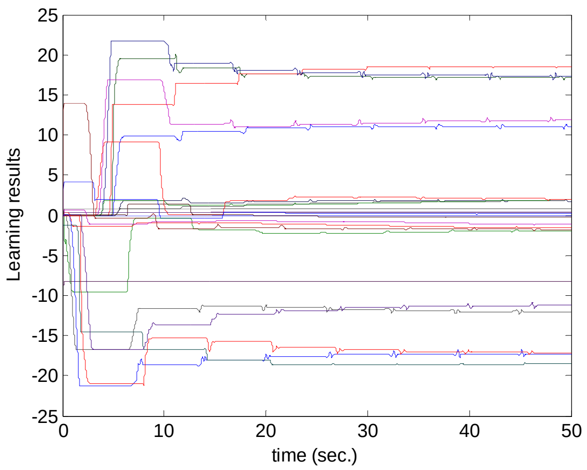

Fig. 11. The learned results ($\boldsymbol{\theta}_s$) of the fuzzy system.

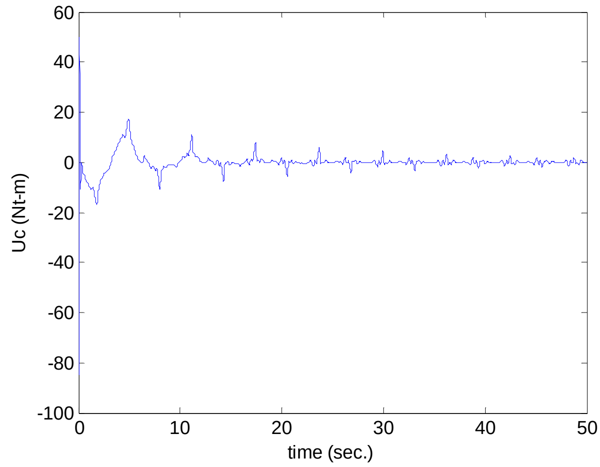

Fig. 12. The compensative control action while the normal adaptive law (24) is used in (36).

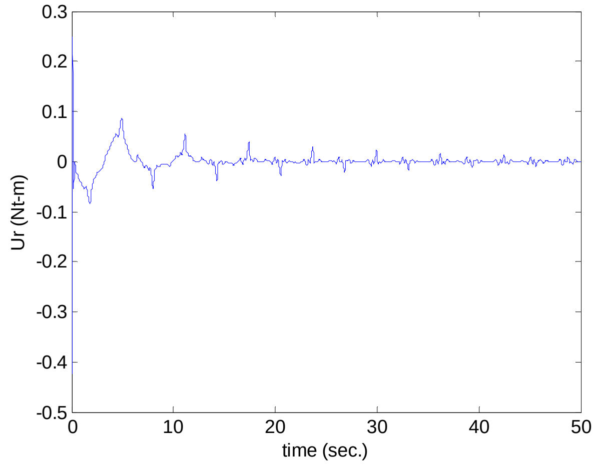

Fig. 13. The robust control action from (35) while the normal adaptive law (24) is used in (36).

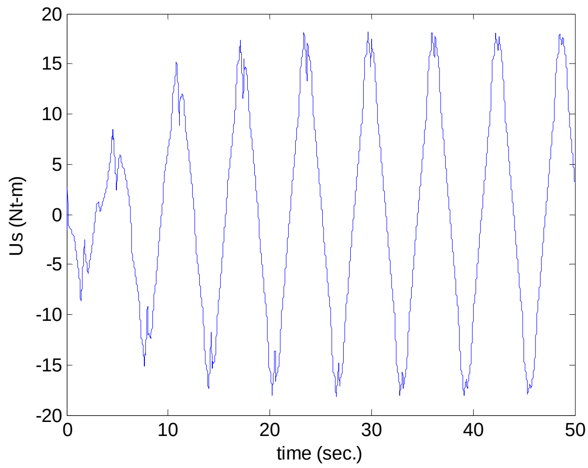

Fig. 14. The control action of the fuzzy system (13) while the normal adaptive law (24) is used in (36).

