

Linear Control and Parallel Distributed Fuzzy Control Design for T-S Fuzzy Time-Delay System

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Abstract

This paper studies the stabilization problem for T-S fuzzy systems with time delay. Based on Lyapunov criterion and Razumikhin theorem, two types of stabilizing controller are designed. One is the linear state feedback controller and the other is the parallel distributed fuzzy controller (PDFC). For each type of the controller, there are some derived sufficient conditions under which the whole closed loop fuzzy time-delay system is asymptotically stable. Moreover, those sufficient conditions can be transformed into the linear matrix inequality (LMI) problem. Then the controllers can be obtained by solving LMIs. Finally, a practical continuous stirred tank reactor (CSTR) system illustrates the control design and its stabilization effectiveness.

Keywords: Time-delay, T-S fuzzy model, Linear control, Parallel distributed compensation (PDC).

1. Introduction

It is well known that time-delay is often encountered in various engineering systems to be controlled. Many researchers (see [1], [2], [7], [8], [11]-[13], [20], [23], and [24]) have paid a great attention to various control methods in time-delay systems. For instance, [7] and [12] study the robust stability and stabilization problems for uncertain time-delay systems. [8] investigates H_∞ state feedback control design for discrete time delay system. [11] proposes the concept of time delay control (TDC) for a discrete time sliding mode system. [20] studies the stability of the decoupling internal model control (IMC) for multiple time delays system. [23] and [24] discuss the stability and stabilization problems for large-scale systems with time-delays. Further, [2] utilizes linear matrix inequalities (LMIs) method to solve stability, stabilization and H_∞ control for discrete time Markovian jump linear system with norm-bounded time-delays.

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The above papers deal with the problem on conventional control systems. Recently, the fuzzy system represented by IF-THEN rules has become one of the useful modeling approaches for complex systems. Fuzzy modeling has capability of modeling complex nonlinear processes to arbitrary degrees of accuracy [3], [19], and [26]. After Takagi and Sugeno et al. proposed the so-called Takagi-Sugeno (T-S) fuzzy model [14]-[16] and [18], the flexibility of fuzzy logic theory and rigorous mathematical theories of linear or nonlinear systems are combined into a unified framework.

In recent, the task of effectively controlling nonlinear system with time-delay described by T-S fuzzy model has been noticed. For uncertain nonlinear fuzzy system with time delays, [9], [10], and [25] discuss the H_∞ control design for T-S fuzzy models. In those papers the concept of the parallel distributed compensation (PDC) and LMIs method are used.

In this paper, we shall investigate the stabilization problem of fuzzy time-delay systems with two types of controllers, linear state feedback control and parallel distributed fuzzy control (PDFC), respectively. Two types of controllers are derived with some sufficient conditions such that the fuzzy time-delay system is asymptotically stable. These sufficient conditions can be easily transformed into the problem of LMIs by using Schur complement. Also, the difference and comparison between the two types of controllers are discussed.

2. System and Problem Description

Suppose there is a T-S fuzzy model for the nonlinear time-delay plant. This model is composed of r rules and each rule is represented as follows

$$R^i : \text{If } z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_g(t) \text{ is } M_{ig}, \quad (1)$$

$$\text{Then } \dot{x}(t) = A_i x(t) + \tilde{A}_i x(t - \tau_i(t)) + B_i u(t),$$

where $x(t) = \psi(t)$, $t \in [-\tau, 0]$, $\tau_i \leq \tau$, $i = 1, 2, \dots, r$. A_i , \tilde{A}_i , and B_i are constant matrices with appropriate dimensions; M_{ij} is the fuzzy set; $x(t) \in R^n$ is the state vector; and $u(t) \in R^m$ is the input vector. $z_1(t)$, $z_2(t)$, \dots , $z_g(t)$ are the premise variables which may be equal to some elements of $x(t)$ or be a function of $x(t)$; and $\psi(t)$ is the initial condition of the state.

By a singleton fuzzifier, minimum fuzzy inference, and central-average defuzzifier, (1) can be inferred as

$$\dot{x}(t) = \frac{\sum_{i=1}^r \omega_i(z(t))[A_i x(t) + \tilde{A}_i x(t - \tau_i(t)) + B_i u(t)]}{\sum_{i=1}^r \omega_i(z(t))} \quad (2)$$

$$= \sum_{i=1}^r \mu_i(z(t))[A_i x(t) + \tilde{A}_i x(t - \tau_i(t)) + B_i u(t)]$$

where $\omega_i(z(t)) = \min_q (M_{iq}(z_q(t)))$, $\mu_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^r \omega_i(z(t))$, $M_{iq}(z_q(t))$ is the membership grade of $z_q(t)$ in M_{iq} . It is seen that $\omega_i(z(t)) \geq 0$, $i = 1, 2, \dots, r$, for all t , and $\sum_{i=1}^r \mu_i(z(t)) = 1$.

