

The h-hemiregular Fuzzy Duo Hemirings

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Abstract

In this paper, the concept of a fuzzy h-bi-ideal, a fuzzy h-quasi-ideal in a hemiring is introduced, and some related properties of such fuzzy h-ideals are investigated. Also, the notions of h-hemiregular hemirings and h-hemiregular fuzzy duo hemirings are given, and their characterizations in terms of h-ideals and fuzzy h-ideals are studied

Keywords: Hemiring, Fuzzy h-ideals, Fuzzy h-bi-ideals, Fuzzy h-quasi-ideals, h-hemiregular, Fuzzy duo.

1. Introduction

There are many concepts of universal algebras generalizing an associative ring $(R, +, \cdot)$. Some of them - in particular, nearrings and several kinds of semirings - have been proven very useful. Nearrings arise from rings by cancelling either the axioms of left or those of right distributivity. The second type of those algebras $(S, +, \cdot)$ called semirings (and sometimes halfrings), share the same properties as a ring except that $(R, +)$ is assumed to be a semigroup rather than a commutative group. Semirings, ordered semirings and hemirings appear in a natural manner in some applications to the theory of automata and formal languages (see [2]). Ideals of semirings play an important role in the structure theory and are useful for many purposes. However, they do not in general coincide with the usual ring ideals if S is a ring and, for this reason, their use is somewhat limited in trying to obtain analogues of ring theorems for semirings. To solve this problem, Henriksen[3] defined a more restricted class of ideals in semirings, which is called k-ideals, with the properties that if the semiring S is a ring then a complex in S is an k-ideal if and only if it is a ring ideal. Iizula[4] gave a still more restricted class ideals in hemirings, which is called h-ideals. La Torre[5] investigated h-ideals and k-ideals in hemirings in an effort to obtain analogous ring theorems for hemirings. Several authors have discussed a fuzzy theorem in semirings (see, e.g., [6]-[9], [11]-[14]). In this paper, we shall introduce the concepts of fuzzy h-bi-ideals and fuzzy

h-quasi-ideals in a hemiring, and give some related properties of such fuzzy h-ideals. Next, we give the concepts of h-hemiregular hemirings and h-hemiregular fuzzy duo hemirings and character them in terms of h-ideals and fuzzy h-ideals.

2. Preliminaries

A semiring is an algebraic system $(S, +, \cdot)$ consisting of a non-empty set S together with two binary operations on S called addition and multiplication (denoted in the usual manner) such that $(S, +)$ and (S, \cdot) are semigroups and the following distributive laws

$$a(b+c) = ab+ac \quad \text{or} \quad (a+b)c = ac+bc$$

are satisfied for all $a, b, c \in S$.

By zero of a semiring $(S, +, \cdot)$ we mean an element $0 \in S$ such that $0 \cdot x = x \cdot 0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$. A semiring with zero and a commutative semigroup $(S, +)$ is called a hemiring.

A subset A of a hemiring S is called a left (resp. right) ideal of S if A is closed under addition and $SA \subseteq A$ (resp. $AS \subseteq A$). A subset A of a hemiring S is called an ideal if it is both a left ideal and a right ideal of S . Moreover, a subset A of a hemiring S is called a bi-ideal if A is closed under addition and multiplication such that $ASA \subseteq A$.

A left ideal A of S is called a left h-ideal if $a, b \in A$, $x, z \in S$ and $x+a+z = b+z$ implies $x \in A$. Right h-ideals, h-ideals and h-bi-ideals are defined similarly.

Definition 2.1: A fuzzy subset μ of a hemiring S is called a fuzzy left h-ideal if for all $x, y, z, a, b \in S$, we have

- (i) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$,
- (ii) $\mu(xy) \geq \mu(y)$,
- (iii) $x+a+z = b+z \rightarrow \mu(x) \geq \min\{\mu(a), \mu(b)\}$.

Fuzzy right h-ideals and fuzzy h-ideals are defined similarly.

Definition 2.2: A fuzzy subset μ of a hemiring S is called a fuzzy h-bi-ideal if for all $x, y, z, a, b \in S$, we have

- (i) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$,
- (ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$,
- (iii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$,
- (iv) $x+a+z = b+z \rightarrow \mu(x) \geq \min\{\mu(a), \mu(b)\}$.

