Importance-Assessing Method with Fuzzy Number-Valued Fuzzy Measures and Discussions on TFNs And TrFNs

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Abstract

The weight is one of the most useful tools to measure the attribute importance when individuals make a decision or evaluate the alternatives. Among the methods which measure the weight, fuzzy measures is are subjective scales for the degrees of fuzziness and widely used to determine the degrees of subjective importance of evaluation items in numerous studies for the time being. The purpose of this study is to use fuzzy number-valued fuzzy measures to determine the attribute importance. Several types of triangular fuzzy numbers and trapezoidal fuzzy numbers were applied in order to observe which type of fuzzy numbers had the best effects in the empirical study. In addition, we took advantage of distance measures to adjust the distance between attribute importance such as Euclidean distance, and Tran and Duckstein's distances. The results indicate that since using different types of fuzzy numbers and distance measures in fuzzy number-valued fuzzy measures have the similar performance, we suggest that triangular fuzzy numbers are better than trapezoidal fuzzy numbers, and Euclidean distance is preferred on account of their simple form to calculate.

Keywords: Attribute importance, Fuzzy number-valued fuzzy measure, Triangular fuzzy number, Trapezoidal fuzzy number, Distance measure.

1. Introduction

In multiple attribute decision-making (MADM) analysis, fuzzy measures have been widely successfully applied in diverse fields such as the hotel industry, computer games, hospital materials etc. [1-3]. A fuzzy measure, introduced by Sugeno [4], is a subjective scale for the degrees of fuzziness and is suitable in analyzing human subjective evaluation processes [5, 6]. When fuzzy measures are used to real world problems, the candidate sets generally correspond to the evaluation items and the values of fuzzy measures stand for the grades of importance of corresponding evaluation items [7]. Fuzzy measures combined with fuzzy integrals are regarded as a useful paradigm for analyzing the human evaluation process and specifying the preference structures [8].

Since the data of human subjective judgment are usually fuzzy and imprecise in nature, fuzzy data can be expressed in linguistic terms or in fuzzy numbers [9]. Thus, we consider the value of fuzzy measures as a linguistic value and then convert linguistic terms to fuzzy numbers. Zhang [10-12] developed the elementary concepts and theorems of a fuzzy number-valued fuzzy measure ((z)fuzzy measure) and a fuzzy number-valued fuzzy integral on the fuzzy set. Fuzzy numbers are often represented in applications by LR fuzzy sets [13] but in particular, triangular and trapezoidal fuzzy sets. Triangular fuzzy numbers (TFNs) and trapezoidal fuzzy numbers (TrFNs) are suggested to solve many practical and complex problems in a great amount of literature [14-16]. There are several types of TFNs and TrFNs in linguistic scales while few studies contrast them and discuss which one of the scales is the best to be applied in the MADM practice. Therefore, this study tries to uses (z)fuzzy measures in different sort of TFNs and TrFNs to calculate the importance of attribute.

However, a critical issue may emerge from the investigation process of respondents' judgment concerning linguistic terms; acquiescence bias results because some respondents tend to give positive connotations to the attribute importance [17]. To control the spurious influence of response bias, we use different fuzzy distance measures to adjust overestimated ratings of attribute importance by using comparison of fuzzy number-valued fuzzy measures based on a fuzzy distance measure. After obtaining each attribute importance which is calculated by (z)fuzzy measures in TFNs and TrFNs and fuzzy distance measures, we use fuzzy integrals to acquire the priority of alternatives. Finally, three performance indices, the spearman rank-order correlation coefficient, the consistency rate of the best alternative, and the consistency rate of better alternatives, are employed to compare the predicted priority and the real priority generated by the decision-makers, and then to gain the best type of fuzzy

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numbers which can be applied in decision-making problems.

2. TFNs and TrFNs

A fuzzy number is a fuzzy subset in real numbers which have two properties, convexity and normality. Fuzzy numbers become more meaningful to quantify a subjective measurement into a range rather than in an exact value [18]. TFNs and TrFNs are special cases of fuzzy numbers. The weights of decisive criteria and the ratings of qualitative criteria which are assessed by the TFNs and TrFNs in linguistic variables can solve lots of uncertain problems and practical applications [1, 19-21]. Plenty of studies developed and employed different types of TFNs and TrFNs in the linguistic scales. Tables 1 and 2 summarize all applications in TFNs and TrFNs, respectively, and present fuzzy numbers associated with linguistic terms.

Table 1. Linguistic scale of TFNs.

Source	The meaning of linguistic scales	Numerical scale	The range
5-point linguist scale			
Chang	1. Importance of various criteria;	(0, 0, 0.25)	Between
and	2. Evaluate the appropriateness of	(0, 0.25, 0.5)	0 and 1
Chen	alternatives	(0.25, 0.5, 0.75)	
[22]	Very importance (VI);	(0.5 ,0.75, 1)	
(A)	Very Low (VL)	(0.75, 1, 1)	
	Importance (I); Low (L)		
	Medium (M); Medium (M)		
	Unimportance (U); High (H)		
	very unimportance (VU);		
	Very High (VH)		
Liou	The rating of criteria represents	(0, 0, 0.3)	Between
and	different aspects of the mental	(0, 0.3, 0.5)	0 and 1
Wang	workload	(0.2, 0.5, 0.8)	
[23]	Very low (VL)	(0.5, 0.7, 1)	
(B)	Low (L)	(0.7, 1, 1)	
	Medium (M)		
	High (H)		
	Very high (VH)		
Chien	1. Consumer's satisfaction degree;	(0 ,0, 2)	Normalized
and	2. Importance degree	(0, 2, 4)	(0, 0, 1/7)
Tsai	Very unsatisfied (VU);	(2, 4, 6)	(0, 1/7, 3/7)
[24]	Very unimportance (VU)	(4, 6, 8)	(1/7, 3/7,
(C)	Unsatisfied (U);	(6, 8, 8)	5/7)
	Unimportance (U)		(3/7, 5/7, 1)
	Fair (F); Fair (F)		(5/7, 1, 1)
	Satisfied (S); Importance (I)		
	Very satisfied (VS);		
	Very importance (VI)		
Lee	1. The weightings of the criteria	(0, 0, 0.2)	Between
et al.	2. The performance of a schedule	(0, 0.2, 0.4)	0 and 1
[25]	Very unimportant (VU);	(0.3, 0.5, 0.7)	
(D)	Very poor (VP)	(0.6, 0.8, 1)	
	Unimportant (U); poor (P)	(0.8, 1, 1)	
	Medium (M); Medium (M)		
	Important (I); Good (G)		
	Very important (VI);		
	Very good (VG)		
Lau	1. The most likely possible value	(0, 0, 0.25)	Between
et al.	2. The most pessimistic value	(0, 0.25, 0.5)	0 and 1
[26]	3. The most optimistic value	(0.25, 0.5, 0.75)	
	Same importance	(0.5, 0.75, 1)	
	Weak importance	(0.75, 1, 1)	
	Strong importance		
	Demonstrated importance		
	Absolute importance		

Table 1. Linguistic scale of TFNs. (Cont.)

