Stability analysis of sampled-data fuzzy-model-based control systems

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Abstract

This paper presents the stability analysis of the sampled-data fuzzy-model-based control systems. To investigate the system stability, a T-S fuzzy model is employed to represent the nonlinear system. A sampled-data fuzzy controller is employed to close the feedback loop. Due to the sampling activity of the sampled-data fuzzy controller, discontinuity is introduced which complicates the stability analysis. Consequently, the existing analysis approach for continuous-time systems cannot be applied. Furthermore, the imperfectly matched grades of membership lead to the favourable technique relaxing stability analysis result not applicable. To facilitate the stability analysis of sampled-data fuzzy-model-based control systems, a membership function condition is proposed to introduce some slack matrices for nonlinearity compensation. LMI-based stability conditions are derived using the Lyapunov-based approach to guarantee the system stability. An application example is given to illustrate the merits of the proposed approach.

1. Introduction

System stability of the fuzzy-model-control system was extensively studied for many years. Flourishing stability results were obtained to guarantee the system stability based on the Lyapunov-based approach. In the study of the fuzzy-model-based control systems, the TS-fuzzy model [1]-[2] plays an important role to carry out the stability analysis. The TS-fuzzy model offers a general and systematic framework to represent the nonlinear plant to facilitate the system analysis and the controller synthesis. In [3]-[4], basic stability conditions in the form of linear matrix inequalities (LMIs) were derived to guarantee the stability of the fuzzy-model-based control systems. By sharing the same antecedent rules between the fuzzy model and fuzzy controller, relaxed stability conditions were derived. Further relaxed stability conditions can be found in [5]-[10].

Sampled-data fuzzy controller offers a beneficial feature that it can be implemented by microcontrollers or digital computers to enhance the design flexibility and lower the implementation cost. It is thus a good alternative to substitute the analog fuzzy controller [3]-[10] in various applications. However, complex dynamics of the sampling activity makes the system stability analysis difficult. Two major difficulties will be faced for investigating the sampled-data fuzzy-model-based control systems. One, the zero-order-hold (ZOH) unit will keep the control signals constant during the sampling period and have the control signals changed at the sampling instant. As a result, discontinuity will be introduced into the system making the system dynamics more complicated. In the stability investigation in [3]-[10], only the pure continuous-time or discrete-time fuzzy-model-based control systems were concerned. Consequently, when the sampled-data fuzzy control systems are considered, the stability conditions in [3]-[10] cannot be applied. Two, due to the existence of the ZOH unit, the controller design criterion, which the fuzzy model and fuzzy controller share the same antecedent, leading to relaxed stability result can not be applied. To investigate the stability of linear sampled-data control system, a descriptor approach [11] was proposed to carry out the stability analysis. By properly representing the linear sampled-data control system in a descriptor form [11], the discontinuity introduced by the sampled-data activity can be handled. In [12], the stability analysis of the output-feedback sampled-data fuzzy control systems was investigated by introducing an equivalent jump system. The jump system is a piecewise continuous-time system during the sampling period and will have a jump at the sampling instant. To investigate the system stability, the dynamics of the piecewise continuous-time system and the dynamics of the system at the sampling instant have to be studied separately. As the analysis is carried out based on the two systems during the sampling period and at the sampling instant, it can be seen that the analysis is quite tedious. The output-feedback sampled-data fuzzy control system [12] is guaranteed to be stable if there exists a periodic time-varying solution to a set of stability conditions. As the stability conditions are not in an LMI form, the solution cannot be solved by taking advantage of the effective convex programming technique.

It has been shown in [11] that the descriptor representation offers a potential approach to investigate the system stability of linear sample-data systems. However, when nonlinear sampled-data systems are concerned, it leads to a conservative stability analysis result as the system nonlinearity is difficulty to be considered. The descriptor representation in [11] was employed to investigate the stability of sampled-data neural-network-based control systems [13] by the same authors. The nonlinearity of the nonlinear can be handled using the desirable characteristics of the TS-fuzzy model.

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However, it is due to the sampling activity the favorable property of sharing the same premises between the fuzzy model and controller [3]-[10] cannot be employed to relax the system analysis result. In this paper, the system stability of the sampled-data fuzzy-model-based control systems is investigated. The descriptor representation in [11] is employed to investigate the system stability. The drawback leading to conservative stability analysis result in [13] during the stability analysis is alleviated. Based on a descriptor representation and the characteristics of membership functions, some free matrices are introduced to produce stability analysis result. Stability conditions are derived using the Lyapunov-based approach and expressed in an LMI form which can be solved effectively and numerically using convex programming techniques.

2. Fuzzy model and sampled-data fuzzy controller

A sampled-data fuzzy-model-based control system formed by a continuous-time fuzzy model and a sampled-data fuzzy controller is introduced.

A. Fuzzy Model

Let p be the number of fuzzy rules describing the nonlinear plant. The i-th rule is of the following format,

Rule i: IF $f_1(\mathbf{x}(t))$ is M_1^i AND … AND $f_\Psi(\mathbf{x}(t))$ is M_Ψ^i THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$ (1)

where M_α^i is a fuzzy term of rule i corresponding to the function $f_\alpha(\mathbf{x}(t))$, α 1, 2, ..., Ψ, i 1, 2, ..., p, Ψ is a positive integer; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$ are the known constant system and input matrices respectively; $\mathbf{x}(t) \in \Re^{n\times 1}$ is the system state vector and $\mathbf{u}(t) \in \Re^{m\times 1}$ is the input vector. The system dynamics are defined as,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))\big(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\big) \quad (2)$$

where

$$\sum_{i=1}^{p} w_i(\mathbf{x}(t)) = 1, \quad w_i(\mathbf{x}(t)) \in [0 \quad 1] \text{ for all } i \quad (3)$$

$$w_i(\mathbf{x}(t)) = \frac{\mu_{\mathrm{M}_1^i}\big(f_1(\mathbf{x}(t))\big)\times\mu_{\mathrm{M}_2^i}\big(f_2(\mathbf{x}(t))\big)\times\cdots\times\mu_{\mathrm{M}_\Psi^i}\big(f_\Psi(\mathbf{x}(t))\big)}{\sum_{k=1}^{p}\Big(\mu_{\mathrm{M}_1^k}\big(f_1(\mathbf{x}(t))\big)\times\mu_{\mathrm{M}_2^k}\big(f_2(\mathbf{x}(t))\big)\times\cdots\times\mu_{\mathrm{M}_\Psi^k}\big(f_\Psi(\mathbf{x}(t))\big)\Big)} \quad (4)$$

is a known nonlinear function of $f_\alpha(\mathbf{x}(t))$. $\mu_{\mathrm{M}_\alpha^i}\big(f_\alpha(\mathbf{x}(t))\big)$, α = 1, 2, …, Ψ, is the grade of membership corresponding to the fuzzy term M_α^i. It should be noted that the grades of membership are uncertain if the nonlinear plant is subject to parameter uncertainties.

