# A Robust Fuzzy Control Approach to Stabilization of Nonlinear Time-delay Systems with Saturating Inputs

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## Abstract

**This paper deals with the stabilization of uncertain nonlinear time-delay systems subject to input saturation. A nonlinear time-delay system is first represented by Takagi-Sugeno (T-S) fuzzy model, a set of fuzzy implications which are used to characterize local dynamics with actuator saturation. Based on a delay independent stability analysis, a domain of attraction in which the admissible initial states are ensured to converge asymptotically to the origin is determined. The fuzzy control law is then developed to maximize the estimation of this domain. The derived conditions are formulated in terms of linear matrix inequalities (LMIs) so that the synthesis of fuzzy controller and the estimation of stability domain can be carried out efficiently. Moreover, a robust stabilization for systems with parameter uncertainties, which are time-varying and norm-bounded, is discussed. Numerical examples of truck-trailer control are provided to demonstrate the effectiveness of the design.**

***Keywords: T-S fuzzy model, robust fuzzy control, domain of attraction.***

## 1. Introduction

The PID controllers have been widely used in the industry because of their simple structures and inexpensive cost on design. Given the three-term functionality, PID control covers treatment to both transient and steady-state responses of practical engineering processes. More than 90% of industrial controllers are of PID type, appeared in a continuous-time or a discrete-time version. In spite of their extensive applications, PID controllers may suffer considerable loss of performance due to integrator windup when used in a system with actuator saturation [1]. The actuator saturation is very common because the actuator cannot deliver unlimited energy to physical plants. The effect of this saturation can range from degradation of system performance to closed-loop instability.

Stabilization of systems with input saturation has drawn much research attention in the past years. Generally, a low gain control strategy is implemented to decrease the output of controller. However, the low-gain controller usually has low levels of performance. The analysis and synthesis of control systems with actuator saturation nonlinearity can be classified into two main approaches. The first one is directly to take the effect of nonlinear saturation in designing controller [2-4]. The second method is developed on the assumption that a controller has been previously designed to satisfy some specifications. Then, an anti-windup loop is included to mitigate the influence of saturation on the system stability and performance [5-7].

When designing controller for the system with input saturation, global stability is of main concern. Several results are given in [8, 9]. However, the stability in their studies is valid only for open-loop stable systems [5]. Instead, local stability of closed-loop saturated system is investigated. That is, a region of attraction, in which the admissible initial states will converge asymptotically to the origin in the presence of saturated input, is determined using designed control strategy.

In [2], the domain of attraction is verified by applying the conventional circle and Popov criteria. Burgat and Tarbouriech [6] introduced an intelligent anti-windup compensator, a linear system with particular saturating control law, for estimation of region of local stability. Moreover, they extended this notion to synthesize a nonlinear state feedback controller to deal with output tracking problems [7], where an anti-windup gain is devised to maximize the region of attraction [5]. Another approach for estimating the domain of attraction is given by Hu et al. [3], where an auxiliary feedback matrix is utilized to relax the conservatism in the estimation. Similar works based on Hu's method are addressed in [10, 11]. Notably, the plant in the aforementioned literatures is linear. As to stabilization of nonlinear systems with constrained input, the existing studies devoted to this subject are very limited. In [12], based on input-output linearization technique, an approach inte-

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grating nonlinear internal model control with anti-windup loop is developed for uncertain nonlinear systems with input saturation. Moreover, Su [13] employed sliding mode control method with an anti-windup compensation to stabilize a class of nonlinear cascade systems.

Time delay is often encountered in many practical systems such as chemical processes, rolling mill, and long transmission lines in pneumatic or hydraulic. Time delay usually is a source of instability and can decrease system performance. Therefore, designing control systems with time-delays is very challenging. To deal with the stabilization of linear time-delay systems with input saturation, an anti-windup control approach based on a modified sector condition is proposed in [14]. The largest stability region can be estimated via the anti-windup controller.

The objective of this paper is to design a fuzzy control law for stabilization of nonlinear time-delay systems with saturating actuators. The nonlinear system is first expressed as Takagi-Sugeno (T-S) fuzzy model [15]. Given a properly chosen Lyapunov-Krasovskii functional candidate, sufficient conditions for ensuring asymptotic stability of closed-loop system in a region are derived. In addition, the fuzzy controller is developed to maximize the region of attraction. The obtained result is then transformed into a convex problem that can be efficiently solved by using the linear matrix inequality (LMI) technique. Extending the design principle, we consider the problem of stabilization of uncertain nonlinear systems with time delay and input saturation, where the uncertainties are assumed to be time-varying and norm-bounded. Finally, computer simulations on truck-trailer control are implemented to demonstrate the effectiveness of proposed method.

*Notations*: $I_n$ denotes the identity matrix in $R^{n\times n}$. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ stand for the maximal and minimal eigenvalues of matrix $P$, respectively. $b_\tau = b([-\tau,0],R^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau,0]$ to $R^n$ with topology of uniform convergence. $\|\phi\|_c = \sup_{-\tau\le t\le 0}\|\phi(t)\|$ presents norm value of a function $\phi\in b_\tau$. $b_\tau^v$ denotes a set defined by $b_\tau^v = \{\phi\in b_\tau ; \|\phi\|_c < v, v>0\}$.

## 2. Preliminary

Consider a nonlinear time-delay system described by

$$\begin{aligned}&\dot{x}(t) = f(x(t), x(t-\tau), v(t)),\\ &x_{t_0}(t) = \varphi(t), t\in[-\tau_0, 0],\end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state vector, $v(t)\in R^m$ is the constrained control input, for a given $t\ge t_0$, $x_t(\cdot)$ denotes the restriction of $x(\cdot)$ to the interval $[t-\tau, t]$ translated to $[-\tau,0]$, i.e. $x_t(\theta)=x(t+\theta)$ for $\theta\in[-\tau,0]$, $\tau(t)$ is a time-varying delay and satisfies $\tau(t)\le\tau_0$, and $\tau_0$ is a real positive constant presenting the upper bound of time-varying delay. It is further assumed that $\dot{\tau}(t)\le\beta<1$, and $\beta$ is a known constant. The constrained input $v(t)$ is expressed as $v(t)=\sigma(u(t))$, where $u(t)$ is the designed controller and $\sigma: R^m\to R^m$ denotes a standard saturation function as

$$\sigma(u) = [\sigma(u_1) \quad \sigma(u_2) \quad \cdots \quad \sigma(u_m)]^T$$

with $\sigma(u_i) = sign(u_i)\min\{1, |u_i|\}$. Notably, the notation $\sigma$ is slightly abused for representing both a scalar function and a vector function. Also, without loss of generality, this study assumes that the saturation level is equal to one. For a non-unity saturation, the level of saturation can be absorbed into the input, $\hat{v}=vU, \hat{u}=U^{-1}u$, where $U = diag(u_{\max,i})$ and $u_{\max,i}$ is the saturation amplitude of the *i*th input.

