Design of a Fuzzified Terminal Guidance Law

Chun-Liang Lin, Tzu-Chiang Kao and Meng-Tzong Wu

Abstract

A new fuzzified terminal guidance law is proposed which employs the versatile feature of the fuzzy logic system to shape a beneficial trajectory for target interception. In the proposed scheme, the guidance law is partitioned into three stages in accordance with the relative range between the interceptor and target. Estimated line-of-sight angle, line-of-sight angle rate, line-of-sight angular acceleration and relative range are used to constitute the rule antecedent of the guidance law to shape an appropriate flight trajectory for engagement. Asymmetric membership functions and guidance rules are developed for satisfactory engagement performance and robustness. Engagement performance is well proven by extensive computer simulations.

Keywords: Fuzzy logic control, missile, guidance system, proportional navigation guidance.

1. Introduction

The popularity of proportional navigation guidance (PNG) law in traditional aerodynamically controlled missiles is based on its simplicity, effectiveness, and ease of implementation [1]. However, such a design approach cannot provide satisfactory engagement performance and performance robustness with respect to advanced fighters. Recently developed neural network algorithms and fuzzy logic theory serve as possible approaches for solving the highly nonlinear flight control problems [2]. However, very limited numbers of papers have addressed the issue of fuzzy guidance design [3]-[5].

Fuzzy control based on fuzzy logic provides a new design paradigm such that a controller can be designed for complex, ill-defined processes without knowing quantitative data regarding the input-output relations, which are required in conventional approaches [6], [7]. The use of fuzzy logic control in the filed of guidance and control is necessitated by the need to deal with the nonlinear dynamics and performance robustness problems. While a few missile guidance designs have applied the fuzzy logic theory [4], [5], these approaches should only be viewed as a direct realization of the traditional PNG in terms of fuzzy logic thinking. In [8], the concept of fuzzy logic control was applied to solve the nonlinear H_∞ guidance design problem. While the approach is theoretically attractive, it heavily relies on complicated numerical computations.

In this paper, a terminal guidance law is developed using flexibility and extensive design freedom of the fuzzy logic system. The fuzzy logic guidance (FLG) law is partitioned into three phases in accordance with the relative range between the interceptor and target; estimated line-of-sight (LOS) angle, LOS angle rate, LOS angular acceleration and relative range are used to constitute the rule antecedent of the guidance law to shape beneficial flight trajectories for efficient engagement. Compared to the traditional PNG, the proposed design gives the missile guidance system with smaller miss distances (MDs), less energy consumption and better performance robustness.

2. Construction of Homing Loop

Referring to [9], [10] a typical guidance system with trim aerodynamics and a perfectly stabilized seeker is constructed as shown Fig. 1 in which the proposed fuzzy logic-based guidance law has been included. The homing loop describing missile-target engagement is also illustrated. In the figure, n is the external noise; T_s, seeker tracking loop time constant; K_s, stabilizing loop gain; K_g, gyro gain; V_c, missile-target closing velocity; T_g, guidance filter time constant; T_α, aerodynamic turning rate time constant; V_m, missile velocity; σ, LOS angle (see Fig. 2 for the definition); ε, boresight error angle; θ_m, missile body angle; RMT, missile-target relative range; V_m, missile velocity; V_t, target, velocity; A_c, acceleration command; A_m, missile lateral acceleration; A_t, target evasive acceleration. The fuzzy guidance law is characterized by

$$A_c \triangleq Fuzz(\sigma, \dot{\sigma}, \ddot{\sigma}, RMT) \qquad (1)$$

Chun-Liang Lin is with Department of Electrical Engineering, National Chung Hsing University, Taichung, Taiwan 402, R.O.C.
E-mail: chunlin@dragon.nchu.edu.tw
Tzu-Chiang Kao and Meng-Tzong Wu are with Department of Electrical Engineering, National Chung Hsing University, Taichung, Taiwan 402, R.O.C.

Details for its design is given in the next section. For the flight control system (FCS), $K_{1,2,3}$, $a_{12,21,22}$ and $b_{11,12}$ are the aerodynamic coefficients, K_a, K_Q and K_A are the autopilot loop gains, W_I is the stabilizing loop gain.

