Nonlinear Robust Control of Fuzzy Time-Delay Systems

Gwo-Ruey Yu and Chun-Sheng You

Abstract

This paper investigates the nano-positioning control for a piezoelectric actuator system with states time-delay via multiple Lyapunov functions. The multiple Lyapunov functions can relax stabilization conditions derived by the traditional single Lyapunov function. First, the Bouc-Wen equation establishes the mathematical model of a piezoelectric actuator system. To reduce the nonlinear hysteresis effect, the Bouc-Wen model is linearized by Takagi-Sugeno (T-S) fuzzy systems. According to the proposed stability theorem, we find the feasible solutions of feedback gains by LMIs and employ the strategy of parallel distributed compensation (PDC) to control the piezoelectric actuator system.

Keywords: Linear matrix inequality, multiple Lyapunov functions, piezoelectric systems.

1. Introduction

The nano-positioning technology has been widely studied in recent years. The piezoelectric ceramic materials are often used as actuators in the nano-positioning control system [1]. Inverse piezoelectric effect is a process in electromechanical energy conversion that relates electrical energy to mechanical displacement. Piezoelectric actuators include stack type, bimorph type, and ring type etc [2]. The stack type of piezoelectric actuators has movement ranges from 1 um to 10 um. The advantages of stack type of piezoelectric actuators are as follows, unlimited resolution, high frequency response, no friction, and high stiffness [3]. Thus, the stack type of piezoelectric actuators has been studied in this paper.

The piezoelectric actuator is a kind of nonlinear system because piezoelectric materials have hysteresis and creep effects. Hysteresis phenomena arise from materials polarization and molecule friction. Memory effect is the main characteristics in hysteresis systems [4]. As input voltages of a piezoelectric actuator alternate between increasing and decreasing, the displacement response diverges from its original path. The positioning errors are about 2%~5% according to different attribution of materials. The input voltages of a piezoelectric actuator also produce polarization and cause the slow movement. The phenomenon is called creep effect. The precision of nano-positioning system will deteriorate due to the nonlinear effects of hysteresis and creep.

There have been many researches in analyzing the hysteresis behavior of piezoelectric actuators. Wei used piecewise polynomial functions to approximate hysteresis curves [5]. M. Goldfard set up a generalized Maxwell's model from the viewpoint of energy [6]. Adriaens set up a mathematic model of hysteresis using differential equations [7]. Xu and Ku applied neural networks to study hysteresis characteristics [8]. Ping and D. Croft used Preisach's model to describe hysteresis phenomena [9]. In this paper, the Bouc-Wen model is analyzed and studied [10].

States time-delay often occur in many dynamical systems like communication transits, mechanical systems, biological systems, and chemical reactions, etc. The phenomena of states time-delay also exist in piezoelectric actuators. It makes the degeneration of control performance. This paper uses the T-S fuzzy model with states time-delay [11] to represent the nonlinear Boun-Wen equation via fuzzy IF- THEN rules.

In the stability analysis of fuzzy control systems, main stability conditions are usually based on a single Lyapunov function $V(x(t)) = x(t)^T Px(t)$. It is necessary to find a common positive definite matrix P. However, it is sometimes difficult to find the matrix P in many cases due to conservative stability conditions. The multiple Lypunov functions [12] could be applied to relax the limits of stability conditions. In fact, the common Lyapunov function is a special case of the multiple Lyapunov functions. In this paper, the multiple Lyapunov function shares the same membership functions with the T-S fuzzy model of piezoelectric actuator systems. The parallel distributed compensation (PDC) approach is applied to design the nonlinear controller. In the PDC concept, each control rule is designed from the corresponding rule of the T-S fuzzy model [13].

The organization of this paper is as follow. Section 2 describes the mathematical model of a piezoelectric actuator. In section 3, we derive the stabilization conditions for open-loop systems and closed-loop sys-

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tems with states time delay. Section 4 shows computer simulations of nano-position control systems.