The stability analysis for time-delay systems is generally classified into two categories, namely, delay-independent criteria and delay-dependent criteria. The delay-independent criteria allow a large time delay and apply to the systems in which no *a priori* knowledge of delay time is available [16]. In contrast, the delay-dependent criteria include information on the delay and can handle the system whose stability depends on the size of the time delay [17, 18]. This investigation is devoted to the delay-independent stabilization.

To describe nonlinear systems, the T-S fuzzy model [15] is regarded as an effective tool because it can approximate a complex or ill defined system on a compact set to arbitrary accuracy. Based on this approach, the T-S fuzzy model for nonlinear time-delay system (1) is expressed as

*Plant rule* $i$:

If $z_1(t)$ is $M_1^i$ and $\cdots$ and $z_j(t)$ is $M_j^i$

$$\begin{aligned}\text{then } &\dot{x}(t) = A_{1i}x(t) + A_{2i}x(t-\tau(t)) + B_i v(t),\\ &x_{t_0}(t) = \varphi(t),\ t\in[-\tau_0,0], i=1,2,\cdots,r,\end{aligned}$$

where $A_{1i}$, $A_{2i}$, and $B_i$ are system matrices of compatible dimensions, $M_j^i$ is the fuzzy set, $z_i(t)$ is the premise variable of fuzzy implication, $r$ is the number of the fuzzy rules. It is assumed that the premise variables do not depend on the input $v(t)$.

The center of gravity defuzzification yields the output of the fuzzy system

$$\dot{x}(t) = \sum_{i=1}^{r} w_i(z)[A_{1i}x(t) + A_{2i}x(t-\tau(t)) + B_i v(t)] \Big/ \sum_{i=1}^{r} w_i(z), \tag{2}$$

where $w_i(z) = \prod_{l=1}^{j} M_l^i(z_l)$ and $M_l^i(z_l)$ denotes the grade of membership function $M_l^i$ corresponding to $z_l(t)$. Let

$$\mu_i(z) = w_i(z) \Big/ \sum_{l=1}^{r} w_l(z). \tag{3}$$

Then the overall fuzzy system can be presented by

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z)[A_{1i}x(t) + A_{2i}x(t-\tau(t)) + B_i v(t)]. \tag{4}$$

Notably, $\sum_{i=1}^{r} \mu_i(z) = 1$ and $\mu_i(z) \ge 0$ for $i = 1, 2, \cdots, r$. Moreover, the fuzzy controller, according to the parallel distributed compensation (PDC), is given by

Control *rule* $i$: If $z_1(t)$ is $M_1^i$ and $\cdots$ and $z_j(t)$ is $M_j^i$
then $u(t) = F_i x(t), i = 1, 2, \cdots, r,$

or, equivalently

$$u(t) = \sum_{i=1}^{r} \mu_i F_i x(t) = \tilde{F}x(t). \tag{5}$$

The global stability conditions of nonlinear time-delay system without actuator saturation are derived in [16], which is based on the delay-independent analysis. In the presence of input saturation, the conditions cannot be applied. The designing work is instead to consider the local stabilization. For the nonlinear time-delay system (4), given the assumption $\|x_t(\theta)\|_c < v,\ v > 0$, the domain of attraction is defined as the region containing all initial conditions $x_t(\theta) \in b_\tau^v$ such that the corresponding state trajectory will converge asymptotically to the origin. Since determination of the exact domain of attraction is practically impossible [14], a problem of interest is to estimate a set of the admissible initial conditions, $\Omega_0 = \{x_t \in b_t^v : \|x_t(\theta)\|_c^2 < \delta\}$, such that the asymptotic stability is ensured. Therefore, the objective of this study is to design the fuzzy controller (5) to maximize the domain of attraction. Notably, similar studies in this field have been reported in [11, 12, 14, 19], where the analyzed system are linear time-invariant, output-delayed, or in a T-S fuzzy model, respectively. The proposed method can be viewed as an extension of their works.

## 3. Stability analysis and design of fuzzy systems

Before proceeding the derivation, some issues related to the development of the proposed method need to be addressed.

Let the *i*th row of a matrix $F \in R^{n\times m}$ be $f_i$. Define the symmetric polyhedron

$$\mathcal{L}(F) = \{x \in R^n : |f_i x| \le 1, i = 1, 2, \cdots, m\}.$$

It can be easily verified that $x \in \bigcap_{l=1}^{r} \mathcal{L}(F_l)$ implies $x \in \mathcal{L}(\tilde{F})$. In particular, $\mathcal{L}(\tilde{F})$ denotes the region where the control $u$ does not saturate. That is, if $x \in \mathcal{L}(\tilde{F})$, then the closed-loop dynamics of (4) become

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j [(A_{1i} + B_i F_j)x(t) + A_{2i}x(t-\tau(t))]. \tag{6}$$

In this case, the stability can be ensured by a properly chosen control law [16].

Let $\mathcal{M}$ be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. For example, suppose $m = 2$, then

$$\mathcal{M} = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

Consequently there are $2^m$ elements in $\mathcal{M}$. Denote each element of $\mathcal{M}$ as $E_i, i = 1, \cdots, m$. Let $E_i^- = I - E_i$, obviously, $E_i^- \in \mathcal{M}$

*Lemma 1* [20]: Given $F, H \in R^{n\times m}$ and $x \in R^n$, if $x \in \mathcal{L}(H)$, then $\sigma(Fx) \in co\ \{E_i F x + E_i^- H x,\ i = 1, \cdots, 2^m\}$, where *co* denotes the convex hull. That is, $\sigma(Fx)$ can be expressed as

$$\sigma(Fx) = \sum_{i=1}^{2^m} \eta_i (E_i F + E_i^- H)x \tag{7}$$

where $0 \le \eta_i \le 1, \sum_{i=1}^{2^m} \eta_i = 1.$

Choose $F$ and $H$ as $F = \tilde{F} = \sum_{l=1}^{r} \mu_l F_l$ and $H = \sum_{l=1}^{r} \mu_l H_l$, respectively. By Lemma 1, the fuzzy control law (5) can be written as

$$\begin{aligned}\sigma\Big(\sum_{l=1}^{r}\mu_l F_l x\Big) &= \sum_{s=1}^{2^m}\eta_s\left(\sum_{l=1}^{r}\mu_l (E_s F_l + E_s^- H_l)x\right)\\ &= \sum_{s=1}^{2^m}\sum_{l=1}^{r}\eta_s\mu_l (E_s F_l + E_s^- H_l)x.\end{aligned} \tag{8}$$

Notably, the condition $x \in \bigcap_{l=1}^{r}\mathcal{L}(H_l)$ also implies $x \in \mathcal{L}(\tilde{H})$. Given a positive definite matrix $P \in R^{n\times n}$ and a positive scalar $\rho$, an ellipsoid is defined as $\mathcal{E}(P,\rho) = \{x \in R^n : x^T P x \le \rho\}$. Moreover, $\mathcal{E}(P,\rho)$ is inside $\mathcal{L}(\tilde{F})$ if and only if the following conditions are satisfied [19]:

$$f_{li}(P/\rho)^{-1} f_{li}^T \le 1, i = 1,\cdots m, l = 1,\cdots,r$$

where $f_{li}$ is the $i$th row of $F_l$.