3. Design of Fuzzified Guidance Law

Proportional navigation guidance is perhaps the most commonly adopted missile guidance law, which is typically described by

$$A_c = NV_c\dot{\sigma} \tag{2}$$

where *N* is the proportional navigation gain. The popularity of this interceptor guidance law is based on its simplicity, effectiveness, and ease of implementation. However, engagement performance provided by PNG can be further improved by fully applying available interceptor-target messages in the design.

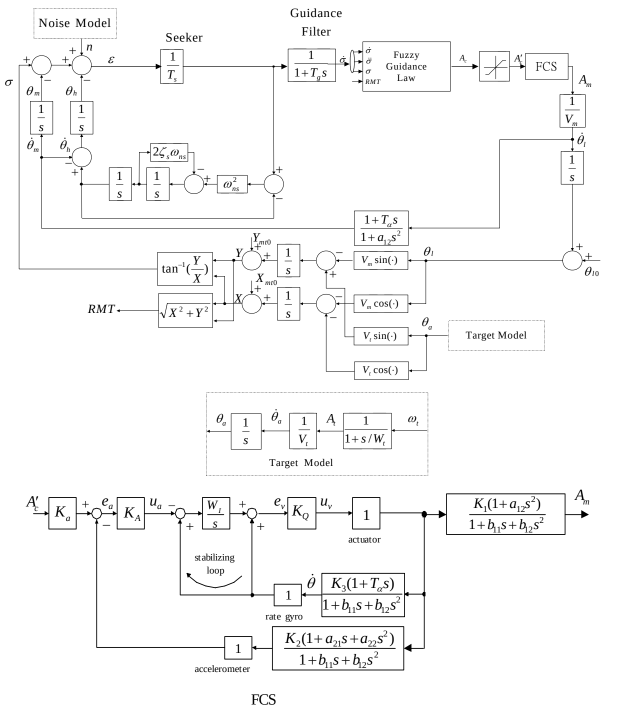

Figure 1. Simplified homing loop and missile guidance and control system

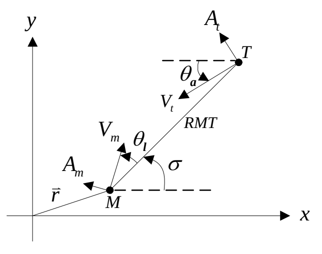

Figure 2. Definition of LOS angle

Theoretically, the PNG issues acceleration commands, perpendicular to the instantaneous missile-target line-of-sight, which are proportional to the line-of-sight rate and closing velocity. From the viewpoint of control theory, this may be viewed as a derivative gain multiplied with the derivative of tracking error. A missile employing PN guidance is not fired at the target but fired in direction lead to the target. In essence, we are firing the missile at the expected interception point. The missile will hit target if both of missile and target continue to fly along a straight-line path at constant velocities. However, this idealized assumption might be voided for a maneuvering target, environmental turbulence or gust.

Our preliminary study has shown that the missile looking down at the target with an appropriate LOS angle is an excellent attitude during the terminal phase since it possesses movable dominance with respect to the moving target. Furthermore, if the missile is on the diving course, less acceleration commands are required for the missile to hit the target. Especially, this type of engagement trajectories can efficiently attack the evasive target. However, the missile-target LOS angle should be in the available seeker field of view is a necessity.

In the present approach, the linguistic variables σ, $\dot{\sigma}$, $\ddot{\sigma}$, and RMT used in the subsequent design are expressed by linguistic sets. Each of the linguistic variables is assumed to take the following linguistic sets: LN $\equiv$ large negative, LP $\equiv$ large positive, MN $\equiv$ medium negative, MP $\equiv$ medium positive, SN $\equiv$ small negative, SP $\equiv$ small positive, ZE $\equiv$ zero, F $\equiv$ far, M $\equiv$ middle, N $\equiv$ near. Term sets described for the variables of the antecedent part, σ, $\dot{\sigma}$, $\ddot{\sigma}$, and RMT, are denoted by

$$T_\sigma \equiv \{\text{LN, SN, ZE, SP, LP}\}$$
$$T_{\dot{\sigma}} \equiv \{\text{LN, MN, SN, ZE, SP, MP, LP}\} \tag{3a}$$

$$T_{\ddot{\sigma}} \equiv \{\text{LN, MN, SN, ZE, SP, MP, LP}\}$$
$$T_{RMT} \equiv \{\text{F,M, N}\} \tag{3b}$$