B. Sampled-Data Fuzzy Controller

A sampled-data fuzzy controller with p fuzzy rules is designed based on the fuzzy model of the nonlinear plant. The j-th rule of the sampled-data fuzzy controller is of the following format:

Rule j: IF $g_1(\mathbf{x}(t_\gamma))$ is N_1^j AND … AND $g_\Omega(\mathbf{x}(t_\gamma))$ is N_Ω^j THEN $\mathbf{u}(t) = \mathbf{G}_j\mathbf{x}(t_\gamma)$, $t_\gamma < t \le t_{\gamma+1}$ (5)

where N_β^j is a fuzzy term of rule j corresponding to the function $g_\beta(\mathbf{x}(t_\gamma))$, β 1, 2, ..., Ω, j 1, 2, ..., p, Ω is a positive integer; $\mathbf{G}_j \in \Re^{m\times n}$ is the feedback gain of rule j to be designed. $t_\gamma = \gamma h_s$, $\gamma = 0, 1, 2, ..., \infty$, denotes a sampling time instant; $h_s = t_{\gamma+1} - t_\gamma$ denotes the constant sampling period. The inferred output of the sampled-data fuzzy controller is given by,

$$\mathbf{u}(t) = \sum_{j=1}^{p} m_j(\mathbf{x}(t_\gamma))\mathbf{G}_j\mathbf{x}(t-\tau(t)),\ t_\gamma < t \le t_{\gamma+1} \quad (6)$$

where $\tau(t) = t - t_\gamma \le h_s$ for $t_\gamma < t \le t_{\gamma+1}$,

$$\sum_{j=1}^{p} m_j(\mathbf{x}(t_\gamma)) = 1, \quad m_j(\mathbf{x}(t_\gamma)) \in [0 \quad 1] \text{ for all } j \quad (7)$$

$$m_j(\mathbf{x}(t_\gamma)) = \frac{\mu_{\mathrm{N}_1^j}(g_1(\mathbf{x}(t_\gamma)))\times\mu_{\mathrm{N}_2^j}(g_2(\mathbf{x}(t_\gamma)))\times\cdots\times\mu_{\mathrm{N}_\Omega^j}(g_\Omega(\mathbf{x}(t_\gamma)))}{\sum_{k=1}^{p}\Big(\mu_{\mathrm{N}_1^k}(g_1(\mathbf{x}(t_\gamma)))\times\mu_{\mathrm{N}_2^k}(g_2(\mathbf{x}(t_\gamma)))\times\cdots\times\mu_{\mathrm{N}_\Omega^k}(g_\Omega(\mathbf{x}(t_\gamma)))\Big)} \quad (8)$$

is a nonlinear function of $\mathbf{x}(t_\gamma)$ and $\mu_{\mathrm{N}_\beta^j}(g_\beta(\mathbf{x}(t_\gamma)))$ is the known grade of membership corresponding to the fuzzy term N_β^j. It can be seen from (6) that $\mathbf{u}(t) = \mathbf{u}(t_\gamma)$ which holds constant value for $t_\gamma < t \le t_{\gamma+1}$.

3. Stability Analysis

In this section, the system stability of the sampled-data fuzzy control system formed by nonlinear plant in the form of (2) and the sampled-data fuzzy controller of (6) will be investigated. In the following analysis, $w_i(\mathbf{x}(t))$ and $m_j(\mathbf{x}(t_\gamma))$ are denoted as w_i and m_j respectively for brevity. From (3) and (7), we have the property that $\sum_{i=1}^{p} w_i = \sum_{j=1}^{p} m_j = \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j = 1$ which is used during the stability analysis. From (6), the sampled-data fuzzy controller can be represented as the following form.

$$\mathbf{u}(t) = \sum_{j=1}^{p} m_j\mathbf{G}_j\mathbf{x}(t) - \sum_{j=1}^{p} m_j\mathbf{G}_j\int_{t-\tau(t)}^{t}\dot{\mathbf{x}}(\varphi)d\varphi \quad (9)$$

From (9), we have the following property which will be used later.

$$\sum_{j=1}^{p} m_j \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} - \sum_{j=1}^{p} m_j \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_j \end{bmatrix} \int_{t-\tau(t)}^{t} \dot{\mathbf{x}}(\varphi) d\varphi = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{10}$$

The system stability of the sampled-data fuzzy control system will be investigated based on the following Lyapunov function candidate.

$$V(t) = V_1(t) + V_2(t) \tag{11}$$

$$\text{where } V_1(t) = \mathbf{x}(t)^{\mathrm{T}} \mathbf{P}_1 \mathbf{x}(t) \tag{12}$$

$$V_2(t) = \int_{-h_s}^{0} \int_{t+\sigma}^{t} \dot{\mathbf{x}}(\varphi)^{\mathrm{T}} \mathbf{R} \dot{\mathbf{x}}(\varphi) d\varphi d\sigma \tag{13}$$

where $\mathbf{P}_1 = \mathbf{P}_1^{\mathrm{T}} \in \Re^{n \times n} > 0$ and $\mathbf{R} \in \Re^{n \times n} > 0$. It will be shown that $\dot{V}(t) \le 0$ (equality holds when x(t) = x(t_γ) = **0**) which implies the asymptotically stability of the sampled-data fuzzy-model-based control system. From (2), (6), (10) and (12), we have,

$$\dot{V}_1(t) = \mathbf{x}(t)^{\mathrm{T}} \mathbf{P}_1 \dot{\mathbf{x}}(t) + \dot{\mathbf{x}}(t)^{\mathrm{T}} \mathbf{P}_1 \mathbf{x}(t)$$