The objective of this paper is to synthesize two types of the control $u(t)$, linear state feedback control and PDFC, such that the overall closed-loop fuzzy time-delay system is asymptotically stable.

3. Control Synthesis

This section proposes the design technique for two types of controllers, linear state feedback control and PDFC, respectively. It should be mentioned that, in the consequent of this paper, any two square matrices A and B with the relationship $A > B$ or $A - B > 0$ means the matrix $A - B$ is positive definite.

3.1 Linear State Feedback Control Synthesis

In this subsection, we consider the first type of controller, linear state feedback control,

$$u(t) = -\sum_{i=1}^r F_i x(t) = -F x(t), \quad (3)$$

where $F = \sum_{i=1}^r F_i$, r is the number of rules in (1). Thus, combining (2) and (3), the global closed-loop fuzzy time-delay system becomes

$$\dot{x}(t) = \sum_{i=1}^r \mu_i(z)[A_i x(t) + \tilde{A}_i x(t - \tau_i(t)) - B_i \sum_{j=1}^r F_j x(t)] \quad (4)$$

$$= \sum_{i=1}^r \mu_i(z)(A_i - B_i \sum_{j=1}^r F_j)x(t) + \sum_{i=1}^r \mu_i(z)\tilde{A}_i x(t - \tau_i(t))$$

Before deriving the main result, an important lemma should be presented first.

Lemma 1 [21]: For any two compatible constant matrices X and Y , there is

$$X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y, \quad (5)$$

where ε is any positive constant.

From (3), the linear feedback control F is the sum of all F_i . In other words, F is a common state feedback gain for each rule in (1). Therefore, by Lyapunov stability criterion [6] and Razumikhin theorem [4], the first result is expressed in the following theorem.

Theorem 1: The T-S fuzzy time-delay system (2) can be stabilized asymptotically by the linear state feedback control (3), if there exist positive a definite matrix P and r matrices $S_i > 0$ such that the following conditions hold.

$$(a) \quad G_{ii}^T P + P G_{ii} + P \tilde{A}_i S_i \tilde{A}_i^T P + P < 0, \text{ for all } i \quad (6a)$$

$$(b) \quad (G_{ij} + G_{ji})^T P + P(G_{ij} + G_{ji}) < 0, \text{ for all } i < j \quad (6b)$$

$$(c) \quad P \geq S_i^{-1} > 0, \text{ for all } i \quad (6c)$$

where $G_{ij} \equiv A_i / r - B_i F_j$.

Proof: Set the Lyapunov function candidate for the closed-loop system (4) to be

$$V(x(t)) = x^T(t) P x(t).$$

Taking the derivative of $V(x(t))$ along (4), we have

$$\dot{V}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i(z) \mu_j(z) x^T(t) \left[\left(\frac{A_i}{r} - B_i F_j \right)^T P + P \left(\frac{A_i}{r} - B_i F_j \right) \right] x(t) \quad (7)$$

$$+ \sum_{i=1}^r \mu_i(z) 2x^T(t) P \tilde{A}_i x(t - \tau_i(t))$$

Replacing $A_i / r - B_i F_j$ by G_{ij} , then

$$\dot{V}(x(t)) = \sum_{i=1}^r \mu_i(z) x^T(t) [G_{ii}^T P + P G_{ii}] x(t)$$

$$+ \sum_{i=1}^r \mu_i(z) 2x^T(t) P \tilde{A}_i x(t - \tau_i(t))$$

$$+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z) \mu_j(z) x^T(t) [G_{ij}^T P + P G_{ij}] x(t)$$

$$= \sum_{i=1}^r \mu_i(z) x^T(t) [G_{ii}^T P + P G_{ii}] x(t)$$

$$+ \sum_{i=1}^r \mu_i(z) 2x^T(t) P \tilde{A}_i S_i^2 S_i^{-2} x(t - \tau_i(t))$$

$$+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z) \mu_j(z) x^T(t) [G_{ij}^T P + P G_{ij}] x(t)$$

where $S_i > 0$. By Lemma 1, it follows

$$\dot{V}(x(t)) \leq \sum_{i=1}^r \mu_i(z) x^T(t) [G_{ii}^T P + P G_{ii} + P \tilde{A}_i S_i \tilde{A}_i^T P] x(t)$$