2. Mathematical Model of Hysteresis Curve

The Bouc-Wen model [10] is used to approximate the hysteresis curve of a piezoelectric actuator. The mathematical equations are

$$\begin{cases} m\ddot{x} + b\dot{x} + kx = k(du - h) + \rho \\ \dot{h} = \alpha d\dot{u} - \beta|\dot{u}|h - \gamma\dot{u}|h| \\ \rho = kx_0 \end{cases} \quad (1)$$

where $\rho = kx_0$ is the pressure amount in advance; m, b, k and d mean the tangent mass, damping, stiffness, and effective piezoelectric coefficients, respectively. The symbol u is the input voltage of piezoelectric actuator. The symbol is the displacement of piezoelectric actuator. The variable h is from the nonlinear hysteresis equation. The α, β and γ constants control the curves of mathematical equation.

To find these parameters, a voltage signal is inputted to the piezoelectric actuator. The voltage signal is a triangular wave with amplitude 100 voltages and frequency 10Hz. After several experiments and computer simulations, the parameters of piezoelectric actuator are obtained and listed in Table 1. Figure 1 shows the real hysteresis curve of piezoelectric actuator and simulation responses of Bouc-Wen model.

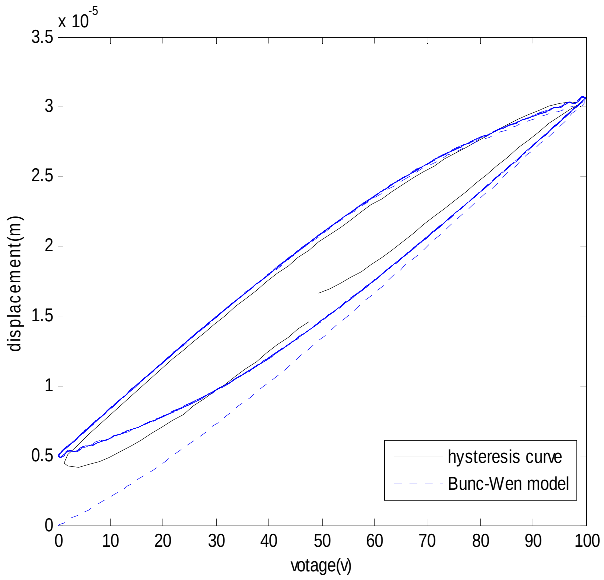

Fig. 1. Hysteresis curves of the Bouc-Wen model.

Table 1. Parameters of Bouc-Wen Model.

m	0.148kg	d	3.5×10^{-7}
B	129.5N-s/m	α	0.5
K	3×10^{6} N/m	β	0.023
ρ	0	γ	0.01

3. Fuzzy Control of States Time-Delay Systems

A. *T-S Fuzzy Model with States Time-Delay*

The Takagi-Sugeno fuzzy model is represented by fuzzy IF-THEN rules. Every rule is a local dynamic subsystem. The subsystem is described by the state equation $A_i x(t) + B_i u(t)$. The T-S fuzzy model combines these subsystems with inference rules to approximate a nonlinear plant. This paper uses the T-S fuzzy model with states time-delay to linearize the nonlinear Bouc-Wen equation. The subsystem is described by the state equation $A_i x(t) + A_{id} x(t-\tau) + B_i u(t)$.

The state-space equation of Bouc-Wen model is defined as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & -\frac{k}{m} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{kd}{m} & 0 \\ 0 & \alpha d \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$+|u_2| \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\beta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma \end{bmatrix} \left| \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right| \quad (2)$$
$$y = [1\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

where $x_1 = x$, $x_2 = \dot{x}_1 = \dot{x}$, $x_3 = h$, input $u_1 = u$, and $u_2 = \dot{u}$.