$$\begin{aligned} &= \frac{1}{\rho} \sum_{i=1}^{p} \sum_{j=1}^{p} w_i w_j \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \left(\mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} \mathbf{P} \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \\ &+ \frac{1}{\rho} \sum_{i=1}^{p} \sum_{j=1}^{p} w_i \left(\rho m_j - w_j \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \left(\mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} \mathbf{P} \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \\ &- 2 \sum_{i=1}^{p} m_i \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_i \end{bmatrix} \int_{t-\tau(t)}^{t} \dot{\mathbf{x}}(\varphi) d\varphi \end{aligned} \tag{14}$$

where $\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{P}_2 & \mathbf{P}_3 \end{bmatrix} \in \Re^{(n+m) \times (n+m)}$, $\mathbf{P}_2 \in \Re^{m \times n}$, $\mathbf{P}_3 \in \Re^{m \times m}$ and ρ > 1 is a positive scalar. With the property that $\sum_{i=1}^{p} w_i$ = $\sum_{j=1}^{p} m_j$ = $\sum_{i=1}^{p} \sum_{j=1}^{p} w_i m_j$ = 1, we have $\sum_{j=1}^{p} \left(\rho m_j - w_j \right)$ = $\sum_{i=1}^{p} \sum_{j=1}^{p} w_i \left(\rho m_j - w_j \right)$ = ρ − 1. Based on this property and from (14), we have,

$$\begin{aligned} \dot{V}_1(t) &= \frac{1}{\rho} \sum_{i=1}^{p} w_i \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \left(\mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_i & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_i & -\mathbf{I} \end{bmatrix} \mathbf{P} - (\rho - 1) \mathbf{\Lambda}_i \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \\ &+ \frac{1}{\rho} \sum_{i=1}^{p} \sum_{j=1}^{p} w_i \left(\rho m_j - w_j \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \left(\mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} \mathbf{P} + \mathbf{\Lambda}_i \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \\ &- \int_{t-\tau(t)}^{t} \sum_{i=1}^{p} m_i 2 \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_i \end{bmatrix} \dot{\mathbf{x}}(\varphi) d\varphi \end{aligned} \tag{15}$$

where $\mathbf{\Lambda}_i = \mathbf{\Lambda}_i^{\mathrm{T}} \in \Re^{(n+m) \times (n+m)}$, i = 1, 2, ..., p, is an arbitrary matrix. From [14], we have the followings.

$$-2\mathbf{a}(t)^{\mathrm{T}} \mathbf{T}_i \dot{\mathbf{x}}(\varphi) \le \begin{bmatrix} \mathbf{a}(t) \\ \dot{\mathbf{x}}(\varphi) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \hat{\mathbf{R}}_i & \mathbf{Y}_i - \mathbf{T}_i \\ \mathbf{Y}_i^{\mathrm{T}} - \mathbf{T}_i^{\mathrm{T}} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{a}(t) \\ \dot{\mathbf{x}}(\varphi) \end{bmatrix} \tag{16}$$

where $\begin{bmatrix} \hat{\mathbf{R}}_i & \mathbf{Y}_i \\ \mathbf{Y}_i^{\mathrm{T}} & \mathbf{R} \end{bmatrix} \ge 0$. Let $\mathbf{a}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}$, $\mathbf{T}_i = \mathbf{Y}_i = \mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_i \end{bmatrix}$, $\hat{\mathbf{R}}_i = \hat{\mathbf{R}}_i^{\mathrm{T}} \in \Re^{(n+m) \times (n+m)}$, i = 1, 2, ..., p. From (16), we have,

$$\begin{aligned} -\int_{t-\tau(t)}^{t} \sum_{i=1}^{p} m_i 2 \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_i \end{bmatrix} \dot{\mathbf{x}}(\varphi) d\varphi &\le h_s \sum_{i=1}^{p} m_i \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \hat{\mathbf{R}}_i \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \\ &+ \int_{t-\tau(t)}^{t} \dot{\mathbf{x}}(\varphi)^{\mathrm{T}} \mathbf{R} \dot{\mathbf{x}}(\varphi) d\varphi \end{aligned} \tag{17}$$

From (15) and (17), we have,

$$\begin{aligned} \dot{V}_1(t) &\le \frac{1}{\rho} \sum_{i=1}^{p} w_i \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \left(\mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_i & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_i & -\mathbf{I} \end{bmatrix} \mathbf{P} - (\rho - 1) \mathbf{\Lambda}_i + h_s \hat{\mathbf{R}}_i \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \\ &+ \frac{1}{\rho} \sum_{i=1}^{p} \sum_{j=1}^{p} w_i \left(\rho m_j - w_j \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \left(\mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} \mathbf{P} + \mathbf{\Lambda}_i + h_s \hat{\mathbf{R}}_j \right) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \\ &+ \int_{t-\tau(t)}^{t} \dot{\mathbf{x}}(\varphi)^{\mathrm{T}} \mathbf{R} \dot{\mathbf{x}}(\varphi) d\varphi \end{aligned} \tag{18}$$

From (2) and (13),

$$\dot{V}_2(t) = h_s \dot{\mathbf{x}}(t)^{\mathrm{T}} \mathbf{R} \dot{\mathbf{x}}(t) - \int_{t-h_s}^{t} \dot{\mathbf{x}}(\varphi)^{\mathrm{T}} \mathbf{R} \dot{\mathbf{x}}(\varphi) d\varphi$$

$$= \sum_{i=1}^{p} \sum_{j=1}^{p} w_i w_j h_s \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i^{\mathrm{T}} \\ \mathbf{B}_i^{\mathrm{T}} \end{bmatrix} \mathbf{R} \begin{bmatrix} \mathbf{A}_j^{\mathrm{T}} \\ \mathbf{B}_j^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} - \int_{t-h_s}^{t} \dot{\mathbf{x}}(\varphi)^{\mathrm{T}} \mathbf{R} \dot{\mathbf{x}}(\varphi) d\varphi \tag{19}$$