*Lemma 2* [17]: For any $x, y \in R^n$ and any positive definite matrix $S \in R^{n\times n}$, the following inequality holds:

$$2x^T y = x^T y + y^T x \le x^T S^{-1} x + y^T S y.$$

*Lemma 3* [21]: Let $A$, $D$, $E$, and $F$ be real matrices of appropriate dimensions with $\|F\| \le 1$. Then, for any positive definite matrix $P$ and any scalar $\varepsilon > 0$ satisfying $\varepsilon I - EPE^T > 0$ the following inequalities hold:

(a) $DFE + E^T F^T D^T \le \varepsilon^{-1} DD^T + \varepsilon E^T E$, for any scalar $\varepsilon > 0$;

(b) $$\begin{aligned}(A + DFE)P(A + DFE)^T &\le APA^T\\ &+ APE^T(\varepsilon I - EPE^T)^{-1} EPA^T + \varepsilon DD^T.\end{aligned}$$

Substituting (8) into (4) yields the closed-loop system dynamics

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{2^m}\mu_i\mu_j\eta_s[A_{1i}x(t) + A_{2i}x(t-\tau)\\ &\quad + B_i(E_sF_j + E_s^- H_j)x(t)]\\ &= \sum_{s=1}^{2^m}\eta_s\{\sum_{i=1}^{r}\mu_i^2[A_{1i}x(t) + A_{2i}x(t-\tau)\\ &\quad + B_i(E_sF_i + E_s^- H_i)x(t)]\\ &\quad + \sum_{i<j}\mu_i\mu_j[A_{1i}x(t) + A_{2i}x(t-\tau)\\ &\quad + B_i(E_sF_j + E_s^- H_j)x(t)\\ &\quad + A_{1j}x(t) + A_{2j}x(t-\tau)\\ &\quad + B_j(E_sF_i + E_s^- H_i)x(t)]\},\end{aligned} \tag{9}$$

where $\sum_{i<j}$ denotes $\sum_{i=1}^{r-1}\sum_{i<j}^{r}$.

Define $G_{ijs} = A_{1i} + B_i(E_sF_j + E_s^- H_j)$. Then

$$\begin{aligned}\dot{x}(t) &= \sum_{s=1}^{2^m}\eta_s\{\sum_{i=1}^{r}\mu_i^2[G_{iis}x(t) + A_{2i}x(t-\tau)]\\ &\quad + \sum_{i<j}\mu_i\mu_j[(G_{ijs} + G_{jis})x(t)\\ &\quad + (A_{2i} + A_{2j})x(t-\tau)]\}\end{aligned} \tag{10}$$

For notational convenience, in the following derivation a function of *t* will be succinctly expressed by dropping its argument. The presented approach is derived as follows.

**Theorem 1.** Consider the time-delay fuzzy system described in (4). Suppose that the positive definite matrices *P*, *S*, auxiliary matrices $H_i, i = 1,\cdots,r$, and a scalar $\rho > 0$ exist, and the following conditions hold:

$$\begin{aligned}&G_{iis}^T P + PG_{iis} + PA_{2i}S^{-1}A_{2i}^T P + S/(1-\beta) < 0,\\ &i = 1,\cdots,r, 1 \le s \le 2^m\end{aligned}; \tag{11}$$

$$\begin{aligned}&(G_{ijs} + G_{jis})^T P + P(G_{ijs} + G_{jis})\\ &+ P(A_{2i} + A_{2j})(2S)^{-1}(A_{2i} + A_{2j})^T P\\ &+ 2S/(1-\beta) < 0,\ 1 \le i < j \le r, 1 \le s \le 2^m,\end{aligned} \tag{12}$$

and $\mathcal{E}(P,\rho) \subset \bigcap_{i=1}^{r}\mathcal{L}(H_i)$. Then, for all initial state $x_t(\theta) \in \Omega_0$ with

$$\delta = (1-\beta)\rho/((1-\beta)\lambda_{\max}(P) + \tau_0\lambda_{\max}(S)), \tag{13}$$

the closed-loop system is guaranteed to be asymptotically stable by the fuzzy control law (8).

**Proof:** Choose the Lyapunov-Krasovskii functional as

$$V(x_t) = x^T(t)Px(t) + \int_{t-\tau(t)}^{t} x^T(\sigma)Sx(\sigma)d\sigma/(1-\beta). \tag{14}$$

Then

$$\begin{aligned}\dot{V}(x_t) &= 2x^T P\dot{x} + x^T Sx/(1-\beta)\\ &\quad - (1-\dot{\tau}(t))x^T(t-\tau)Sx(t-\tau)/(1-\beta)\\ &\le 2x^T P\dot{x} + x^T Sx/(1-\beta) - x^T(t-\tau)Sx(t-\tau).\end{aligned} \tag{15}$$

Substituting (10) into (15) gives

$$\dot{V}(x_t) \le \sum_{s=1}^{2^m}\eta_s\left[\sum_{i=1}^{r}\mu_i^2 x^T(G_{iis}^T P + PG_{iis} + S/(1-\beta))x\right.$$

$$
\begin{aligned}
&+\sum_{i<j}\mu_i\mu_j x^T((G_{ijs}+G_{jis})^T P+P(G_{ijs}+G_{jis})\\
&+2S/(1-\beta))x+\sum_{i=1}^{r}\mu_i^2\, 2x^T PA_{2i}x(t-\tau)\\
&\left.+\sum_{i<j}\mu_i\mu_j\, 2x^T P(A_{2i}+A_{2j})x(t-\tau)\right]\\
&-x^T(t-\tau)Sx(t-\tau).
\end{aligned}
\tag{16}
$$

By Lemma 2, the following relations hold

$$
\begin{aligned}
&2x^T PA_{2i}x(t-\tau)\le x^T PA_{2i}S^{-1}A_{2i}^T Px\\
&+x^T(t-\tau)Sx(t-\tau),\\
&2x^T P(A_{2i}+A_{2j})x(t-\tau)\\
&\le x^T P(A_{2i}+A_{2j})(2S)^{-1}(A_{2i}+A_{2j})^T Px\\
&+x^T(t-\tau)2Sx(t-\tau).
\end{aligned}
\tag{17}
$$

Then

$$
\begin{aligned}
\dot V(x_t)\le \sum_{s=1}^{2^m}\eta_s\Bigg[&\sum_{i=1}^{r}\mu_i^2 x^T(G_{iis}^T P+PG_{iis}\\
&+PA_{2i}S^{-1}A_{2i}^T P+S/(1-\beta))x\\
&+\sum_{i<j}\mu_i\mu_j x^T((G_{ijs}+G_{jis})^T P\\
&+P(G_{ijs}+G_{jis})+2S/(1-\beta)\\
&+P(A_{2i}+A_{2j})(2S)^{-1}(A_{2i}+A_{2j})^T P)x\Bigg].
\end{aligned}
\tag{18}
$$