This paper uses the Gaussian functions to establish the membership functions of T-S fuzzy model. Figure 2 shows the Gaussian membership functions. The state $x_1(t)$ is chosen as the premise variable. The inference rules and local models are designed as follows

Model Rule 1 (M1)

IF $x_1(t)$ is about $1.5\times10^{-5} m$

THEN

$$\begin{aligned} \dot{x}(t) &= h_1(x_1(t))(A_1(t) + A_{1d}(t-\tau) + B_1 u(t)) \\ y(t) &= [1 \quad 0 \quad 0] x(t) \end{aligned} \quad (3)$$

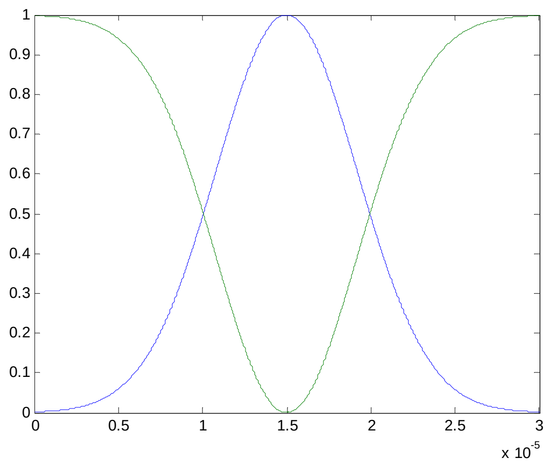

Fig. 2. Membership functions of T-S fuzzy model.

Model Rule 2 (M2)

IF $x_1(t)$ is about 0 or $3\times10^{-5}m$

THEN

$$\begin{aligned}\dot{x}(t) &= h_2(x_1(t))(A_2(t)+A_{2d}(t-\tau)+B_2u(t)) \\ y(t) &= [1 \quad 0 \quad 0]x(t)\end{aligned} \tag{4}$$

where

$$h_1(x_1(t)) = \exp[-(x_1(t)-1.5\times10^{-5})/2\times(0.42\times10^{-5})]$$

$$h_2(x_1(t)) = 1-\exp[-(x_1(t)-1.5\times10^{-5})/2\times(0.42\times10^{-5})]$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ -\dfrac{k}{m} & -\dfrac{b}{m} & -\dfrac{k}{m} \\ 0 & \dfrac{\alpha d u_2}{x_2(t)} & -(\beta|u_2|+\delta u_2) \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ -\dfrac{k}{m} & -\dfrac{b}{m} & -\dfrac{k}{m} \\ 0 & \dfrac{2\alpha d u_2}{x_2(t)} & -2(\beta|u_2|+\delta u_2) \end{bmatrix},$$

$$A_{1d} = A_{2d} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ \dfrac{kd}{m} \\ 0 \end{bmatrix}$$

B. *Stability Analysis via Multiple Lyapunov Functions Approach*

Let the multiple Lyapunov functions for T-S fuzzy system with states time-delay be

$$V(x) = \sum_{\rho=1}^{r} h_\rho(z(t))x^T(t)P_\rho x(t) + \sum_{\rho=1}^{r}\int_{t-\tau}^{t} h_\rho(z(s))x^T(s)R_\rho x(s)ds \tag{5}$$

A sufficient stability condition of open-loop system with states time-delay is given as follows:

Theorem 1: Assume that

$$\left|\dot{h}_\rho(z(t))\right| \le \phi_\rho \tag{6}$$

where $\phi_\rho \ge 0$ for $\rho = 1,2,\ldots,r-1$. The open-loop T-S fuzzy system with states time-delay is asymptotically stable if there exist $\phi_1,\phi_2,\ldots,\phi_{r-1}$, positive definite matrices $P_1,P_2,\ldots,P_r$, and $R_1,R_2,\ldots,R_r$ such that

$$P_\rho \ge P_r$$

$$\begin{aligned}&\sum_{\rho=1}^{r-1}\phi_\rho(P_\rho-P_r)+\frac{1}{2}\sum_{\rho=1}^{r}\sum_{i=1}^{r}A_i^T P_\rho + P_\rho A_i + A_\rho^T P_i + P_i A_\rho \\ &+ \mathrm{P}_\rho A_{id}R_\rho^{-1}A_{id}^T\mathrm{P}_\rho + \mathrm{P}_i A_{\rho d}R_\rho^{-1}A_{\rho d}^T\mathrm{P}_i + R_\rho < 0\end{aligned} \tag{7}$$

