On a product of fuzzy H_v-submodules

B. Davvaz and V. Leoreanu-Fotea

Abstract

This paper deals with the hyperstructures called H_v-modules. The main tools concerning the class of H_v-modules with the ordinary modules are the fundamental relations. The fundamental relations in H_v -modules and fuzzy H_v-submodules are studied in this paper. In particular, the product structure of fuzzy sets on H_v-modules is studied.

Keywords: hyperstructure, H_v-module, fundamental relation, fundamental module, fuzzy set, fuzzy direct product.

1. Introduction and preliminaries

The concept of a hyperstructure was introduced in 1934 by Marty [9] at the 8th Congress of Scandinavian Mathematicians. Hyperstructures have many applications in several branches of both pure and applied sciences. A short review of the theory of hyperstructures appears in [2] and a recent book [1] contains a wealth of applications. This paper deals with the hyperstructures called H_v-structures. The theory of H_v-structures introduced and studied by Vougiouklis [14,15] and Davvaz [3] surveyed the theory of Hv-structures, also see [16-18]. The concept of H_v-structures not only constitutes a generalization of the well-known hyperstructures, but it also comprises a very interesting and deep algebraic theory. Actually, some axioms concerning the classical algebraic hyperstructures (hypergroups, hyperrings, hypermodules and so on) are replaced by the corresponding weak ones (weak associativity, weak distributivity and so on). The main tools concerning the class of H_v-structures with the classical algebraic structures are the fundamental relations. The fundamental relations, especially in H_v-modules are studied in this paper.

A hyperstructure is a non-empty set H together with a function $\circ : H \times H \rightarrow \mathcal{P}^*(H)$ called a hyperoperation, where $\mathcal{P}^*(H)$ is the set of all non-empty subsets of H. A hyperstructure $(H,\circ)$ is called an H_v-group [15] if the following axioms hold:

(i) $(x \circ y) \circ z \cap x \circ (y \circ z) \neq \emptyset$ for all $x, y, z \in H$,

(ii) $a \circ H = H \circ a = H$ for all $a \in H$.

(ii) is called the reproduction axiom. If $(H,\circ)$ satisfies only the first axiom, then it is called an H_v-semigroup. In the above definition, if $x \in H$ and A, B are non-empty subsets of H then

$$A \circ B = \bigcup_{\substack{a\in A\\ b\in B}} a \circ b, \quad x \circ B = \{x\} \circ B, \quad A \circ x = A \circ \{x\}.$$

An H_v-ring [15] is a multi-valued system $(R,+,\cdot)$ which satisfies the ring-like axioms in the following way:

(i) $(R,+,\cdot)$ is an H_v-group,

(ii) $(R,\cdot)$ is an H_v-semigroup,

(iii) $(\cdot)$ is weak distributive with respect to $(+)$,
i.e., for all $x, y, z \in R$ we have
$x \cdot (y + z) \cap ((x \cdot y) + (x \cdot z)) \neq \emptyset$ and
$(x + y) \cdot z \cap ((x \cdot z) + (y \cdot z)) \neq \emptyset$.

A non-empty set M is an H_v-module over an H_v-ring R [15] if $(M, +)$ is a weak commutative H_v-group and there exists the map $\cdot : R \times M \rightarrow \mathcal{P}^*(M)$ by $(r, x) \rightarrow r \cdot x$ such that for all $a, b \in R$ and $x, y \in M$, we have $a\cdot(x+y)\cap(a\cdot x+a\cdot y) \neq \emptyset$, $(a+b)\cdot x\cap(a\cdot x+b\cdot x) \neq \emptyset$ and $(ab) \cdot x \cap a \cdot (b \cdot x) \neq \emptyset$.

Let M be an H_v-module over an H_v-ring R. A non-empty subset N of M is called an H_v-submodule if the following conditions hold: i) $(N, +)$ is an H_v-subgroup of $(M, +)$, ii) $R\cdot N\subseteq\_N$.

2. Fuzzy H_v-submodules andfuzzy fundamental submodules

The concept of a fuzzy subset of a non-empty set was introduced by Zadeh in 1965 [19]. Let X be a non-empty set. A mapping $A : X \rightarrow [0; 1]$ is called a fuzzy subset of X. Rosenfeld [13] applied the concept of fuzzy sets to the theory of groups and defined the concept of fuzzy subgroups of a group. Since then, many papers concerning various fuzzy algebraic structures have appeared in the literature, for example, Osmer [11] and Ray [12] investigated this concept. The concept of fuzzy modules was introduced by Negoita and Ralescu in [10]. Since then, several authors have studied fuzzy modules.