The term set described for the variable A_c of the consequent part is characterized by

$$T_{A_c} \equiv \{\text{LN, MN, SN, ZE, SP, MP, LP}\} \tag{4}$$

The linguistic variable RMT is partitioned into three parts denoted F, M, and N over the homing range $[-10000, 0]$ km, shown as in Fig. 3.

At the first stage of the terminal guidance, *RMT* is F, the guidance law is devoted to drive the missile to the altitude higher than the target so that it can possess favorable attacking attitude while entering the third stage (*RMT* is N). To this aim, an asymmetric guidance rule base is developed. Since our purpose is to have a beneficial altitude rather than directly attacking the target, the information of LOS angle σ alone is enough to construct the rules. Membership functions of σ are shown in Fig. 4 with the range [-0.02, 0.02] rad and the guidance rule is listed in Table 1. Furthermore, since the position error is not the major concern and from the viewpoint of conserving the control energy, the fuzzy sets chosen are widely separated from the zero LOS angle.

Table 1. Fuzzy-PD Guidance rule for σ when "RMT is F"

RMT is F	σ	LN	SN	ZE	SP	LP
	A_c (g)	SN	ZE	SP	SP	LP

Table 2. Fuzzy-PD Guidance rule for $\dot{\sigma}$ when "RMT is M"

RMT is M	$\dot{\sigma}$	LN	MN	SN	ZE	SP	MP	LP
	A_c (g)	SN	SN	ZE	ZE	SP	SP	SP

Table 3. Numerated guidance rules with respect to $\dot{\sigma}$ and $\ddot{\sigma}$ when "RMT is N"

RMT is N	$\dot{\sigma}$	LN	MN	SN	ZE	SP	MP	LP
	A_c (g)	-9	-6	-3	0	3	6	9

RMT is N	$\ddot{\sigma}$	LN	MN	SN	ZE	SP	MP	LP
	A_c (g)	-3	-2	-1	0	1	2	3

Table 4. Numerated guidance rule base

A_c (g)		$\ddot{\sigma}$						
		LN	MN	SN	ZE	SP	MP	LP
$\dot{\sigma}$	LN	-12	-11	-10	-9	-8	-7	-6
	MN	-9	-8	-7	-6	-5	-4	-3
	SN	-6	-5	-4	-3	-2	-1	0
	ZE	-3	-2	-1	0	1	2	3
	SP	0	1	2	3	4	5	6
	MP	3	4	5	6	7	8	9
	LP	6	7	8	9	10	11	12

For the second stage, RMT is M, the missile only needs to keep the attitude and ensure that the guidance commands issued from the first stage would be transferred smoothly to the final phase. The guidance rule adopted is shown in Table 2, where $\dot{\sigma}$ remains as the unique premise variable. Consider three situations describing movements of the missile from position A to B (see Fig. 5). If $\dot{\sigma} > 0$, then the positive acceleration commands are exerted, which make the missile fly upward to keep its beneficial altitude. Conversely, if $\dot{\sigma} < 0$ or $\ddot{\sigma} << 0$ then the negative acceleration commands should be exerted. In this stage, the maximal and minimal acceleration commands are restricted to the fuzzy sets SN and SP to conserve energy and avoid large changes to the flight trajectory.