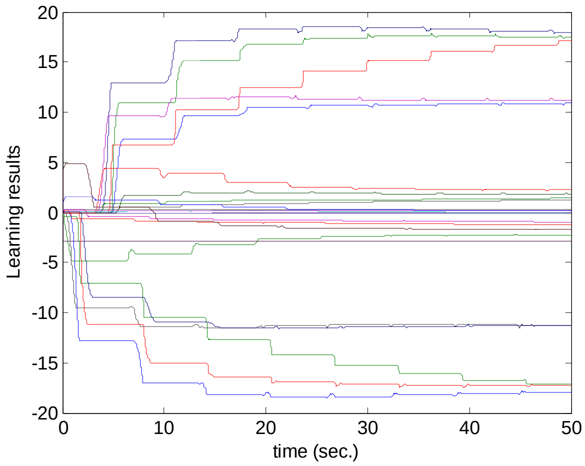

Fig. 15. The learned results ($\boldsymbol{\theta}_s$) of the fuzzy system while the normal adaptive law (24) is used in (36).

5. Discussions

In this paper, a novel state error feedback sliding controller is proposed. Since the control gain required in the optimal state error feedback sliding controller is unknown, thus, in our study, an adaptive fuzzy system ($\hat{u}_s$) together with a state error feedback compensative controller (u_c) is employed to approximate the controller. It can be found that u_c is also an approximate error estimator. In the proposed adaptive law as (32), the approximate error of the fuzzy system can immediately be considered in the learning process and as a result, the learning speed becomes fast as shown in our simulation. As a matter of fact, by using (32), a more negative value of the Lyapunov energy derivative is obtained. It can be easily verified that the system stability of the proposed approach not only is guaranteed, but also becomes more stable.

References

[1] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice Hall, 1991.
[2] W. Perruquetti and J. P. Barbot, *Sliding Mode Control in Engineering*, Marcel Dekker, 2002.
[3] V. Utkin, J. Guldner, and J. X. Shi, *Sliding Mode Control in Electromechanical Systems*. London, U.K., Taylor and Francis, 1999.
[4] S. R. Hebertt and S. O. Ramón, *Control Design Techniques in Power Electronics Devices*, London, U.K. Springer, 2006.
[5] J. A. Farrell, M. M. Polycarpou, *Adaptive Approximation Based Control – Unifying Neural, Fuzzy and Traditional Adaptive Approximation Approaches*. John Wiley & Sons, 2006.
[6] L. X. Wang, *A Course in Fuzzy Systems and Control*. Prentice-Hall, Englewood Cliffs, NJ, 1997.
[7] V. Utkin, "Variable structure systems with sliding modes," *IEEE Trans on Automat. Control*, vol. 22, no. 2, pp. 212-222, April 1977.
[8] L. X. Wang, "Fuzzy systems are universal approximators," *IEEE Int'l Conf. On Fuzzy Systems*, pp. 1163-1170, 1992.
[9] B. Kosko, "Fuzzy systems as universal approximates," *IEEE Int'l Conf. on Fuzzy Systems*, pp. 1153-1162, 1992.
[10] L. X. Wang, "Stable adaptive fuzzy control of nonlinear systems," *IEEE Trans. Fuzzy Systems*, vol. 1, pp. 146-155, May 1993.
[11] L. X. Wang, "Stable adaptive fuzzy controllers with application to inverted pendulum tracking," *IEEE Trans. Systems, Man, and Cybernetics, Part B*, vol. 26, no. 5, pp. 677-691, Oct. 1996.
[12] R. Shahnazi, H. M. Shanechi, and N. Pariz, "Position control of induction and DC servomotors: a novel adaptive fuzzy PI sliding mode control," *IEEE Trans. Energy Conversion*, vol. 23, no. 1, March 2008.
[13] F. Geng and X. Zhu, "A novel adaptive fuzzy control of the inverted pendulum system," *IEEE Int'l Conf. On Control and Automation*, China, pp. 284-288, 2007.
[14] L. K. Wong, F. H. F. Leung, P. K. S. Tam, "A fuzzy sliding controller for nonlinear systems," *IEEE Trans on Industrial Electronics*, vol. 48, no. 1, pp. 32-37, Feb. 2001.
[15] L. K. Wang, F. H. F. Lenug, and P. K. S. Tam, "A chattering elimination algorithm for sliding mode control of uncertain non-linear systems," *Mechatronics*, vol.8, pp. 765-775, 1998.
[16] R. Shahnazi, H. M. Shanechi, and N. Pariz, "Position control of induction and DC servomotors: a novel adaptive fuzzy PI sliding mode control," *IEEE Trans. on Energy Conversion*, vol. 23, no. 1, pp.138-147, March 2008.
[17] Y. T Kim and Z. Z. Bien, "Robust adaptive fuzzy control in the presence of external disturbance and approximation error," *Fuzzy sets and systems*, vol. 148, pp.377-393, 2004.
[18] P. Kachroo and M. Tomizuka, "Chattering reduction and error convergence in the sliding- mode control of a class of nonlinear systems," *IEEE Trans. Automatic Control*, vol. 41, no. 7, pp. 1063-1068, July 1996.
[19] P. Kachroo, "Existence of solutions to a class of nonlinear convergent chattering-free sliding mode control systems," *IEEE Trans. on Automatic Control*, vol. 44, no. 8, pp.1620-1624, Aug. 1999.