Source	The meaning of linguistic scales	Numerical scale	The range
5-point linguist scale			
Yang	1. Evaluate the importance of	(0, 0.1, 0.25)	Between
and	attributes;	(0.15, 0.3, 0.45)	0 and 1
Hung	2. To illustrate the proposed	(0.35, 0.5, 0.65)	
[27]	fuzzy TOPSIS method;	(0.55, 0.7, 0.85)	
(E)	3. To benchmark the empirical	(0.75, 0.9, 1)	
	results with other precise value		
	methods		
	Very Low (VL)		
	Low (L)		
	Medium (M)		
	High (H)		
	Very High (VH)		
6-point linguist scale			
Chan et	Pessimistic value and optimistic	(3, 5, 5)	Between
al.	value	(1, 3, 5)	1/5 and 5
[18]	Very high (VH)	(1/3, 1, 3)	
	High (H)	(1, 1, 1)	
	Medium (M)	(1/5, 1/3 ,1)	
	Exactly equal (EQ)	(1/5, 1/5, 1/3)	
	Low (L)		
	Very low (VL)		
Ertay et	1. Importance degree of customer	(1, 1, 1)	Between
al.	needs;	(1/2, 1, 3/2)	1/2 and 7/2
[28]	2. Difficulty degree of product	(1, 3/2, 2)	
	technical requirements	(3/2, 2, 5/2)	
	Just equal (JI)	(2, 5/2, 3)	
	Equally important (EI)	(5/2, 3, 7/2)	
	Weakly more important (WMI)		
	Strongly more important (SMI)		
	Very strongly more		
	important(VSMI)		
	Absolutely more important (AMI)		
7-point linguist scale			
Chen	The location suitability and	(0, 0, 0.1)	Between
[29]	determine the best selection	(0, 0.1, 0.3)	0 and 1
	Very low (VL)	(0.1, 0.3, 0.5)	
	Low (L)	(0.3, 0.5, 0.7)	
	Medium low (ML)	(0.5, 0.7, 0.9)	
	Medium (M)	(0.7, 0.9, 1)	
	Medium high (MH)	(0.9, 1, 1)	
	High (H)		
	Very high (VH)		
7-point linguist scale			
Herrera	Preference degree of the alternative	(0.83, 1, 1)	Between 0
et al.	Perfect (P)	(0.67, 0.83, 1)	and 1
[15]	Very high (VH)	(0.5, 0.67, 0.83)	
	High (H)	(0.33, 0.5, 0.67)	
	Medium (M)	(0.17, 0.33, 0.5)	
	Low (L)	(0, 0.17, 0.33)	
	Very low (VL)	(0, 0, 0.17)	
	None (N)		
Chen	The importance of criteria and	(0, 0, 0.2)	Between 0
et al.	evaluate the ratings of candidates	(0.1, 0.2, 0.3)	and 1
[19]	with respect	(0.2, 0.35, 0.5)	
	to each criterion	(0.4, 0.5, 0.6)	
	Very low (VL)	(0.5, 0.65, 0.8)	
	Low (L)	(0.7, 0.8, 0.9)	
	Medium low (ML)	(0.8, 0.9, 1)	
	Medium (M)		
	Medium high (MH)		
	High (H)		
	Very high (VH)		
Kuo	The importance of criteria and	(0, 0, 1)	Normalized
et al.	select the most suitable location	(1, 1, 2)	(0, 0, 0.1)

[30]	Very poor (VP)	(3, 2, 2)	(0.1, 0.1, 0.2)
	Poor (P)	(5, 2, 2)	(0.3, 0.2, 0.2)
	Medium poor (MP)	(7, 2, 2)	(0.5, 0.2, 0.2)
	Fair (F)	(9, 2, 1)	(0.7, 0.2, 0.2)
	Medium good (MG)	(10, 1, 0)	(0.9, 0.2, 0.1)
	Good (G), Very good (VG)		(1, 0.1, 0)

Table 2. Linguistic scale of TrFNs.

Source	The meaning of linguistic scales	Numerical scale	The range
5-point linguist scale			
Ding	Describing the subjective	(0, 0, 0.2, 0.3)	Between
et al.	assessments of the importance	(0.2, 0.3, 0.4, 0.5)	0 and 1
[31]	of all criteria in	(0.4, 0.5, 0.6, 0.7)	
(F)	choosing a suitable CSP	(0.6, 0.7, 0.8, 0.9)	
	Very low (VL)	(0.8, 0.9, 1, 1)	
	Low (L)		
	Medium (M)		
	H igh (H)		
	Very high (VH)		
Xia	Weighting of qualitative and	(0, 0.1, 0.2, 0.3)	Between
et al.	quantitative attributes	(0.1, 0.2, 0.3, 0.4)	0 and 1
[32]	Very low (VL)	(0.3, 0.4, 0.5, 0.6)	
(G)	Low (L)	(0.5, 0.6, 0.7, 0.8)	
	Medium (M)	(0.7, 0.8, 0.9, 1)	
	High (H)		
	Very high (VH)		
Designed by this study **(H)**		(0, 0.05, 0.15, 0.25) (0.15, 0.25, 0.35, 0.45) (0.35, 0.45, 0.55, 0.65) (0.55, 0.65, 0.75, 0.85) (0.75, 0.85, 0.95, 1)	Between 0 and 1
7-point linguist scale			
Cheng	1. Mobility capability;	(0, 0, 0.1, 0.2)	Between
and	2. Communication and control	(0.1, 0.2, 0.2, 0.3)	0 and 1
Lin	capability;	(0.2, 0.3, 0.4, 0.5)	
[20]	3. self-defense capability;	(0.4, 0.5, 0.6, 0.7)	
	4. attack capability	(0.5, 0.6, 0.7, 0.8)	
	Very low (VL)	(0.6, 0.7, 0.8, 0.9)	
	Low (L)	(0.8, 0.9, 1, 1)	
	Medium low (ML)		
	Medium (M)		
	Medium high (MH)		
	High (H)		
	Very high (VH)		
Chen	The importance of the criteria	(0, 0, 0.1, 0.2)	Between
et al.	Very low (VL)	(0.1, 0.2, 0.2, 0.3)	0 and 1
[19]	Low (L)	(0.2, 0.3, 0.4, 0.5)	
	Medium low (ML)	(0.4,0.5,0.5, 0.6)	
	Medium (M)	(0.5,0.6,0.7,0.8)	
	Medium high (MH)	(0.7,0.8,0.8, 0.9)	
	High (H)	(0.8,0.9,1,1)	
	Very high (VH)		
9-point linguist scale			
Liou	The weight of "task difficulty",	(0, 0, 0, 0.2)	Between
and	"time pressure" and	(0, 0, 0.2, 0.4)	0 and 1
Wang	"performance"	(0, 0.2, 0.2, 0.4)	
[23]	Unimportant (U)	(0, 0.2, 0.5, 0.7)	
	Between U and SL	(0.3, 0.5, 0.5, 0.7)	
	(B.U&SL)	(0.3, 0.5, 0.8, 1)	
	Slightly important (SL)	(0.6, 0.8, 0.8, 1)	
	Between SL and VH	(0.6, 0.8, 1, 1)	
	(B.SL&VH)	(0.8, 1, 1, 1)	
	Moderately important (M)		
	Between M and SE (B.M&SE)		
	Seriously important (SE)		
	Between SE and VSE		
	(B.SE&VSE)		
	Very seriously important (VSE)		
Liang	The attention degree of subjects	(0, 0, 0, 0.2)	Between
et al.	companies under each of the	(0, 0, 0.2, 0.4)	0 and 1
[16]	management strategies	(0, 0.2, 0.2, 0.4)	
	Very low (VL)	(0, 0.2, 0.5, 0.7)	
	Between very low and low	(0.3, 0.5, 0.5, 0.7)	
	(B.VL&L)	(0.3, 0.5, 0.8, 1)	
	Low (L)	(0.6, 0.8, 0.8, 1)	

Table 2. Linguistic scale of TrFNs. (Cont.)