For the final stage, RMT is N, a PD-liked fuzzy rule base is developed with asymmetric fuzzy sets and fuzzy rules governing $\dot{\sigma}$, $\ddot{\sigma}$, and A_c. From Fig. 6 one easily observes that under the same moving range and time, there will be larger variations of σ, $\dot{\sigma}$, and $\ddot{\sigma}$ for the target moving to position A and then to position B. Therefore, it is quite natural to have an asymmetric guidance rule and asymmetric term sets. Development for the guidance rule base is summarized in the follows.

i) At the beginning of the homing phase $\dot{\sigma}$ is placed with higher weight than $\ddot{\sigma}$. The latter is used to predict the target movement and thus to early correct the guidance command. Correspondingly, we pose the former with larger acceleration commands (suppose that the weight is three times to the latter). The numerated guidance rules designed with respect to the premise variables $\dot{\sigma}$ and $\ddot{\sigma}$ are listed in Table 3.
ii) Completion of the whole rule base on Table 3 gives the numerated guidance rules, see Table 4.
iii) As mentioned previously, the target moves upward or downward resulting in asymmetric variations of $\dot{\sigma}$ and $\ddot{\sigma}$. Based on this fact, the guidance rules are developed, as in Table 5(a) for the head-on cases and Table 5(b) for the cases where target is moving upward.

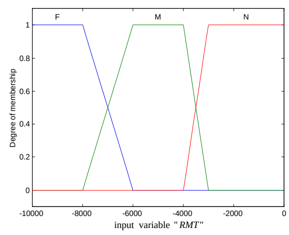

Figure 3. Membership functions describing for *RMT*

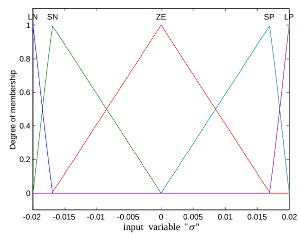

Figure 4. Membership functions describing for σ

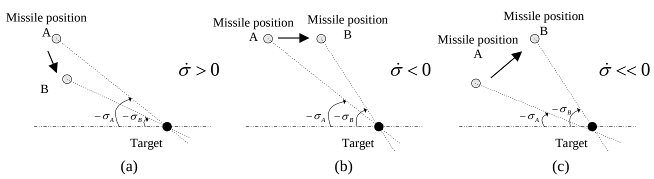

Figure 5. Relative position between interceptor and target

during the second phase

Table 5. Conversion of numerical acceleration commands to fuzzy representations

(a) Head-on

RMT is N	A_c (g)	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
	$\tilde{A}_c$	LN	LN	LN	LN	LN	MN	MN	MN	SN	SN	SN	ZE	ZE

(b) Target move upward

RMT is N	A_c (g)	12	11	10	9	8	7	6	5	4	3	2	1	0
	$\tilde{A}_c$	LP	LP	LP	LP	LP	LP	MP	MP	MP	SP	SP	SP	ZE

Table 6. Fuzzy-PD guidance rule table for "RMT is N"

A_c		$\ddot{\sigma}$						
		LN	MN	SN	ZE	SP	MP	LP
$\dot{\sigma}$	LN	LN	LN	LN	LN	LN	MN	MN
	MN	LN	LN	MN	MN	MN	SN	SN
	SN	MN	MN	SN	SN	SN	ZE	ZE
	ZE	SN	SN	ZE	ZE	SP	SP	SP
	SP	ZE	SP	SP	SP	MP	MP	MP
	MP	SP	MP	MP	MP	LP	LP	LP
	LP	MP	LP	LP	LP	LP	LP	LP

iv) Table 6 summarizes the fuzzy representation of the converted numerical values given in Table 4 with the help of Table 5.

v) It can be observed from Fig. 6 that when the target moves upward or downward, change of $\dot{\sigma}$ is not symmetric, correspondingly, the fuzzy sets describing $\dot{\sigma}$ are chosen to be asymmetric as well, see Fig. 7, with the domain [-0.05,0.05] rad/sec.

vi) The purpose of adopting $\ddot{\sigma}$ is to further predict target movement and thus to correct the guidance command in advance. As depicted above when the target moves upward or downward the corresponding missile's motion should be consistent with the target to keep its favorable attitude. Comparing to the cases of the target accelerating upward and downward with the same amount, the missile needs higher sensitivity of response for the latter case. Thus asymmetric membership functions are chosen as in Fig. 8 with the domain of $\ddot{\sigma}$ being [-2.5, 2.5] rad/sec.

vii) The fuzzy sets described for A_c are chosen as in Fig. 9 where the densely distributed membership functions are dispersed from the "ZE" fuzzy set to increase the guidance sensitivity with respect to high g target maneuvers. For the simulation purpose, assume that the admissible domain of A_c is [-245, 245] m/sec^2.