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The h-closure \bar{A} of A in a hemiring S is defined as $\bar{A} = \{x \in S \mid x + a + z = a' + z \text{ for some } a, a' \in A, z \in S\}$

A subset A of a hemiring S is called a h-quasi-ideal of S if A is closed under addition, $\overline{SA} \cap \overline{AS} \subseteq A$ and $x + a + z = b + z$ implies $x \in A$ for all $a, b \in A, x, y \in S$.

Lemma 2.3 [14]: For a hemiring S , we have

- (1) $A \in \bar{A}$ for $\forall A \subseteq S$.
- (2) If $A \subseteq B \subseteq S$, then $\bar{A} \subseteq \bar{B}$.
- (3) $\overline{\bar{A}} = \bar{A}$ for $\forall A \subseteq S$
- (4) $\overline{AB} = \overline{\bar{A} \cdot \bar{B}}$ for $\forall A, B \subseteq S$.
- (5) For any h-left (resp. h-right) ideal, h-bi-ideal or h-quasi-ideal A of S , we have $A = \bar{A}$.

Definition 2.4: Let μ and ν be fuzzy subsets of a hemiring S . Then the h-product of μ and ν is defined by

$$(\mu \circ_h \nu)(x) = \sup_{x+a_1b_1+z=a_2b_2+z} (\min\{\mu(a_1), \mu(a_2), \nu(b_1), \nu(b_2)\})$$

and $(\mu \circ_h \nu)(x) = 0$ if x cannot be expressed as $x + a_1b_1 + z = a_2b_2 + z$.

For any subset A of a hemiring S , we denote χ_A the characteristic function of A . In particular, we note that the hemiring S can be considered a fuzzy subset of itself and we write $S = \chi_S$.

Lemma 2.5: Let S be a hemiring and $A, B \subseteq S$. Then we have

- (1) $\chi_A \subseteq \chi_B$ if and only if $A \subseteq B$.
- (2) $\chi_A \cap \chi_B = \chi_{A \cap B}$.
- (3) $\chi_A \circ_h \chi_B = \chi_{\overline{AB}}$.

Proof. The proof of (1) and (2) is clear. We only show

(3). Let $x \in S$. If $\chi_A \circ_h \chi_B(x) \neq 0$, then there exist elements a_1, a_2, b_1, b_2 and z of S such that $x + a_1a_2 + z = a_2b_2 + z$ and $\min\{\chi_A(a_1), \chi_A(a_2), \chi_B(b_1), \chi_B(b_2)\} \neq 0$. Hence we have $\chi_A(a_1) = \chi_A(a_2) = \chi_B(b_1) = \chi_B(b_2) = 1$ and so $a_1, a_2 \in A$ and $b_1, b_2 \in B$. Thus $x \in \overline{AB}$ and so $\chi_{\overline{AB}}(x) = 1 \geq (\chi_A \circ_h \chi_B)(x)$, this implies that $\chi_A \circ_h \chi_B \subseteq \chi_{\overline{AB}}$.

Definition 2.6: A fuzzy subset μ of a hemiring S is called

a fuzzy h-quasi-ideal if for all $x, y, z, a, b \in S$, we have

- (i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$,
- (ii) $(\mu \circ_h S) \cap (S \circ_h \mu) \subseteq \mu$,
- (iii) $x + a + z = b + z \rightarrow \mu(x) \geq \min\{\mu(a), \mu(b)\}$

Note that if μ is a fuzzy h-quasi-ideal of a hemiring S , then $\mu(0) \geq \mu(x)$ for all $x \in S$.

Lemma 2.7: A fuzzy subset μ of a semiring S is a fuzzy left (resp. right) h-ideal of S if and only if for any $x, y, z,$

- $a, b \in S$, we have (1) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$; (2) $S \circ_h \mu \subseteq \mu$ (resp. $\mu \circ_h S \subseteq \mu$); (3) $x + a + z = b + z \rightarrow \mu(x) \geq \min\{\mu(a), \mu(b)\}$.