Source	The meaning of linguistic scales	Numerical scale	The range
Liang	Between low and medium	(0.6, 0.8, 1, 1)	Between
et al.	(B.L&M)	(0.8, 1, 1, 1)	0 and 1
[16]	Medium (M)		
	Between medium and high		
	(B.M&H)		
	High (H)		
	Between high and very high		
	(B.H&VH)		
	Very high (VH)		

This study focused on the five-point linguistic scales in TFNs and TrFNs because the five-point scales are not only easy to be used by the respondents but also have the most types in TFNs and TrFNs. Therefore, we selected five types of five-point linguistic scales in TFNs ((A): Chang and Chen [22]; (B): Liou and Wang [23]; (C): Chien and Tsai [24]; (D): Lee et al. [25]; (E): Yang and Hung [27] and two types of five-point linguistic scales in TrFNs (F): Ding et al. [31]; (G): Xia et al. [32]. There are two notes. First, because the linguistic scale in TFNs introduced by Lau et al. [26] is the same with the scale proposed by Chang and Chen [22], we only adopted Chang and Chen's scale. Second, we hoped there were at least three types of linguistic scales in TrFNs; hence, we designed another type of five-point linguistic scales in TrFNs which are labeled as (H).

3. Methods

A. *Fuzzy measures*

The fuzzy measure is a measure for representing the membership degrees of an object to candidate sets. Let X be a universal set and $P(X)$ be the power set of X. A fuzzy measure, g, is defined as follows:

$$g: P(X) \rightarrow [0,1]. \quad (1)$$

The axioms of fuzzy measures satisfy boundary conditions $(g(\phi)=0$ and $g(X)=1)$, monotonicity (for every A, $B \in P(X)$, if $A \subseteq B$, then $g(A) \leq g(B)$), and continuity.

The λ-fuzzy measure is a special case of fuzzy measures. With the λ-fuzzy measure, the interaction between each information element is characterized by a single parameter λ. Hence, the full power of fuzzy measure is not utilized. The λ-fuzzy measures satisfy the following additional property: for all A, $B \subset X$, $A \cap B = \phi$,

$$g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B), \quad (2)$$

where $\lambda \in (-1, \infty)$. The λ indicates the degree of interrelation between A and B. Let X be a finite set and be equal to $\{x_1, x_2, ..., x_{n-1}, x_n\}$, where n is the total number of attributes, and $g_i = g_\lambda(\{x_i\})$ $((1 \le i \le n))$ is called a fuzzy density. Then, $g_\lambda(\{x_1, x_2, ..., x_s\})$ $((1 \le s \le n))$ is computed as follows.

$$g_\lambda(\{x_1, x_2, \cdots, x_n\}) = \frac{1}{\lambda}[\prod_{i=1}^{n}(1+\lambda g_i) - 1]. \quad (3)$$

B. *(z)Fuzzy Measures*

Zhang [10-12] developed the elementary concepts and theorems of a fuzzy number-valued fuzzy measure and a fuzzy number-valued fuzzy integral on the fuzzy set. Continuing his work, Zhang [33] discussed a series of structural characteristics for the fuzzy number-valued fuzzy measure on the fuzzy σ-algebra. Zhang [34] also gave some definitions of the convergence of a sequence of fuzzy number-valued fuzzy measurable functions.

Let X be a nonempty set and $F(X)=\{T;\ T: X\rightarrow[0,1]\}$. $F^*(X)\subset F(X)$ is called a fuzzy σ-algebra if $F^*(X)$ satisfies: (i) ϕ, $X\in F^*(X)$, where $\phi(x)=0$, $X(x)=1$, for every $x\in X$; (ii) if $\{T_n\}\subset F^*(X)$, then $\cup_{n=1}^{\infty} T_n \in F^*(X)$; and (iii) if $T\in F^*(X)$, then $\overline{T} \in F^*(X)$. Let F^* be the set of all fuzzy numbers and $A\in F^*$. An α-level interval of fuzzy number A is denoted as $[{}^\alpha A^-, {}^\alpha A^+]$. Then, $A = \underset{\alpha\in[0,1]}{\cup} \alpha[{}^\alpha A^-, {}^\alpha A^+]$ for $A\in F^*$ by the decomposition theorem of a fuzzy set. Let $F_+^* = \{A; A \ge 0, A \in F^*\}$. A fuzzy distance of fuzzy numbers A and B is denoted as $D(A,B)$, where $D(A,B)=$ $\underset{\alpha\in[0,1]}{\cup} \alpha\left[\left|{}^1A^- - {}^1B^-\right|, \sup_{\alpha\le\eta\le1}\left|{}^\eta A^- - {}^\eta B^-\right| \vee \left|{}^\eta A^+ - {}^\eta B^+\right|\right]$.

A fuzzy number-valued fuzzy measure (i.e., a (z)fuzzy measure) on $F^*(X)$ is a fuzzy number-valued fuzzy set function μ: $F^*(X)\rightarrow F_+^*$, with the following properties: (i) $\mu(\phi)=0$; (ii) if T_1, $T_2\in F^*(X)$, $T_1 \subset T_2$, then $\mu(T_1) \le \mu(T_2)$;(iii) if $T_1 \subset T_2 \subset\cdots$, $\{T_n\}\subset F^*(X)$, then $(D)\lim_{n\to\infty}\mu(T_n) = \mu(\cup_{n=1}^{\infty} T_n)$; and (iv) if $T_1\supset T_2\supset\cdots$, $\{T_n\}\subset F^*(X)$, and there exists n_0 such that $\mu(T_{n_0}) \ne \infty$, then $(D)\lim_{n\to\infty}\mu(T_n) = \mu(\cap_{n=1}^{\infty} T_n)$. $(X,F^*(X))$ is called a fuzzy measurable space, and $(X,F^*(X),\mu)$ is a fuzzy number-valued fuzzy measure space ((z)fuzzy measure space). Since we apply (z)fuzzy measures to characterize attribute salience, the set X is designated a discrete and finite set, $\{x_1, x_2, \cdots, x_n\}$, throughout this paper.

Let $\mathcal{R}$ denote the set of all real numbers. For $\beta\in\mathcal{R}$, let $F_\beta = \{x; f(x) \ge \beta\}$. A mapping f: $X\rightarrow\mathcal{R}$ is called a fuzzy measurable function, if $\chi_{F_\beta} \in F^*(X)$ and

$$\chi_{F_\beta}(x) = \begin{cases} 1 & \text{iff } x \in F_\beta, \\ 0 & \text{iff } x \notin F_\beta. \end{cases} \quad (4)$$

The set of all fuzzy measurable functions is denoted by M^*. In addition, let M_+^* denote the set of all non-negative fuzzy measurable functions.