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Definition 2.1. Let M be a module over a ring R and A be a fuzzy subset of M. Then A is called a fuzzy submodule if it satisfies:
(i) $A(0) = 1$,
(ii) $\min\{A(x), A(y)\} \le A(x-y)$ for all $x, y \in M$,
(iii) $A(x) \le A(rx)$ for all $x \in M$, $r \in R$.

In [5], Davvaz applied the concept of a fuzzy set to the algebraic hyperstructures, in particular in [4] is defined the concept of a fuzzy H_v-module of an H_v-module, which is a generalization of the concept of fuzzy submodule. Further investigations are contained in [6-8,20]. In [8], Davvaz and et. al. applied the concept of an intuitionistic fuzzy set to H_v-modules and defined the notion of an intuitionistic fuzzy H_v-submodule of an H_v-module. In [7], Davvaz and Corsini introduced a new type of fuzzy submodule of an H_v-module called $(\in, \in \vee q)$-fuzzy H_v-submodules. Also they introduced the concept of a fuzzy H_v-submodule with thresholds. Now, we recall the following definition from [4].

Definition 2.2. Let M be an H_v-module over an H_v-ring R and let A be a fuzzy subset of M. Then A is said to be a fuzzy H_v-submodule of M if the following axioms hold:
(i) $\min\{A(x), A(y)\} \le \inf_{\alpha \in x+y} \{A(\alpha)\}$ for all $x, y \in M$,
(ii) for all $x, a \in M$, there exists $y \in M$, such that $x \in a+y$ and $\min\{A(a), A(x)\} \le A(y)$,
(iii) for all $x, a \in M$ there exists $z \in M$ such that $x \in z+a$ and $\min\{A(a), A(x)\} \le A(z)$,
(iv) $A(x) \le \inf_{z \in r\cdot x} \{A(z)\}$ for all $r \in R, x \in M$.
(ii) is called the left fuzzy reproduction axiom and (iii) is called the right fuzzy reproduction axiom.

Let A be a fuzzy subset of a non-empty set X and let $t \in [0; 1]$. The set $A_t = \{x \in X \mid A(x) \ge t\}$ is called a level subset of A. For a fuzzy subset A of an H_v-module M, the following statements are equivalent:
(i) A is a fuzzy H_v-submodule of M.
(ii) A_t $(t \in \operatorname{Im} A)$ is an H_v-submodule of M [4].

Let A and B be fuzzy subsets of a nonempty set X. The Cartesian cross-product $A \times B$ is usually defined by: $(A \times B)(x, y) = \min\{A(x), B(y)\}$ for all $x, y \in X$. Let M_1, M_2 be two H_v-modules over an H_v-ring R and let A, B be fuzzy H_v-submodules of M_1, M_2 respectively. Then $A \times B$ is a fuzzy H_v-submodule of $M_1 \times M_2$ [15]. If A, B are two fuzzy H_v-submodules of an H_v-module M, then $(A \times B)_t = A_t \times B_t$.

Consider the H_v-module M over an H_v-ring R. The relation γ^* is the smallest equivalence relation on R such that the quotient R / γ^* is a ring. γ^* is called the fundamental equivalence relation on R and R / γ^* is called the fundamental ring, see [15]. The fundamental relation ε^* on M over R is the smallest equivalence relation such that M / ε^* is a module over the ring R / γ^*. According to [16] $\mathcal{U}$ denotes the set of all expressions consisting either finite hyperoperations on R and M or the external hyperoperation applied on finite sets of R and M. Then a relation ϵ can be defined on M as follows: $x \varepsilon y$ if and only if $\{x, y\} \subseteq u$ for some $u \in \mathcal{U}$. The relation ε^* is the transitive closure of the relation ε, see [15]. Suppose that $\gamma^*(r)$ is the equivalence class containing $r \in R$ and $\varepsilon^*(x)$ is the equivalence class containing $x \in M$. The sum $\oplus$ and the external product $\odot$ on M / ε^* using the γ^* classes in R, are defined as follows:

$$\varepsilon^*(x) \oplus \varepsilon^*(y) = \varepsilon^*(c) \text{ for all } c \in \varepsilon^*(x) + \varepsilon^*(y)$$

$$\gamma^*(r) \odot \varepsilon^*(x) = \varepsilon^*(d) \text{ for all } d \in \gamma^*(r) \cdot \varepsilon^*(x).$$

