Fuzzy-model-based Control Systems Using Fuzzy Combination Techniques

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Abstract

This paper presents a fuzzy combined model and a fuzzy combined controller to handle nonlinear systems. A fuzzy combination of some local fuzzy models is employed to represent a nonlinear system. Based on this fuzzy combined model, a fuzzy controller combining some local fuzzy controllers is proposed to control the nonlinear system. Conditions are derived to guarantee the system stability. By using the fuzzy combination technique, the stability analysis is reduced to investigating only the stability of the local fuzzy control systems. The system that combines the local fuzzy systems will then be guaranteed stable. This property reduces the difficulty of finding the solution to the stability conditions even though the fuzzy combined model has effectively distributed the complex nonlinearity over the local fuzzy models. Furthermore, any sharp changes in the system states and control signals caused by the switching activities between local fuzzy control systems are smoothed out by the fuzzy combined controller. An application example will be given to show the merits of the proposed approach.

1. Introduction

Fuzzy-model-based control is a popular approach to handle complex nonlinear systems. By using a TS-fuzzy model [1], [2], a nonlinear system can be represented as a weighted sum of some linear sub-systems. This approach gives a general framework for modelling nonlinear systems. The linear characteristic of the nonlinear system is extracted and shown explicitly in the fuzzy model. Furthermore, the fuzzy model exhibits outstanding approximation and generalization abilities. These properties facilitate the stability analysis and control of the nonlinear systems. Different stability conditions and relaxed conditions [3]-[15] have been derived to guarantee the system stability. In [3]-[15], the fuzzy controller is a weighted sum of linear sub-controllers. Consequently, the closed-loop system is represented as a weighted sum of some linear control systems. In general, the system stability is guaranteed by finding a common solution to some Lyapunov inequalities. The advantage of this approach is that the stability conditions can be represented as some linear matrix inequalities (LMIs) of which the solution can be solved numerically using some software tool. However, when the nonlinearities become complex, these stability conditions may not be solved.

The switching control approach is effective in handling systems with complex nonlinearity. In general, it can be categorized into two classes [16], namely the autonomous and controlled switching approaches. For the autonomous switching approach, the switching actions of the switching components are activated when the system states hit some boundaries. For the controlled switching approach, the switching actions of the switching components are activated subject to some switching laws. Both of the switching approaches were applied in some fuzzy models to represent nonlinear systems. An autonomous switching fuzzy model, which consists of some local fuzzy models, was proposed in [17]. The local fuzzy models will switch among one another to represent the dynamics of the nonlinear system based on the information of the system states. A local fuzzy controller can be designed corresponding to a local fuzzy model. During the operation, the local fuzzy controllers will switch among one another to handle the nonlinear control system. A controlled switching fuzzy model was proposed in [18]. The switching action is controlled by some switching laws that will single out a linear model to represent the nonlinear system at any instant. A linear controller corresponding to the selected linear system is then applied to handle the nonlinear system. In general, both classes of switching fuzzy models distribute the nonlinearity over the local models. This property makes the stability conditions to be satisfied more easily as the nonlinearities of the local models are less complex when compared to those of the nonlinear system. However, switching actions will generate an undesired chattering effect and sharp changes in the system states and control signals. A smoothing approach [17] was proposed to alleviate this problem by assigning closer values to the feedback gains at the boundaries. Though the undesired effects due to switching are reduced, the problem is

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not fully solved. Furthermore, this smoothing approach restricts the design flexibility of the controller.

To eliminate the undesired side-effects of the switching actions while keeping the advantages of the switching fuzzy control approach, a fuzzy combined model and a fuzzy combined controller are proposed. The fuzzy combined model involves a fuzzy combination of some local fuzzy models to represent the nonlinear system. Based on this fuzzy combined model, a fuzzy combined controller combining some local fuzzy controllers is proposed to control the nonlinear system. By using the fuzzy combination technique, the switching activities are smoothed out such that the chattering effects and sharp changes appearing in the system states and control signals are eliminated. Stability conditions will be derived to guarantee the system stability. With the favorable properties of the fuzzy combined model and controller, the system stability is governed only by the local fuzzy control systems individually.

This paper is organized as follows. In section 2, the fuzzy combined model and the fuzzy combined controller are presented. In section 3, the stability of the fuzzy combined control system will be analyzed. Stability conditions will be derived based on the Lyapunov stability theory. In section 4, an application example will be presented to illustrate the design procedure and the merits of the proposed approach. A conclusion will be drawn in section 5.

2. Fuzzy Combined Model and Controller

The fuzzy combined model and the fuzzy combined controller will be discussed in this section.

A. *Local Fuzzy Plant Model*

Let the union of the operating domains of Φ local fuzzy models be denoted by $\mathbf{X}$, which is the operating domain of the nonlinear system to be handled. The rules of the i-th local fuzzy model are defined as,

Rule j: IF $f_1^i(\mathbf{x}(t))$ is $\mathrm{M}_{j,1}^i$ AND … AND $f_\Psi^i(\mathbf{x}(t))$ is $\mathrm{M}_{j,\Psi}^i$ THEN $\dot{\mathbf{x}}(t)=\mathbf{A}_{ij}\mathbf{x}(t)+\mathbf{B}_{ij}\mathbf{u}(t)$ for $\mathbf{x}(t)\in\mathbf{X}_i$; $i = 1, 2, ..., \Phi$ (1)

where $\mathrm{M}_{j,\alpha}^i$ is the fuzzy term of the j-th rule of the i-th local fuzzy model corresponding to the function $f_\alpha^i(\mathbf{x}(t))$; $\alpha = 1, 2, \ldots, \Psi$; $i = 1, 2, \ldots, \Phi$; $j = 1, 2, \ldots, p$; p and Ψ denote the numbers of rules and fuzzy terms respectively; $\mathbf{A}_{ij}\in\Re^{n\times n}$ and $\mathbf{B}_{ij}\in\Re^{n\times m}$ are the known constant system and input matrices of rule j for the i-th local fuzzy model respectively; $\mathbf{x}(t)\in\Re^{n\times 1}$ is the system state vector and $\mathbf{u}(t)\in\Re^{m\times 1}$ is the input vector, $\mathbf{X}_i$ denotes the operating domain of the local fuzzy model and $\mathbf{X}=\bigcup_{i=1}^{\Phi}\mathbf{X}_i$. The inferred i-th local fuzzy model is defined as,

$$\dot{\mathbf{x}}(t)=\sum_{j=1}^{p} w_{ij}(\mathbf{x}(t))\left(\mathbf{A}_{ij}\mathbf{x}(t)+\mathbf{B}_{ij}\mathbf{u}(t)\right) \tag{2}$$

where

$$w_{ij}(\mathbf{x}(t))=\frac{\prod_{\alpha=1}^{\Psi}\mu_{\mathrm{M}_{j,\alpha}^i}\left(f_\alpha^i(\mathbf{x}(t))\right)}{\sum_{k=1}^{p}\prod_{\zeta=1}^{\Psi}\mu_{\mathrm{M}_{k,\zeta}^i}\left(f_\zeta^i(\mathbf{x}(t))\right)} \tag{3}$$

$$\sum_{j=1}^{p} w_{ij}(\mathbf{x}(t))=1,\quad w_{ij}(\mathbf{x}(t))\in\begin{bmatrix}0 & 1\end{bmatrix}\ \text{for all } i \text{ and } j \tag{4}$$

and $\mu_{\mathrm{M}_{j,\alpha}^i}(f_\alpha^i(\mathbf{x}(t)))$ is the grade of membership function corresponding to the fuzzy term $\mathrm{M}_{j,\alpha}^i$. $w_{ij}(\mathbf{x}(t))$ is the normalized grade of membership which can be regarded as a weighting function. Its value indicates the firing strength of rule j in the i-th local fuzzy model. It can be seen from (2) that the value of $w_{ij}(\mathbf{x}(t))$ determines the contribution of the j-th linear sub-system in the i-th local fuzzy model to the nonlinear plant model. If $w_{ij}(\mathbf{x}(t))$ has a large value, the system dynamics of the i-th fuzzy model will be dominated by the j-th linear sub-system.