Proof. It is sufficient to show that the condition (2) is equivalent to the condition (ii) of Definition 2.1. First assume that μ is a fuzzy left h-ideal of S . Let $x \in S$. Then $(S \circ_h \mu)(x) = 0 \leq \mu(x)$ if x cannot be expressed as $x + a_1b_1 = a_2b_2$. Otherwise, we have

$$\begin{aligned} (S \circ_h \mu)(x) &= \sup_{x+a_1b_1+z=a_2b_2+z} (\min\{\mu(b_1), \mu(b_2)\}) \\ &\leq \sup_{x+a_1b_1+z=a_2b_2+z} (\min\{\mu(a_1b_1), \mu(a_2b_2)\}) \\ &\leq \sup_{x+a_1b_1+z=a_2b_2+z} \mu(x) = \mu(x) \end{aligned}$$

and so $S \circ_h \mu \subseteq \mu$.

Conversely, assume that the given condition holds. Let $x, y \in S$. Then, we have

$$\begin{aligned} \mu(xy) &\geq (S \circ_h \mu)(xy) = \sup_{xy+a_1b_1+z=a_2b_2+z} (\min\{\mu(b_1), \mu(b_2)\}) \\ &\geq \mu(y) \text{ (since } xy + 0y + 0 = xy + 0) \end{aligned}$$

This implies that the condition (ii) of definition 2.1 is valid and so μ is a fuzzy left h-ideal of S . The case for fuzzy right h-ideal can be similarly disposed of.

Lemma 2.8: Any fuzzy h-quasi-ideal of a hemiring S is a fuzzy h-bi-ideal of S .

Proof. Let μ be any fuzzy h-quasi-ideal of S . It sufficient to show that

$$\mu(xyz) \geq \min\{\mu(x), \mu(z)\} \text{ for all } x, y, z \in S.$$

In fact, by the assumption, we have

$$\begin{aligned} \mu(xyz) &\geq ((\mu \circ_h S) \cap (S \circ_h \mu))(xyz) \\ &= \min\{(\mu \circ_h S)(xyz), (S \circ_h \mu)(xyz)\} \\ &= \min\left\{ \sup_{xyz+a_1b_1+z'+a_2b_2+z'} (\min\{\mu(a_1), \mu(a_2)\}), \right. \\ &\quad \left. \sup_{xyz+a_1b_1+z'+a_2b_2+z'} (\min\{\mu(b_1), \mu(b_2)\}) \right\} \end{aligned}$$

(since $xyz + 0 \cdot 0 + 0 = (xy)z + 0$ and $xyz + 0 \cdot 0 + 0 = x(yz) + 0$)
 $\geq \min\{\min\{\mu(0), \mu(z)\}, \min\{\mu(x), \mu(0)\}\}$
 $= \min\{\mu(x), \mu(z)\}$.

This completes the proof.

Lemma 2.9 :Let S be a hemiring and $A \subseteq S$. Then the following conditions hold.

- (1) A is a left (resp. right) h-ideal of S if and only if χ_A is a fuzzy left (resp. right) h-ideal of S.
- (2) A is a h-bi-ideal of S if and only if χ_A is a fuzzy h-bi-ideal of S.
- (3) A is a h-quasi-ideal of S if and only if χ_A is a fuzzy h-quasi-ideal of S.

Proof. The proof is straightforward.

3. Regular fuzzy duo ordered semigroup

Definition 3.1 : [12, Definition 3.1] A hemiring S is said to be h-hemiregular if for each $x \in S$, there exist $a, a', z \in S$ such that $x + xax + z = xa'x + z$.

Lemma 3.2 : [12, Lemma 3.5] A hemiring S is h-regular if and only if for any right h-ideal A and left h-ideal B we have $\overline{AB} = A \cap B$.

Theorem 3.3: [12, Theorem 3.6] A hemiring S is h-hemiregular if and only if for every fuzzy right h-ideal μ and every fuzzy left h-ideal ν , we have $\mu \circ_h \nu = \mu \cap \nu$.

A hemiring S is called duo if every one-side h-ideal of it is a h-ideal of S. A hemiring S is called fuzzy duo if every fuzzy one-side h-ideal of it is a fuzzy h-ideal of S. A hemiring S is called h-hemiregular fuzzy duo if it is both h-hemiregular and fuzzy duo.

Lemma 3.4 :Let S be a hemiring. Then the following conditions are equivalent.