Proof: The derivative of *V(x)*

$$\begin{aligned}\dot{V}(x) &= \sum_{\rho=1}^{r}\dot{h}_\rho(z(t))x^T(t)P_\rho x(t) \\ &+ \sum_{\rho=1}^{r}h_\rho(z(t))\dot{x}^T(t)P_\rho x(t) + \sum_{i=1}^{r}h_\rho(z(t))x^T(t)P_\rho\dot{x}(t) \\ &+ \sum_{\rho=1}^{r}h_\rho(z(t))x^T(t)R_\rho x(t) - \sum_{\rho=1}^{r}h_\rho(z(t))x^T(t-\tau)R_\rho x(t-\tau)\end{aligned}$$

The form of open-loop system with states time-delay is

$$\dot{x}(t) = \sum_{i=1}^{r}h_i(z(t))\left[A_i x(t) + A_{id}x(t-\tau)\right]$$

$$\begin{aligned}\dot{V}(x) &= \sum_{\rho=1}^{r-1}\dot{h}_\rho(z(t))x^T(t)P_\rho x(t) + h_r(z(t))\dot{x}^T(t)P_\rho x(t) + \frac{1}{2} \\ &\sum_{\rho=1}^{r}\sum_{i=1}^{r}h_\rho(z(t))h_i(z(t))x^T(t)\left[A_i^T P_\rho + P_\rho A_i + A_\rho^T P_i + P_i A_\rho\right]x(t) \\ &+ 2\sum_{\rho=1}^{r}\sum_{i=1}^{r}h_\rho(z(t))h_i(z(t))x^T(t)P_\rho A_{id}x(t-\tau) \\ &+ \sum_{\rho=1}^{r}h_\rho(z(t))x^T(t)R_\rho x(t) - \sum_{\rho=1}^{r}h_\rho(z(t))x^T(t-\tau)R_\rho x(t-\tau)\end{aligned}$$

Using the fact that

$$\begin{aligned}2x^T(t)P_\rho A_{id}x(t-\tau) \le{}& \dot{x}^T(t)PA_{id}R_\rho^{-1}A_{id}^T Px(t) \\ &+ x^T(t-\tau)R_\rho x(t-\tau)\end{aligned}$$

We have

$$\dot{V}(x) \le \sum_{\rho=1}^{r-1} \dot{h}_\rho(z(t)) x^T(t) P_\rho x(t) + h_r(z(t)) \dot{x}^T(t) P_\rho x(t)$$
$$+\frac{1}{2}\sum_{\rho=1}^{r}\sum_{i=1}^{r} h_\rho(z(t)) h_i(z(t)) x^T(t)$$
$$[A_i^T P_\rho + P_\rho A_i + A_\rho^T P_i + P_i A_\rho$$
$$+ P_\rho A_{id} R_\rho^{-1} A_{id}^T P_\rho + P_i A_{\rho d} R_\rho^{-1} A_{\rho d}^T P_i] x(t)$$

Assume that

$$P_\rho \ge P_r$$

From the relaxed stability conditions [11]

$$\sum_{\rho=1}^{r} \dot{h}_\rho(z(t)) = 0 \quad \forall z(t)$$
$$\Rightarrow \dot{h}_r(z(t)) = -\sum_{\rho=1}^{r-1} \dot{h}_\rho(z(t)) \tag{8}$$

Thus, the stability condition of open-loop system with state time-delay is

$$\dot{V}(x(t)) \le \sum_{i=1}^{r} h_i(z(t)) x^T(t) \{ \sum_{\rho=1}^{r-1} \phi_\rho (P_\rho - P_r) +$$
$$\frac{1}{2}\sum_{\rho=1}^{r}\sum_{i=1}^{r} A_i^T P_\rho + P_\rho A_i + A_\rho^T P_i + P_i A_\rho + \tag{9}$$
$$\mathrm{P}_\rho A_{id} R_\rho^{-1} A_{id}^T \mathrm{P}_\rho + \mathrm{P}_i A_{\rho d} R_\rho^{-1} A_{\rho d}^T \mathrm{P}_i + R_\rho \} x(t) < 0$$