B. *Fuzzy Combined Model*

The fuzzy combined model of the plant involves a fuzzy combination of the local fuzzy plant models for describing the dynamics of the nonlinear plant. The rule of the fuzzy combined model is defined as,

Rule l: IF $g_1(\mathbf{x}(t))$ is N_1^l AND ... AND $g_\Omega(\mathbf{x}(t))$ is N_Ω^l

THEN $\dot{\mathbf{x}}(t)=\sum_{j=1}^{p} w_{lj}(\mathbf{x}(t))\left(\mathbf{A}_{lj}\mathbf{x}(t)+\mathbf{B}_{lj}\mathbf{u}(t)\right)$ for $\mathbf{x}(t)\in\mathbf{X}_l$, $l = 1, 2, ..., \Phi$ (5)

where N_β^l is the fuzzy term of the l-th rule corresponding to $g_\beta(\mathbf{x}(t))$; $l = 1, 2, ..., \Phi$; $\beta = 1, 2, \ldots, \Omega$. The fuzzy combined model is defined as,

$$\dot{\mathbf{x}}(t)=\sum_{l=1}^{\Phi} v_l(\mathbf{x}(t))\sum_{j=1}^{p} w_{lj}(\mathbf{x}(t))\left(\mathbf{A}_{lj}\mathbf{x}(t)+\mathbf{B}_{lj}\mathbf{u}(t)\right) \tag{6}$$

where

$$v_l(\mathbf{x}(t))=\frac{\prod_{\beta=1}^{\Omega}\mu_{\mathrm{N}_\beta^l}\left(g_\beta(\mathbf{x}(t))\right)}{\sum_{k=1}^{\Phi}\prod_{\xi=1}^{\Omega}\mu_{\mathrm{N}_\xi^k}\left(g_\xi(\mathbf{x}(t))\right)} \tag{7}$$

$$\sum_{l=1}^{\Phi} v_l(\mathbf{x}(t))=1,\quad v_l(\mathbf{x}(t))\in\begin{bmatrix}0 & 1\end{bmatrix} \tag{8}$$

and $\mu_{N_\beta^l}\left(g_\beta(\mathbf{x}(t))\right)$ is the membership function for N_β^l. With the fuzzy combination technique, the transition between sub-operating regions of the fuzzy combined model can be ensured to be smooth and continuous.

C. *Local Fuzzy Controller*

A local fuzzy controller can be designed based on the *i*-th local fuzzy plant model operating in $\mathbf{X}_i$. The *j*-th rule is of the following format:

Rule *j*: IF $f_1^i(\mathbf{x}(t))$ is $M_{j,1}^i$ AND … AND $f_\Psi^i(\mathbf{x}(t))$ is $M_{j,\Psi}^i$ THEN $\mathbf{u}(t)=\mathbf{G}_{ij}\mathbf{x}(t)$ for $\mathbf{x}(t)\in\mathbf{X}_i$, i = 1, 2, ..., Φ (9)

where $\mathbf{G}_{ij}\in\Re^{m\times n}$ denotes the feedback gain for the *j*-th rules of the fuzzy controller of the *i*-th local fuzzy model. The control law of the *i*-th local fuzzy controller is defined as,

$$\mathbf{u}(t)=\sum_{j=1}^{p}w_{ij}(\mathbf{x}(t))\mathbf{G}_{ij}\mathbf{x}(t) \quad 10)$$

D. *Fuzzy Combined Controller*

The fuzzy combined controller involves a fuzzy combination of the local fuzzy controllers. The rule of the fuzzy combined controller is defined as,

Rule *l*: IF $g_1(\mathbf{x}(t))$ is N_1^l AND ... AND $g_\Omega(\mathbf{x}(t))$ is N_Ω^l THEN $\mathbf{u}(t)=\sum_{j=1}^{p}w_{lj}(\mathbf{x}(t))\mathbf{G}_{lj}\mathbf{x}(t)$ for $\mathbf{x}(t)\in\mathbf{X}_l$; l = 1, 2, ..., Φ (11)

The control law of the fuzzy combined controller is defined as,

$$\mathbf{u}(t)=\sum_{l=1}^{\Phi}v_l(\mathbf{x}(t))\sum_{j=1}^{p}w_{lj}(\mathbf{x}(t))\mathbf{G}_{lj}\mathbf{x}(t) \quad (12)$$

The membership functions of the fuzzy combined model are designed such that 1) $v_l(\mathbf{x}(t))\neq 0$ when the system states are inside the operating domain $\mathbf{X}_l$, 2) $\mu_{N_\beta^l}\left(g_\beta(\mathbf{x}(t))\right)\mu_{N_\beta^k}\left(g_\beta(\mathbf{x}(t))\right)=0$ for $l\neq k$ and $\mu_{N_\beta^l}\left(g_\beta(\mathbf{x}(t))\right)$ and $\mu_{N_\beta^k}\left(g_\beta(\mathbf{x}(t))\right)$ are not adjacent membership functions. The membership functions shown in Fig. 1 are designed subject to these conditions. For instance, referring to Fig. 1, $\mu_{NE}\left(g_1(\mathbf{x}(t))\right)$ and $\mu_{ZE}\left(g_1(\mathbf{x}(t))\right)$ are adjacent membership functions but $\mu_{NE}\left(g_1(\mathbf{x}(t))\right)$ and $\mu_{PS}\left(g_1(\mathbf{x}(t))\right)$ are not. Hence, $\mu_{NE}\left(g_1(\mathbf{x}(t))\right)\mu_{ZE}\left(g_1(\mathbf{x}(t))\right)>0$ at the overlapping operating domain and $\mu_{NE}\left(g_1(\mathbf{x}(t))\right)\mu_{PS}\left(g_1(\mathbf{x}(t))\right)=0$. As the transition between local fuzzy controllers is smooth and continuous, the undesired chattering effect due to hard switching between local fuzzy controllers can be avoided.

Remark 1: $\mathbf{X}_i$ is designed to have a small part of overlapping between adjacent operating domains of the local fuzzy models. As a result, those adjacent local fuzzy models at their common overlapping operating domain are equivalent to each other as they represent the same system inside this overlapping operating domain. With this property, $\sum_{j=1}^{p}w_{ij}(\mathbf{x}(t))\left(\mathbf{A}_{ij}\mathbf{x}(t)+\mathbf{B}_{ij}\mathbf{u}(t)\right)$ = $\sum_{j=1}^{p}w_{kj}(\mathbf{x}(t))\left(\mathbf{A}_{kj}\mathbf{x}(t)+\mathbf{B}_{kj}\mathbf{u}(t)\right)$ at the overlapping operating domain of which $v_i(\mathbf{x}(t))v_k(\mathbf{x}(t))>0$. It should be noted that $v_i(\mathbf{x}(t))v_k(\mathbf{x}(t))=0$ outside the overlapping operating domains, and $\sum_{j=1}^{p}w_{ij}(\mathbf{x}(t))\left(\mathbf{A}_{ij}\mathbf{x}(t)+\mathbf{B}_{ij}\mathbf{u}(t)\right)$ and $\sum_{j=1}^{p}w_{kj}(\mathbf{x}(t))\left(\mathbf{A}_{kj}\mathbf{x}(t)+\mathbf{B}_{kj}\mathbf{u}(t)\right)$ will not contribute to the fuzzy combined model. It is an important property that will be used in the stability analysis. This property is shown in the following example.

Example 1: In this example, the idea of the fuzzy combined model and the fuzzy combined controller will be shown. A fuzzy combined model having 6 rules is shown as follows.

Rule *i*: IF $g_1(\mathbf{x}(t))$ is N_1^i AND $g_2(\mathbf{x}(t))$ is N_2^i THEN $\dot{\mathbf{x}}(t)=\sum_{j=1}^{p}w_{ij}(\mathbf{x}(t))\left(\mathbf{A}_{ij}\mathbf{x}(t)+\mathbf{B}_{ij}\mathbf{u}(t)\right)$ for $\mathbf{x}(t)\in\mathbf{X}_i$, i = 1, 2, ..., 6 where N_1^i is set to be NE, NE, ZE, ZE, PS and PS respectively for i = 1, 2, ..., 6; N_2^i is set to be NE, PS, NE, PS, NE and PS for i = 1, 2, ..., 6; NE (negative), ZE (zero) and PS (positive) are fuzzy terms. 3 and 2 fuzzy terms are assigned to the functions $g_1(\mathbf{x}(t))$ and $g_2(\mathbf{x}(t))$ respectively. The fuzzy combined model, from (6) is given by,

$$\dot{\mathbf{x}}(t)=\sum_{l=1}^{6}v_l(\mathbf{x}(t))\sum_{j=1}^{p}w_{lj}(\mathbf{x}(t))\left(\mathbf{A}_{lj}\mathbf{x}(t)+\mathbf{B}_{lj}\mathbf{u}(t)\right) \quad (13)$$