- (1) S is h-hemiregular duo hemiring.
- (2) $L \cap R = \overline{LR}$ for every left h-ideal L and every right h-ideal R of S.
- (3) $A \cap Q = \overline{AQA}$ for every h-ideal A and every h-quasi-ideal of S.

Proof. (1) \Leftrightarrow (2) Assume that (1) holds. Let L and R be any left h-ideal and any right h-ideal of S, respectively. Then since S is duo hemiring, we have both L and R are h-ideals of S. Moreover, since S is regular, it follows

from Lemma 3.2 that $L \cap R = \overline{LR}$ and so (1) implies (2). Conversely, assume that (2) holds. Let L and R be any left h-ideal and any right h-ideal of S, respectively. Since S itself is an h-ideal of S, we have $L \cap S = \overline{LS} \supseteq LS$ and $R = R \cap S = SR \supseteq SR$. Hence both L and R are h-ideals of S and so S is duo. Now, by the assumption, it is easy to see that $R \cap L = \overline{RL}$ for every right h-ideal R and every left h-ideal L of S. Hence S is h-hemiregular and so (2) implies (1).

(1) \Leftrightarrow (3) Assume that (1) holds. Let A and Q be any h-ideal and any h-quasi-ideal of S, respectively. Then $\overline{AQA} \subseteq \overline{A} = A$ and $\overline{AQA} \subseteq \overline{AQ \cap QA} \subseteq \overline{AQ} \cap \overline{QA} \subseteq \overline{SQ} \cap \overline{QS} \subseteq Q$, hence $\overline{AQA} \subseteq A \cap Q$. Now, let $x \in A$. Then since S is h-hemiregular, there exist $a, a', z \in S$ such that $x + xax + z = xa'x + z$, then we have $xax + xaxax + zax = xa'xax + zax$ and $xa'x + xaxa'x + za'x = xa'xa'x + za'x$, $x + xax + z + xaxax + xaxa'x + zax + za'x = xa'x + z + xaxax + xaxa'x + zax + za'x$, and so $x + xa'xax + xaxa'x + z' = xaxax + xa'xa'x + z'$, where $z' = z + zax + za'x$. Since A is an h-ideal of S, we have $xa, xa' \in A$ and $ax, a'x \in S$, and in consequence, $(xa)x(a'x) + (xa')x(ax)$, $xaxax + xa'xa'x \in AQA$ and so $x \in \overline{AQA}$. Hence $A \cap Q = \overline{AQA}$ and so (1) implies (3). Conversely, assume that (3) holds. Let L and R be any left h-ideal and any right h-ideal of S, respectively. Then both L and R are h-quasi-ideals of S. Since S itself is an h-ideal of S, by the assumption, we have $\overline{SLS} = S \cap L = L$ and $\overline{SRS} = S \cap R = R$. Hence $\overline{LS} = \overline{SLS} \subseteq \overline{SLSS} \subseteq \overline{SLS} = L$ and $\overline{SR} = \overline{SRS} \subseteq \overline{SSRS} \subseteq \overline{SRS} = R$, and so both L and R are h-ideals of S, that is, S is duo. Moreover, by the assumption, we have $R \cap L = \overline{RLR} \subseteq \overline{RL} \subseteq \overline{R} \cap \overline{L} = R \cap L$ and so S is h-hemiregular by Lemma 3.2. Hence S is h-hemiregular duo and so (3) implies (1).

Lemma 3.5 Let S be an h-hemiregular hemiring. Then S is duo if and only if S is fuzzy duo.

Proof. Assume that S is duo. Let μ be any fuzzy left h-ideal of S and let x and y be any elements of S. Then, since the set \overline{Sx} is a left h-ideal of S, it is an h-ideal of S by the assumption. And since S is h-hemiregular, we have

$$xy \in \overline{xSx}y = \overline{\overline{x}Sx}y \subseteq \overline{\overline{x}SxS} = \overline{\overline{x}SxS} \subseteq \overline{Sx} = \overline{Sx}.$$

this implies that there exist $a, a', z \in S$ such that $xy + ax + z = a'x + z$. Since μ is a fuzzy left h-ideal of S, we have

$$\mu(xy) \geq \min\{\mu(ax), \mu(a'x)\} \geq \mu(x)$$

and so μ is a fuzzy right h-ideal of S. It can be seen in a similar way that any fuzzy right h-ideal of S is a fuzzy h-ideal of S. Thus S is fuzzy duo. Conversely, assume that S is fuzzy duo. Let L be any left h-ideal of S, then the characteristic function χ_L of L is a fuzzy left h-ideal of S. By the assumption, we have χ_L is a fuzzy h-ideal of S and so L is an h-ideal of S. Similarly, we can show that any right h-ideal of S is an h-ideal of S. Thus S is duo. This completes the proof.