Q.E.D

C. *PDC Stabilization Controller Designs via LMIs*

For the closed-loop system with states time-delay, a stability condition is proposed. The linear matrix inequalities (LMIs) are derived by Schur complement to get the optimal feedback gains matrices F_i of parallel distributed compensation (PDC) controller.

Theorem 2: The closed-loop T-S fuzzy system with states time-delay is asymptotically stable if there exist $\varepsilon > 0$, $s_1, s_2, ..., s_r$, positive-definite matrices $P_1, P_2, ..., P_r$ and matrices $F_1, F_2, ..., F_r$ such that

$$P_\rho \ge s_\rho I, \quad s_\rho \ge 1, \quad \rho = 1,2,....,r-1$$

$$\begin{bmatrix} M & \Omega_v & \Lambda_v \\ \Omega_v^T & -6(r-1)I & 0 \\ \Lambda_v^T & 0 & -6(r-1)R_k I \end{bmatrix} \le 0 \tag{10}$$
$$i \le j \le k, \quad \rho = 1,2,....,r-1$$

where

$$M = \phi(P_\rho - P_r) - \frac{1}{3\varepsilon^2 (r-1)}(s_\rho + s_i + s_j) I_{n\times n} + 6R_\rho$$
$$\Omega = \left[\Omega_{ijk} \quad \Omega_{ikj} \quad \Omega_{jik} \quad \Omega_{jki} \quad \Omega_{kij} \quad \Omega_{kji} \right]$$
$$\Omega_{ijk} = \varepsilon \left(A_i - B_i F_j \right)^T + \frac{1}{\varepsilon} P_k$$
$$\Lambda_v = \left[\Lambda_{ij} \quad \Lambda_{ik} \quad \Lambda_{ji} \quad \Lambda_{jk} \quad \Lambda_{ki} \quad \Lambda_{kj} \right]$$
$$\Lambda_{ij} = P_j A_{id}$$

Proof: The derivative of $V(x)$

$$\dot{V}(x) = \sum_{\rho=1}^{r} \dot{h}_\rho(z(t)) x^T(t) P_\rho x(t)$$
$$+ \sum_{\rho=1}^{r} h_\rho(z(t)) \dot{x}^T(t) P_\rho x(t) + \sum_{\rho=1}^{r} h_\rho(z(t)) x^T(t) P_\rho \dot{x}(t)$$
$$+ \sum_{\rho=1}^{r} h_\rho(z(t)) \dot{x}^T(t) R_\rho x(t) - \sum_{\rho=1}^{r} h_\rho(z(t)) \dot{x}^T(t-\tau) P_\rho x(t-\tau)$$

The closed-loop system is represented as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \left[A_i x(t) + A_{id} x(t-\tau) + B_i u(t) \right]$$
$$u(t) = -\sum_{i=1}^{r} h_i z(t) F_i x(t)$$