The kernel of the canonical map $\varphi : M \rightarrow M / \varepsilon^*$ is called the core of M and is denoted by ω_M. Here we also denote by ω_M the unit element of the group $(M / \varepsilon^*, \oplus)$. Let M_1, M_2 be two H_v-modules over an H_v-ring R and let ε_1^*, ε_2^* and ε^* be the fundamental relations on M_1, M_2 and $M_1 \times M_2$ respectively, then $(x_1, x_2)\ \varepsilon^*\ (y_1, y_2)$ if and only if $x_1\ \varepsilon_1^*\ y_1$ and $x_2\ \varepsilon_2^*\ y_2$ for all $(x_1, x_2), (y_1, y_2) \in M_1 \times M_2$ [15,16] and it is easy to see that

$$(M_1 \times M_2) / \varepsilon^* \cong M_1 / \varepsilon_1^* \times M_2 / \varepsilon_2^*.$$

Definition 2.3. Let M be an H_v-module over an H_v-ring R and let A be a fuzzy subset of M. The fuzzy subset A_{ε^*} on M / ε^* is defined as follows:

$$A_{\varepsilon^*} : M / \varepsilon^* \rightarrow [0; 1],$$

$$A_{\varepsilon^*}(\varepsilon^*(x)) = \begin{cases} \sup_{a \in \varepsilon^*(x)} \{A(a)\} & \text{if } \varepsilon^*(x) \neq \omega_M, \\ 1 & \text{if } \varepsilon^*(x) = \omega_M. \end{cases}$$

Theorem 2.4. Let M be an H_v-module over an H_v-ring R and A be a fuzzy H_v-submodule of M. Then A_{ε^*} is a fuzzy submodule of M / ε^*.

Proof. See [4].

Now, we study the fundamental modules with respect to the product of two fuzzy subsets.

Theorem 2.5. Suppose that
(1) M_1, M_2 are H_v-modules over an H_v-ring R,
(2) ε_1^*, ε_2^* and ε^* are the fundamental equivalence relations on M_1, M_2 and $M_1 \times M_2$, respectively,
(3) A, B are fuzzy H_v-submodules of M_1, M_2.
Then we have

$(A\times B)_{\varepsilon^*}(\varepsilon^*(x, y)) = (A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})(\varepsilon_1^*(x), \varepsilon_2^*(y))$.

Proof. By Condition (3), we conclude that $A \times B$ is a fuzzy H_v-submodule of $M_1 \times M_2$. So by Theorem 2.4, we have $(A \times B)_{\varepsilon^*}$ is a fuzzy submodule of the fundamental module $(M_1 \times M_2)/\varepsilon^*$. Now, assume that $x \in M_1$ and $y \in M_2$. If $\varepsilon^*(x, y) = \omega_{M_1\times M_2}$, then we obtain $(A \times B)_{\varepsilon^*}(\omega_{M_1\times M_2}) = 1 = (A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})(\omega_{M_1}, \omega_{M_2})$. If $\varepsilon^*(x, y) \neq \omega_{M_1\times M_2}$ (= 0), then

$$
\begin{aligned}
&(A\times B)_{\varepsilon^*}(\varepsilon_1^*(x, y))\\
&= \sup_{(a,b)\in\varepsilon^*(x,y)} \{(A \times B)(a, b)\}\\
&= \sup_{(a,b)\in\varepsilon^*(x,y)} \{\min\{(A(a), B(b)\}\}\\
&= \sup_{\substack{a\in\varepsilon_1^*(x)\\ b\in\varepsilon_2^*(y)}} \{\min\{(A(a), B(b)\}\}\\
&= \min\left\{\sup_{a\in\varepsilon_1^*(x)}\{A(a)\},\ \sup_{b\in\varepsilon_2^*(x)}\{B(b)\}\right\}\\
&= \min\left\{A_{\varepsilon_1^*}(\varepsilon_1^*(x)), B_{\varepsilon_2^*}(\varepsilon_2^*(y))\right\}\\
&= (A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})(\varepsilon_1^*(x),\ \varepsilon_2^*(y)).
\end{aligned}
$$

Theorem 2.6. Let M_1, M_2 be two H_v-modules over an H_v-ring R and let C be a fuzzy subset of $M_1 \times M_2$. Then A, B are fuzzy submodules of M_1/ε_1^*, M_2/ε_2^* respectively, where

$A(\varepsilon_1^*(x)) = C_{\varepsilon^*}(\varepsilon_1^*(x), \omega_{M_2})$ for all $\varepsilon_1^*(x) \in M_1/\varepsilon_1^*$,

$B(\varepsilon_2^*(y)) = C_{\varepsilon^*}(\omega_{M_1}, \varepsilon_2^*(y))$ for all $\varepsilon_2^*(y) \in M_2/\varepsilon_2^*$.