If the conditions (11) and (12) are satisfied, then $\dot V(x_t)<0$. This implies $x^T Px\le V(x_t)\le V(x_{t_0})\le\kappa\|x_t\|_c^2\le\rho$ for a positive scalar $\rho$, where $\kappa=\lambda_{\max}(P)+\lambda_{\max}(S)\tau_0/(1-\beta)$. Hence, it follows that $\delta=\rho/\kappa$ as presented in (13). For any $x_t(\theta)\in\Omega_0,\theta\in[-\tau_0,0]$, the trajectory is confined in $\mathcal{E}(P,\rho)$. Based on Krasovskii Theorem [22], one may conclude that for any initial condition belonging to $\Omega_0$ the system dynamics (10) is asymptotically stable. This completes the proof. □

Notably, the condition $\mathcal{E}(P,\rho)\subset\bigcap_{i=1}^{r}\mathcal{L}(H_i)$ provides a basis for the stability analysis and the convex expression of $\sigma(u)$. When the input is unsaturated, $\mathcal{L}(\tilde F)$ turns out to be a domain of attraction, given a properly designed control. This kind of estimation, $\mathcal{L}(\tilde F)$, is conservative, particularly for the saturating situation. To overcome the deficiency, a less conservative algorithm for estimation of the region of attraction is introduced in [3, 19], where an auxiliary matrix $H$ is utilized to determine if the devised ellipsoid is contractive. It is proved that the estimated region of attraction can be enlarged.

In order to apply LMI optimization technique for synthesizing fuzzy controller, the stability constrains in Theorem 1 are transformed into a linear matrix inequality problem. By Schur complement, the conditions (11) and (12) are equivalent to the following forms:

$$
\begin{bmatrix}\mathbf{T}_{ii} & X\\ * & -(1-\beta)W\end{bmatrix}<0,1\le i\le r,1\le s\le 2^m,
\tag{19}
$$

$$
\begin{bmatrix}\mathbf{T}_{ij} & X\\ * & -\frac{1}{2}(1-\beta)W\end{bmatrix}<0,1\le i<j\le r,1\le s\le 2^m,
\tag{20}
$$

where

$$
\begin{aligned}
\mathbf{T}_{ii}&=XA_{1i}^T+A_{1i}X+M_i^T E_s B_i^T+B_iE_sM_i+N_i^T E_s^- B_i^T\\
&\quad+B_iE_s^-N_i+A_{2i}WA_{2i},\\
\mathbf{T}_{ij}&=XA_{1i}^T+A_{1i}X+M_j^T E_s B_i^T+B_iE_sM_j+N_j^T E_s^- B_i^T\\
&\quad+B_iE_s^-N_j\\
&\quad+XA_{1j}^T+A_{1j}X+M_i^T E_s B_j^T+B_jE_sM_i+N_i^T E_s^- B_j^T\\
&\quad+B_jE_s^-N_i+(A_{2i}+A_{2j})W(A_{2i}+A_{2j})^T/2,
\end{aligned}
$$

$X=P^{-1}$, $M_i=F_iX$, $N_i=H_iX$, $W=S^{-1}$, and * denotes the transposed elements in the symmetric positions. From the S-procedure, the condition that the ellipsoid $\mathcal{E}(P,\rho)$ belongs to $\bigcap_{i=1}^{r}\mathcal{L}(H_i)$ can be depicted as

$$
\begin{bmatrix}X & Z_{li}^T\\ * & \gamma\end{bmatrix}\ge 0,1\le i\le m,1\le l\le r,
\tag{21}
$$

where $\gamma=\rho^{-1}$, $Z_{li}$ is the $i$th row of $Z_l$ and $Z_l=H_lX$. Recall that objective of this study is to design a fuzzy controller to maximize the domain of attraction. To this end, an optimization criterion is introduced in the context of Theorem 1. Since $\delta$ presents a measured size of the domain of stability, smaller values of $\lambda_{\max}(P)$ and $\lambda_{\max}(S)$ will provide a larger value of $\delta$ or, equivalently, a larger domain of attraction. Hence, the optimization problem is stated as follows [14]:

$$\min\{c_1m_1+c_2m_2\}$$

*subject to* the constraints (19)-(21),

$$
\begin{bmatrix}m_1I_n & I_n\\ * & X\end{bmatrix}\ge 0\text{ ,and}\begin{bmatrix}m_2I_n & I_n\\ * & W\end{bmatrix}\ge 0,
\tag{22}
$$

where $c_1$ and $c_2$ are tuning factors.

Motivated by the result of Theorem 1, the design principle is extended to the T-S fuzzy system with uncertainties. Consider the T-S fuzzy model

$$\begin{aligned}&R^l: \text{If } z_1(t) \text{ is } M_1^l \text{ and} \cdots \text{and } z_j(t) \text{ is } M_j^l\\ &\text{then } \dot{x}(t) = (A_{1l} + \Delta A_{1l})x(t) + (A_{2l} + \Delta A_{2l})x(t-\tau(t))\\ &\qquad + (B_l + \Delta B_l)v(t),\\ &\qquad x_{t_0}(t) = \varphi(t),\ t \in [-\tau_0, 0], l = 1, 2, \cdots, r.\end{aligned}$$

where $\Delta A_{1l}, \Delta A_{2l}$, and $\Delta B_l$ are corresponding uncertainties that represent parameter variations. The dynamics of the fuzzy system is expressed as

$$\begin{aligned}\dot{x}(t) = \sum_{l=1}^{r} \mu_l(z)[(A_{1l} + \Delta A_{1l})x(t)\\ + (A_{2l} + \Delta A_{2l})x(t-\tau(t)) + (B_l + \Delta B_l)v(t)].\end{aligned} \tag{23}$$

In addition, the uncertainties are expressed in the form of

$$[\Delta A_{1l} \quad \Delta A_{2l} \quad \Delta B_l] = D_l \Xi_l(t)[E_{1l} \quad E_{2l} \quad E_{bl}],$$

where $D_l$, $E_{1l}$, $E_{2l}$, and $E_{bl}$ are known real matrices of appropriate dimensions and $\Xi_l(t)$ denotes unknown time-varying matrix functions satisfying $\Xi_l^T(t)\Xi_l(t) \le I$. With the fuzzy control law (8), the closed-loop system can be written as

$$\begin{aligned}\dot{x}(t) &= \sum_{s=1}^{2^m} \eta_s \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j [(A_{1i} + \Delta A_{1i})x(t)\\ &\quad + (A_{2i} + \Delta A_{2i})x(t-\tau)\\ &\quad + (B_i + \Delta B_i)(E_s F_j + E_s^{-} H_j)x(t)]\\ &= \sum_{s=1}^{2^m} \eta_s \left\{ \sum_{i=1}^{r} \mu_i^2 [(G_{iis} + \Delta A_i + \Delta B_i \overline{E}_{si})x(t) \right.\\ &\quad + (A_{2i} + \Delta A_{2i})x(t-\tau)]\\ &\quad + \sum_{i<j} \mu_i \mu_j [(G_{ijs} + G_{jis} + \Delta A_{1i} + \Delta A_{1j}\\ &\quad + \Delta B_i \overline{E}_{sj} + \Delta B_j \overline{E}_{si})x(t)\\ &\quad \left. + (A_{2i} + A_{2j} + \Delta A_{2i} + \Delta A_{2j})x(t-\tau)] \right\},\end{aligned} \tag{24}$$

where $\overline{E}_{si} = E_s F_i + E_s H_i$.