The membership functions for the fuzzy terms and the local fuzzy models are shown in Fig. 1. Referring to this figure, it can be seen that $v_1(\mathbf{x}(t))=\mu_{NE}\left(g_1(\mathbf{x}(t))\right)\mu_{NE}\left(g_2(\mathbf{x}(t))\right)$, $v_2(\mathbf{x}(t))=\mu_{NE}\left(g_1(\mathbf{x}(t))\right)\mu_{PS}\left(g_2(\mathbf{x}(t))\right)$, $v_3(\mathbf{x}(t))=\mu_{ZE}\left(g_1(\mathbf{x}(t))\right)\mu_{NE}\left(g_2(\mathbf{x}(t))\right)$, $v_4(\mathbf{x}(t))=\mu_{ZE}\left(g_1(\mathbf{x}(t))\right)\mu_{PS}\left(g_2(\mathbf{x}(t))\right)$, $v_5(\mathbf{x}(t))=\mu_{PS}\left(g_1(\mathbf{x}(t))\right)\mu_{NE}\left(g_2(\mathbf{x}(t))\right)$, $v_6(\mathbf{x}(t))=\mu_{PS}\left(g_1(\mathbf{x}(t))\right)\mu_{PS}\left(g_2(\mathbf{x}(t))\right)$; $\sum_{l=1}^{6}v_l(\mathbf{x}(t))=1$. Referring to Fig. 1, region l (characterized by $v_l(\mathbf{x}(t))>0$) is the operating domain of the local plant model l; l = 1, 2, ..., 6. The areas in shade denote the overlapping operating domains. For instance, the area A is the overlapping operating domain of the local fuzzy models 1 and 2. Hence, in this area, $v_1(\mathbf{x}(t))v_2(\mathbf{x}(t)) > 0$. Furthermore, local fuzzy models 1 and 2 are equivalent inside the area A as they are both valid local fuzzy models in this area, i.e., $\dot{\mathbf{x}}(t) = \sum_{j=1}^{p} w_{1j}(\mathbf{x}(t))\left(\mathbf{A}_{1j}\mathbf{x}(t) + \mathbf{B}_{1j}\mathbf{u}(t)\right)$ = $= \sum_{j=1}^{p} w_{2j}(\mathbf{x}(t))\left(\mathbf{A}_{2j}\mathbf{x}(t) + \mathbf{B}_{2j}\mathbf{u}(t)\right)$. Take the area C as another example, the local fuzzy models 1 to 4 are equivalent inside this area ($v_1(\mathbf{x}(t))v_2(\mathbf{x}(t)) > 0$, $v_1(\mathbf{x}(t))v_3(\mathbf{x}(t)) > 0$, $v_2(\mathbf{x}(t))v_4(\mathbf{x}(t)) > 0$ and $v_3(\mathbf{x}(t))v_4(\mathbf{x}(t)) > 0$). In short, all local fuzzy models included in a common overlapping operating domain are equivalent. A fuzzy combined controller with the following 6 rules is designed for the fuzzy combined model.

Rule i: IF $g_1(\mathbf{x}(t))$ is N_1^i AND $g_2(\mathbf{x}(t))$ is N_2^i THEN $\mathbf{u}(t) = \sum_{j=1}^{p} w_{ij}(\mathbf{x}(t))\mathbf{G}_{ij}\mathbf{x}(t)$ for $\mathbf{x}(t) \in \mathbf{X}_i$, i = 1, 2, ..., 6. The fuzzy combined controller, from (12), is given by,

$$\mathbf{u}(t) = \sum_{l=1}^{6} v_l(\mathbf{x}(t)) \sum_{j=1}^{p} w_{lj}(\mathbf{x}(t))\mathbf{G}_{lj}\mathbf{x}(t) \tag{14}$$

In the following, for simplicity, $w_{lj}(\mathbf{x}(t))$ and $v_l(\mathbf{x}(t))$ are written as w_{ij} and v_l respectively. Furthermore, the property that $\sum_{l=1}^{\Phi} v_l = \sum_{j=1}^{p} w_{ij} = \sum_{j=1}^{p}\sum_{l=1}^{\Phi} w_{ij} v_l = 1$ for all i will be used. From (13) and (14), the closed-loop system is given by.

$$\dot{\mathbf{x}}(t) = \sum_{l=1}^{6} v_l \sum_{j=1}^{p} w_{lj} \left(\mathbf{A}_{lj}\mathbf{x}(t) + \mathbf{B}_{lj} \left(\sum_{k=1}^{6} v_k \sum_{q=1}^{p} w_{kq} \mathbf{G}_{kq} \mathbf{x}(t) \right) \right)$$
$$= \sum_{l=1}^{6} \sum_{k=1=1}^{6} \sum_{j=1}^{p} \sum_{q=1}^{p} v_l v_k w_{lj} w_{kq} \left(\mathbf{A}_{lj} + \mathbf{B}_{lj}\mathbf{G}_{kq} \right)\mathbf{x}(t) \tag{15}$$

Referring to Fig. 1, we have $v_1(\mathbf{x}(t))v_5(\mathbf{x}(t)) = v_5(\mathbf{x}(t))v_1(\mathbf{x}(t)) = 0$, $v_1(\mathbf{x}(t))v_6(\mathbf{x}(t)) = v_6(\mathbf{x}(t))v_1(\mathbf{x}(t)) = 0$, $v_2(\mathbf{x}(t))v_5(\mathbf{x}(t)) = v_5(\mathbf{x}(t))v_2(\mathbf{x}(t)) = 0$, $v_2(\mathbf{x}(t))v_6(\mathbf{x}(t)) = v_6(\mathbf{x}(t))v_2(\mathbf{x}(t)) = 0$ at any instant. Hence, (15) can be reduced to

$$\dot{\mathbf{x}}(t) = \sum_{l=1}^{6} \sum_{j=1}^{p} \sum_{q=1}^{p} v_l v_l w_{lj} w_{kq} \left(\mathbf{A}_{lj} + \mathbf{B}_{lj}\mathbf{G}_{kq} \right)\mathbf{x}(t)$$
$$+ \underbrace{\sum_{l=1}^{6} \sum_{k=1}^{6} \sum_{j=1}^{p} \sum_{q=1}^{p} v_l v_k w_{lj} w_{kq} \left(\mathbf{A}_{lj} + \mathbf{B}_{lj}\mathbf{G}_{kq} \right)\mathbf{x}(t)}_{l \neq k \text{ and } v_l v_k > 0} \tag{16}$$

The second term on the right hand side of (16) contains all the terms of $\sum_{j=1}^{p}\sum_{q=1}^{p} v_l v_k w_{lj} w_{kq} \left(\mathbf{A}_{lj} + \mathbf{B}_{lj}\mathbf{G}_{kq} \right)\mathbf{x}(t)$ with $l \neq k$ and $v_l(\mathbf{x}(t))v_k(\mathbf{x}(t)) > 0$. From (16),

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \sum_{l=1}^{6}\sum_{j=1}^{p}\sum_{q=1}^{p} v_l v_l w_{lj} w_{lq} \left(\mathbf{A}_{lj} + \mathbf{B}_{lj}\mathbf{G}_{lq} \right)\mathbf{x}(t) \\
&+ v_1 v_2 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{1j} w_{2q} \left(\mathbf{A}_{1j} + \mathbf{B}_{1j}\mathbf{G}_{2q} \right)\mathbf{x}(t) + v_2 v_1 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{2j} w_{1q} \left(\mathbf{A}_{2j} + \mathbf{B}_{2j}\mathbf{G}_{1q} \right)\mathbf{x}(t) \\
&+ v_1 v_3 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{1j} w_{3q} \left(\mathbf{A}_{1j} + \mathbf{B}_{1j}\mathbf{G}_{3q} \right)\mathbf{x}(t) + v_3 v_1 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{3j} w_{1q} \left(\mathbf{A}_{3j} + \mathbf{B}_{3j}\mathbf{G}_{1q} \right)\mathbf{x}(t) \\
&+ v_1 v_4 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{1j} w_{4q} \left(\mathbf{A}_{1j} + \mathbf{B}_{1j}\mathbf{G}_{4q} \right)\mathbf{x}(t) + v_4 v_1 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{4j} w_{1q} \left(\mathbf{A}_{4j} + \mathbf{B}_{4j}\mathbf{G}_{1q} \right)\mathbf{x}(t) \\
&+ v_2 v_3 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{2j} w_{3q} \left(\mathbf{A}_{2j} + \mathbf{B}_{2j}\mathbf{G}_{3q} \right)\mathbf{x}(t) + v_3 v_2 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{3j} w_{2q} \left(\mathbf{A}_{3j} + \mathbf{B}_{3j}\mathbf{G}_{2q} \right)\mathbf{x}(t) \\
&+ v_2 v_4 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{2j} w_{4q} \left(\mathbf{A}_{2j} + \mathbf{B}_{2j}\mathbf{G}_{4q} \right)\mathbf{x}(t) + v_4 v_2 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{4j} w_{2q} \left(\mathbf{A}_{4j} + \mathbf{B}_{4j}\mathbf{G}_{2q} \right)\mathbf{x}(t) \\
&+ v_3 v_4 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{3j} w_{4q} \left(\mathbf{A}_{3j} + \mathbf{B}_{3j}\mathbf{G}_{4q} \right)\mathbf{x}(t) + v_4 v_3 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{4j} w_{3q} \left(\mathbf{A}_{4j} + \mathbf{B}_{4j}\mathbf{G}_{3q} \right)\mathbf{x}(t) \\
&+ v_3 v_5 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{3j} w_{5q} \left(\mathbf{A}_{3j} + \mathbf{B}_{3j}\mathbf{G}_{5q} \right)\mathbf{x}(t) + v_5 v_3 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{5j} w_{3q} \left(\mathbf{A}_{5j} + \mathbf{B}_{5j}\mathbf{G}_{3q} \right)\mathbf{x}(t) \\
&+ v_3 v_6 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{3j} w_{6q} \left(\mathbf{A}_{3j} + \mathbf{B}_{3j}\mathbf{G}_{6q} \right)\mathbf{x}(t) + v_6 v_3 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{6j} w_{3q} \left(\mathbf{A}_{6j} + \mathbf{B}_{6j}\mathbf{G}_{3q} \right)\mathbf{x}(t) \\
&+ v_4 v_5 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{4j} w_{5q} \left(\mathbf{A}_{4j} + \mathbf{B}_{4j}\mathbf{G}_{5q} \right)\mathbf{x}(t) + v_5 v_4 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{5j} w_{4q} \left(\mathbf{A}_{5j} + \mathbf{B}_{5j}\mathbf{G}_{4q} \right)\mathbf{x}(t) \\
&+ v_4 v_6 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{4j} w_{6q} \left(\mathbf{A}_{4j} + \mathbf{B}_{4j}\mathbf{G}_{6q} \right)\mathbf{x}(t) + v_6 v_4 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{6j} w_{4q} \left(\mathbf{A}_{6j} + \mathbf{B}_{6j}\mathbf{G}_{4q} \right)\mathbf{x}(t) \\
&+ v_5 v_6 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{5j} w_{6q} \left(\mathbf{A}_{5j} + \mathbf{B}_{5j}\mathbf{G}_{6q} \right)\mathbf{x}(t) + v_6 v_5 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{6j} w_{5q} \left(\mathbf{A}_{6j} + \mathbf{B}_{6j}\mathbf{G}_{5q} \right)\mathbf{x}(t)
\end{aligned} \tag{17}$$