Moreover, we have $A \times B \subseteq C_{\varepsilon^*}$.

Proof. The proof is straightforward.

Theorem 2.7. Let M_1, M_2 be two H_v-modules over an H_v-ring R and let A, B be fuzzy subsets of $M_1 \times M_2$ respectively. If $A \times B$ is a fuzzy H_v-submodule of $A \times B$, then at least one of the following two statements holds:

(1) $B_{\varepsilon_2^*}(\omega_{M_2}) \geq A_{\varepsilon_1^*}(\varepsilon_1^*(a))$ for all a $\in M_1$,

(2) $A_{\varepsilon_1^*}(\omega_{M_1}) \geq B_{\varepsilon_2^*}(\varepsilon_2^*(b))$ for all b $\in M_2$.

Proof. Suppose $A \times B$ is a fuzzy H_v-submodule of $M_1 \times M_2$. Then by Theorem 2.4, $(A \times B)_{\varepsilon^*}$ is a fuzzy submodule of $(M_1 \times M_2)/\varepsilon^*$. Using Theorem 2.5, we have $(A \times B)_{\varepsilon^*} = A_{\varepsilon_1^*} \times B_{\varepsilon_2^*}$. By contraposition, suppose that none of the statements (1) and (2) holds. Then, we can find $a_0 \in M_1$ and $b_0 \in M_2$ such that

$$A_{\varepsilon_1^*}(\varepsilon_1^*(a_0)) > B_{\varepsilon_2^*}(\omega_{M_2})$$

and

$$B_{\varepsilon_2^*}(\varepsilon_2^*(b_0)) > A_{\varepsilon_1^*}(\omega_{M_1}).$$

Now, we have

$$
\begin{aligned}
&(A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})(\varepsilon_1^*(a_0),\ \varepsilon_2^*(b_0))\\
&= \min\{A_{\varepsilon_1^*}(\varepsilon_1^*(a_0), B_{\varepsilon_2^*}(\varepsilon_2^*(b_0))\}\\
&> \min\{A_{\varepsilon_1^*}(\omega_{M_1}), B_{\varepsilon_2^*}(\omega_{M_2})\}\\
&= (A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})(\omega_{M_1}, \omega_{M_2}).
\end{aligned}
$$

On the other hand, by using the definition of a fuzzy submodule, a fuzzy submodule of a module attains its supremum at zero element, and so we have

$$
\begin{aligned}
&(A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})(\omega_{M_1}, \omega_{M_2})\\
&\geq (A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})(\varepsilon_1^*(a_0),\ \varepsilon_2^*(b_0)).
\end{aligned}
$$

Thus $A_{\varepsilon_1^*}\times B_{\varepsilon_2^*}$ is not a fuzzy submodule of $M_1/\varepsilon_1^* \times M_2/\varepsilon_2^*$. Therefore, either $B_{\varepsilon_2^*}(\omega_{M_2}) \geq A_{\varepsilon_1^*}(\varepsilon_1^*(a))$ for all $a \in M_1$ or $A_{\varepsilon_1^*}(\omega_{M_1}) \geq B_{\varepsilon_2^*}(\varepsilon_2^*(b))$ for all $b \in M_2$.

Theorem 2.8. Let M_1, M_2 be two H_v-modules over an H_v-ring R and let A, B be fuzzy subsets of M_1, M_2 respectively, such that $A \times B$ is a fuzzy H_v-submodule of $M_1 \times M_2$. If $A_{\varepsilon_1^*}(\varepsilon_1^*(a)) \leq B_{\varepsilon_2^*}(\omega_{M_2})$ for all $a \in M_1$, then we have $A_{\varepsilon_1^*}$ is a fuzzy submodule of M_1/ε_1^*.