[20] K. J. Kim, J. B. Park, and Y. H. Choi, "Chattering free sliding mode control," *SICE-ICASE, Inter'l Joint Conference*, pp. 732-735, Oct, 2006.
[21] W. D. Chang, R. C. Hwang, and J. G. Hsieh, "Application of an auto-tuning neuron to sliding mode control," *IEEE Trans. on Systems, Man, and Cybernetics - Part C: Applications and Reviews*, vol. 32, no. 4, pp. 517-522, Nov. 2002.
[22] J. K. Liu, "Global sliding mode control with adaptive fuzzy chattering free method for nonlinear system," *IMACS Multiconference on Computational Engineering in Systems Applications*, vol. 1, pp. 541-546, Oct. 2006.
[23] C. Y. Liang1 and S. W. Tan, "A new approach to chattering reduction in the sliding mode controls," *Second International Conference on Innovative Computing, Information and Control*, pp. 334-337, Sept. 2007.
[24] G. D. Buckner, "Intelligent bounds on modeling uncertainty: applications to sliding mode control," *IEEE Trans. on Systems, Man, and Cybernetics - Part C: Applications and Reviews*, vol. 32, no. 2, May 2002.
[25] Y. J. Huang, T. C. Kuo, and S. H. Chang, "Adaptive sliding-mode control for nonlinear systems with uncertain parameters," *IEEE Trans. on Systems, Man, and Cybernetics: Part B: Cybernetics*, vol. 38, no. 2, pp. 534-539, April 2008.
[26] R. J. Wai, C. M. Lin, and C. F. Hsu, "Adaptive fuzzy sliding-mode control for electrical servo drive," *Fuzzy Sets and Systems*, vol. 143, pp. 295-310, 2004.
[27] P. C. Chen and A. C. Huang, "Adaptive sliding control of active suspension systems with uncertain hydraulic actuator dynamics," *Vehicle System Dynamics*, vol. 44, no. 5, pp. 357-368, May 2006.
[28] A. C. Huang and Y.S. Kuo, "Sliding control of non-linear systems containing time-varying uncertainties with unknown bounds," *Int. J. Control*, vol. 74, no. 3, pp.252-264, 2001.
[29] C. M. Lin and C. F. Hsu, "Adaptive Fuzzy Sliding-Mode Control for Induction Servomotor Systems," *IEEE Trans. on Energy Conversion*, vol. 19, no. 2, June 2004.
[30] E. Iglesias, Y. García, M. Sanjuan, O. Camacho, and C. Smith, "Fuzzy surface-based sliding mode control," *ISA Transactions*, vol.46, pp.73-83, 2007.
[31] A. Boubakir, F. Boudjema, C. Boubakir, and S. Labiod, "A fuzzy sliding mode controller using nonlinear sliding surface applied to the coupled tanks system," *International Journal of Fuzzy Systems*, vol. 10, no. 2, June 2008.
[32] J. Yu, K. Zhang, and S. Fei, "Adaptive fuzzy tracking control of a class of stochastic nonlinear systems with unknown dead-zone input," *International Journal of Fuzzy Systems*, vol. 10, no. 1, March 2008.
[33] C. C. Chuang, "Annealing robust fuzzy neural networks for modeling of molecular autoregulatory feedback loop systems," *International Journal of Fuzzy Systems*, vol. 10, no. 1, March 2008.
[34] W.-Y. Wang, Y.-H. Chien, Y.-G. Leu, and T.-T. Lee, "On-line adaptive T-S fuzzy-neural control for a class of general multi-link robot manipulators," *International Journal of Fuzzy Systems*, vol. 10, no. 4, December 2008.

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