Outputs of the linguistic rules must be defuzzified before feeding to the plant. The crisp control action is calculated here using the center-of-gravity (COA) defuzzification procedure [7]. The criterion generates defuzzified output with better continuity. From a practical viewpoint, it is more practical than other defuzzification techniques for plants which are highly sensitive to the command quality.

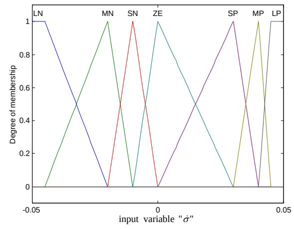

Figure 7. Membership functions describing for $\dot{\sigma}$

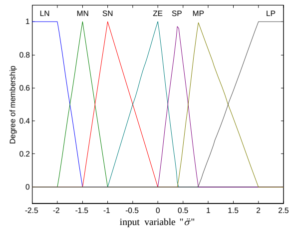

Figure 8. Membership functions describing for $\ddot{\sigma}$

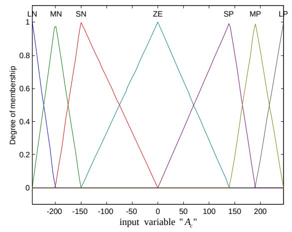

Figure 9. Membership functions describing for A_c

It should finally be noted that the number of membership functions in the presented guidance design are not very restricted. Section of an appropriate number and the shape of membership functions are usually a compromise between guidance accuracy and computation complexity. The ones adopted here are the trade-off between the two factors while covering missile guidance expertise.

4. Results and Analysis

Nominal values of all parameters adopted in the simulation studies are listed in Table 7. The missile's height is assumed to be 5 Km. Figure 10(a) shows that the missile always flies over the target for both of FLG and PNG (with the navigation gain *N*=4). Figure 10(b) shows that the target possesses higher attitude at the middle section. This figure clearly indicates that the proposed FLG consume less control energy.

Next consider performance robustness of the proposed design. Figure 11 shows lateral separations for the traditional PNG and the proposed FLG with 200% and 50%

variations to the nominal turning rate time constant T_α (=0.32 sec). It can be found from this figure that the MD profile has larger variations when there is explicit slope change on the flight trajectory. However, the proposed FLG possesses better engagement performance robustness than the traditional PNG; that is, the final MD is less sensitive to the aerodynamic variations.

Table 7. Nominal parameters

N	4	a_{12}	-5.30×10^{-4}
V_c	1200 m/s	a_{21}	6.45×10^{-4}
RR	0.02	a_{22}	-3.20×10^{-4}
ζ_s	1	b_{11}	0.01
ω_{ns}	100 rad/s	b_{12}	0.003
T_s	0.1 s	W_I	18.4 rad/s
T_g	0.12 s	W_t	2 rad/s
T_α	0.32 s	X_{mt0}	-10000 m
K_a	2.94	Y_{mt0}	0 m
K_Q	0.35	V_m	900 m/s
K_A	0.38	V_t	300 m/s
K_1	-1.57		
K_2	-1.57		
K_3	-1.16		

Finally, engagement accuracy of the proposed design against 30 different maneuvering targets (by exerting different target evasive accelerations) is fully examined. The RMS values of MDs obtained for the traditional PNG and the proposed FLG were obtained, respectively, as 5.11 and 3.7 m. The results statistically confirm superiority of the presented approach.