Proof. Suppose $x, y \in M_1$, then we have

$$
\begin{aligned}
&A_{\varepsilon_1^*}(\varepsilon_1^*(x) \oplus \varepsilon_2^*(y)\\
&= \min\{A_{\varepsilon_1^*}(\varepsilon_1^*(x) \oplus \varepsilon_1^*(y)),\ B_{\varepsilon_2^*}(\omega_{M_2} \oplus \omega_{M_2})\}\\
&= (A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})((\varepsilon_1^*(x), \omega_{M_2}) \oplus (\varepsilon_1^*(y), \omega_{M_2}))\\
&\geq \min\{(A_{\varepsilon_1^*}\times B_{\varepsilon_1^*})(\varepsilon_1^*(x), \omega_{M_2}),\\
&\qquad (A_{\varepsilon_1^*}\times B_{\varepsilon_2^*})(\varepsilon_1^*(y), \omega_{M_2})\}\\
&= \min\{\min\{A_{\varepsilon_1^*}(\varepsilon_1^*(x)),\ B_{\varepsilon_2^*}(\omega_{M_2})\}\\
&\qquad \min\{A_{\varepsilon_1^*}(\varepsilon_1^*(y),\ B_{\varepsilon_2^*}(\omega_{M_2})\}\}\\
&= \min\{A_{\varepsilon_1^*}(\varepsilon_1^*(x)),\ A_{\varepsilon_1^*}(\varepsilon_1^*(y))\}.
\end{aligned}
$$

Also, we have

$A_{\varepsilon_1^*}(-\varepsilon_1^*(x)) = \min\{A_{\varepsilon_1^*}(-\varepsilon_1^*(x)),\ B_{\varepsilon_2^*}(\omega_{M_2})\}$

$= (A_{\varepsilon_1^*} \times B_{\varepsilon_2^*})(-\varepsilon_1^*(x), \omega_{M_2})$

$\geq (A_{\varepsilon_1^*} \times B_{\varepsilon_2^*})(\varepsilon_1^*(x), \omega_{M_2})$

$= \min\{A_{\varepsilon_1^*}(\varepsilon_1^*(x)),\ B_{\varepsilon_2^*}(\omega_{M_2})\} = A_{\varepsilon_1^*}(\varepsilon_1^*(x))$.

Similarly, for any $r \in R$ and $x \in M_1$, we have

$A_{\varepsilon_1^*}(\gamma^*(r) \odot \varepsilon_1^*(x))$

$= \min\{A_{\varepsilon_1^*}(\gamma^*(r) \odot \varepsilon_1^*(x)),\ B_{\varepsilon_2^*}(\gamma^*(r) \odot \omega_{M_2})\}$

$= (A_{\varepsilon_1^*} \times B_{\varepsilon_2^*})(\gamma^*(r) \odot (\varepsilon_1^*(x), \omega_{M_2})\}$

$\geq (A_{\varepsilon_1^*} \times B_{\varepsilon_2^*})(\varepsilon_1^*(x), \omega_{M_2})$

$= \min\{A_{\varepsilon_1^*}(\varepsilon_1^*(x)),\ B_{\varepsilon_2^*}(\omega_{M_2})\} = A_{\varepsilon_1^*}(\varepsilon_1^*(x))$.

Therefore $A_{\varepsilon_1^*}$ is a fuzzy submodule of M_1/ε_1^*.

Corollary 2.9. Let M_1, M_2 be two H_v-modules over an H_v-ring R and let A, B be fuzzy subsets of M_1, M_2 respectively, such that $A \times B$ is a fuzzy H_v-submodule of $M_1 \times M_2$. If $B_{\varepsilon_2^*}(\varepsilon_2^*(b)) \leq A_{\varepsilon_1^*}(\omega_{M_1})$ for all $b \in M_2$, then $B_{\varepsilon_2^*}$ is a fuzzy submodule of M_2/ε^*.

Proof. The proof is similar to the proof of Theorem 2.8.

3. Conclusions

The theory of H_v-structures was introduced by Vougiouklis and the theory of fuzzy H_v-structures was studied by Davvaz. This paper deals with H_v-modules. The main tools concerning the class of H_v-modules with the ordinary modules are the fundamental relations. So we studied the connections between fuzzy H_v-submodules and fundamental relations. The focus of the work was the product structure of fuzzy sets on H_v-modules. This may lead to a new direction of research in different algebraic hyperstructures.

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