Thus

$$\dot{V}(x) = \sum_{\rho=1}^{r} \dot{h}_\rho(z(t)) x^T(t) P_\rho x(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} h_i(z(t)) h_j(z(t)) \cdot$$
$$h_k(z(t)) (x^T(t) A_i^T - x^T(t) F_j^T B_i^T + x^T(t-\tau) A_{id}^T) P_k x(t)$$
$$+ \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} h_i(z(t)) h_j(z(t)) h_k(z(t)) x^T(t) \cdot$$
$$P_k (A_i x(t) - B_i F_j x(t) + A_{id} x(t-\tau)) + \sum_{\rho=1}^{r} h_\rho(z(t)) \cdot$$
$$\dot{x}^T(t) R_\rho x(t) - \sum_{\rho=1}^{r} h_\rho(z(t)) \dot{x}^T(t-\tau) P_\rho x(t-\tau)$$
$$= \sum_{\rho=1}^{r} \dot{h}_\rho(z(t)) x^T(t) P_\rho x(t) \ + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} h_i(z(t)) h_j(z(t)) \cdot$$
$$h_k(z(t)) x^T(t) ((A_i^T - F_j^T B_i^T) P_k + P_k (A_i - B_i F_j)) x(t)$$
$$+ \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} h_i(z(t)) h_j(z(t)) h_k(z(t)) \cdot (x^T(t-\tau) A_{id}^T P_k x(t)$$
$$+ x^T(t) P_k A_{id} x(t-\tau)) + \sum_{\rho=1}^{r} h_\rho(z(t)) x^T(t) R_\rho x(t)$$
$$- \sum_{\rho=1}^{r} h_\rho(z(t)) x^T(t-\tau) P_\rho x(t-\tau)$$

Using the fact that for any vector x_1, x_2 and matrix Y

$$x_1^T Y x_2 + x_2^T Y^T x_1 \le x_1^T Y R^{-1} Y^T x_1 + x_2^T R x_2$$

where R is a positive matrix. From the last inequality and (6), (8), we have

$$\dot{V}(x) \le \sum_{\rho=1}^{r-1} \phi_\rho x^T(t) P_\rho x(t) - \phi_\rho x^T(t) P_r x(t)$$
$$+ \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} h_i(z(t)) h_j(z(t)) h_k(z(t)) x^T(t) \cdot$$
$$((A_i^T - F_j^T B_i^T) P_k + P_k (A_i - B_i F_j) +$$
$$P_k A_{id} R_k^{-1} A_{id}^T P_k + R_k) x(t)$$
$$\dot{V}(x) \le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} h_i(z(t)) h_j(z(t)) h_k(z(t)) x^T(t) \cdot$$
$$\{\phi_\rho (P_\rho - P_r) + \frac{1}{6(r-1)} \hat{W}_{ijk} + \frac{1}{6(r-1)} \Lambda_v + R_k\} x(t)$$

where

$$G_{ij} = A_i - B_i F_j$$
$$W_{ijk} = G_{ij}^T P_k + P_k G_{ij}$$
$$\hat{W}_{ijk} = W_{ijk} + W_{ikj} + W_{jik} + W_{jki} + W_{kij} + W_{kji}$$

The derivative of multiple Lyapunov functions is negative if there exist $\varepsilon > 0$ such that

$$\begin{aligned} &\phi_\rho (P_\rho - P_r) + \frac{1}{6(r-1)} (\hat{W}_{ijk} + \varepsilon^2 G_{ij}^T G_{ij} + \varepsilon^2 G_{ji}^T G_{ji} \\ &+ \varepsilon^2 G_{jk}^T G_{jk} + \varepsilon^2 G_{kj}^T G_{kj} + \varepsilon^2 G_{ik}^T G_{ik} + \varepsilon^2 G_{ki}^T G_{ki}) \\ &+ \frac{1}{6(r-1)} \Lambda_v + R_\rho \le 0 \end{aligned} \quad (11)$$

The left-hand can be rewritten as

$$\begin{aligned} &\phi_\rho (P_\rho - P_r) \\ &+ \frac{1}{6(r-1)} (S_{ijk} + S_{ikj} + S_{jik} + S_{jki} + S_{kij} + S_{kji}) \\ &+ \frac{1}{6(r-1)} \Lambda_v + R_\rho \le 0 \end{aligned} \quad (12)$$

where

$$S_{ijk} = U_{ijk} - \frac{1}{\varepsilon^2} P_k P_k$$
$$U_{ijk} = \left(\varepsilon G_{ij}^T + \frac{1}{\varepsilon} P_k\right)\left(\varepsilon G_{ij}^T + \frac{1}{\varepsilon} P_k\right)^T$$