From (11), (18), (19) and with the fact that $\tau(t) \le h_s$ leading to $\int_{t-\tau(t)}^{t} \dot{\mathbf{x}}(\varphi)^{\mathrm{T}} \mathbf{R} \dot{\mathbf{x}}(\varphi) d\varphi \le \int_{t-h_s}^{t} \dot{\mathbf{x}}(\varphi)^{\mathrm{T}} \mathbf{R} \dot{\mathbf{x}}(\varphi) d\varphi$, we have,

$$\dot{V}(t) \le \frac{1}{\rho} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \mathbf{Q} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} + \frac{1}{\rho} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \overline{\mathbf{Q}} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \tag{20}$$

where

$$\mathbf{Q} = \sum_{i=1}^{p} \sum_{j=1}^{p} w_i w_j \left(\begin{aligned} &\mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_i & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_i & -\mathbf{I} \end{bmatrix} \mathbf{P} - (\rho - 1) \mathbf{\Lambda}_i \\ &+ h_s \hat{\mathbf{R}}_i + \rho h_s \begin{bmatrix} \mathbf{A}_i^{\mathrm{T}} \\ \mathbf{B}_i^{\mathrm{T}} \end{bmatrix} \mathbf{R} \begin{bmatrix} \mathbf{A}_j^{\mathrm{T}} \\ \mathbf{B}_j^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \end{aligned} \right) \tag{21}$$

$$\overline{\mathbf{Q}} = \sum_{i=1}^{p} \sum_{j=1}^{p} w_i \left(\rho m_j - w_j \right) \left(\mathbf{P}^{\mathrm{T}} \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{G}_j & -\mathbf{I} \end{bmatrix} \mathbf{P} + \mathbf{\Lambda}_i + h_s \hat{\mathbf{R}}_j \right) \tag{22}$$

Let $\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} \\ \mathbf{X}_2 & \mathbf{X}_3 \end{bmatrix} = \mathbf{P}^{-1} \in \Re^{(n+m) \times (n+m)}$, $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}} \in \Re^{n \times n}$, $\mathbf{X}_2 \in \Re^{m \times n}$, $\mathbf{X}_3 \in \Re^{m \times m}$. It is a assumed that there exists a scalar value of ρ such that $\rho m_j - w_j > 0$, j = 1, 2, ..., p. From (22), we have,

$$\dot{V}(t) \le \frac{1}{\rho} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \mathbf{X}^{-1^{\mathrm{T}}} \mathbf{X}^{\mathrm{T}} \mathbf{Q} \mathbf{X} \mathbf{X}^{-1} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} + \frac{1}{\rho} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix}^{\mathrm{T}} \mathbf{X}^{-1^{\mathrm{T}}} \mathbf{X}^{\mathrm{T}} \overline{\mathbf{Q}} \mathbf{X} \mathbf{X}^{-1} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \tag{23}$$

From (23), it can be seen that $\dot{V}(t) < 0$ which implies the asymptotically stability of the sampled-data fuzzy-model-based control system when $\mathbf{X}^{\mathrm{T}} \mathbf{Q} \mathbf{X} < 0$ and $\mathbf{X}^{\mathrm{T}} \overline{\mathbf{Q}} \mathbf{X} < 0$. Let $\mathbf{M} = \mathbf{R}^{-1}$, $\mathbf{M}_i = \mathbf{M}_i^{\mathrm{T}} = \mathbf{X}^{\mathrm{T}} \hat{\mathbf{R}}_i \mathbf{X} = \begin{bmatrix} \mathbf{M}_i^{(11)} & \mathbf{M}_i^{(21)^{\mathrm{T}}} \\ \mathbf{M}_i^{(21)} & \mathbf{M}_i^{(22)} \end{bmatrix} \in \Re^{(n+m) \times (n+m)}$, $\mathbf{M}_i^{(11)} = \mathbf{M}_i^{(11)^{\mathrm{T}}} \in \Re^{n \times n}$, $\mathbf{M}_i^{(21)} \in \Re^{m \times n}$, $\mathbf{M}_i^{(22)} = \mathbf{M}_i^{(22)^{\mathrm{T}}} \in \Re^{m \times m}$, $\mathbf{V}_i = \mathbf{V}_i^{\mathrm{T}} = \mathbf{X}^{\mathrm{T}} \mathbf{\Lambda}_i \mathbf{X} = \begin{bmatrix} \mathbf{V}_i^{(11)} & \mathbf{V}_i^{(21)^{\mathrm{T}}} \\ \mathbf{V}_i^{(21)} & \mathbf{V}_i^{(22)} \end{bmatrix} \in \Re^{(n+m) \times (n+m)}$, $\mathbf{V}_i^{(11)} = \mathbf{V}_i^{(11)^{\mathrm{T}}} \in \Re^{n \times n}$, $\mathbf{V}_i^{(21)} \in \Re^{m \times n}$,

$\mathbf{V}_i^{(22)} = \mathbf{V}_i^{(22)\mathrm{T}} \in \Re^{m\times m}$, $\mathbf{G}_i = \mathbf{N}_i\mathbf{X}_1^{-1}$ and $\mathbf{N}_i \in \Re^{m\times n}$ for i = 1, 2, ..., p. From (21), we have,

$$\mathbf{X}^{\mathrm{T}}\mathbf{Q}\mathbf{X} = \sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j \left(\begin{bmatrix} \mathbf{\Xi}_i^{(11)} & \mathbf{\Xi}_i^{(21)\mathrm{T}} \\ \mathbf{\Xi}_i^{(21)} & \mathbf{\Xi}_i^{(22)} \end{bmatrix} + \rho h_s \begin{bmatrix} \mathbf{X}_1\mathbf{A}_i^{\mathrm{T}} + \mathbf{X}_2^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} \\ \mathbf{X}_3^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} \end{bmatrix} \mathbf{R} \begin{bmatrix} \mathbf{X}_1\mathbf{A}_j^{\mathrm{T}} + \mathbf{X}_2^{\mathrm{T}}\mathbf{B}_j^{\mathrm{T}} \\ \mathbf{X}_3^{\mathrm{T}}\mathbf{B}_j^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \right) \quad (24)$$

where

$$\mathbf{\Xi}_i^{(11)} = \mathbf{A}_i\mathbf{X}_1 + \mathbf{X}_1\mathbf{A}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{X}_2 + \mathbf{X}_2^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} - (\rho - 1)\mathbf{V}_i^{(11)} + h_s\mathbf{M}_i^{(11)} \quad (25)$$

$$\mathbf{\Xi}_i^{(21)} = \mathbf{X}_3^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} + \mathbf{N}_i - \mathbf{X}_2 - (\rho - 1)\mathbf{V}_i^{(21)} + h_s\mathbf{M}_i^{(21)} \quad (26)$$

$$\mathbf{\Xi}_i^{(22)} = -\mathbf{X}_3 - \mathbf{X}_3^{\mathrm{T}} - (\rho - 1)\mathbf{V}_i^{(22)} + h_s\mathbf{M}_i^{(22)} \quad (27)$$