**Theorem 2.** For the uncertain fuzzy system described in (23), suppose that positive definite matrices *P*, *S*, auxiliary matrices $H_i, i = 1, \cdots, r,$ and positive scalars $\rho$, $\varepsilon_{0i}, \varepsilon_{1i}$, and $\varepsilon_{2i}$ exist such that the following conditions hold:

$$\begin{aligned}&G_{iis}^T P + P G_{iis} + (\varepsilon_{0i} + \varepsilon_{1i} + \varepsilon_{2i}) P D_i D_i^T P\\ &+ \varepsilon_{1i}^{-1} E_{1i}^T E_{1i} + \varepsilon_{2i}^{-1} \overline{E}_{si}^T \overline{E}_{si} + P \Gamma_i P\\ &+ S/(1-\beta) < 0,\ i = 1, \cdots, r, s = 1, \cdots, 2^m;\end{aligned} \tag{25}$$

$$\begin{aligned}&(G_{ijs} + G_{jis})^T P + P(G_{ijs} + G_{jis})\\ &+ (\varepsilon_{0i} + \varepsilon_{1i} + \varepsilon_{2i}) P D_i D_i^T P\\ &+ (\varepsilon_{0j} + \varepsilon_{1j} + \varepsilon_{2j}) P D_j D_j^T P\\ &+ \varepsilon_{2i}^{-1} \overline{E}_{sj}^T E_{bi}^T E_{bi} \overline{E}_{sj} + \varepsilon_{2j}^{-1} \overline{E}_{si}^T E_{bj}^T E_{bj} \overline{E}_{si}\\ &+ \varepsilon_{1i}^{-1} E_{1i}^T E_{1i} + \varepsilon_{1j}^{-1} E_{1j}^T E_{1j} + P(\Gamma_i + \Gamma_j)P\\ &+ 2S/(1-\beta) < 0, 1 \le i < j \le r, 1 \le s \le 2^m,\end{aligned} \tag{26}$$

where
$\Gamma_i = A_{2i} S^{-1} A_{2i}^T + A_{2i} S^{-1} E_{2i}^T (\varepsilon_{0i} I - E_{2i} S^{-1} E_{2i}^T)^{-1} E_{2i} S^{-1} A_{2i}^T$,
and $\mathcal{E}(P, \rho) \subset \bigcap_{i=1}^{r} \mathcal{L}(H_i)$. Then, for all initial state $x_t(\theta) \in \Omega_0$ with

$$\delta = (1-\beta)\rho \big/ ((1-\beta)\lambda_{\max}(P) + \tau_0 \lambda_{\max}(S)), \tag{27}$$

the asymptotic stability of the closed-loop system is guaranteed by the fuzzy control law (8) .

**Proof.** The Lyapunov-Krasovskii functional candidate is chosen as

$$V(x_t) = x^T(t)Px(t) + \int_{t-\tau(t)}^{t} x^T(\sigma)Sx(\sigma)d\sigma \Big/ (1-\beta). \tag{28}$$

Then

$$\dot{V}(x_t) \le 2x^T P\dot{x} + x^T Sx/(1-\beta) - x^T(t-\tau)Sx(t-\tau). \tag{29}$$

Using Lemma 2 and Lemma 3, one can verify the following relations:

$$\begin{aligned}&2x^T P \Delta A_{1i} x \le \varepsilon_{1i} x^T P D_i D_i^T P x + \varepsilon_{1i}^{-1} x^T E_{1i}^T E_{1i} x,\\ &2x^T P \Delta B_i \overline{E}_{sj} x \le \varepsilon_{2i} x^T P D_i D_i^T P x + \varepsilon_{2i}^{-1} x^T \overline{E}_{sj}^T E_{bi}^T E_{bi} \overline{E}_{sj} x,\\ &2x^T P(A_{2i} + \Delta A_{2i})x(t-\tau)\\ &\quad \le x^T P \Gamma_i P x + \varepsilon_{0i} x^T P D_i D_i^T P x + x^T(t-\tau)Sx(t-\tau),\end{aligned}$$

$\forall i, j \in [1, r]$. Substituting (24) into (29) and applying the above relations give

$$\begin{aligned}\dot{V}(x_t) &\le \sum_{s=1}^{2^m} \eta_s \left\{ \sum_{i=1}^{r} \mu_i^2 x^T [G_{iis}^T P + P G_{iis} + \varepsilon_{1i}^{-1} E_{1i}^T E_{1i} \right.\\ &+ (\varepsilon_{0i} + \varepsilon_{1i} + \varepsilon_{2i}) P D_i D_i^T P + \varepsilon_{2i}^{-1} \overline{E}_{si}^T \overline{E}_{si} + P \Gamma_i P\\ &+ S/(1-\beta)]x + \sum_{i<j} \mu_i \mu_j x^T [(G_{ijs} + G_{jis})^T P\\ &+ P(G_{ijs} + G_{jis}) + (\varepsilon_{0i} + \varepsilon_{1i} + \varepsilon_{2i}) P D_i D_i^T P\\ &+ (\varepsilon_{0j} + \varepsilon_{1j} + \varepsilon_{2j}) P D_j D_j^T P + \varepsilon_{2i}^{-1} \overline{E}_{sj}^T E_{bi}^T E_{bi} \overline{E}_{sj}\\ &+ \varepsilon_{2j}^{-1} \overline{E}_{si}^T E_{bj}^T E_{bj} \overline{E}_{si} + \varepsilon_{1i}^{-1} E_{1i}^T E_{1i}\\ &\left. + \varepsilon_{1j}^{-1} E_{1j}^T E_{1j} + P(\Gamma_i + \Gamma_j)P + 2S/(1-\beta)]x \right\}.\end{aligned} \tag{30}$$

If the conditions (25) and (26) are satisfied, then $\dot{V}(x_t) < 0$. Therefore, $V(x_t)$ is bounded and decreasing. That is, $x^T P x \le V(x_t) \le V(x_{t_0}) \le \kappa \|x_t\|_c^2 \le \rho$, $\rho > 0$. Based on the Krasovskii Theorem [22], the asymptotic stability is guaranteed for any initial condition belonging to $\Omega_0$. □