$$+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z) \mu_j(z) x^T(t) [G_{ij}^T P + P G_{ij}] x(t)$$

$$+ \sum_{i=1}^r \mu_i(z) x^T(t - \tau_i(t)) S_i^{-1} x(t - \tau_i(t)).$$

According to (6c), we have

$$\dot{V}(x(t)) \leq \sum_{i=1}^r \mu_i(z) x^T(t) [G_{ii}^T P + P G_{ii} + P \tilde{A}_i S_i \tilde{A}_i^T P] x(t)$$

$$+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z) \mu_j(z) x^T(t) [G_{ij}^T P + P G_{ij}] x(t)$$

$$+ \sum_{i=1}^r \mu_i(z) x^T(t - \tau_i(t)) P x(t - \tau_i(t))$$

$$= \sum_{i=1}^r \mu_i(z) x^T(t) [G_{ii}^T P + P G_{ii} + P \tilde{A}_i S_i \tilde{A}_i^T P] x(t)$$

$$+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z) \mu_j(z) x^T(t) [G_{ij}^T P + P G_{ij}] x(t)$$

$$+ \sum_{i=1}^r \mu_i(z) V[x(t - \tau_i(t))].$$

By using Razumikhin theorem [4], if there exists a real $\delta > 1$ such that

$$V(x(t-\theta)) < \delta V(x(t)) \text{ for } \theta \in [0, \tau],$$

then

$$\begin{aligned} \dot{V}(x(t)) &< \sum_{i=1}^r \mu_i(z)x^T(t)[G_{ii}^T P + PG_{ii} + P\tilde{A}_i S_i \tilde{A}_i^T P]x(t) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)x^T(t)[G_{ij}^T P + PG_{ij}]x(t) \\ &+ \sum_{i=1}^r \mu_i(z) \delta V[x(t)] \\ &= \sum_{i=1}^r \mu_i(z)x^T(t)[G_{ii}^T P + PG_{ii} + P\tilde{A}_i S_i \tilde{A}_i^T P + \delta P]x(t) \\ &+ \sum_{i=1}^r \sum_{j < i}^r \mu_i(z)x^T(t)[(G_{ij} + G_{ji})^T P + P(G_{ij} + G_{ji})]x(t) \end{aligned}$$

Obviously, $\dot{V}(x(t)) < 0$ if

$$\Xi(\delta) \equiv G_{ii}^T P + PG_{ii} + P\tilde{A}_i S_i \tilde{A}_i^T P + \delta P < 0, \text{ for all } i,$$

and

$$(G_{ij} + G_{ji})^T P + P(G_{ij} + G_{ji}) < 0, \text{ for all } i < j,$$

hold, then the overall closed-loop system is asymptotically stable. If (6) holds, that is, $\Xi(1) < 0$, then by continuity, there is a $\delta = 1 + \alpha$ with $\alpha > 0$ sufficiently small such that $\Xi(\delta) < 0$ for all i . The proof is completed.

By Schur complement, stability conditions of Theorem 1 are equivalent to the following LMIs,

$$X \frac{A_i^T}{r} + \frac{A_i}{r} X + X - Y_i^T B_i^T - B_i Y_i + \tilde{A}_i S_i \tilde{A}_i^T < 0, \text{ for all } i, \quad (9a)$$

$$X \frac{A_i^T}{r} + \frac{A_i}{r} X + X \frac{A_j^T}{r} + \frac{A_j}{r} X - Y_j^T B_j^T - B_j Y_j - Y_i^T B_j^T - B_j Y_i < 0$$

for all $i < j$, (9b)

$$S_i \geq X, \text{ for all } i, \quad (9c)$$

where $X = P^{-1}$ and $F_i = Y_i X^{-1}$. Therefore, if we can find X and Y_i to satisfy LMI (9a)-(9c), F_i is yielded and the linear feedback gain F is the sum of all F_i , $i = 1, 2, \dots, r$.

Remark 1: We have to point out that it is difficult to find or design the common F without dividing the gain F into r subgain F_i 's. Based on the concept of parallel distributed compensation, the form of F as the sum of all F_i 's make $\dot{V}(x(t)) < 0$ much easier.

3.2 PDFC Synthesis

It is noted in the last section, the found F is a common feedback controller for all rules in (1). Now, we like to know what is the benefit if the common feedback controller is replaced by PDFC. It is well known that PDFC has r fuzzy rules corresponding to the rule in the fuzzy plant (1). The PDFC is as below

$$C^i : \text{If } z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_g(t) \text{ is } M_{ig}, \quad (10)$$

$$\text{Then } u(t) = -F_i x(t), \quad i = 1, 2, \dots, r.$$

The weight $\mu_i(z(t))$ is the same defined as in (2).