By Schur complement, $\mathbf{X}^{\mathrm{T}}\mathbf{Q}\mathbf{X} < 0$ is equivalent to the following inequality.

$$\sum_{i=1}^{p} w_i \mathbf{W}_i < 0 \quad (28)$$

where

$$\mathbf{W}_i = \begin{bmatrix} \mathbf{\Xi}_i^{(11)} & \mathbf{\Xi}_i^{(21)\mathrm{T}} & \rho h_s(\mathbf{A}_i\mathbf{X}_1 + \mathbf{X}_2\mathbf{B}_i)^{\mathrm{T}} \\ \mathbf{\Xi}_i^{(21)} & \mathbf{\Xi}_i^{(22)} & \rho h_s(\mathbf{B}_i\mathbf{X}_3)^{\mathrm{T}} \\ \rho h_s(\mathbf{A}_i\mathbf{X}_1 + \mathbf{X}_2\mathbf{B}_i) & \rho h_s(\mathbf{B}_i\mathbf{X}_3) & -\rho h_s\mathbf{M} \end{bmatrix} \quad (29)$$

It can be seen from (28) that $\sum_{i=1}^{p} w_i \mathbf{W}_i < 0$ if $\mathbf{W}_i < 0$, i = 1, 2, ..., p. Similarly, from (22) and (23), we have,

$$\mathbf{X}^{\mathrm{T}}\overline{\mathbf{Q}}\mathbf{X} = \sum_{i=1}^{p}\sum_{j=1}^{p} w_i(\rho m_j - w_j)\overline{\mathbf{W}}_{ij} \quad (30)$$

where

$$\overline{\mathbf{W}}_{ij} = \begin{bmatrix} \mathbf{\Theta}_{ij}^{(11)} & \mathbf{\Theta}_{ij}^{(21)\mathrm{T}} \\ \mathbf{\Theta}_{ij}^{(21)} & \mathbf{\Theta}_{ij}^{(22)} \end{bmatrix} \quad (31)$$

$$\mathbf{\Theta}_{ij}^{(11)} = \mathbf{A}_i\mathbf{X}_1 + \mathbf{X}_1\mathbf{A}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{X}_2 + \mathbf{X}_2^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} + \mathbf{V}_i^{(11)} + h_s\mathbf{M}_j^{(11)} \quad (32)$$

$$\mathbf{\Theta}_{ij}^{(21)} = \mathbf{X}_3^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} + \mathbf{N}_j - \mathbf{X}_2 + \mathbf{V}_i^{(21)} + h_s\mathbf{M}_j^{(21)} \quad (33)$$

$$\mathbf{\Theta}_{ij}^{(22)} = -\mathbf{X}_3 - \mathbf{X}_3^{\mathrm{T}} + \mathbf{V}_i^{(22)} + h_s\mathbf{M}_j^{(22)} \quad (34)$$

From (30), it can be seen that $\mathbf{X}^{\mathrm{T}}\overline{\mathbf{Q}}\mathbf{X} < 0$ if $\overline{\mathbf{W}}_{ij} < 0$, i, j = 1, 2, ..., p and $\rho m_j - w_j \geq 0$, j = 1, 2, ..., p. Furthermore, the property of (17) holds when $\begin{bmatrix} \hat{\mathbf{R}}_i & \mathbf{Y}_i \\ \mathbf{Y}_i^{\mathrm{T}} & \mathbf{R} \end{bmatrix} \geq 0$ which is equivalent to the followings.

$$\begin{bmatrix} \mathbf{X}^{\mathrm{T}}\mathbf{R}_i\mathbf{X} & \mathbf{X}^{\mathrm{T}}\mathbf{Y}_i\mathbf{X}_1 \\ \mathbf{X}_1\mathbf{Y}_i^{\mathrm{T}}\mathbf{X} & \mathbf{X}_1\mathbf{M}^{-1}\mathbf{X}_1 \end{bmatrix} \geq 0, \; i = 1, 2, ..., p \quad (35)$$

It should be noted that the inequalities of (35) are not LMI conditions due to the existence of the term $\mathbf{X}_1\mathbf{M}^{-1}\mathbf{X}_1$. With the property that $\mathbf{M} = \mathbf{M}^{\mathrm{T}} > 0$, we consider the following inequality,

$$(\mathbf{X}_1 - \zeta\mathbf{M})^{\mathrm{T}}\mathbf{M}^{-1}(\mathbf{X}_1 - \zeta\mathbf{M}) = \mathbf{X}_1^{\mathrm{T}}\mathbf{M}^{-1}\mathbf{X}_1 - \zeta\mathbf{X}_1^{\mathrm{T}} - \zeta\mathbf{X}_1 + \zeta^2\mathbf{M} > 0$$

$$\Rightarrow \mathbf{X}_1\mathbf{M}^{-1}\mathbf{X}_1 \geq 2\zeta\mathbf{X}_1 - \zeta^2\mathbf{M} \quad (36)$$

where ζ is a non-zero positive scalar. From (35) and (36), the holding of the following LMIs imply the holding of (35)

$$\begin{bmatrix} \mathbf{M}_i^{(11)} & \mathbf{M}_i^{(21)\mathrm{T}} & \mathbf{0} \\ \mathbf{M}_i^{(21)} & \mathbf{M}_i^{(22)} & \mathbf{N}_i \\ \mathbf{0} & \mathbf{N}_i^{\mathrm{T}} & 2\zeta\mathbf{X}_1 - \zeta^2\mathbf{M} \end{bmatrix} \geq 0, \; i = 1, 2, ..., p \quad (37)$$

The stability conditions of the sampled-data fuzzy control system are summarized in the following Theorem.