For $T\in F^*(X)$, $f\in M_+^*$, the fuzzy number-valued fuzzy integral (i.e., (z)fuzzy integral) of f on T with respect to μ is defined by

$$\int_T f\,\mathrm{d}\mu = \underset{\alpha\in[0,1]}{\cup} \alpha\left[\sup_{\beta\in[0,\infty)} \beta \wedge^\alpha \left(\mu(T\cap\chi_{F_\beta})\right)^-, \sup_{\beta\in[0,\infty)} \beta \wedge^\alpha \left(\mu(T\cap\chi_{F_\beta})\right)^+\right], \quad (5)$$

where $F_\beta = \{x; f(x) \ge \beta\}, \beta \in [0, \infty)$. Zhang [12] has proved a series of elementary properties inherited by (z)fuzzy integrals.

In this study, we apply (z)fuzzy measures to determine the salience of decision attributes. The concept of linguistic variables can reasonably approximate human judgment in subjective importance of evaluation attributes. Thus, in the investigation stage, we use linguistic terms to collect the data of respondents' subjective judgments concerning attribute importance. The linguistic terms are further transformed into fuzzy numbers, so we can construct (z)fuzzy measures to develop an importance-assessing method. Moreover, synthetic evaluations can be derived using (z)fuzzy integrals.

C. *(z)Fuzzy Integrals*

The fuzzy integral represents the weights of criteria with non-additive characteristics as the fuzzy measure, and the fuzzy measure is often used with fuzzy integrals in order to aggregate the information for evaluation [4, 7, 35]. The Choquet integral with respect to a fuzzy measure proposed by Murofushi and Sugeno [36] is a basic tool for the subjective evaluation and the decision analysis. The most important properties of the Choquet integral are comonotonically additivity and monotone. The Choquet fuzzy integral is a fuzzy integral based on λ-fuzzy measure that provides alternative computational scheme for aggregating information. Assume that $f(x_1)$, $f(x_2)$, …, $f(x_n)$ are a collection of input sources of f, and g is a λ-fuzzy measure, the Choquet fuzzy integral can be computed as follows:

$$E_g(f) = f(x_n)g(X_n) + [f(x_{n-1}) - f(x_n)]g(X_{n-1}) + ... + [f(x_1) - f(x_2)g(X_1)]. \quad (6)$$

where $f(x_1) \le f(x_2) \le \ldots \le f(x_n)$, and $f(x_o)=0$.

D. *An Importance-Assessing Method*

This study develops an importance-assessing method by comparison of (z)fuzzy measures using a fuzzy distance measure. In the light of the positive leniency problem of investigation data, we do not ask respondents their direct explication on attribute importance, but investigate the relative importance instead. That is, respondents are asked for performing the psychological importance distance of an attribute to the most important one. We apply a similarity scale introduced by Triantaphyllou [37, 38] to require a respondent to rate attribute salience in comparison with the highest degree of importance explicitly used as a frame of reference. Through a simple approach using fuzzy distance measures, the fuzzy number for each attribute will be explicated and attribute importance, expressed in terms of (z)fuzzy measures, can then be ascertained.

A fuzzy set A is a generalized left right fuzzy number (GLRFN) [13, 39] if and only if there exists a closed interval $[a_1, a_2] \neq \phi$ such that,

$$A(t)=\begin{cases} L\left(\dfrac{a_2-t}{a_2-a_1}\right) & \text{for } a_1 \le t \le a_2, \\ 1 & \text{for } a_2 \le t \le a_3, \\ R\left(\dfrac{t-a_3}{a_4-a_3}\right) & \text{for } a_3 \le t \le a_4, \\ 0 & \text{else.} \end{cases} \tag{7}$$

Then we denote A as (a_1, a_2, a_3, a_4). L and R are strictly decreasing functions defined on [0,1] and satisfying two conditions: (i)$L(t)=R(t)=1$ if $t\le 0$; (ii)$L(t)=R(t)=0$ if $t\ge 1$. When $a_2=a_3$, a GLRFN is a left right fuzzy number defined by Dubois and Prade [13]. When $L(t)=R(t)=1-t$, a GLRFN is a TrFN. When $L(t)=R(t)=1-t$ and $a_2=a_3$, a GLRFN is a TFN.

The Euclidean distance calculates the distance between two triangular fuzzy numbers. If $A=(a_1,a_2,a_3)$ and $B=(b_1,b_2,b_3)$ be two TFNs then the equation can be written as follows:

$$D(A,B)=\sqrt{\frac{1}{6}\left[(a_1-b_1)^2+4(a_2-b_2)^2+(a_3-b_3)^2\right]}. \tag{8}$$

If $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$ be two TrFNs then the equation can be written as follows:

$$D(A,B)=\sqrt{\frac{1}{6}\left[(a_1-b_1)^2+2(a_2-b_2)^2+2(a_3-b_3)^2+(a_4-b_4)^2\right]}. \tag{9}$$

Tran and Duckstein [40] introduced a new approach for ranking fuzzy numbers based on a distance measure. Tran and Duckstein gave the equations to compute distances for fuzzy numbers with two different weighting functions: $w(\alpha)=1$ and $w(\alpha)=\alpha$. Given two TFNs that $A=(a_1,a_2,a_3)$ and $B=(b_1,b_2,b_3)$. When $w(\alpha)=1$, the distance between A and B is as follows:

$$\begin{aligned} D_{\mathrm{TFN}}(A,B,1) &= (a_2-b_2)^2+\frac{1}{2}(a_2-b_2)[(a_3+a_1) \\ &-(b_3+b_1)]+\frac{1}{9}[(a_3-a_2)^2+(a_2-a_1)^2+(b_3-b_2)^2 \\ &+(b_2-b_1)^2]-\frac{1}{9}[(a_2-a_1)(a_3-a_2)+(b_2-b_1)\cdot \\ &(b_3-b_2)]+\frac{1}{6}(2a_2-a_1-a_3)(2b_2-b_1-b_3). \end{aligned} \tag{10}$$

When $w(\alpha)=\alpha$, the distance between A and B is:

$$\begin{aligned} D_{\mathrm{TFN}}(A,B,\alpha) &= (a_2-b_2)^2+\frac{1}{3}(a_2-b_2)[(a_3+a_1) \\ &-(b_3+b_1)]+\frac{1}{18}[(a_3-a_2)^2+(a_2-a_1)^2+(b_3-b_2)^2 \\ &+(b_2-b_1)^2]-\frac{1}{18}[(a_2-a_1)(a_3-a_2)+(b_2-b_1)\cdot \\ &(b_3-b_2)]-\frac{1}{12}(2a_2-a_1-a_3)(2b_2-b_1-b_3). \end{aligned} \tag{11}$$