Assume that $s_\rho I \le P_\rho$, and $1 \le s_i \quad \forall i$. Then, we can get that (12) must smaller than (13)

$$\begin{aligned} &\phi_\rho (P_\rho - P_r) - \left\{\frac{1}{3\varepsilon^2 (r-1)} (s_i + s_j + s_k)\right\} I + 6(r-1) R_k \\ &+ \frac{1}{6(r-1)} (U_{ijk} + U_{ikj} + U_{jik} + U_{jki} + U_{kij} + U_{kji}) + \frac{1}{6(r-1)} \Lambda_v \end{aligned} \quad (13)$$

The (13) can be written as (10) by Schur complement. If condition (10) holds, then $\dot{V}(x(t)) < 0$.
Q.E.D

4. Computer Simulation

The optimal feedback gains of PDC controller could be obtained through the solutions of LMIs (10). Let the reference command be a step function with amplitude 10 μm. Set $\phi_1 = \phi_2 = 0.835$ and we get feedback gains as follows:

$$F_1 = 10^8 \times [7.21503 \quad 0.004074 \quad -0.0633]$$
$$F_2 = 10^8 \times [5.21387 \quad 0.003977 \quad -0.0248]$$

The positive matrices are

$$P_1 = \begin{bmatrix} 22.9996 & 0.0241 & 2.2372 \\ 0.0241 & 0.1425 & -0.0001 \\ 2.2372 & -0.0001 & 0.5014 \end{bmatrix} > 0$$
$$P_2 = \begin{bmatrix} 19.9297 & 0.0182 & 2.4519 \\ 0.0182 & 0.3897 & -0.001 \\ 2.4519 & -0.001 & 0.8066 \end{bmatrix} > 0$$
$$R_1 = \begin{bmatrix} 16.3464 & 0.0962 & 1.6067 \\ 0.0962 & 1.3404 & -0.0260 \\ 1.6067 & -0.0260 & 15.0502 \end{bmatrix} > 0$$
$$R_2 = \begin{bmatrix} 15.162 & -0.0973 & 0.1196 \\ -0.0973 & 0.0535 & 0.0587 \\ 0.1196 & 0.0587 & 8.1673 \end{bmatrix} > 0$$

Fig. 3 shows the step response of the piezoelectric actuator system with states time-delay $\tau = 1$ ms. The dotted line represents the response using the novel fuzzy controller design developed in this paper. The PDC controller can follow the reference command and the nano-positioning error is smaller than 100 nm. The steady-state error is 3.514×10^{-8} m. For comparison, we have also carried out the multiple Lyapunov functions design without considering the states time-delay. The dashed line represents the response using the fuzzy controller developed in the literature [12]. It is obvious that the performance of the multiple Lyapunov functions with states time-delay design is better.

Fig. 4 shows the tracking response of the piezoelectric control system. The reference command is a sinusoidal signal with amplitude 5 μm. The time period is 0.1 sec. The dotted line represents the response using the multiple Lyapunov functions with states time-delay design developed in this paper. The novel fuzzy controller can track the reference command very well. The dashed line represents the response using the fuzzy controller developed in the literature [12]. Simulation results demonstrate the effectiveness of this novel design. Fig. 5 compares the performance of the piezoelectric act uator system with different delay time. Fig. 6 shows the staircase response of the piezoelectric actuator system.

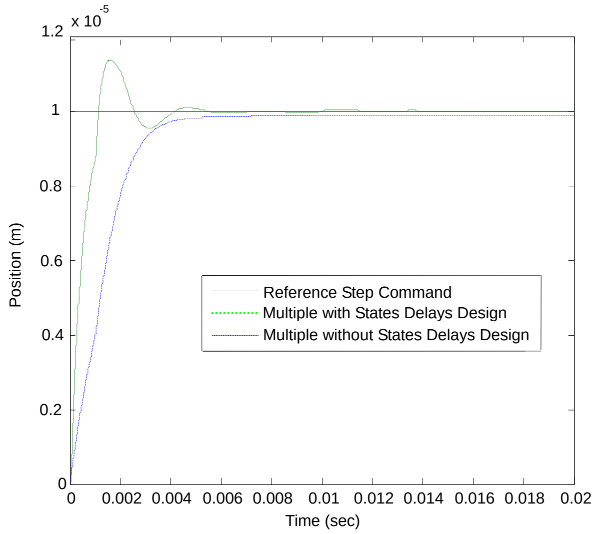

Fig. 3. Step response of the piezoelectric system with states time-delay.