Lemma 3.6 : Let S be an h-hemiregular fuzzy duo ordered hemiring. Then every fuzzy h-bi-ideal of S is a fuzzy h-ideal of S.

Proof. Let f be any fuzzy h-bi-ideal of S, x and y be any element of S. Then, since the set \overline{Sx} is a left h-ideal of S, it is an h-ideal of S by the assumption. And since S is h-hemiregular, we have

$$xy \in \overline{xSxy} = \overline{\overline{xS}xy} \subseteq \overline{\overline{xS}xS} = \overline{\overline{xS}xS} \subseteq \overline{xSx} = xSx.$$

this implies that there exist $a, a', z \in S$ such that $xy + xax + z = xa'x + z$. Since μ is a fuzzy h-bi-ideal of S, we have

$$\mu(xy) \geq \min\{\mu(xax), \mu(xa'x)\} \geq \mu(x)$$

and so μ is a fuzzy right h-ideal of S. It can be seen in a similar way that μ is fuzzy left h-ideal of S and so μ is a fuzzy h-ideal of S.

Theorem 3.7 : Let S be a hemiring. Then the following conditions are equivalent.

- (1) S is an h-hemiregular duo hemiring.
- (2) S is an h-hemiregular fuzzy duo hemiring.
- (3) $\mu \cap \nu = \mu \circ_h \nu$ for all fuzzy h-bi-ideal μ and ν of S.
- (4) $\mu \cap \nu = \mu \circ_h \nu$ for every fuzzy h-bi-ideal μ and every fuzzy h-quasi-ideal ν of S.
- (5) $\mu \cap \nu = \mu \circ_h \nu$ for every fuzzy h-bi-ideal μ and every fuzzy right h-ideal ν of S.
- (6) $\mu \cap \nu = \mu \circ_h \nu$ for every fuzzy h-quasi-ideal μ and every fuzzy h-bi-ideal ν of S.
- (7) $\mu \cap \nu = \mu \circ_h \nu$ for all fuzzy h-quasi-ideal μ and ν of S
- (8) $\mu \cap \nu = \mu \circ_h \nu$ for every fuzzy h-quasi-ideal μ and every fuzzy right h-ideal ν of S.
- (9) $\mu \cap \nu = \mu \circ_h \nu$ for every fuzzy left h-ideal μ and every fuzzy h-bi-ideal ν of S.
- (10) $\mu \cap \nu = \mu \circ_h \nu$ for every fuzzy left h-ideal μ and every fuzzy right h-ideal ν of S.

Proof. It follows from Lemma 3.5 that (1) and (2) are equivalent. Assume that (2) holds. Let μ and ν be any fuzzy h-bi-ideals of S. Then it follows from Lemma 3.6 that μ is a fuzzy right h-ideal of S and ν is a fuzzy left h-ideal of S. Since S is h-hemiregular, it follows from Lemma 3.2 that $\mu \cap \nu = \mu \circ_h \nu$ and so (2) implies (3). It is clear that (3) \Rightarrow (4) \Rightarrow (5) \Rightarrow (8) \Rightarrow (10), (3) \Rightarrow (6) \Rightarrow (7) \Rightarrow (8) \Rightarrow (10) and (6) \Rightarrow (9) \Rightarrow (10). Assume that (10) holds. Let L and R be any left h-ideal and any right h-ideal of S, respectively. Then the characteristic functions χ_L and χ_R of L and R, respectively, is a fuzzy left h-ideal of S and a fuzzy right h-ideal of S. By the assumption and Lemma 2.5, we have

$$\chi_{L \cap R} = \chi_L \cap \chi_R = \chi_L \circ_h \chi_R = \chi_{LR}$$

and so $L \cap R = LR$. Hence it follows Lemma 3.4 that S is an h-hemiregular duo hemiring and so (10) implies (1). This completes the proof.