Analogous to (2), the final output of the fuzzy controller C^i is

$$u(t) = -\sum_{i=1}^r \mu_i(z) F_i x(t) \quad (11)$$

Thus, combining (2) and (11), the global closed-loop fuzzy time-delay system becomes

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \mu_i(z)[A_i x(t) + \tilde{A}_i x(t - \tau_i(t)) + B_i u(t)] \\ &= \sum_{i=1}^r \mu_i(z)[A_i x(t) + \tilde{A}_i x(t - \tau_i(t)) - B_i \sum_{j=1}^r \mu_j(z) F_j x(t)] \end{aligned} \quad (12)$$

Before proceeding to the main result, we have to present a useful lemma again.

Lemma 2[22]: Tchebyshev's inequality holds for any matrix $H_i \in R^{n \times n}$, that is,

$$\left[\sum_{i=1}^m H_i \right]^T \left[\sum_{i=1}^m H_i \right] \leq m \sum_{i=1}^m H_i^T H_i \quad (13)$$

Thus, the following theorem presents the stabilization conditions for the fuzzy time-delay system (2) with the PDFC (10).

Theorem 2: The fuzzy time-delay system (2) can be stabilized asymptotically by the PDFC (10), if there exist a positive definite matrix P , r matrices $S_i > 0$, and a symmetric matrix R , such that the following conditions hold,

$$(a) \quad \bar{G}_{ii}^T P + P \bar{G}_{ii} + \hat{r} P \tilde{A}_i S_i \tilde{A}_i^T P + \hat{r} P < R_{ii}, \text{ for all } i, \quad (14a)$$

$$(b) \quad (\bar{G}_{ij} + \bar{G}_{ji})^T P + P(\bar{G}_{ij} + \bar{G}_{ji}) < R_{ij} + R_{ji}, \text{ for all } i < j, \quad (14b)$$

$$(c) \quad R \equiv \begin{bmatrix} R_{11} & \dots & R_{1r} \\ \vdots & \ddots & \vdots \\ R_{r1} & \dots & R_{rr} \end{bmatrix} < 0, \text{ and } P \geq S_i^{-1} > 0, \text{ for all } i \quad (14c)$$

where $\bar{G}_{ij} \equiv A_i - B_i F_j$, and \hat{r} is the maximal number of the fired rules by any instant input $[z_1(t), z_2(t), \dots, z_g(t)]$.

Proof: Set the Lyapunov function candidate for the closed-loop system (12) to be

$$V(x(t)) = x^T(t) P x(t).$$

Taking the derivative of $V(x(t))$ along (12), we have

$$\begin{aligned} \dot{V}(x(t)) &= \sum_{i=1}^r \sum_{j=1}^r \mu_i(z) \mu_j(z) x^T(t) [(A_i - B_i F_j)^T P + P(A_i - B_i F_j)] x(t) \\ &+ \sum_{i=1}^r \mu_i(z) 2x^T(t) P \tilde{A}_i x(t - \tau_i(t)). \end{aligned} \quad (15)$$

Set $\bar{G}_{ij} \equiv A_i - B_i F_j$, we obtain

$$\begin{aligned} \dot{V}(x(t)) &= \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii}]x(t) \\ &+ \sum_{i=1}^r \mu_i(z)2x^T(t)P\tilde{A}_i x(t-\tau_i(t)) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)\mu_j(z)x^T(t)[\bar{G}_{ij}^T P + P\bar{G}_{ij}]x(t) \\ &= \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii}]x(t) \\ &+ \sum_{i=1}^r \sum_{j=1}^r \mu_i(z)\mu_j(z)2x^T(t)P\tilde{A}_i S_i^{-\frac{1}{2}} S_i^{-\frac{1}{2}} x(t-\tau_i(t)) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)\mu_j(z)x^T(t)[\bar{G}_{ij}^T P + P\bar{G}_{ij}]x(t) \end{aligned}$$

According to Lemma 2, then

$$\begin{aligned} \dot{V}(x(t)) &\leq \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii}]x(t) \\ &+ \sum_{i=1}^r \mu_i^2(z)2\hat{r}x^T(t)P\tilde{A}_i S_i^{-\frac{1}{2}} S_i^{-\frac{1}{2}} x(t-\tau_i(t)) \quad (16) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)\mu_j(z)x^T(t)[\bar{G}_{ij}^T P + P\bar{G}_{ij}]x(t) \end{aligned}$$

Based on Lemma 2, inequality (16) is true if \hat{r} is replaced by r . It is known that if any input fires the premise of fuzzy rules, not all rules are fired. Only $\hat{r} \leq r$ rules are fired. Therefore \hat{r} in (16) can replace the total rules' number r .