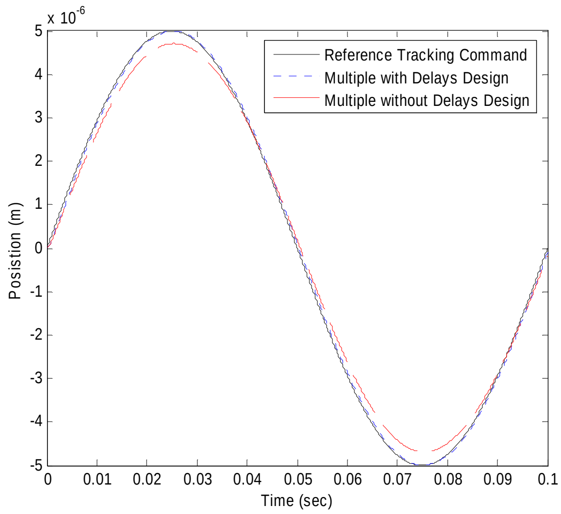

Fig. 4. Step response of the piezoelectric system with states time-delay.

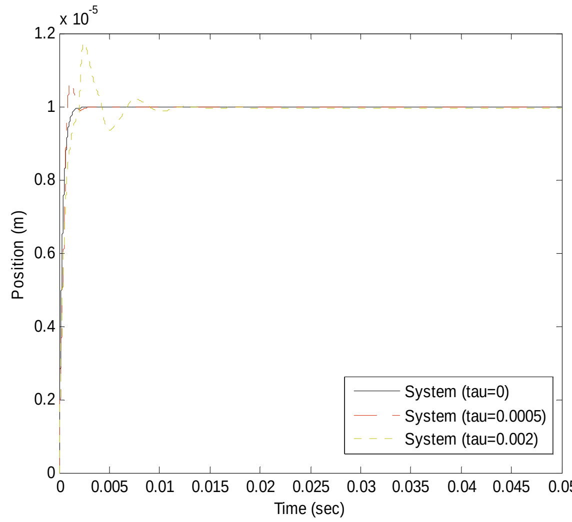

Fig. 5. Comparison of different state time-delay.

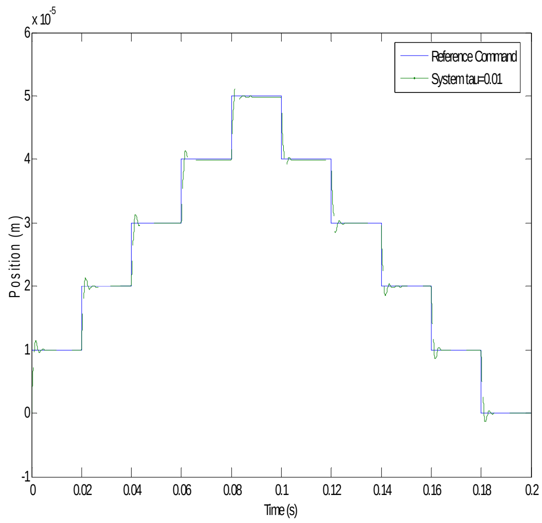

Fig. 6. Staircase response of the piezoelectric system with states time-delay.

5. Conclusions

In this paper, we have proposed stability conditions for Takagi-Sugeno fuzzy system with states time-delay via the multiple Lyapunov function. Computer simulations illustrate that the PDC controller can track the reference command when the piezoelectric actuator system owns states time-delay. The error is smaller than 100 nm which conforms to nano-positioning specifications.

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