Given two TrFNs that $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$. When $w(\alpha)=1$, the distance between A and B is as follows:

$$\begin{aligned} D_{TrFN}(A,B,1) &= \left(\frac{a_2+a_3}{2}-\frac{b_2+b_3}{2}\right)^2+\frac{1}{6}\left(\frac{a_2+a_3}{2}-\right. \\ &\left.\frac{b_2+b_3}{2}\right)\left[(a_4-a_3)-(a_2-a_1)-(b_4-b_3)-(b_2-b_1)\right] \\ &\cdot\left(\frac{a_3-a_2}{2}\right)^2+\frac{1}{6}\left(\frac{a_3-a_2}{2}\right)\left[(a_4-a_3)+(a_2-a_1)\right] \\ &+\frac{1}{3}\left(\frac{b_3-b_2}{2}\right)^2+\frac{1}{6}\left(\frac{b_3-b_2}{2}\right)\left[(b_4-b_3)+(b_2-b_1)\right] \\ &+\frac{1}{9}\left[(a_4-a_3)^2+(a_2-a_1)^2+(b_4-b_3)^2+(b_2-b_1)^2\right] \\ &-\frac{1}{9}\left[(a_2-a_1)(a_4-a_3)+(b_2-b_1)(b_4-b_3)\right] \\ &+\frac{1}{6}\left[(a_4-a_3)(b_2-b_1)+(a_2-a_1)(b_4-b_3)-\right. \\ &\left.(a_4-a_3)(b_4-b_3)-(a_2-a_1)(b_2-b_1)\right]. \end{aligned} \tag{12}$$

When $w(\alpha)=\alpha$, the distance between A and B is:

$$\begin{aligned} D_{TrFN}(A,B,\ 1) &= \left(\frac{a_2+a_3}{2}-\frac{b_2+b_3}{2}\right)^2+\frac{2}{9}\left(\frac{a_2+a_3}{2}\right. \\ &\left.-\frac{b_2+b_3}{2}\right)\cdot\left[(a_4-a_3)-(a_2-a_1)-(b_4-b_3)-\right. \\ &(b_2-b_1)]\cdot\left(\frac{a_3-a_2}{2}\right)^2+\frac{1}{9}\left(\frac{a_3-a_2}{2}\right)\left[(a_4-a_3)+\right. \end{aligned}$$

$$
\begin{aligned}
&(a_2-a_1)\big]+\frac{2}{3}\left(\frac{b_3-b_2}{2}\right)^2+\frac{1}{9}\left(\frac{b_3-b_2}{2}\right)\big[(b_4-b_3)\\
&+(b_2-b_1)\big]+\frac{1}{18}\big[(a_4-a_3)^2+(a_2-a_1)^2+(b_4-b_3)^2\\
&+(b_2-b_1)^2\big]-\frac{1}{18}\big[(a_2-a_1)(a_4-a_3)+(b_2-b_1)\cdot\\
&(b_4-b_3)\big]+\frac{1}{12}\big[(a_4-a_3)(b_2-b_1)+(a_2-a_1)\\
&(b_4-b_3)-(a_4-a_3)(b_4-b_3)-(a_2-a_1)(b_2-b_1)\big].
\end{aligned}
\tag{13}
$$

With the above equations and the similarity scale, as long as investigating respondents' grade of the most important attribute, we can calculate the other grades of attributes. We propose an algorithm of calculating process for the priority of alternatives, as shown in Figure 1.

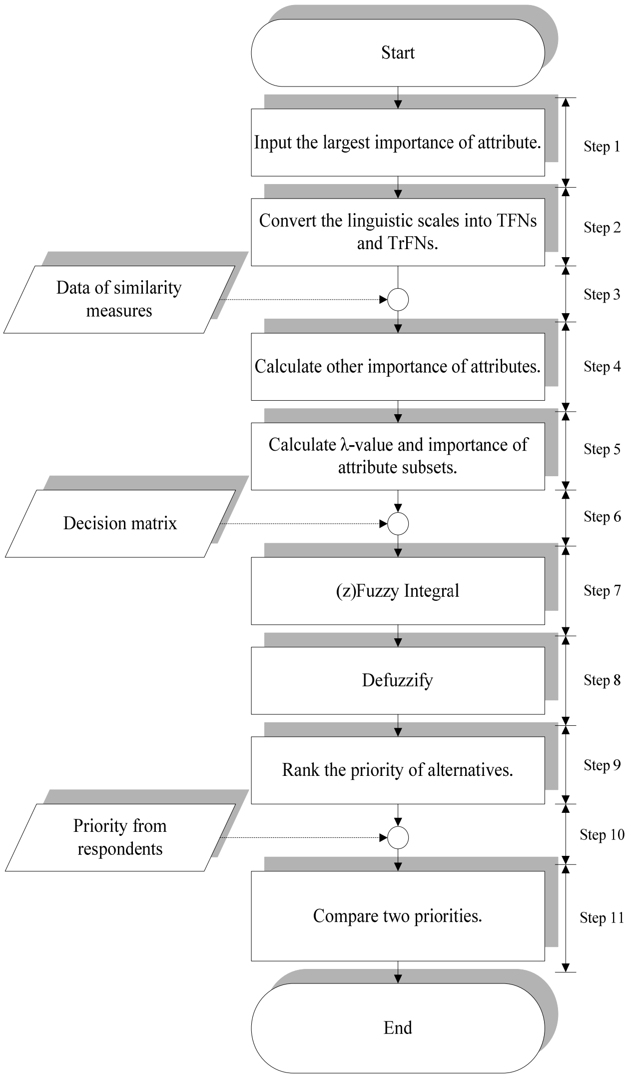

Figure 1. The flowchart of the proposed algorithm.

The presented algorithm can be summed up as a series of successive steps:

Step 1. Choose the largest grade of attribute importance as the input data through questionnaire survey.

Step 2. Convert the linguistic terms into TFNs and TrFNs for the largest importance of attribute**.** There are five types of TFNs and three types of TrFNs which we employ in this study. The numerical scales for TFNs are showed in Table 1 as (A), (B), (C), (D), and (E). The numerical scales for TrFNs are presented in Table 2 as (F), (G), and (H).

Step 3. Prepare the data of similarity measures to obtain the distance between the largest grade of attribute importance and other attribute importance through the questionnaire survey.

Step 4. Calculate the TFNs of other attribute importance by (8), (10), and (11). Calculate the TrFNs of other attribute importance by (9), (12), and (13).

Step 5. Calculate λ-value and importance of attribute subsets by (2) and (3).

Step 6. Prepare decision matrix for fuzzy integral.

Step 7. Use (z)fuzzy integrals to represent alternative computational scheme for aggregating information by (6).

Step 8. Defuzzify TFNs and TrFNs into crisp numbers by centre of gravity method.

Step 9. Rank the priority of alternatives.

Step 10. Prepare the priority of alternatives obtained by respondents in the questionnaire survey.

Step 11. The spearman rank-order correlation coefficient, the consistency rate of the best alternative, and the consistency rate of the better alternative are used to compare the difference between the priority we calculate and the priority derived from the respondents.

4. Empirical Study

A. *Stimulus Product*

Because of consumer decision making is not a single process, we choose the method of category proposed by Assael [41]. Figure 2 presents a typology of consumer purchasing decisions based on two dimensions: (i) the extent of decision making and (ii) the degree of involvement in the purchase. The notebook computer, the cell phone, the athletic shoes, and the tissue paper are chosen to be our stimulus products.