Applying Lemma 1 leads to

$$\begin{aligned} \dot{V}(x(t)) &\leq \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii}]x(t) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)\mu_j(z)x^T(t)[\bar{G}_{ij}^T P + P\bar{G}_{ij}]x(t) \\ &+ \sum_{i=1}^r \mu_i^2(z)\hat{r}x^T(t)P\tilde{A}_i S_i \tilde{A}_i^T P x(t) \\ &\quad + x^T(t-\tau_i(t))S_i^{-1}x(t-\tau_i(t)) \\ &= \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii} + \hat{r}P\tilde{A}_i S_i \tilde{A}_i^T P]x(t) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)\mu_j(z)x^T(t)[\bar{G}_{ij}^T P + P\bar{G}_{ij}]x(t) \\ &+ \sum_{i=1}^r \mu_i^2(z)\hat{r}x^T(t-\tau_i(t))S_i^{-1}x(t-\tau_i(t)). \end{aligned}$$

If $P \geq S_i^{-1} > 0$ holds, then

$$\begin{aligned} \dot{V}(x(t)) &\leq \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii} + \hat{r}P\tilde{A}_i S_i \tilde{A}_i^T P]x(t) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)\mu_j(z)x^T(t)[\bar{G}_{ij}^T P + P\bar{G}_{ij}]x(t) \\ &+ \sum_{i=1}^r \mu_i^2(z)\hat{r}x^T(t-\tau_i(t))P x(t-\tau_i(t)) \\ &= \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii} + \hat{r}P\tilde{A}_i S_i \tilde{A}_i^T P]x(t) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)\mu_j(z)x^T(t)[\bar{G}_{ij}^T P + P\bar{G}_{ij}]x(t) \\ &+ \sum_{i=1}^r \mu_i^2(z)\hat{r}V[x(t-\tau_i(t))] \end{aligned}$$

By using Razumikhin theorem, if there exists a real

$\delta > 1$ such that

$$V[x(t-\theta)] < \delta V[x(t)] \quad \text{for } \theta \in [0, \tau],$$

then

$$\begin{aligned} \dot{V}(x(t)) &< \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii} + \hat{r}P\tilde{A}_i S_i \tilde{A}_i^T P]x(t) \\ &+ \sum_{i=1}^r \sum_{j \neq i}^r \mu_i(z)\mu_j(z)x^T(t)[\bar{G}_{ij}^T P + P\bar{G}_{ij}]x(t) \\ &+ \sum_{i=1}^r \mu_i^2(z)\hat{r}\delta V[x(t)] \\ &= \sum_{i=1}^r \mu_i^2(z)x^T(t)[\bar{G}_{ii}^T P + P\bar{G}_{ii} + \hat{r}P\tilde{A}_i S_i \tilde{A}_i^T P + \hat{r}\delta P]x(t) \\ &+ \sum_{i=1}^r \sum_{j < i}^r \mu_i(z)\mu_j(z)x^T(t)[(\bar{G}_{ij} + \bar{G}_{ji})^T P + P(\bar{G}_{ij} + \bar{G}_{ji})]x(t) \quad (17) \end{aligned}$$

Substituting (14a) and (14b) into (17), we obtain

$$\begin{aligned} \dot{V}(x(t)) &< \sum_{i=1}^r \mu_i^2(z)x^T(t)R_{ii}x(t) \\ &+ \sum_{i=1}^r \sum_{j < i}^r \mu_i(z)\mu_j(z)x^T(t)(R_{ij} + R_{ji})x(t) \\ &= \begin{bmatrix} \mu_1(z)x(t) \\ \vdots \\ \mu_r(z)x(t) \end{bmatrix}^T \begin{bmatrix} R_{11} & \cdots & R_{1r} \\ \vdots & \ddots & \vdots \\ R_{r1} & \cdots & R_{rr} \end{bmatrix} \begin{bmatrix} \mu_1(z)x(t) \\ \vdots \\ \mu_r(z)x(t) \end{bmatrix}. \end{aligned}$$

Therefore, if (14c) hold, $\dot{V}(x(t)) < 0$, then the fuzzy time-delay system (2) is asymptotically stable. By Razumikhin theorem, there exists a $\delta > 1$ such that the above inequalities hold. The proof is completed.