From Remark 1, those local fuzzy models at an overlapping operating domain are equivalent. Taking l = 1 and 2 for instance, $\dot{\mathbf{x}}(t) = \sum_{j=1}^{p} w_{1j}(\mathbf{x}(t))\left(\mathbf{A}_{1j}\mathbf{x}(t) + \mathbf{B}_{1j}\mathbf{u}(t)\right)$ = $\sum_{j=1}^{p} w_{2j}(\mathbf{x}(t))\left(\mathbf{A}_{2j}\mathbf{x}(t) + \mathbf{B}_{2j}\mathbf{u}(t)\right)$ at the overlapping operating domain. Hence, taking k = 1 and 2, $\sum_{j=1}^{p}\sum_{q=1}^{p} w_{1j} w_{2q} \left(\mathbf{A}_{1j} + \mathbf{B}_{1j}\mathbf{G}_{2q} \right)\mathbf{x}(t)$ = $\sum_{j=1}^{p}\sum_{q=1}^{p} w_{2j} w_{2q} \left(\mathbf{A}_{2j} + \mathbf{B}_{2j}\mathbf{G}_{2q} \right)\mathbf{x}(t)$ and $\sum_{j=1}^{p}\sum_{q=1}^{p} w_{2j} w_{1q} \left(\mathbf{A}_{2j} + \mathbf{B}_{2j}\mathbf{G}_{1q} \right)\mathbf{x}(t)$ = $\sum_{j=1}^{p}\sum_{q=1}^{p} w_{1j} w_{1q} \left(\mathbf{A}_{1j} + \mathbf{B}_{1j}\mathbf{G}_{1q} \right)\mathbf{x}(t)$ at the overlapping operating domain. When the system is working outside the overlapping operating domain, the interactive local fuzzy control systems $v_1 v_2 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{1j} w_{2q} \left(\mathbf{A}_{1j} + \mathbf{B}_{1j}\mathbf{G}_{2q} \right)\mathbf{x}(t)$ = $v_1 v_2 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{2j} w_{2q} \left(\mathbf{A}_{2j} + \mathbf{B}_{2j}\mathbf{G}_{2q} \right)\mathbf{x}(t)$ = $v_2 v_1 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{2j} w_{1q} \left(\mathbf{A}_{2j} + \mathbf{B}_{2j}\mathbf{G}_{1q} \right)\mathbf{x}(t)$ = $v_1 v_1 \sum_{j=1}^{p}\sum_{q=1}^{p} w_{1j} w_{1q} \left(\mathbf{A}_{1j} + \mathbf{B}_{1j}\mathbf{G}_{1q} \right)\mathbf{x}(t)$ = 0 as $v_1 v_2 = v_2 v_1 = 0$. Hence, the closed-loop system is reduced to,

$$\dot{\mathbf{x}}(t)=\sum_{l=1}^{6}\sum_{j=1}^{p}\sum_{q=1}^{p}v_l v_l w_{lj}w_{lq}\left(\mathbf{A}_{lj}+\mathbf{B}_{lj}\mathbf{G}_{lq}\right)\mathbf{x}(t)+\underbrace{\sum_{l=1}^{6}\sum_{k=1}^{6}\sum_{j=1}^{p}\sum_{q=1}^{p}v_l v_k w_{kj}w_{kq}\left(\mathbf{A}_{kj}+\mathbf{B}_{kj}\mathbf{G}_{kq}\right)\mathbf{x}(t)}_{l\neq k \text{ and } v_l v_k>0}$$

$$=\sum_{l=1}^{6}\sum_{j=1}^{p}\sum_{q=1}^{p}v_l v_l w_{lj}w_{lq}\left(\mathbf{A}_{lj}+\mathbf{B}_{lj}\mathbf{G}_{lq}\right)\mathbf{x}(t)+\underbrace{\sum_{k=1}^{6}\sum_{l=1}^{6}\sum_{j=1}^{p}\sum_{q=1}^{p}v_k v_l w_{lj}w_{lq}\left(\mathbf{A}_{lj}+\mathbf{B}_{lj}\mathbf{G}_{lq}\right)\mathbf{x}(t)}_{l\neq k \text{ and } v_l v_k>0} \quad (18)$$

Thanks to the particular design of the membership functions of the fuzzy combined model and controllers, the fuzzy control systems formed by the k-th and l-th local fuzzy models and fuzzy controllers can be related to the l-th local fuzzy control system only as given by (18). Consequently, the closed-loop system contains only the l-th local fuzzy control systems formed by the l-th local fuzzy model and the l-th local fuzzy controller, i.e. $\dot{\mathbf{x}}(t)=\sum_{j=1}^{p}\sum_{q=1}^{p}w_{lj}w_{lq}\left(\mathbf{A}_{lj}+\mathbf{B}_{lj}\mathbf{G}_{lq}\right)\mathbf{x}(t)$. These favorable properties simplify the stability analysis to investigating only the stability of the local fuzzy control systems.

3. Stability Analysis

The stability of the closed-loop system formed by (6) and (12) will be investigated. From (6) and (12),

$$\dot{\mathbf{x}}(t)=\sum_{l=1}^{\Phi}v_l\sum_{j=1}^{p}w_{lj}\left(\mathbf{A}_{lj}\mathbf{x}(t)+\mathbf{B}_{lj}\left(\sum_{k=1}^{\Phi}v_k\sum_{q=1}^{p}w_{kq}\mathbf{G}_{kq}\mathbf{x}(t)\right)\right)$$
$$=\sum_{l=1}^{\Phi}\sum_{k=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_l v_k w_{lj}w_{kq}\left(\mathbf{A}_{lj}+\mathbf{B}_{lj}\mathbf{G}_{kq}\right)\mathbf{x}(t) \quad (19)$$