Theorem 3.8 : Let S be a hemiring. The following conditions are equivalent.

- (1) S is an h-hemiregular duo hemiring.
- (2) $\mu \cap \nu = \mu \circ_h \nu \circ_h \mu$ for every fuzzy h-ideal μ and every fuzzy h-bi-ideal ν of S.
- (3) $\mu \cap \nu = \mu \circ_h \nu \circ_h \mu$ for every fuzzy h-ideal μ and every fuzzy h-quasi-ideal ν of S.

Proof. (1) \Rightarrow (2) Assume that (1) holds. Let μ and ν be any fuzzy h-ideal and any fuzzy h-bi-ideal of S, respectively. Then we have

$$\mu \circ_h \nu \circ_h \mu \subseteq (\mu \circ_h S) \circ_h S \subseteq \mu \circ_h S \subseteq \mu.$$

On the other hand, since S is h-hemiregular duo, it follows from Lemma 3.6 that ν is a fuzzy h-ideal of S. Thus we have

$$\mu \circ_h \nu \circ_h \mu \subseteq (S \circ_h \nu) \circ_h S \subseteq \nu \circ_h S \subseteq \nu,$$

and so

$$\mu \circ_h \nu \circ_h \mu \subseteq \mu \cap \nu.$$

Now let x be any element of S. Then since S is h-hemiregular, there exist $a, a', z \in S$ such that $x + xax + z = xa'x + z$ and that $x + xa'xax + xaxa'x + z' = xaxax + xa'xa'x + z'$, where $z' = z + zax + za'x$. Hence we have

$$\begin{aligned} & (\mu \circ_h \nu \circ_h \mu)(x) \\ &= \sup_{x+a_1b_1+z=a_2b_2+z} (\min\{(\mu \circ_h \nu)(a_1), (\mu \circ_h \nu)(a_2), \mu(b_1), \mu(b_2)\}) \\ &\geq \min\{(\mu \circ_h \nu)(x), \mu(a'xax + axa'x), \mu(axax + a'xa'x)\} \\ &\min\{ \sup_{x+a_1b_1+z=a_2b_2+z} (\{\mu(a_1), \mu(a_2), \nu(b_1), \nu(b_2)\}), \mu(a'xax + axa'x), \\ &\mu(axax + a'xa'x)\} \\ &\geq \min\{\mu(xa), \mu(xa'), \nu(x), \mu(a'xax + axa'x), \mu(axax + a'xa'x)\} \end{aligned}$$

$$\geq \min\{\mu(x), \nu(x)\} = (\mu \cap \nu)(x),$$

and so $\mu \cap \nu \subseteq \mu \circ_h \nu \circ_h \mu$. Hence $\mu \cap \nu = \mu \circ_h \nu \circ_h \mu$ and so (1) implies (2).

(2) \Rightarrow (3) is clear by Lemma 2.8.

(3) \Rightarrow (1) Assume that (3) holds. Let A and Q be any h-ideal and any h-quasi-ideal of S, respectively. Then the characteristic functions χ_A and of χ_Q of A and Q is a fuzzy h-ideal of S and a fuzzy h-quasi-ideal of S, respectively. By the assumption and Lemma 2.5, we have

$$\chi_{A \cap Q} = \chi_A \cap \chi_Q = \chi_A \circ_h \chi_Q \circ_h \chi_A = \chi_{\overline{AQA}}$$

and so $A \cap Q = \overline{AQA}$. Hence it follows from Lemma 3.4 that S is an h-hemiregular duo hemiring and so (3) implies (1).

4. Conclusions

In this paper, our aim is to promote research and the development of fuzzy technology by studying the fuzzy hemirings. The goal is to explain new methodological development in fuzzy hemirings which will also be of growing importance in the future. Also, through this paper, we may see that the basic results of fuzzy sets in hemirings are similar, but not identical, with the corresponding results for rings. So it is important for us to study different types of fuzzy sets in hemirings. Our future work on this topic will focus on studying of intuitionistic or interval-valued fuzzy sets in hemirings and other algebraic constructions of hemirings.

7. References

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