Similar to Theorem 1, by Schur complement, (14a)-(14c) can be easily transformed into the following LMI's form, too.

$$XA_i^T + A_i X + \hat{r}X - Y_i^T B_i^T - B_i Y_i + \hat{r}\tilde{A}_i S_i \tilde{A}_i^T < \bar{R}_{ii}, \quad \text{for all } i, \quad (18a)$$

$$\begin{aligned} &XA_i^T + A_i X + XA_j^T + A_j X - Y_j^T B_j^T \\ &\quad - B_j Y_j - Y_i^T B_j^T - B_j Y_i < \bar{R}_{ij} + \bar{R}_{ji}, \quad \text{for all } i < j, \quad (18b) \end{aligned}$$

$$\bar{R} \equiv \begin{bmatrix} \bar{R}_{11} & \cdots & \bar{R}_{1r} \\ \vdots & \ddots & \vdots \\ \bar{R}_{r1} & \cdots & \bar{R}_{rr} \end{bmatrix} < 0, \quad \text{and } S_i \geq X > 0, \quad \text{for all } i \quad (18c)$$

where $X = P^{-1}$, $F_i = Y_i X^{-1}$, and $\bar{R}_{ij} = X R_{ij} X$.

Remark 2: Comparing (14a)-(14c) to (6a)-(6c), respectively, it is found that there are some relations between them. If we multiply r into each term of (6a), then (6a) becomes

$$(A_i - rB_i F_i)^T P + P(A_i - rB_i F_i) + rP\tilde{A}_i S_i \tilde{A}_i^T P + rP < 0 \quad (19)$$

It is seen that each gain F_i in (6a) is amplified to be rF_i in (19). But in (14a) F_i is not amplified. The same observation can be found on the comparison of (6b) and (14b). Furthermore, we may say that the common feedback gain F is the sum of all F_i (see (3)) which

should be much larger than PDFC in (11). The reason is that the common controller (3) needs to stabilize all subsystems of (1) no matter what rules are fired and how heavy of rules is fired. Nevertheless, PDFC has parallel distributed compensation capability. The controller gain as (11) can be adjusted based on the fired rule weights.

Remark 3: We compare Theorem 1 with Theorem 2 from the viewpoint of stabilization again. Obviously, LMIs (14a)-(14c) have an extra matrix $R < 0$ to be solved than LMIs (6a)-(6c) does, but LMIs (14a)-(14c) are less conservative than LMIs (6a)-(6c). The reason is that the right sides of (6a) and (6b) are only “zero,” but the right sides of (14a) and (14b) are R_{ii} and $R_{ij} + R_{ji}$, respectively. Besides, the parameter r in (6a) is replaced by a smaller \hat{r} in (14a). This replacement also contributes to relax the stabilization conditions.

4. An Illustrative Example

Consider a first order irreversible exothermic reaction $A \rightarrow B$ which occurs in a well-mixed continuous stirred tank reactor (CSTR) as in [5]. By using the analytical approach of [17] the reaction $A \rightarrow B$ can be modeled as the following fuzzy model with operating points (x_d, u_d) , which is a stationary point of the nonlinear system.

Rule 1: If the temperature $x_2(t)$ is low, then

$$\dot{e}(t) = \begin{bmatrix} -1.4274 & 0.0757 \\ -1.4189 & -0.9442 \end{bmatrix} e(t) + \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} e(t-\tau) + \begin{bmatrix} 0 \\ 0.3 \end{bmatrix} \hat{u}(t),$$

Rule 2: If the temperature $x_2(t)$ is middle, then

$$\dot{e}(t) = \begin{bmatrix} -2.0508 & 0.3958 \\ -6.4066 & 1.6168 \end{bmatrix} e(t) + \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} e(t-\tau) + \begin{bmatrix} 0 \\ 0.3 \end{bmatrix} \hat{u}(t),$$

Rule 3: If the temperature $x_2(t)$ is high, then

$$\dot{e}(t) = \begin{bmatrix} -4.5279 & 0.3167 \\ -26.2228 & 0.9837 \end{bmatrix} e(t) + \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} e(t-\tau) + \begin{bmatrix} 0 \\ 0.3 \end{bmatrix} \hat{u}(t),$$

where $e(t) = x(t) - x_d$, $e(t-\tau) = x(t-\tau) - x_d$, and $\hat{u}(t) = u(t) - u_d$. $x(t) = [x_1(t) \ x_2(t)]^T$, $x_1(t)$ corresponds to the conversion rate of the reaction $0 \leq x_1(t) \leq 1$; $x_2(t)$ is the dimensionless temperature. The membership functions of $x_2(t)$ are shown in Fig 1.