Based on Remark 1,

$$\dot{\mathbf{x}}(t)=\sum_{l=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_l v_l w_{lj}w_{lq}\left(\mathbf{A}_{lj}+\mathbf{B}_{lj}\mathbf{G}_{lq}\right)\mathbf{x}(t)+\underbrace{\sum_{k=1}^{\Phi}\sum_{l=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_k v_l w_{lj}w_{lq}\left(\mathbf{A}_{lj}+\mathbf{B}_{lj}\mathbf{G}_{lq}\right)\mathbf{x}(t)}_{l\neq k \text{ and } v_l v_k>0}$$

$$=\sum_{l=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_l v_l w_{lj}w_{lq}\mathbf{H}_{ljlq}\mathbf{x}(t)+\underbrace{\sum_{k=1}^{\Phi}\sum_{l=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_k v_l w_{lj}w_{lq}\mathbf{H}_{ljlq}\mathbf{x}(t)}_{l\neq k \text{ and } v_l v_k>0} \quad (20)$$

where

$$\mathbf{H}_{ljlq}=\mathbf{A}_{lj}+\mathbf{B}_{lj}\mathbf{G}_{lq} \quad (21)$$

To investigate the stability of (20), the following Lyapunov function is employed,

$$V=\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{x}(t) \quad (22)$$

where $\mathbf{P}\in\Re^{n\times n}$ is a symmetric positive definite matrix. From (20) and (22),

$$\dot{V}=\dot{\mathbf{x}}(t)^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)+\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\dot{\mathbf{x}}(t)$$
$$=-\sum_{l=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_l v_l w_{lj}w_{lq}\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_{ljlq}\mathbf{x}(t)-\underbrace{\sum_{k=1}^{\Phi}\sum_{l=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_k v_l w_{lj}w_{lq}\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_{ljlq}\mathbf{x}(t)}_{l\neq k \text{ and } v_l v_k>0} \quad (23)$$

where

$$\mathbf{Q}_{ljlq}=-\left(\mathbf{H}_{ljlq}{}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{H}_{ljlq}\right) \quad (24)$$

From (23),

$$\dot{V}=-\sum_{l=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_l v_l w_{lj}w_{lq}\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_{ljlq}\mathbf{x}(t)-\underbrace{\sum_{k=1}^{\Phi}\sum_{l=1}^{\Phi}\sum_{j=1}^{p}\sum_{q=1}^{p}v_k v_l w_{lj}w_{lq}\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_{ljlq}\mathbf{x}(t)}_{l\neq k \text{ and } v_l v_k>0} \quad (25)$$

$$=-\sum_{l=1}^{\Phi}v_l v_l\sum_{j=1}^{p}w_{lj}w_{lj}\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_{ljlj}\mathbf{x}(t)-\sum_{l=1}^{\Phi}v_l v_l\sum_{j<q}^{p}w_{lj}w_{lq}\mathbf{x}(t)^{\mathrm{T}}\left(\mathbf{Q}_{ljlq}+\mathbf{Q}_{lqlj}\right)\mathbf{x}(t)$$
$$-\underbrace{\sum_{k=1}^{\Phi}\sum_{l=1}^{\Phi}v_k v_l\sum_{j=1}^{p}w_{lj}w_{lj}\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_{ljlj}\mathbf{x}(t)}_{l\neq k \text{ and } v_l v_k>0}-\underbrace{\sum_{k=1}^{\Phi}\sum_{l=1}^{\Phi}v_k v_l\sum_{j<q}^{p}w_{lj}w_{lq}\mathbf{x}(t)^{\mathrm{T}}\left(\mathbf{Q}_{ljlq}+\mathbf{Q}_{lqlj}\right)\mathbf{x}(t)}_{l\neq k \text{ and } v_l v_k>0} \quad (26)$$

Let

$$\mathbf{Q}_{ljlj}-\mathbf{P}_{ljlj}>0,\ l=1,2,\ldots,\Phi;\ j=1,2,\ldots,p \quad (27)$$

$$\mathbf{Q}_{ljlq}+\mathbf{Q}_{lqlj}-\mathbf{P}_{ljlq}-\mathbf{P}_{lqlj}\geq 0,\ l=1,2,\ldots,\Phi;\ j=1,2,\ldots,p;\ q=1,2,\ldots,p;\ j<q \quad (28)$$

where $\mathbf{P}_{ljlq}=\mathbf{P}_{lqlj}{}^{\mathrm{T}}$. From (26) to (28),

$$\dot{V}\leq-\sum_{l=1}^{\Phi}v_l v_l\begin{bmatrix}w_{l1}\mathbf{x}(t)\\ w_{l2}\mathbf{x}(t)\\ \vdots\\ w_{lp}\mathbf{x}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{P}_{l1l1} & \mathbf{P}_{l1l2} & \cdots & \mathbf{P}_{l1lp}\\ \mathbf{P}_{l2l1} & \mathbf{P}_{l2l2} & \cdots & \mathbf{P}_{l2lp}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{P}_{lpl1} & \mathbf{P}_{lpl2} & \cdots & \mathbf{P}_{lplp}\end{bmatrix}\begin{bmatrix}w_{l1}\mathbf{x}(t)\\ w_{l2}\mathbf{x}(t)\\ \vdots\\ w_{lp}\mathbf{x}(t)\end{bmatrix}$$
$$-\underbrace{\sum_{k=1}^{\Phi}\sum_{l=1}^{\Phi}v_k v_l\begin{bmatrix}w_{l1}\mathbf{x}(t)\\ w_{l2}\mathbf{x}(t)\\ \vdots\\ w_{lp}\mathbf{x}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{P}_{l1l1} & \mathbf{P}_{l1l2} & \cdots & \mathbf{P}_{l1lp}\\ \mathbf{P}_{l2l1} & \mathbf{P}_{l2l2} & \cdots & \mathbf{P}_{l2lp}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{P}_{lpl1} & \mathbf{P}_{lpl2} & \cdots & \mathbf{P}_{lplp}\end{bmatrix}\begin{bmatrix}w_{l1}\mathbf{x}(t)\\ w_{l2}\mathbf{x}(t)\\ \vdots\\ w_{lp}\mathbf{x}(t)\end{bmatrix}}_{l\neq k \text{ and } v_l v_k>0}$$
$$=-\sum_{l=1}^{\Phi}v_l v_l\mathbf{z}_l(t)^{\mathrm{T}}\overline{\mathbf{P}}_l\mathbf{z}_l(t)-\underbrace{\sum_{k=1}^{\Phi}\sum_{l=1}^{\Phi}v_k v_l\mathbf{z}_l(t)^{\mathrm{T}}\overline{\mathbf{P}}_l\mathbf{z}_l(t)}_{l\neq k \text{ and } v_l v_k>0} \quad (29)$$

where $\mathbf{z}(t)_l=\begin{bmatrix}w_{l1}\mathbf{x}(t)\\ w_{l2}\mathbf{x}(t)\\ \vdots\\ w_{lp}\mathbf{x}(t)\end{bmatrix}$ and $\overline{\mathbf{P}}_l=\begin{bmatrix}\mathbf{P}_{l1l1} & \mathbf{P}_{l1l2} & \cdots & \mathbf{P}_{l1lp}\\ \mathbf{P}_{l2l1} & \mathbf{P}_{l2l2} & \cdots & \mathbf{P}_{l2lp}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{P}_{lpl1} & \mathbf{P}_{lpl2} & \cdots & \mathbf{P}_{lplp}\end{bmatrix}$.

If there exist $\mathbf{P}>0$ and $\overline{\mathbf{P}}_l>0$, $l=1,2,\ldots,\Phi$, such that the LMIs of (27) and (28) are satisfied, we have $\dot{V}\leq 0$ (equality holds when $\mathbf{x}(t)=\mathbf{0}$). It can be concluded that the closed-loop system is asymptotically stable, which implies $\mathbf{x}(t)\rightarrow\mathbf{0}$ as $t\rightarrow\infty$. The analysis result can be summarized by the following lemma.

Lemma 1: The closed-loop nonlinear system represented by the fuzzy combined model of (6) and the fuzzy combined controller of (12) is guaranteed to be asymptotically stable if there exist symmetric positive definite matrices, $\mathbf{P}$ and $\overline{\mathbf{P}}_l$ such that the following LMIs are satisfied.

$\mathbf{P} > 0$; $\overline{\mathbf{P}}_l = \begin{bmatrix} \mathbf{P}_{l1l1} & \mathbf{P}_{l1l2} & \cdots & \mathbf{P}_{l1lp} \\ \mathbf{P}_{l2l1} & \mathbf{P}_{l2l2} & \cdots & \mathbf{P}_{l2lp} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{lpl1} & \mathbf{P}_{lpl2} & \cdots & \mathbf{P}_{lplp} \end{bmatrix} > 0$, $l = 1, 2, ..., \Phi$;

$\mathbf{Q}_{ljlj} - \mathbf{P}_{ljlj} > 0$, *l = 1, 2, ..., Φ; j = 1, 2, ..., p;*

$\mathbf{Q}_{ljlq} + \mathbf{Q}_{lqlj} - \mathbf{P}_{ljlq} - \mathbf{P}_{lqlj} \geq 0$, *l = 1, 2, ..., Φ; j = 1, 2, ..., p; q = 1, 2, ..., p; j < q where* $\mathbf{P}_{ljlq} = \mathbf{P}_{lqlj}^{\mathrm{T}}$.

The stability conditions in Lemma 1 are reduced to the stability conditions in [6]-[8] when $l = 1$.