From Schur complement, the linear matrix inequalities (25) and (26) can be transformed into the following forms:

$$\begin{bmatrix} U_{ii} & XE_{1i}^T & Y_{is}^T E_{bi}^T & A_{2i} W E_{2i}^T & X \\ * & -\varepsilon_{1i} I & 0 & 0 & 0 \\ * & * & -\varepsilon_{2i} I & 0 & 0 \\ * & * & * & -\Psi_i & 0 \\ * & * & * & * & -(1-\beta)W \end{bmatrix} < 0, \tag{31}$$

$i = 1, \cdots, r, s = 1, \cdots, 2^m$,

$$\begin{bmatrix} U_{ij} & XE_{1i}^T & XE_{1j}^T & Y_{js}^T E_{bi}^T & Y_{is}^T E_{bj}^T & A_{2i} W E_{2i}^T & A_{2j} W E_{2j}^T & X \\ * & -\varepsilon_{1i} I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_{1j} I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{2i} I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{2j} I & 0 & 0 & 0 \\ * & * & * & * & * & -\Psi_i & 0 & 0 \\ * & * & * & * & * & * & -\Psi_j & 0 \\ * & * & * & * & * & * & * & \frac{1}{2}(1-\beta)W \end{bmatrix} < 0,$$

$1 \le i < j \le r, s = 1, \cdots, 2^m$, (32)

where

$$\begin{aligned} U_{ii} &= XA_{1i}^T + A_{1i}X + M_i^T E_s B_i^T + B_i E_s M_i + N_i^T E_s^- B_i^T \\ &\quad + B_i E_s^- N_i + (\varepsilon_{0i} + \varepsilon_{1i} + \varepsilon_{2i}) D_i D_i^T + A_{2i} W A_{2i}^T, \\ U_{ij} &= XA_{1i}^T + A_{1i}X + M_j^T E_s B_i^T + B_i E_s M_j + N_j^T E_s^- B_i^T \\ &\quad + B_i E_s^- N_j + (\varepsilon_{0i} + \varepsilon_{1i} + \varepsilon_{2i}) D_i D_i^T + A_{2i} W A_{2i}^T \\ &\quad + XA_{1j}^T + A_{1j}X + M_i^T E_s B_j^T + B_j E_s M_i + N_i^T E_s^- B_j^T \\ &\quad + B_j E_s^- N_i + (\varepsilon_{0j} + \varepsilon_{1j} + \varepsilon_{2j}) D_j D_j^T + A_{2j} W A_{2j}^T, \\ Y_{is} &= E_s M_i + E_s^- N_i, \quad \Psi_i = \varepsilon_{0i} I - E_{2i} W E_{2i}^T. \end{aligned}$$

Accordingly, the maximum domain of attraction is obtained by a feasible solution of the optimization problem

$$\min\{c_1 m_1 + c_2 m_2\}$$

*subject to* the constraints (21), (31), (32),

$$\begin{bmatrix} m_1 I_n & I_n \\ * & X \end{bmatrix} \ge 0, \text{ and } \begin{bmatrix} m_2 I_n & I_n \\ * & W \end{bmatrix} \ge 0, \tag{33}$$

where $c_1$ and $c_2$ are tuning factors.

## 4. Numerical example

In this section, computer simulation on backing up control of a truck-trailer [17] is applied to demonstrate the effectiveness of the proposed algorithm.

The time-delay truck-trailer model is described by

$$\begin{aligned} \dot{x}_1(t) &= -a x_1(t) q\bar{t} / \big((L + \Delta L(t)) t_0\big) \\ &\quad - (1-a) x_1(t-\tau) q\bar{t} / \big((L + \Delta L(t)) t_0\big) \\ &\quad + 10 v(t) q\bar{t} / (l + \Delta l(t)) t_0, \\ \dot{x}_2(t) &= a x_1(t) q\bar{t} / \big((L + \Delta L(t)) t_0\big) \\ &\quad + (1-a) x_1(t-\tau) q\bar{t} / \big((L + \Delta L(t)) t_0\big), \\ \dot{x}_3(t) &= t_0^{-1} q\bar{t} \sin[x_2(t) + a x_1(t) q\bar{t} / \big(2(L + \Delta L(t))\big) \\ &\quad + (1-a) x_1(t-\tau) q\bar{t} / \big(2(L + \Delta L(t))\big)], \end{aligned}$$

$$A_{11} = \begin{bmatrix} -a\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ -a\dfrac{q^2\bar{t}^2}{2Lt_0} & \dfrac{q\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -(1-a)\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{q^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} \dfrac{10q\bar{t}}{lt_0} & 0 & 0 \end{bmatrix}^T,$$

$$A_{12} = \begin{bmatrix} -a\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ -a\dfrac{dq^2\bar{t}^2}{2Lt_0} & \dfrac{dq\bar{t}}{t_0} & 0 \end{bmatrix}, A_{22} = \begin{bmatrix} -(1-a)\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{dq^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

where $a = 0.7$, $q = -1.0$ , $\bar{t} = 2.0$, $t_0 = 0.5$, $L = 5.5$, $l = 2.8$, $\Delta L(t) = 0.0138 + 0.2757\sin(\pi t/2)$, $\Delta l(t) = 0.007 + 0.1404\sin(\pi t/2)$ . The control input $v(t)$ is assumed to be confined within unity saturation level, $|v(t)| \le 1$. From the given values of the uncertainties, it is verified that $-0.2619 \le \Delta L(t) \le 0.2895$ and $-0.1333 \le \Delta l(t) \le 0.1474$ or, equivalently, $0.95(1/L) \le 1/(L + \Delta L(t)) \le 1.05(1/L)$ and $0.95(1/l) \le 1/(l + \Delta l(t)) \le 1.05(1/l)$.

The T-S fuzzy model for this system is [17]:

*Rule 1*: IF

$$\theta(t) = x_2(t) + a(q\bar{t}/2L)x_1(t) + (1-a)(q\bar{t}/2L)x_1(t-\tau)$$

is about 0, THEN

$$\dot{x}(t) = (A_{11} + \Delta A_{11})x(t) + (A_{21} + \Delta A_{21})x(t-\tau) + (B_1 + \Delta B_1)v(t).$$

*Rule 2*: IF

$$\theta(t) = x_2(t) + a(q\bar{t}/2L)x_1(t) + (1-a)(q\bar{t}/2L)x_1(t-\tau)$$

is about $\pi$ or $-\pi$ THEN

$$\dot{x}(t) = (A_{12} + \Delta A_{12})x(t) + (A_{22} + \Delta A_{22})x(t-\tau) + (B_2 + \Delta B_2)v(t).$$