Theorem 1: *The sampled-data fuzzy-model-based control system formed by the nonlinear plant in the form of* (2) *and the sampled-data fuzzy controller of* (6) *is asymptotically stable if there exist constant non-zero positive scalars,* h_s, $\rho > 1$, *and* ζ *such that* $\rho m_j(\mathbf{x}(t_\gamma)) - w_j(\mathbf{x}(t)) > 0$, *j = 1, 2, ..., p, and constant matrices,* $\mathbf{X}_1 \in \Re^{n\times n}$, $\mathbf{X}_2 \in \Re^{m\times n}$, $\mathbf{X}_3 \in \Re^{m\times m}$, $\mathbf{M} \in \Re^{n\times n}$, $\mathbf{M}_i^{(11)} = \mathbf{M}_i^{(11)\mathrm{T}} \in \Re^{n\times n}$, $\mathbf{M}_i^{(21)} \in \Re^{m\times n}$, $\mathbf{M}_i^{(22)} = \mathbf{M}_i^{(22)\mathrm{T}} \in \Re^{m\times m}$, $\mathbf{N}_j \in \Re^{m\times n}$, $\mathbf{V}_i^{(11)} = \mathbf{V}_i^{(11)\mathrm{T}} \in \Re^{n\times n}$, $\mathbf{V}_i^{(21)} \in \Re^{m\times n}$ *and* $\mathbf{V}_i^{(22)} = \mathbf{V}_i^{(22)\mathrm{T}} \in \Re^{m\times m}$ *such that the following LMIs hold:* $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}} > 0$ *;* $\mathbf{X}_3 = \mathbf{X}_3^{\mathrm{T}} > 0$ *;* $\mathbf{M} = \mathbf{M}^{\mathrm{T}} > 0$ *;* (29), (31) *and* (37)*. The feedback gains are defined as* $\mathbf{G}_i = \mathbf{N}_i\mathbf{X}_1^{-1}$, *i = 1, 2, ..., p.*

Remark 1: The above analysis is valid if **X** is invertible. Referring to the Theorem 1, if there exists a solution to the stability conditions in Theorem 1, it implies that $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}} > 0$ and $-\mathbf{X}_3 - \mathbf{X}_3^{\mathrm{T}} < 0$. These are the sufficient conditions for $\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} \\ \mathbf{X}_2 & \mathbf{X}_3 \end{bmatrix}$ to be a non-singular matrix to guarantee that **X** is invertible.

4. Application Example

An application example on stabilizing a cart-pole inverted pendulum [15] using the proposed fuzzy controller will be given in this section.

Step I) The dynamical equations of the inverted pendulum on the cart [15] is given by,

$$\dot{x}_1(t) = x_2(t) \quad (38)$$

$$\dot{x}_2(t) = \frac{\begin{pmatrix} -F_1(M+m)x_2(t) - m^2l^2x_2(t)^2\sin x_1(t)\cos x_1(t) + F_0mlx_4(t)\cos x_1(t) \\ +(M+m)mgl\sin x_1(t) - ml\cos x_1(t)u(t) \end{pmatrix}}{(M+m)(J+ml^2) - m^2l^2(\cos x_1(t))^2} \quad (39)$$

$$\dot{x}_3(t) = x_4(t) \quad (40)$$

$$\dot{x}_4(t) = \frac{\begin{pmatrix} F_1mlx_2(t)\cos x_1(t) + (J+ml^2)mlx_2(t)^2\sin x_1(t) - F_0(J+ml^2)x_4(t) \\ -m^2gl^2\sin x_1(t)\cos x_1(t) + (J+ml^2)u(t) \end{pmatrix}}{(M+m)(J+ml^2) - m^2l^2(\cos x_1(t))^2} \quad (41)$$

where $x_1(t)$ and $x_2(t)$ denote the angular displacement (rad) and the angular velocity (rad/s) of the pendulum from vertical respectively, $x_3(t)$ and $x_4(t)$ denote the displacement (m) and the velocity (m/s) of the cart respectively, $g = 9.8$ m/s^2 is the acceleration due to gravity, $m = 0.22$ kg is the mass of the pendulum, $M = 1.3282$ kg is the mass of the cart, $l = 0.304$ m is the length from the center of mass of the pendulum to the shaft axis, $J = ml^2/3$ kgm^2 is the moment of inertia of the pendulum around the center of mass, $F_0 = 22.915$ N/m/s and $F_1 = 0.007056$ N/rad/s are the friction factors of the cart and the pendulum respectively, and $u(t)$ is the force (N) applied to the cart. The objective is to design the proposed fuzzy controller to control the nonlinear plant such that $x_1(t) = x_3(t) = 0$ at steady state. The nonlinear plant can be represented by a fuzzy plant model with two fuzzy rules [15]. The i-th rule is given by,

Rule i: IF $x_1(t)$ is M_1^i

THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t)$ for $i = 1, 2$ (42)

The system dynamics are described by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{2} w_i\left(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t)\right) \tag{43}$$

where $\mathbf{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t)\end{bmatrix}^{\mathrm{T}}$;

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (M+m)mgl/a_1 & -F_1(M+m)/a_1 & 0 & F_0 ml/a_1 \\ 0 & 0 & 1 & 0 \\ -m^2gl^2/a_1 & F_1 ml/a_1 & 0 & -F_0(J+ml^2)/a_1 \end{bmatrix};$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{3\sqrt{3}}{2\pi}(M+m)mgl/a_2 & -F_1(M+m)/a_2 & 0 & F_0 ml\cos(\pi/3)/a_2 \\ 0 & 0 & 1 & 0 \\ -\frac{3\sqrt{3}}{2\pi}m^2gl^2\cos(\pi/3)/a_2 & F_1 ml\cos(\pi/3)/a_2 & 0 & -F_0(J+ml^2)/a_1 \end{bmatrix};$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ -ml/a_1 \\ 0 \\ (J+ml^2)/a_1 \end{bmatrix}; \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ -ml\cos(\pi/3)/a_2 \\ 0 \\ (J+ml^2)/a_2 \end{bmatrix};$$

$a_1 = (M+m)(J+ml^2) - m^2l^2$, $a_2 = (M+m)(J+ml^2) - m^2l^2\cos(\pi/3)^2$. The membership functions are defined as $w_1(x_1(t)) = \mu_{\mathrm{M}_1^1}(x_1(t)) = \left(1 - \frac{1}{1+e^{-7(x_1(t)-\pi/6)}}\right)\frac{1}{1+e^{-7(x_1(t)+\pi/6)}}$ and $w_2(x_1(t)) = \mu_{\mathrm{M}_1^2}(x_1(t)) = 1 - \mu_{\mathrm{M}_1^1}(x_1(t))$.