4. Application Example

An application example on stabilizing an inverted pendulum will be given in this section to show the design procedure and the merits of the proposed approach. The proposed fuzzy combined model will be employed to represent the inverted pendulum. Based on the fuzzy combined model, a fuzzy combined controller will be designed to control the inverted pendulum.

Step I) The dynamic equation governing the inverted pendulum is given by,

$$\ddot{\theta}(t) = \frac{g\sin(\theta(t)) - am_p L\dot{\theta}(t)^2 \sin(2\theta(t))/2 - a\cos(\theta(t))u(t)}{4L/3 - am_p L\cos^2(\theta(t))} \quad (30)$$

where θ is the angular displacement of the pendulum, g = 9.8m/s^2 is the acceleration due to gravity, $a = 1/(m_p + M_c)$, m_p = 2kg is the mass of the pendulum, M_c = 8kg is the mass of the cart, $2L$ = 1m is the length of the pendulum, and u is the force applied to the cart. The objective is to design a fuzzy combined controller to stabilize the inverted pendulum such that $\theta = 0$ at steady state. Let $\mathbf{x}(t) = [x_1(t) \quad x_2(t)]^{\mathrm{T}} = [\theta(t) \quad \dot{\theta}(t)]^{\mathrm{T}}$. The operating domain of the inverted pendulum is characterized by $\theta(t) \in [\theta_{\min} \quad \theta_{\max}] = \left[-\frac{22\pi}{45} \quad \frac{22\pi}{45}\right]$ and $\dot{\theta}(t) \in [\dot{\theta}_{\min} \quad \dot{\theta}_{\max}] = [-5 \quad 5]$.

Step II) The operating domain of the nonlinear plant is divided into 3 operating sub-domains as shown in Fig. 2. Three local fuzzy models will be employed to describe the inverted pendulum operating in these three sub-domains respectively. The operating sub-domains are defined as $|x_1(t)| \in \left[0 \quad \frac{11\pi}{45}\right]$ and $|x_2(t)| \in [0 \quad 5]$ for $\mathbf{X}_1$, $|x_1(t)| \in \left[\frac{11\pi}{45} - 0.1 \quad \frac{11\pi}{36} + 0.1\right]$ and $|x_2(t)| \in [0 \quad 5]$ for $\mathbf{X}_2$,and $|x_1(t)| \in \left[\frac{11\pi}{36} \quad \frac{22\pi}{45}\right]$ and $|x_2(t)| \in [0 \quad 5]$ for $\mathbf{X}_3$. Referring to (1), four fuzzy rules in the following format are employed for each local fuzzy model,

Rule j: IF $f_1^i(\mathbf{x}(t))$ is $\mathrm{M}_{j,1}^i$ AND $f_2^i(\mathbf{x}(t))$ is $\mathrm{M}_{j,2}^i$

THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_{ij}\mathbf{x}(t) + \mathbf{B}_{ij}u(t)$ for $\mathbf{x}(t) \in \mathbf{X}_i$, i = 1, 2, 3; j = 1, 2, 3, 4 (31)

where $f_1^i(\mathbf{x}(t))$ and $f_2^i(\mathbf{x}(t))$ are the system-related nonlinear functions corresponding to the fuzzy terms $\mathrm{M}_{j,1}^i$ and $\mathrm{M}_{j,2}^i$ of the j-th rule in the i-th local fuzzy model, respectively. The fuzzy terms are characterized by the membership function defined later. $\mathbf{A}_{ij} \in \Re^{2\times 2}$ and $\mathbf{B}_{ij} \in \Re^{2\times 1}$ are the constant system and input matrices respectively of the j-th linear sub-system in the i-th local fuzzy model operating in the $\mathbf{X}_i$ operating region. Referring to (6), the inferred i-th local fuzzy model is given by,

$$\dot{\mathbf{x}}(t) = \sum_{j=1}^{4} w_{ij}(\mathbf{x}(t))\left(\mathbf{A}_{ij}\mathbf{x}(t) + \mathbf{B}_{ij}u(t)\right) \quad (32)$$

The local system dynamics of the inverted pendulum operating in $\mathbf{X}_i$ operating region can be described by the local fuzzy model of (32), which is determined by 4 linear sub-systems. The contribution of each linear sub-system to the system modelling is governed by the normalized grades of membership, $w_{ij}(\mathbf{x}(t))$. The system-related nonlinear functions are defined as follows. $f_1^1(\mathbf{x}(t))$ = $f_1^2(\mathbf{x}(t))$ = $f_1^3(\mathbf{x}(t))$ = $\frac{g - am_p L x_2(t)^2\cos(x_1(t))}{4L/3 - am_p L\cos^2(x_1(t))}\left(\frac{\sin(x_1(t))}{x_1(t)}\right)$; $f_2^1(\mathbf{x}(t))$ = $f_2^2(\mathbf{x}(t))$ = $f_2^3(\mathbf{x}(t))$ = $-\frac{a\cos(x_1(t))}{\left(4L/3 - am_p L\cos^2(x_1(t))\right)}$. The constant system and input matrices are defined as $\mathbf{A}_1^i$ = $\mathbf{A}_2^i$ = $\begin{bmatrix} 0 & 1 \\ f_{1_{\min}}^i & 0 \end{bmatrix}$ and $\mathbf{A}_3^i$ = $\mathbf{A}_4^i$ = $\begin{bmatrix} 0 & 1 \\ f_{1_{\max}}^i & 0 \end{bmatrix}$; $\mathbf{B}_1^i$ = $\mathbf{B}_3^i$ = $\begin{bmatrix} 0 \\ f_{2_{\min}}^i \end{bmatrix}$ and $\mathbf{B}_2^i$ = $\mathbf{B}_4^i$ = $\begin{bmatrix} 0 \\ f_{2_{\max}}^i \end{bmatrix}$ where $f_{1_{\min}}^1$ = 11.7707, $f_{1_{\max}}^1$ = 17.2941, $f_{2_{\min}}^1$ = –0.1765, $f_{2_{\max}}^1$ = –0.1170; $f_{1_{\min}}^2$ = 10.9829, $f_{1_{\max}}^2$ = 15.0197, $f_{2_{\min}}^2$ = –0.1298, $f_{2_{\max}}^2$ = –0.0761; $f_{1_{\min}}^3$ = 9.4817, $f_{1_{\max}}^3$ = 13.1953, $f_{2_{\min}}^3$ = –0.0905 and $f_{2_{\max}}^3$ = –5.2359×10^{-3}. The normalized membership function is defined as $w_{ij} = \frac{\prod_{\alpha=1}^{2}\mu_{\mathrm{M}_{j,\alpha}^i}(f_\alpha^i(\mathbf{x}(t)))}{\sum_{l=1}^{4}\prod_{\zeta=1}^{2}\mu_{\mathrm{M}_{l,\zeta}^i}(f_\zeta^i(\mathbf{x}(t)))}$, where $\mu_{\mathrm{M}_{\beta,1}^i}(f_1^i(\mathbf{x}(t)))$ = $\frac{-f_1^i(\mathbf{x}(t)) + f_{1_{\max}}^i}{f_{1_{\max}}^i - f_{1_{\min}}^i}$ for β = 1, 2; $\mu_{\mathrm{M}_{\delta,1}^i}(f_1^i(\mathbf{x}(t)))$ =

$1-\mu_{\mathrm{M}_{1,1}^i}(f_1^i(\mathbf{x}(t)))$ for δ = 3, 4; $\mu_{\mathrm{M}_{\kappa,2}^i}(f_2^i(\mathbf{x}(t))) = \dfrac{-f_2^i(\mathbf{x}(t))+f_{2_{\max}}^i}{f_{2_{\max}}^i - f_{2_{\min}}^i}$ for κ = 1, 3; $\mu_{\mathrm{M}_{\phi,2}^i}(f_2^i(\mathbf{x}(t))) = 1-\mu_{\mathrm{M}_{1,2}^i}(f_2^i(\mathbf{x}(t)))$ for ϕ = 2, 4, are the membership functions of the i-th local fuzzy model for i = 1, 2, 3.