The system matrices are given below:

$$A_{11} = \begin{bmatrix} -a\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{q^2\bar{t}^2}{2Lt_0} & \dfrac{q\bar{t}}{t_0} & 0 \end{bmatrix},\ A_{21} = \begin{bmatrix} -(1-a)\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{q^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} \dfrac{10q\bar{t}}{lt_0} & 0 & 0 \end{bmatrix}^T,$$

$$A_{12} = \begin{bmatrix} -a\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{dq^2\bar{t}^2}{2Lt_0} & \dfrac{dq\bar{t}}{t_0} & 0 \end{bmatrix},\ A_{22} = \begin{bmatrix} -(1-a)\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{q\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{dq^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$\Delta A_{11} = 0.05\delta(t)\begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ 0.5091 & 0 & 0 \end{bmatrix},$$

$$\Delta A_{21} = 0.05\delta(t)\begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.2182 & 0 & 0 \end{bmatrix},\ \Delta B_1 = \begin{bmatrix} -14.29 \\ 0 \\ 0 \end{bmatrix},$$

$$\Delta A_{12} = 0.05\delta(t)\begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ 0.8107 & 0 & 0 \end{bmatrix},$$

$$\Delta A_{22} = 0.05\delta(t)\begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.3474 & 0 & 0 \end{bmatrix},\ \Delta B_2 = \begin{bmatrix} -14.29 \\ 0 \\ 0 \end{bmatrix},$$

where $d = 10t_0/\pi$ and $|\delta(t)| \le 1$. The membership functions are characterized by

$$\mu_1 = \big(1 - 1/\big(1 + \exp(-3(\theta(t) - 0.5\pi))\big)\big) \times \big(1/\big(1 + \exp(-3(\theta(t) + 0.5\pi))\big)\big),$$

$$\mu_2 = 1 - \mu_1.$$

The fuzzy control law is given by $u(t) = \sum_{l=1}^{2} \mu_l F_l x(t)$.

In the nominal condition, the feasible solution of (22) is obtained as follows:

$$F_1 = [1.4666 \quad -2.2916 \quad 0.2052],$$

$$F_2 = [1.4588 \quad -2.2745 \quad 0.2026],$$

$$H_1 = [0.1713 \quad -0.2595 \quad 0.0179],$$

$$H_2 = [0.1731 \quad -0.2822 \quad 0.0193],$$

$$P = \begin{bmatrix} 0.401 & -0.6149 & 0.0573 \\ -0.6149 & 1.6751 & -0.1271 \\ 0.0573 & -0.1271 & 0.0192 \end{bmatrix},$$

$$S = \begin{bmatrix} 0.111 & -0.0019 & 0.0049 \\ -0.0019 & 0.0063 & -0.0002 \\ 0.0049 & -0.0002 & 0.003 \end{bmatrix},$$

$$\rho = 11.01, \delta = 5.35, \tau = 1, \beta = 0.1.$$

The initial state is set at $x_0 = [1.45\pi \quad 0.75\pi \quad -1]^T$ and $\|x_0\| = 5.2252$. Hence, $x_0 \in \Omega_0$. Moreover, the proposed controller is compared with the fuzzy controller developed by Cao's method [16], in which the feedback gains are $F_1 = [-0.8686 \quad 3.4824 \quad -0.57]$ and $F_2 = [-0.4568 \quad 1.808 \quad -0.2975]$. Figures 1-4 show the closed-loop system performance; the solid lines present the performance of the proposed approach and the dashed lines refer to the system response by using Cao's method. The simulation results reveal that both controllers can stabilize the truck-trailer in backing up control. However, the controller based on Cao's method does not include the effect of the input saturation, so its output reaches the saturation level, as indicated in Fig. 4. This condition is undesirable because the input saturation might degrade the system performance even cause the closed-loop system unstable. In contrast, the proposed controller renders a satisfactory performance and confines its control value within the unity. The stability domain, as estimated, is ensured by the proposed controller.

The perturbed condition is then included in the simulation. Solve the optimization problem (33) and yield the following results:

$$F_1 = \begin{bmatrix} 2.0067 & -3.1053 & 0.1651 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} 1.8867 & -2.9174 & 0.1551 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 0.2185 & -0.3418 & 0.0146 \end{bmatrix},$$
$$H_2 = \begin{bmatrix} 0.2196 & -0.342 & 0.0144 \end{bmatrix},$$

$$P = \begin{bmatrix} 2.1788 & -3.3621 & 0.1806 \\ -3.3621 & 7.8203 & -0.366 \\ 0.1806 & -0.366 & 0.0302 \end{bmatrix},$$

$$S = \begin{bmatrix} 0.4462 & -0.0067 & 0.0097 \\ -0.0067 & 0.0128 & -0.0005 \\ 0.0097 & -0.0005 & 0.003 \end{bmatrix},$$

$$\rho = 41.4345, \delta = 4.1844.$$

The simulation results regarding the initial state $x_0 = \begin{bmatrix} \pi & 0.75\pi & -1 \end{bmatrix}^T$ are plotted in Figs. 5-8, where the solid lines denote the system response of the proposed approach and the dashed lines indicate the system response based on Cao's method. Obviously, the control value is still within the range of unity, and the asymptotic stability is guaranteed. The effectiveness and correctness of proposed method is demonstrated.

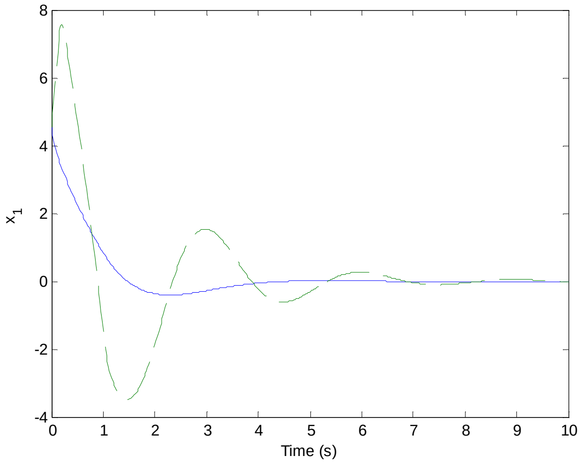

Figure 1. State response of $x_1(t)$.

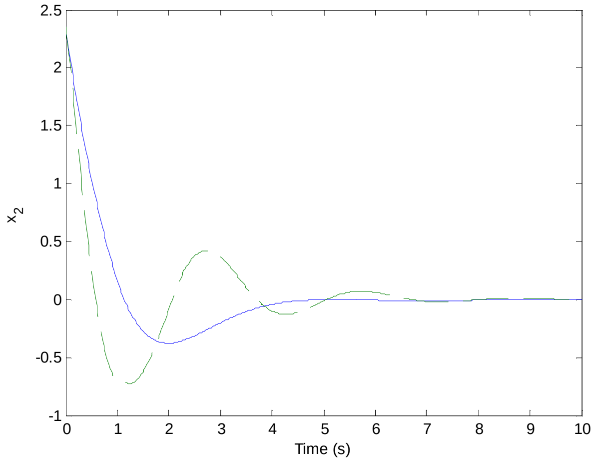

Figure 2. State response of $x_2(t)$.