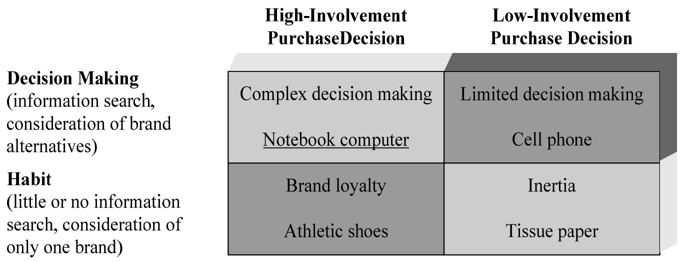

Figure 2. Consumer decision making.

In order to make sure whether the stimulus products were appropriate for each quadrant, a pretest was held. There were 30 college students in this pretest (18 males and 12 females) for respondents. The respondent have to choose which one of descriptions is the most adequate for what they feel when they buy the four stimulus products. The descriptions are "I will carefully gather and compare the information of every brand and then I will make a decision" labeled as the quadrant of complex decision making, "I will probably gather the information of every brand and then I will make a decision" labeled as the quadrant of limited decision making,. "I am a brand loyalty person" labeled as the quadrant of brand loyalty, and "I will take the brand which is the lowest price or on sale into consideration first" labeled as the quadrant of inertia. The question helps us know the respondents' degree of involvement and decision making when they buy the products. The pretest results indicate that the selected stimulus products for each quadrant are very appropriate.

B. *Questionnaire Survey*

The purpose of the pilot study is to acquire the specific attributes and brands for the stimulus products in the formal study. We asked the respondents, which were 60 college students in Taiwan (34 males and 26 females), to write down the attributes and brands which they would consider when buying four stimulus products as possible as they could by means of a thought-listing. With a descriptive statistics, the attributes and, which exceed the number of half respondents consider, will become the designed attributes in the formal study. The top four major brands which the respondents like also will be used in the formal study. The detailed attributes and brands for each stimulus product are as follows. (a) Notebook computer: price, style, stable, hard disk, practicability, weight, and brand image for attributes; Acer, ASUS, HP, and Sony for brands; (b) Cell phone: Price, style, color, stable, function, and design for attributes; Nokia, Motorola, Sony Ericsson, and Samsung for brands; (c) Athletic shoes: Style, price, comfort, and color for attributes; Nike, Adidas, Puma, and Reebok for brands; (d) Tissue paper: Price, softy, and quantity for attributes; Sujay, Mayflower, Tender feelings, and Andante for brands.

The respondents were gathered from 60 college students in Taiwan (36 males and 24 females). Besides investigating respondents' basic information, four main questions were designed.

Question 1: The respondents score every attribute of four brands for stimulus products between 0 and 100. If respondents feel more satisfied with the attributes, they give higher scores to the attributes.

Question 2: The respondents rank the four given brands for stimulus products.

Question 3: The respondents choose the most important attribute for each stimulus product.

Question 4: With the similarity scale, the respondents determine the distance between the most important attribute and other attributes.

C. *Empirical Results*

We use three performance indices, which are Spearman rank-order correlation coefficient, the consistency of the best alternative and the consistency of the better alternative, to observe the results among the different use of TFNs and TrFNs, as well as the different use of distance measures. The category of fuzzy numbers includes five types of TFNs and three types of TrFNs. Furthermore, an importance-assessing method uses distance measures and the similarity scale to obtain other attribute importance. The distance measures include the Tran and Duckstein's distance with $w(\alpha)=1$ and $w(\alpha)=\alpha$, the Euclidean distance.

Table 3 presents the results of the first performance index- Spearman rank-order correlation coefficient. The results indicate that in TFNs, using TFNs (B) and the Tran and Duckstein's distance with $w(\alpha)=1$ for the notebook computer can obtain larger correlation coefficient. In addition, using TFNs (D) and the Tran and Duckstein's distance with $w(\alpha)=1$ for the athlete shoes also can have larger correlation coefficient. However, using TFNs (E) and the Tran and Duckstein's distance with $w(\alpha)=\alpha$ for the cell phone get relatively smaller correlation coefficient. In addition, using TFNs (E) and the Euclidean distance for the cell phone also get relatively smaller correlation coefficient. In TrFNs, using TrFNs (F) and the Euclidean distance for the athlete shoes can obtain relatively larger correlation coefficient. Nevertheless, using TrFNs (G) and the Tran and Duckstein's distance with $w(\alpha)=1$ for the cell phone get relatively smaller correlation coefficient.

Table 3. The results of Spearman rank-order correlation coefficients.

Distance	Product	TFNs (A)	TFNs (B)	TFNs (C)	TFNs (D)	TFNs (E)	Mean
(1)	Computer	0.67 (0.25)	**0.71 (0.25)**	0.67 (0.25)	0.65 (0.26)	0.67 (0.25)	0.67
	Cell phone	0.57 (0.29)	0.52 (0.34)	0.53 (0.34)	0.53 (0.37)	0.50 (0.38)	0.53
	Athletic shoes	0.69 (0.29)	0.66 (0.36)	0.69 (0.29)	**0.71 (0.24)**	0.69 (0.29)	0.69
	Tissue paper	0.52 (0.40)	0.55 (0.36)	0.54 (0.36)	0.52 (0.40)	0.53 (0.40)	0.53
(2)	Computer	0.68 (0.25)	0.68 (0.26)	0.69 (0.25)	0.68 (0.24)	0.68 (0.24)	0.68
	Cell phone	0.51 (0.33)	0.50 (0.36)	0.51 (0.35)	0.54 (0.32)	**0.49 (0.38)**	0.51
	Athletic shoes	0.69 (0.29)	0.69 (0.29)	0.69 (0.29)	0.69 (0.29)	0.66 (0.34)	0.68
	Tissue paper	0.53 (0.40)	0.54 (0.36)	0.54 (0.36)	0.52 (0.40)	0.54 (0.39)	0.53
(3)	Computer	0.61 (0.28)	0.63 (0.33)	0.60 (0.32)	0.61 (0.29)	0.62 (0.28)	0.61
	Cell phone	0.50 (0.34)	0.51 (0.34)	0.51 (0.34)	0.50 (0.33)	**0.49 (0.37)**	0.50
	Athletic shoes	0.60 (0.30)	0.60 (0.36)	0.61 (0.29)	0.61 (0.30)	0.61 (0.29)	0.60
	Tissue paper	0.52 (0.39)	0.53 (0.37)	0.52 (0.38)	0.53 (0.39)	0.52 (0.39)	0.52

(1) Tran and Duckstein's distance with w(α)=1
(2) Tran and Duckstein's distance with w(α)=α
(3) Euclidean distance
Note: the numbers in the brackets are the standard deviation.

Table 3. The results of Spearman rank-order correlation coefficients. (Cont.)