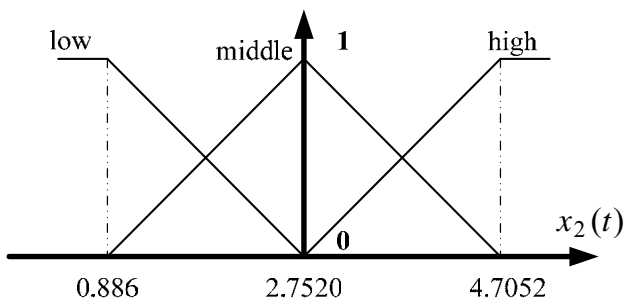


Fig 1. Membership functions of $x_2(t)$ in CSTR fuzzy model.

First, we design a common linear state feedback control to achieve the stabilization of CSTR.

$$\hat{u}(t) = -(F_1 + F_2 + F_3)e(t) = -Fe(t).$$

By solving LMIs in Theorem 1, we have

$$X = \begin{bmatrix} 2699 & -7546 \\ -7546 & 25274 \end{bmatrix}, \quad P = \begin{bmatrix} 0.00224 & 0.00067 \\ 0.00067 & 0.00024 \end{bmatrix},$$

$$F_1 = [12.2766 \ 5.5713], \quad F_2 = [45.4616 \ 19.9933],$$

$$F_3 = [70.8777 \ 33.4419],$$

$$S_1 = \begin{bmatrix} 3299 & -7507 \\ -7507 & 45131 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 15821 & -6775 \\ -6775 & 45143 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} 20800 & -6829 \\ -6829 & 45047 \end{bmatrix}.$$

Therefore, the common linear feedback gain is

$$F = F_1 + F_2 + F_3 = [128.6160 \ 59.0065] \tag{20}$$

Theorem 1 confirms that CSTR with linear feedback gain (20) is asymptotically stable. Fig. 2 shows the simulation result of the closed-loop system under different initial conditions $x(0)$.

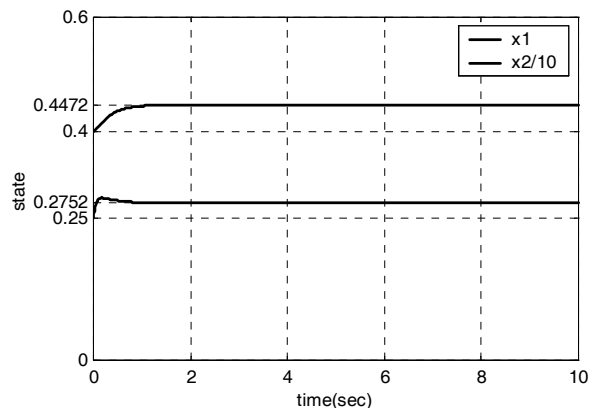


Fig. 2(a). The state responses of CSTR with linear control under operating point $x_d = [0.4472 \ 2.7520]^T$ and initial state $x(0) = [0.4 \ 2.5]^T$.

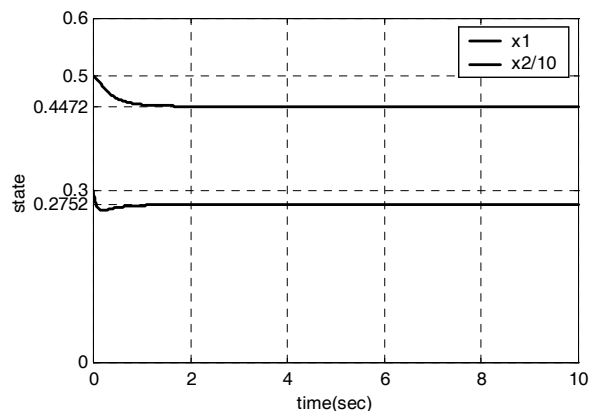


Fig. 2(b). The state responses of CSTR with linear control under operating point $x_d = [0.4472 \ 2.7520]^T$ and initial state $x(0) = [0.5 \ 3]^T$.

Next, a PDFC is synthesized as below.