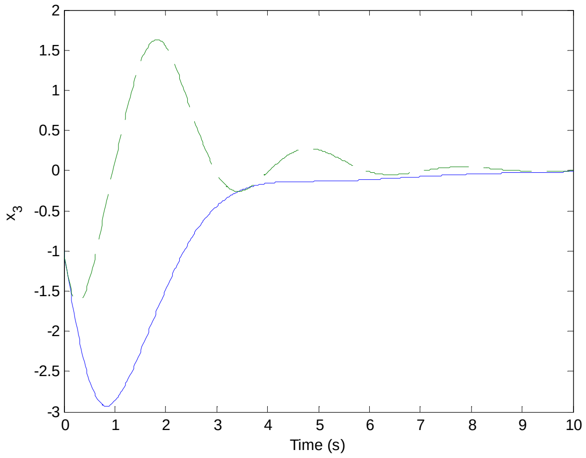

Figure 3. State response of $x_3(t)$.

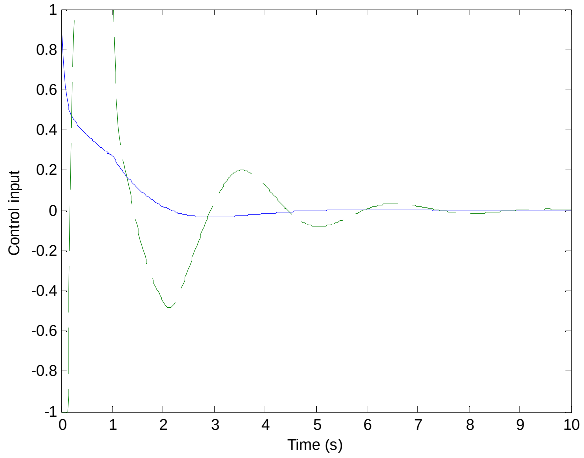

Figure 4. Time response of the control input.

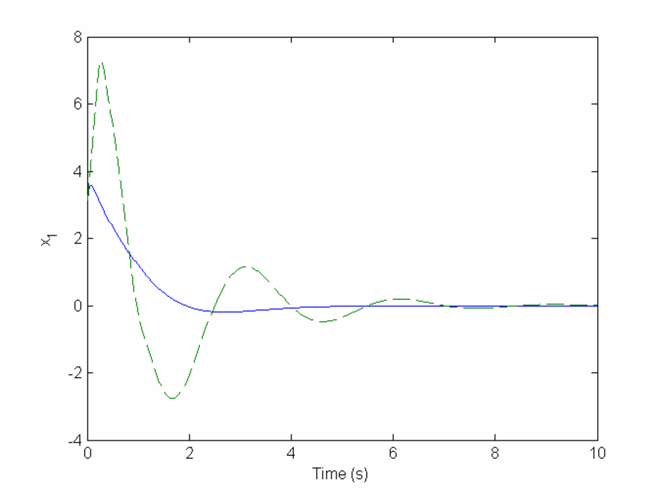

Figure 5. State response of $x_1(t)$ under the perturbed condition.

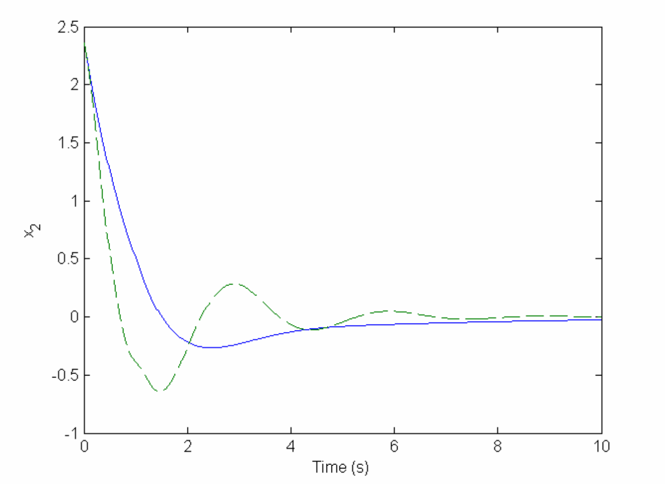

Figure 6. State response of $x_2(t)$ under the perturbed condition.

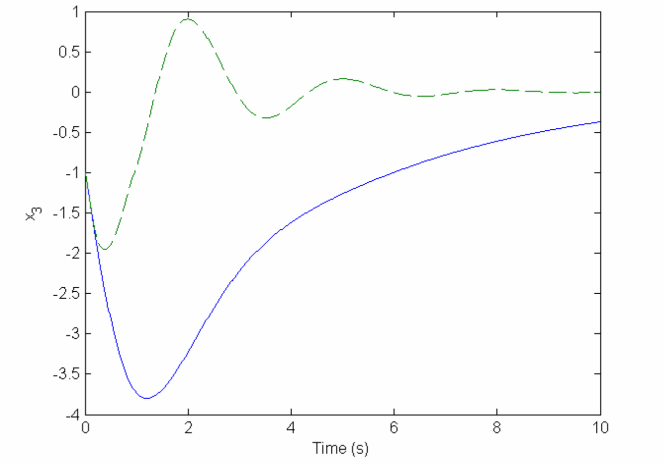

Figure 7. State response of $x_3(t)$ under the perturbed condition.

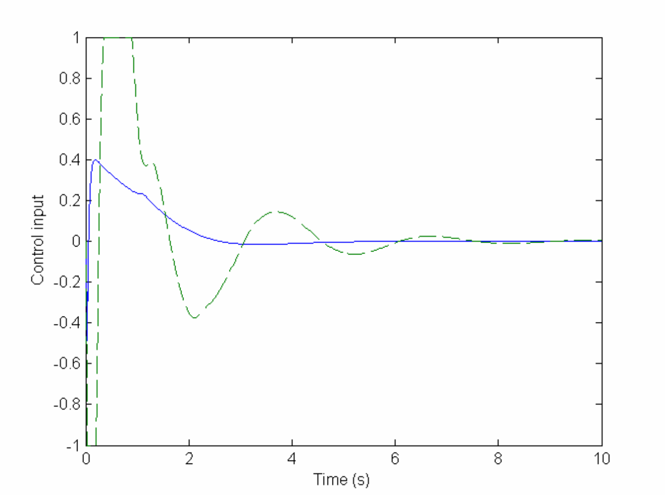

Figure 8. Time response of the control input under the perturbed condition.

## 5. Conclusion

This investigation presents a fuzzy control scheme for stabilization of nonlinear time-delay systems subject to input saturation. The controlled system is analyzed and designed based on the T-S fuzzy model. A convex expression for saturating input is formulated by introducing an auxiliary feedback matrix. Based on the proposed approach, sufficient conditions that ensure the closed-loop system to be asymptotical stable are derived through Lyapunov-Krasovskii functional analysis. Moreover, the problem of estimating the largest region of attraction over the designed control law is solved by using linear matrix inequality technique. Finally, the effectiveness of proposed method is demonstrated by computer simulations on a truck-trailer control.

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