Step II) A two-rule sampled-data fuzzy controller will be employed for the nonlinear plant. The rules are of the following format,

Rule j: IF $x_1(t_\gamma)$ is N_1^j

THEN $u(t) = \mathbf{G}_j\mathbf{x}(t_\gamma)$, $t_\gamma < t \le t_{\gamma+1}$ (44)

The sampled-data fuzzy controller is defined as

$$u(t) = \sum_{j=1}^{2} m_j(x_1(t_\gamma))\mathbf{G}_j\mathbf{x}(t_\gamma) \tag{45}$$

where the membership functions are defined as $m_1(x_1(t_\gamma)) = \mu_{\mathrm{N}_1^1}(x_1(t_\gamma)) = 0.8e^{-\frac{x_1(t_\gamma)^2}{2\times1.8^2}}$ and $m_2(x_1(t_\gamma)) = \mu_{\mathrm{N}_1^2}(x_1(t_\gamma)) = 1 - \mu_{\mathrm{N}_1^1}(x_1(t_\gamma))$.

Step III) Based on Theorem 1, with $h_s = 0.01$s and $\zeta = 2$, we obtain $\mathbf{G}_1$ = [388.9898 33.9790 0.4527 32.0910] and $\mathbf{G}_2$ = [534.2409 41.6278 0.5549 34.3607]. In this example, it is assumed that $\dot{x}_1(t) = x_2(t) \in [-15 \quad 15]$. With this information and considering $t_\gamma \le t \le t_\gamma + h_s$, we have $x_1(t) = x_1(t_\gamma) + \int_{t_\gamma}^{t_\gamma+h_s} x_2(t)dt$ which offer the lower and upper bounds as $x_1(t_\gamma) - 15\int_{t_\gamma}^{t_\gamma+h_s} dt = x_1(t_\gamma) - 15h_s$ and $x_1(t) = x_1(t_\gamma) + 15\int_{t_\gamma}^{t_\gamma+h_s} dt = x_1(t_\gamma) + 15h_s$ respectively. Consequently, for any sampling instant t_γ, the value of $x_1(t)$ on or before the next sampling instant is in the range of $x_1(t_\gamma) - 15h_s \le x_1(t) \le x_1(t_\gamma) + 15h_s$ for $t_\gamma \le t \le t_\gamma + h_s$. It can be shown that this information ensures the satisfaction of the conditions of $\rho m_j(x_1(t_\gamma)) - w_j(x_1(t)) > 0$, $j = 1, 2$, with $\rho = 1.55$.

The sampled-data fuzzy controller of (45) will be employed to control the nonlinear plant of (38) to (41). Fig. 1 shows the system state responses of the sampled-data fuzzy control system under the initial system state conditions of $\mathbf{x}(t) = \begin{bmatrix} \frac{\pi}{3} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ and $\mathbf{x}(t) = \begin{bmatrix} \frac{\pi}{6} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ for $t \in [-h_s \quad 0]$ respectively. It can be seen that the nonlinear plant can be stabilized by the sampled-data fuzzy controller. The control signals of the sampled-data fuzzy controller are shown in Fig. 2. Referring to this figure, it can be seen that control signal is a stepwise signal which value is held constant during the sampling period.

5. Conclusion

The system stability of the sampled-data fuzzy-model-based control systems has been investigated. A sampled-data fuzzy controller has been proposed to control the nonlinear plant. Based on the Lyapunov-based approach, LMI-based stability conditions have been derived to guarantee the system stability of the sampled-data fuzzy-model-based control systems. An application example has been given to illustrate the effectiveness of the proposed approach.

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References

[1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Sys., Man., Cybern.*, vol. smc-15 no. 1, pp. 116-132, Jan. 1985.

[2] M. Sugeno and G.T. Kang, "Structure identification of fuzzy model," *Fuzzy sets and systems*, vol. 28, pp. 15-33, 1988.

[3] C.L. Chen, P.C. Chen, C.K. Chen, "Analysis and design of fuzzy control system," *Fuzzy Sets and Systems*, vol. 57, no 2, 26, pp. 125-140, Jul. 1993.

[4] H.O. Wang, K. Tanaka, and M.F. Griffin, "An approach to fuzzy control of nonlinear systems: stability and the design issues," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 14-23, Feb., 1996.

[5] K. Tanaka, T. Ikeda and H.O. Wang, "Fuzzy regulator and fuzzy observer: Relaxed stability conditions and LMI-based designs," *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 2, pp. 250-265, 1998.

[6] W.J. Wang, S.F. Yan and C.H. Chiu, "Flexible stability criteria for a linguistic fuzzy dynamic system," *Fuzzy Sets and Systems*, vol. 105, no. 1, pp. 63-80, Jul. 1999.

[7] E. Kim, "New approaches to relaxed quadratic stability conditions of fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 5, pp. 523-534, 2000.

[8] Xiaodong Liu and Qingling Zhang, "New approaches to H_∞ controller designs based on fuzzy observers for T-S fuzzy systems via LMI," *Automatica*, vol. 39, no. 9, pp. 1571-1582, Sep. 2003.

[9] M.C.M. Teixeira, E. Assunção and R.G. Avellar, "On relaxed LMI-based designs for fuzzy regulators and fuzzy observers," *IEEE Trans. on Fuzzy Systems*, vol. 11, no. 5, pp. 613-623, Oct. 2003.

[10] C.H. Fang, Y.S. Liu, S.W. Kau, L. Hong and C.H. Lee, "A new LMI-based approach to relaxed quadratic stabilization of T-S fuzzy control systems," *IEEE Trans. on Fuzzy Systems*, vol. 14, no. 3, pp. 386-397, June. 2006.