A fuzzy combined model with 3 rules is employed to represent the inverted pendulum. The rules of the fuzzy combined model are defined as follows,

Rule 1: IF $|x_1(t)|$ is ZE THEN

$$\dot{\mathbf{x}}(t) = \sum_{j=1}^{p} w_{1j}(\mathbf{x}(t))\left(\mathbf{A}_{1j}\mathbf{x}(t)+\mathbf{B}_{1j}u(t)\right) \text{ for } \mathbf{x}(t) \in \mathbf{X}_1 \quad (33)$$

Rule 2: IF $|x_1(t)|$ is ME THEN

$$\dot{\mathbf{x}}(t) = \sum_{j=1}^{p} w_{2j}(\mathbf{x}(t))\left(\mathbf{A}_{2j}\mathbf{x}(t)+\mathbf{B}_{2j}u(t)\right) \text{ for } \mathbf{x}(t) \in \mathbf{X}_2 \quad (34)$$

Rule 3: IF $|x_1(t)|$ is LA THEN

$$\dot{\mathbf{x}}(t) = \sum_{j=1}^{p} w_{3j}(\mathbf{x}(t))\left(\mathbf{A}_{3j}\mathbf{x}(t)+\mathbf{B}_{3j}u(t)\right) \text{ for } \mathbf{x}(t) \in \mathbf{X}_3 \quad (35)$$

where ZE (zero), ME (medium) and LA (large) are fuzzy terms. The membership functions $\mu_{\mathrm{ZE}}(|x_1(t)|)$, $\mu_{\mathrm{ME}}(|x_1(t)|)$ and $\mu_{\mathrm{PS}}(|x_1(t)|)$ corresponding to the fuzzy terms are shown in Fig. 2. The fuzzy combined model is defined as,

$$\dot{\mathbf{x}}(t) = \sum_{l=1}^{3} v_l(\mathbf{x}(t)) \sum_{j=1}^{4} w_{lj}(\mathbf{x}(t))\left(\mathbf{A}_{lj}\mathbf{x}(t)+\mathbf{B}_{lj}u(t)\right) \quad (36)$$

where $v_1(\mathbf{x}(t)) = \mu_{\mathrm{ZE}}(|x_1(t)|)$, $v_2(\mathbf{x}(t)) = \mu_{\mathrm{ME}}(|x_1(t)|)$ and $v_3(\mathbf{x}(t)) = \mu_{\mathrm{PS}}(|x_1(t)|)$.

Step III) Three local fuzzy controllers having four rules are designed corresponding to the three local fuzzy models respectively. The rule is of the following format:

Rule j: IF $f_1^i(\mathbf{x}(t))$ is $\mathrm{M}_{j,1}^i$ AND $f_2^i(\mathbf{x}(t))$ is $\mathrm{M}_{j,2}^i$

THEN $u(t) = \mathbf{G}_{ij}\mathbf{x}(t)$ for $\mathbf{x}(t) \in \mathbf{X}_i$, i = 1, 2, 3; j = 1, 2, 3, 4 (37)

The i-th local fuzzy controller is defined as,

$$u(t) = \sum_{j=1}^{4} w_{ij}(\mathbf{x}(t))\mathbf{G}_{ij}\mathbf{x}(t) \quad (38)$$

The feedback gains of the local fuzzy controllers are designed using the parallel-distribution-compensation (PDC) design approach [4] such that the eigenvalues of $\mathbf{H}_{1j1j}$, $\mathbf{H}_{2j2j}$ and $\mathbf{H}_{3j3j}$, j = 1, 2, 3, 4, are assigned to be [–10 –15], [–5 –15] and [–1 –15] respectively. Hence, we have $\mathbf{G}_{11}$ = [916.6983 141.6663], $\mathbf{G}_{12}$ = [1382.8798 213.7099], $\mathbf{G}_{13}$ = [947.9980 141.6663], $\mathbf{G}_{14}$ = [1430.0965 213.7099], $\mathbf{G}_{21}$ = [662.6204 154.1283], $\mathbf{G}_{22}$ = [1130.3580 262.9261], $\mathbf{G}_{23}$ = [693.7295 154.1283], $\mathbf{G}_{24}$ = [1183.4268 262.9261], $\mathbf{G}_{31}$ = [270.5075 176.7897], $\mathbf{G}_{32}$ = [4675.6576 3055.7678], $\mathbf{G}_{33}$ = [311.5403 176.7897] and $\mathbf{G}_{34}$ = [5384.8999 3055.7678]. The local fuzzy controllers are combined by the following 3 rules:

Rule 1: IF $|x_1(t)|$ is ZE

THEN $u(t) = \sum_{j=1}^{4} w_{1j}(\mathbf{x}(t))\mathbf{G}_{1j}\mathbf{x}(t)$ for $\mathbf{x}(t) \in \mathbf{X}_1$ (39)

Rule 2: IF $|x_1(t)|$ is ME

THEN $u(t) = \sum_{j=1}^{4} w_{2j}(\mathbf{x}(t))\mathbf{G}_{2j}\mathbf{x}(t)$ for $\mathbf{x}(t) \in \mathbf{X}_2$ (40)

Rule 3: IF $|x_1(t)|$ is LA

THEN $u(t) = \sum_{j=1}^{4} w_{3j}(\mathbf{x}(t))\mathbf{G}_{3j}\mathbf{x}(t)$ for $\mathbf{x}(t) \in \mathbf{X}_3$ (41)

The fuzzy combined controller is defined as,

$$u(t) = \sum_{l=1}^{3} v_l(\mathbf{x}(t)) \sum_{j=1}^{4} w_{lj}(\mathbf{x}(t))\mathbf{G}_{lj}\mathbf{x}(t) \quad (42)$$

Step IV) Lemma 1 will be employed to ensure the closed-loop system stability of the fuzzy combined control system. By using the MATLAB LMI toolbox, $\mathbf{P} = \begin{bmatrix} 21.6514 & 0.5300 \\ 0.5300 & 0.3238 \end{bmatrix}$, $\mathbf{P}_{l1l1} = \begin{bmatrix} 5.2160 & -1.3544 \\ -1.3544 & -1.3355 \end{bmatrix}$, $\mathbf{P}_{l1l2} = \begin{bmatrix} -0.3215 & 0.4128 \\ 0.4359 & -0.2183 \end{bmatrix}$, $\mathbf{P}_{l1l3} = \begin{bmatrix} -0.3672 & 0.5388 \\ 0.5374 & 0.0216 \end{bmatrix}$, $\mathbf{P}_{l1l4} = \begin{bmatrix} -0.2484 & 0.3919 \\ 0.3721 & -0.2355 \end{bmatrix}$, $\mathbf{P}_{l2l2} = \begin{bmatrix} 5.2320 & -1.3653 \\ -1.3653 & 1.3298 \end{bmatrix}$, $\mathbf{P}_{l2l3} = \begin{bmatrix} -0.3112 & 0.4262 \\ 0.4059 & -0.2089 \end{bmatrix}$, $\mathbf{P}_{l2l4} = \begin{bmatrix} -0.3619 & 0.5094 \\ 0.5676 & 0.0223 \end{bmatrix}$, $\mathbf{P}_{l3l3} = \begin{bmatrix} 5.2132 & -1.3491 \\ -1.3491 & 1.3287 \end{bmatrix}$, $\mathbf{P}_{l3l4} = \begin{bmatrix} -0.2384 & 0.3889 \\ 0.3663 & -0.2272 \end{bmatrix}$ and $\mathbf{P}_{l4l4} = \begin{bmatrix} 5.2132 & -1.3491 \\ -1.3491 & 1.3287 \end{bmatrix}$, l = 1, 2, 3, are obtained such that the stability conditions of Lemma 1 are satisfied. It can be concluded that the fuzzy combined control system is asymptotically stable.

Fig. 3 shows the system responses and control signals of the fuzzy combined control systems under the initial state of $\mathbf{x}(0) = \begin{bmatrix} \frac{22\pi}{45} & 0 \end{bmatrix}^{\mathrm{T}}$ $\mathbf{x}(0) = \begin{bmatrix} \frac{11\pi}{45} & 0 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{x}(0) = \begin{bmatrix} -\frac{11\pi}{45} & 0 \end{bmatrix}^{\mathrm{T}}$ and $\mathbf{x}(0) = \begin{bmatrix} -\frac{22\pi}{45} & 0 \end{bmatrix}^{\mathrm{T}}$. It can be seen that the inverted pendulum can be stabilized successfully. For comparison purpose, the published switching fuzzy controller in [17] is used to control the inverted pendulum. The partitions of the state space are shown in Fig. 4. Fig. 5 shows the system responses and control signals of the switching fuzzy control system under the initial state of $\mathbf{x}(0) = \begin{bmatrix} \frac{22\pi}{45} & 0 \end{bmatrix}^{\mathrm{T}}$. It can be seen that the proposed fuzzy combination approach is slightly better than the published switching approach in terms of faster transient responses.

Furthermore, referring to Fig. 5(b) and Fig. 5(c), sharp changes are seen in $x_2(t)$ and $u(t)$ on using the switching approach, which are smoothed out by the proposed fuzzy combination approach.