Rule 1: If the temperature $x_2(t)$ is low, then

$$\hat{u}(t) = -F_1 e(t),$$

Rule 2: If the temperature $x_2(t)$ is middle, then

$$\hat{u}(t) = -F_2 e(t),$$

Rule 3: If the temperature $x_2(t)$ is high, then

$$\hat{u}(t) = -F_3 e(t).$$

The membership functions of $x_2(t)$ are shown in Fig 1. By solving the LMIs in Theorem 2, we obtain the following solution matrices:

$$X = \begin{bmatrix} 90.4688 & -3.5581 \\ -3.5581 & 91.3853 \end{bmatrix}, \quad P = \begin{bmatrix} 0.0111 & 0.0004 \\ 0.0004 & 0.0110 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 459.71 & -3.52 \\ -3.52 & 490.63 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 473.66 & -3.58 \\ -3.58 & 490.63 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 511.27 & -3.6 \\ -3.6 & 490.63 \end{bmatrix}, \quad R_{11} = \begin{bmatrix} -73.09 & -0.69 \\ -0.69 & -129.24 \end{bmatrix},$$

$$R_{22} = \begin{bmatrix} -132.9 & -0.33 \\ -0.33 & -245.23 \end{bmatrix}, \quad R_{33} = \begin{bmatrix} -380.18 & 1.12 \\ 1.12 & -245.23 \end{bmatrix},$$

$$R_{12} = R_{21} = \begin{bmatrix} -21.57 & 0.57 \\ 0.57 & -92.11 \end{bmatrix},$$

$$R_{13} = R_{31} = \begin{bmatrix} -56.89 & 0.73 \\ 0.73 & -92.1 \end{bmatrix},$$

$$R_{23} = R_{32} = \begin{bmatrix} -74.18 & 0.34 \\ 0.34 & -92.09 \end{bmatrix}.$$

Therefore, the feedback gains of PDFC are

$$F_1 = [-3.9521 \quad 7.1270], \quad F_2 = [-19.4535 \quad 15.7075],$$

$$F_3 = [-85.5504 \quad 13.5955]. \quad (21)$$

According to Theorem 2, CSTR is asymptotically stable by the PDFC with the gain (21) in each rule. The simulation result of CSTR under different initial condition $x(0)$ is shown in Fig.3. It is noted that the feedback gains (21) of PDFC is much smaller than the gain (20) of the common linear feedback controller. This coincides with the expression in Remark 2.

5. Conclusions

This paper has designed a linear feedback control and a PDFC to stabilize the T-S fuzzy time-delay system, respectively. Based on Lyapunov stability criterion and Razumikhin theorem, some sufficient conditions are presented in Theorem 1 and Theorem 2, respectively, under which the closed-loop fuzzy time-delay system is asymptotically stable. By solving a set of LMIs, both types of the controllers can be obtained. Moreover, Remark 2 points out that the plant with fuzzy rule base model can be stabilized by a large gain common feedback controller, and the PDFC can achieve the stabiliza-

tion with much smaller gain. Also, the differences between the solutions in Theorem 1 and Theorem 2 have been analyzed in Remark 3.

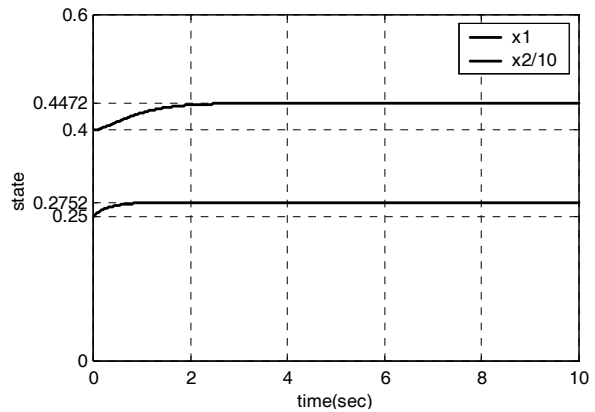


Fig. 3(a). The state responses of CSTR with PDFC under operating point $x_d = [0.4472 \quad 2.7520]^T$ and initial state $x(0) = [0.4 \quad 2.5]^T$.

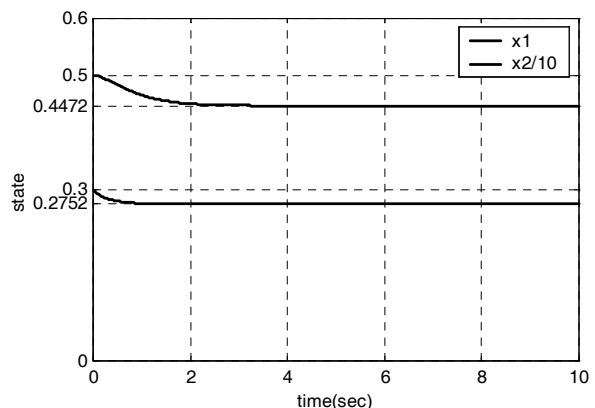


Fig. 3(b). The state responses of CSTR with PDFC under operating point $x_d = [0.4472 \quad 2.7520]^T$ and initial state $x(0) = [0.5 \quad 3]^T$.

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