[11] E. Fridman, A. Seuret and J.P. Richard, "Robust sampled-data stabilization of linear systems: an input delay approach," *Automatica*, vol. 40, pp.1441-1446, 2004.

[12] H. Katayama and A. Ichikawa, "H_∞ control for sampled-data nonlinear systems described by Takagi-Sugeno fuzzy systems," *Fuzzy Sets and Systems*, vol. 148, no. 3, pp. 431-452, Dec. 2004.

[13] H.K. Lam and F.H.F. Leung, "Design and stabilization of sampled-data neural-network-based control systems," *IEEE Trans. Syst., Man and Cybern, Part B: Cybernetics*, vol. 36, no. 5, pp. 995-1005, Oct. 2006.

[14] Y.S. Moon, P. Park, W.H. Kwon and Y.S. Lee, "Delay-dependent robust stabilization of uncertain state-delayed systems," *Int. Journal of Control*, vol. 74, no. 14, pp.1447-1455, 2001.

[15] X.J. Ma and Z.Q. Sun, "Analysis and design of fuzzy reduced-dimensional observer and fuzzy functional observer," *Fuzzy Sets and Systems*, vol. 120, pp. 35-63, 2001.

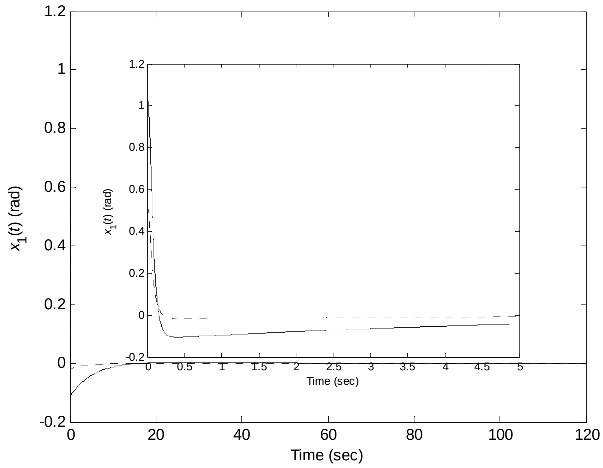

Fig. 1(a). $x_1(t)$.

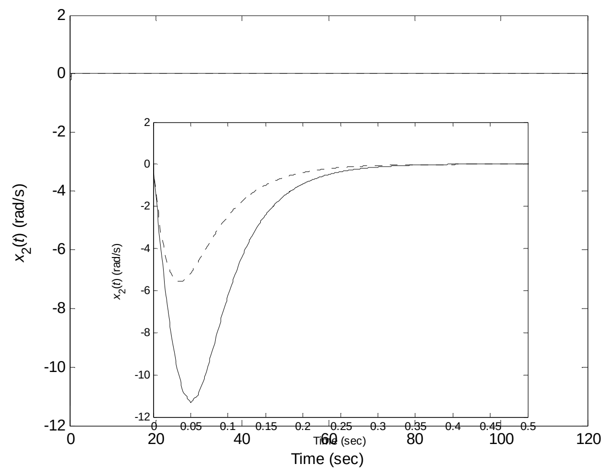

Fig. 1(b). $x_2(t)$.

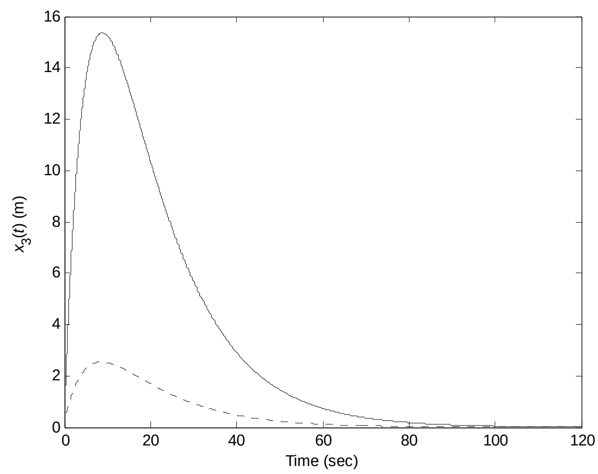

Fig. 1(c). $x_3(t)$ for $0 \le t \le 30$s.

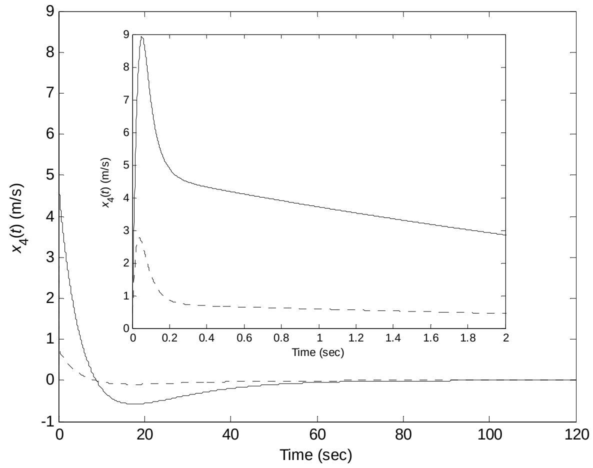

Fig. 1(d). $x_4(t)$.

Fig. 1. System state responses of the sampled-data fuzzy-model-based control system under different initial system state conditions.

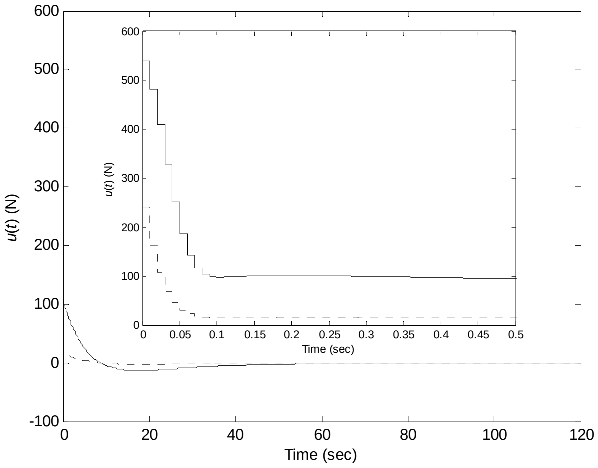

Fig. 2. Control signals of the sampled-data fuzzy controller for the nonlinear plant with different initial system state conditions.