To show the stabilizability of the proposed approach over the linear control approach, a linear state-feedback controller is used to control the inverted pendulum based on the following linearized model around $\mathbf{x}(t) = \mathbf{0}$.

$$\dot{\mathbf{x}}(t) = \tilde{\mathbf{A}}\mathbf{x}(t) + \tilde{\mathbf{B}}u(t) \tag{43}$$

where $\tilde{\mathbf{A}} = \begin{bmatrix} 0 & 1 \\ \frac{M_c + m_p}{M_c L} g & 0 \end{bmatrix}$ and $\tilde{\mathbf{B}} = \begin{bmatrix} 0 \\ -\frac{1}{M_c L} \end{bmatrix}$. The linear state-feedback controller is given by,

$$u(t) = \mathbf{G}\mathbf{x}(t) \tag{44}$$

where $\mathbf{G}$ denotes the feedback gain. The feedback gain is designed as $\mathbf{G} = [418.0000 \quad 81.3333]$ such that the eigenvalues of $\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{G}$ are located at –5.3333 and –15 respectively which are the average eigenvalues used in the PDC design approach for the three local fuzzy control systems. The linear state-feedback controller of (44) will be employed to control the inverted pendulum of (30). Fig. 6 shows the system state responses of the inverted pendulum under the initial state conditions of $\mathbf{x}(0) = \begin{bmatrix} \frac{22\pi}{45} & 0 \end{bmatrix}^T$ $\mathbf{x}(0) = \begin{bmatrix} \frac{11\pi}{45} & 0 \end{bmatrix}^T$, $\mathbf{x}(0) = \begin{bmatrix} -\frac{11\pi}{45} & 0 \end{bmatrix}^T$ and $\mathbf{x}(0) = \begin{bmatrix} -\frac{22\pi}{45} & 0 \end{bmatrix}^T$. Referring to this figure, it can be seen that the linear state-feedback controller cannot stabilize the nonlinear system over a large operating region.

In the above example, the feedback gains are designed based on the PDC approach proposed in [4] under the consideration of system stability only. In general, the design of the feedback gains subject to both stability and performance considerations can be cast as an LMI- or genetic-algorithm (GA)-based problem. On using the LMI-based design approach [5]-[10], the stability and performance conditions will serve as the constraints for finding the feedback gains using some convex programming techniques. Similarly, if the GA-based approach [11] is used, the values of the controller parameters will be tuned subject to the stability and performance conditions. One of the advantages of the GA-based approach is that the stability and performance constraints are not necessary to be expressed in LMI forms. Hence, it is more suitable to deal with nonlinear systems subject to nonlinear stability and performance constraints. Based on these two design approaches, the controller parameters can be obtained in a systematic way. As both algorithms are detailed in [5]-[11] respectively, they are not reiterated in this paper.

5. Conclusion

A fuzzy combined model has been proposed to combine different local fuzzy models to represent nonlinear systems. With the fuzzy combined model, a fuzzy combined controller has been proposed to control the system. Stability conditions have been derived to guarantee the system stability. A favorable property is offered by the fuzzy combination technique that the system stability is related to the local fuzzy control system only. The sharp changes or the chattering effect appearing in the system states and control signals due to the switching actions among the local fuzzy controllers have been smoothed out by the fuzzy combination technique. Furthermore, the advantages of the switching fuzzy control systems are retained in the proposed fuzzy combined control systems. An application example has been given to show the design procedure and the merits of the proposed approach.

With the proposed fuzzy combined model, the nonlinear dynamics of the system can be described by some local fuzzy models, and the stability analysis of the overall system can be reduced to investigating the system stability of the local fuzzy control systems only. As the local fuzzy model is operating in a sub-domain, it is more feasible to have a stable local fuzzy controller than a conventional fuzzy controller that covers the full operating domain. Consequently, a relaxed stability analysis can be done. Furthermore, the proposed fuzzy combination approach has offered a general framework for analysis and design. It can be extended to various types of local models and controllers, apart from the combinations of linear sub-systems and controllers. The proposed methodology offers a systematic way to deal with highly nonlinear and complex systems. Definitely, tedious steps to prepare the local fuzzy models are required. This limitation can be alleviated by developing a software package for the system modeling process.

6. Acknowledgment

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7. References

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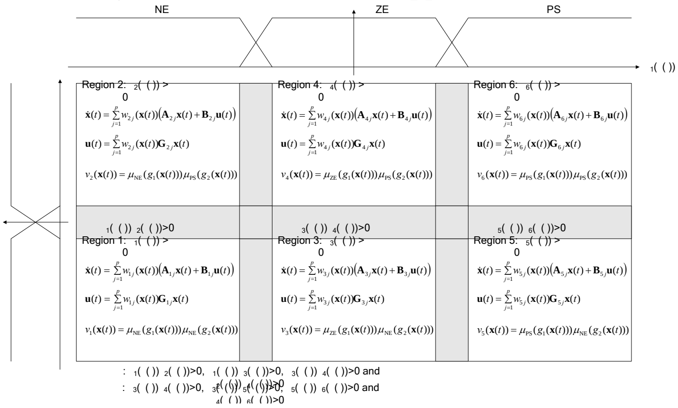

Fig. 1. Membership functions, local fuzzy models and local fuzzy controllers for Example 1.

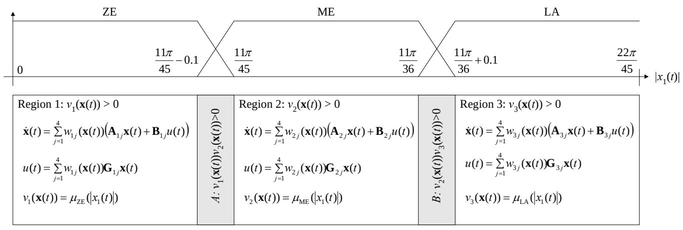

Fig. 2. Membership functions of the fuzzy combined model of the inverted pendulum.

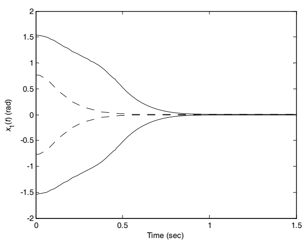

Fig. 3(a). $x_1(t)$.

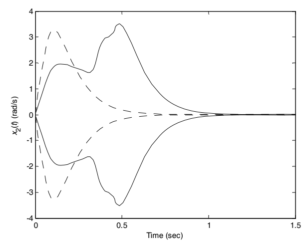

Fig. 3(b). $x_2(t)$.

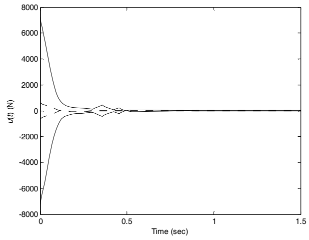

Fig. 3(c). $u(t)$.

Fig. 3. System responses and control signals of the fuzzy combined control system.

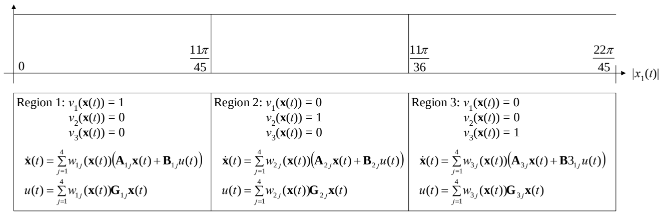

Fig. 4. Partitions of state space for the switching fuzzy model of the inverted pendulum.

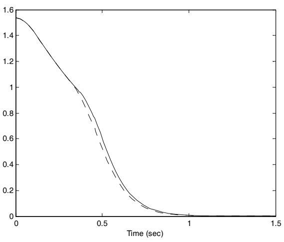

Fig. 5(a). $x_1(t)$.

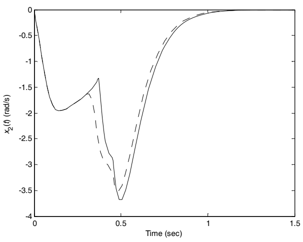

Fig. 5(b). $x_2(t)$.

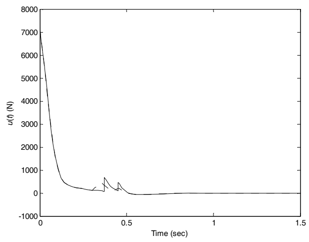

Fig. 5(c). $u(t)$.

Fig. 5. System responses and control signals of the fuzzy combined control system (dotted lines) and the switching fuzzy control system (solid lines).

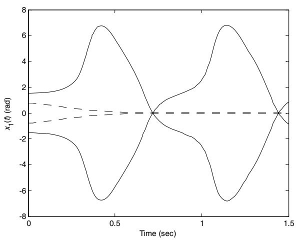

Fig. 6(a). $x_1(t)$.

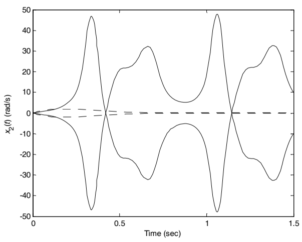

Fig. 6(b). $x_2(t)$.

Fig. 6. System responses and control signals of the inverted pendulum system using the linear state-feedback controller.

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