Distance	Product	TrFNs (F)	TrFNs (G)	TrFNs (H)	Mean
(1)	Computer	0.59 (0.42)	0.52 (0.43)	0.53 (0.45)	0.55
	Cell phone	0.47 (0.38)	**0.41 (0.40)**	0.46 (0.41)	0.45
	Athletic shoes	0.59 (0.42)	0.55 (0.44)	0.58 (0.42)	0.57
	Tissue paper	0.56 (0.39)	0.56 (0.39)	0.55 (0.39)	0.56
(2)	Computer	0.64 (0.37)	0.54 (0.43)	0.54 (0.42)	0.57
	Cell phone	0.48 (0.38)	0.45 (0.41)	0.46 (0.40)	0.46
	Athletic shoes	0.62 (0.38)	0.58 (0.42)	0.60 (0.39)	0.60
	Tissue paper	0.55 (0.39)	0.55 (0.39)	0.55 (0.39)	0.55
(3)	Computer	0.68 (0.27)	0.66 (0.29)	0.65 (0.29)	0.66
	Cell phone	0.51 (0.37)	0.47 (0.41)	0.46 (0.42)	0.48
	Athletic shoes	**0.69 (0.29)**	0.66 (0.33)	0.65 (0.34)	0.67
	Tissue paper	0.53 (0.37)	0.55 (0.39)	0.52 (0.39)	0.53

(1) Tran and Duckstein's distance with w(α)=1
(2) Tran and Duckstein's distance with w(α)=α
(3) Euclidean distance
Note: the numbers in the brackets are the standard deviation.

We employ the one-way ANOVA to test whether using different types of TFNs and TrFNs or different distance measures can have unequal correlation coefficient. Table 4 and Table 5 show the results of ANOVA for the conditions under different types of fuzzy numbers and distance measures, respectively. From the two tables, the results indicate that the average value of correlation coefficients for five types of TFNs have no significant differences. In other words, there is no significantly satisfied TFNs to be used in (z)fuzzy measures. It has the same results for TrFNs. Three types of TrFNs have no salient difference in the average value of correlation coefficients.

Table 4. The results of ANOVA for TFNs and TrFNs.

Product	Distance	Five types of TFNs	Three types of TrFNs
Notebook computer	w(α)=1	0.992	0.856
	w(α)=α	1.000	0.391
	Euclidean	0.982	0.992
Cell phone	w(α)=1	0.866	0.919
	w(α)=α	0.964	0.971
	Euclidean	0.999	0.879
Athletic shoes	w(α)=1	0.950	0.795
	w(α)=α	0.984	0.697
	Euclidean	0.985	0.808
Tissue papers	w(α)=1	0.995	1.000
	w(α)=α	0.996	1.000
	Euclidean	1.000	0.946

Note: the numbers in the table are p-value.

Table 5. The results of ANOVA for distance measures.

Fuzzy number	Notebook computer	Cell phone	Athletic shoes	Tissue papers
TFNs (A)	0.990	0.455	0.959	1.000
TFNs (B)	0.776	0.965	0.797	0.973
TFNs (C)	0.720	0.961	0.976	0.910
TFNs (D)	0.851	0.859	0.869	0.987
TFNs (E)	0.908	0.975	0.893	0.970
TrFNs (F)	0.377	0.952	0.505	0.994
TrFNs (G)	0.130	0.937	0.483	1.000
TrFNs (H)	0.213	0.973	0.409	0.932

Note: the numbers in the table are p-value.

On the average, although the average values of correlation coefficient for TFNs and TrFNs have no significant difference, the use of TFNs can get relatively higher performance than the use of TrFNs. As to different distance measures, three kinds of distance measures also have no salient difference in the average values of correlation coefficient. In addition to the viewpoints of fuzzy numbers and distance measures, we find some information from the category of products. In Table 3, any kind of fuzzy numbers and distance measures for the notebook computer and the athlete shoes can obtain higher correlation coefficient than for the cell phone and the tissue paper. In particular, the notebook computer and the athlete shoes belong to the high-involvement products. That is, using (z)fuzzy measures in the high- involvement products can calculate more coincident and accurate priority of alternatives than in the low- involvement products.

The second performance index is the consistency rate of the best alternative. The aim of this index is to find the probability of the best alternative. In many real world problems, decision makers usually have a best selection in their mind when purchasing products. They can use some different characteristics such as attributes, decision criteria, or objectives to rank all alternatives.

Table 6 presents the results of the consistency rate of the best alternative. We only predict a half accuracy of the best alternative by means of (z)fuzzy measures in TFNs and TrFNs for four products. We can easily figure out that even if using different types of TFNs and TrFNs and different distance measures, we also get almost the same consistency rate of the best alternative. We consider the reason which the consistency rate of the best alternative is not high is when decision maker makes a decision, he/she does not usually select the most favorable alternative. The decision maker could compromise for the second choice rather than the best alternative, especially when the number of competent alternative systems is very large and it is impossible to simulate all alternatives. In other words, when each alternative has conflict between attributes, decision criteria, or objectives, the decision maker may select better alternative. Sometimes, the decision makers could prefer one attribute and give it more weight than others, so the decision could be changed the rank of all alternative.

Table 6. The results of the consistency rate of the best alternative.

Distance	Product	TFNs (A)	TFNs (B)	TFNs (C)	TFNs (D)	TFNs (E)
(1)	Computer	0.47 (0.50)	0.48 (0.50)	0.48 (0.50)	0.47 (0.50)	0.48 (0.50)
	Cell phone	0.35 (0.48)	0.31 (0.47)	0.35 (0.48)	0.37 (0.49)	0.37 (0.49)
	Athletic shoes	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)
	Tissue paper	0.48 (0.50)	0.50 (0.50)	0.50 (0.50)	0.48 (0.50)	0.48 (0.50)
(2)	Computer	0.47 (0.50)	0.45 (0.50)	0.47 (0.50)	0.48 (0.50)	0.48 (0.50)
	Cell phone	0.32 (0.48)	0.32 (0.47)	0.32 (0.47)	0.35 (0.48)	0.30 (0.46)
	Athletic shoes	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)
	Tissue paper	0.48 (0.50)	0.50 (0.50)	0.50 (0.50)	0.48 (0.50)	0.47 (0.50)
(3)	Computer	0.48 (0.50)	0.48 (0.50)	0.48 (0.50)	0.48 (0.50)	0.48 (0.50)
	Cell phone	0.33 (0.48)	0.33 (0.48)	0.33 (0.48)	0.33 (0.48)	0.33 (0.46)
	Athletic shoes	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)	0.48 (0.50)	0.48 (0.50)
	Tissue paper	0.50 (0.50)	0.52 (0.50)	0.50 (0.50)	0.50 (0.50)	0.48 (0.50)

Distance	Product	TrFNs (F)	TrFNs (G)	TrFNs (H)
(1)	Computer	0.42 (0.49)	0.35 (0.48)	0.40 (0.49)
	Cell phone	0.33 (0.48)	0.33 (0.48)	0.33 (0.48)
	Athletic shoes	0.45 (0.50)	0.45 (0.50)	0.45 (0.50)
	Tissue paper	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)
(2)	Computer	0.42 (0.49)	0.38 (0.49)	0.40 (0.49)
	Cell phone	0.33 (0.48)	0.33 (0.48)	0.33 (0.48)
	Athletic shoes	0.45 (0.50)	0.45 (0.50)	0.45 (0.50)
	Tissue paper	0.50 (0.50)	0.50 (0.50)	0.50 (0.50)
(3)	Computer	0.48 (0.49)	0.48 (0.49)	0.48 (0.49)
	Cell phone	0.33 (0.48)	0.33 (0.48)	0.33 (0.48)
	Athletic shoes	0.48 (0.50)	0.48 (0.50)	0.48 (0.50)
	Tissue paper	0.50 (0.50)	0.52 (0.50)	0.50 (0.50)

(1) Tran and Duckstein's distance with w(α)=1
(2) Tran and Duckstein's distance with w(α)=α
(3) Euclidean distance
Note: the numbers in the brackets are the standard deviation.