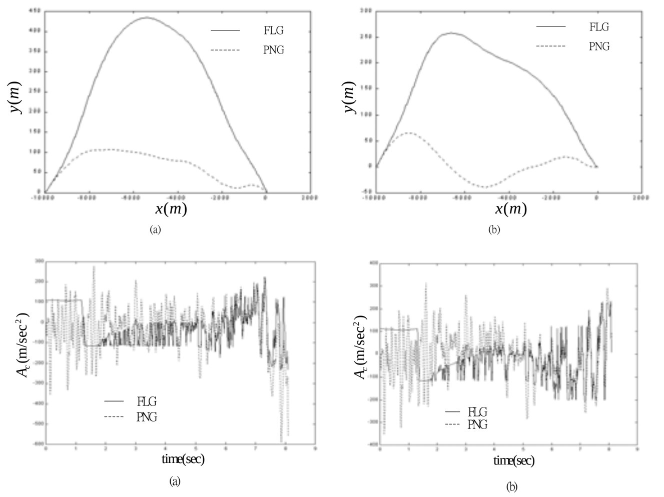

Figure 10. Comparison of lateral separation and acceleration command for the cases: (a) target head on interceptor, (b) target fly upward

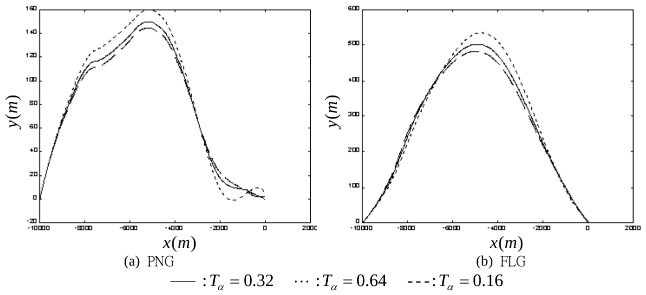

Figure 11. Comparison of performance robustness for (a) traditional PNG and (b) FLG

5. Conclusions

Asymmetric fuzzy guidance rules and fuzzy sets in the design of a missile terminal guidance law are proposed. Based on the shifting trend of the LOS angle and angle rate from the horizontal axis, the particular setting provides missiles with beneficial trajectory to attack targets. Effectiveness and performance robustness of the proposed design was numerically verified with a variety of engagement scenarios.

6. References

[1] P. Zarchan, *Tactical and Strategic Missile Guidance*, 2nd ed., AIAA, Washington, DC, 1994.

[2] C. Huang, J. Tylock, S. Engel, and J. Whitson, “Comparison of neural-network-based, fuzzy-logic-based, and numerical nonlinear inverse flight controls,” *AIAA Guidance Navigation Control Conference*, Pt. 3, pp. 922-929, 1994.

[3] C.L. Lin, H.Z. Hung, Y.Y. Chen and B.S. Chen, “Development of an integrated fuzzy-logic-based missile guidance law against high speed target,” *IEEE Transactions on Fuzzy Systems*, vol. 12, no. 2, pp. 157-169, 2004.

[4] C. M. Lin and Y. J. Mon, “Fuzzy-logic-based guidance law design for missile systems,” *In Proc. of IEEE Conference on Control Applications*, pp. 421-426, Aug., 1999.

[5] K. Mishra, I. G. Sarma and K. N. Swamy, “Performance evaluation of two fuzzy-logic-based homing guidance schemes,” *AIAA Journal of Guidance Control Dynamic*, vol. 17, no. 6, pp. 1389-1391, 1994.

[6] C. C. Lee, “Fuzzy logic in control systems: fuzzy logic controller-Parts I and II,” *IEEE Transactions on System Man Cybernetics*, vol. 20, no. 2, pp. 404-435, 1990.

[7] C. T. Lin and C. S. Lee, *Neural Fuzzy Systems*,

Englewood Cliffs, NJ: Prentice Hall, 1996.
[8] B. S. Chen, Y. Y. Chen and C. L. Lin, "Nonlinear fuzzy H_∞ guidance law with saturation of actuators against maneuvering targets," *IEEE Transactions on Control System Technology*, vol. 10, no. 6, pp. 769-779, 2002.
[9] C. F. Lin, *Modern Navigation, Guidance, and Control Processing*, Englewood Cliffs, NJ: Prentice Hall, 1991.
[10] F. W. Nesline and M. L. Nesline, "How autopilot requirements constrain the aerodynamic design of homing missiles," In *Proc. of American Control Conference*, San Diego, CA, pp. 176-730, Jun. 1984.