The third performance index is the consistency rate of the better alternative. Table 7 shows the results. Although the rate is also not very high, it is higher than the consistency rate of the best alternative. In spite of using different types of fuzzy numbers or distance measures, we also gain similar predicted performance without salient difference.

Table 7. The results of the consistency rate of the better alternative.

Distance	Product	TFNs (A)	TFNs (B)	TFNs (C)	TFNs (D)	TFNs (E)
(1)	Computer	0.67 (0.27)	0.68 (0.27)	0.66 (0.27)	0.67 (0.27)	0.68 (0.27)
	Cell phone	0.51 (0.30)	0.51 (0.30)	0.51 (0.30)	0.51 (0.30)	0.51 (0.30)
	Athletic shoes	0.71 (0.31)	0.70 (0.65)	0.71 (0.31)	0.72 (0.29)	0.71 (0.31)
	Tissue paper	0.65 (0.31)	0.65 (0.31)	0.65 (0.31)	0.65 (0.31)	0.66 (0.31)
(2)	Computer	0.68 (0.27)	0.68 (0.27)	0.67 (0.27)	0.68 (0.27)	0.68 (0.27)
	Cell phone	0.51 (0.30)	0.49 (0.30)	0.50 (0.30)	0.51 (0.30)	0.51 (0.30)
	Athletic shoes	0.71 (0.31)	0.71 (0.65)	0.71 (0.31)	0.71 (0.29)	0.71 (0.31)
	Tissue paper	0.66 (0.31)	0.65 (0.31)	0.65 (0.31)	0.66 (0.31)	0.67 (0.31)
(3)	Computer	0.68 (0.27)	0.68 (0.27)	0.67 (0.27)	0.68 (0.27)	0.68 (0.27)
	Cell phone	0.51 (0.30)	0.51 (0.30)	0.51 (0.30)	0.51 (0.30)	0.52 (0.30)
	Athletic shoes	0.72 (0.31)	0.70 (0.65)	0.71 (0.31)	0.72 (0.29)	0.71 (0.31)
	Tissue paper	0.65 (0.31)	0.66 (0.31)	0.65 (0.31)	0.65 (0.31)	0.66 (0.31)

Distance	Product	TrFNs (F)	TrFNs (G)	TrFNs (H)
(1)	Computer	0.66 (0.27)	0.65 (0.27)	0.65 (0.27)
	Cell phone	0.53 (0.29)	0.54 (0.28)	0.54 (0.28)
	Athletic shoes	0.70 (0.31)	0.68 (0.33)	0.66 (0.33)
	Tissue paper	0.66 (0.31)	0.66 (0.31)	0.66 (0.31)
(2)	Computer	0.69 (0.27)	0.65 (0.27)	0.65 (0.27)
	Cell phone	0.53 (0.29)	0.55 (0.28)	0.54 (0.28)
	Athletic shoes	0.71 (0.31)	0.67 (0.33)	0.70 (0.33)
	Tissue paper	0.66 (0.31)	0.66 (0.31)	0.66 (0.31)
(3)	Computer	0.68 (0.27)	0.68 (0.27)	0.68 (0.27)
	Cell phone	0.51 (0.29)	0.53 (0.28)	0.52 (0.28)
	Athletic shoes	0.71 (0.31)	0.69 (0.33)	0.70 (0.33)
	Tissue paper	0.66 (0.31)	0.66 (0.31)	0.66 (0.31)

(1) Tran and Duckstein's distance with w(α)=1
(2) Tran and Duckstein's distance with w(α)=α
(3) Euclidean distance
Note: the numbers in the brackets are the standard deviation.

5. Conclusions

The multiple attribute decision-making problems are always happening in our life. We anticipate finding whether there is better method to obtain and enhance the accuracy of measurements. In this study, we introduce and compare diverse types of TFNs and TrFNs in (z)fuzzy measures. Moreover, in order to avoid the positive leniency in the investigation, we employ an importance-assessing method which use distance measures and the similarity scale to calculate the importance of attribute. The (z)fuzzy measures combining the Choquet integral can calculate the priority of alternatives. We use three performance indices to examine the difference among the different fuzzy numbers which are inclusive of five types of TFNs and three types of TrFNs. No matter what the Spearman rank-order correlation coefficient, the consistency rate of the best alternative, and the consistency rate of the better alternative analyze, all of the results indicate that the selection of fuzzy numbers and distance measures seems not very essential because they have no significant difference in the performance according to the ANOVA. Since the different types of fuzzy numbers and distance measures receive the similar results, we suggest that (z)fuzzy measures can adopt simple form of fuzzy numbers like the TFNs and the distance measures can choose the easy equation like the Euclidean distance.

However, we can obtain the relatively good performance, but not absolutely and significantly. For example, using TFNs (B), proposed by Liou and Wang [23], and the Tran and Duckstein's distance with w(α)=1 for the notebook computer can acquire better predicted ability. Using TFNs (D), proposed by Lee et al. [25], and the Tran and Duckstein's distance with w(α)=1 for the athlete shoes can also obtain relatively higher performance. In TrFNs, using TrFNs (F), proposed by Ding et al. [31], and the Euclidean distance for the athlete shoes can gain much better effects. On the average, the TFNs receive a little bit better performance than the TrFNs.

A typology of consumer purchasing decisions introduced by Assael [41] is applied in this study because consumers have unequal decision-making process based on the category of products. According to the three performance indices, all results obviously show that using (z)fuzzy measures in any TFNs and TrFNs for the notebook computer and the athlete shoes which belong to the high-involvement products can have higher effects than low-involvement products. It is because highly involved consumers may execute an extensive decision-making process that consumers possess the broad choice process behaviors in order to maximize their expected satisfaction such as brands comparisons, time spending, and multiple attributes using [42, 43]. In other words, consumers would evaluate each attribute in detail under high involvement and consider that each attribute is unique and different. Therefore, the priority of alternatives is highly correlated with the evaluative attributes. As to the low-involvement products, consumers usually focus on one or two attributes and regard the rest attributes as the equivalent evaluation. Due to the reason that the most importance of attributes is equal for low-involvement products, the priority of alternatives would usually depends on the most importance attribute instead of an overall evaluation. This is why the performance indices in the low-involvement products do not gain good results.

To sum up, according to an empirical study, we suggest that since using different types of fuzzy numbers have the same results, the selection of TFNs is better than TrFNs on account of the simple form. Concerning the distance measures, the Euclidean distance is preferred.

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