H_∞ Fuzzy Control Design for Nonlinear Stochastic Fuzzy Systems

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Abstract

This paper describes the robust output feedback H_∞ fuzzy control design for a class of nonlinear stochastic systems. The system dynamic is modelled by $\text{It}\hat{o}-\text{type}$ stochastic differential equations. For general nonlinear stochastic systems, the H_∞ ontrol can be obtained by solving a second-order nonlinear Hamilton-Jacobi inequality. In general, it is difficult to solve the second-order nonlinear Hamilton-Jacobi inequality. In this paper, using fuzzy approach (T-S fuzzy model), the H_∞ fuzzy control design for the nonlinear stochastic systems can be given via solving linear matrix inequalities (LMIs) instead of a second-order Hamilton-Jacobi inequality. Simulation example is provided to illustrate the design procedure and expected performance.

Keywords: Output feedback, H_∞ fuzzy control, T-S fuzzy model, nonlinear stochastic systems, $\text{It}\hat{o}-\text{type}$ stochastic differential equations, second-order nonlinear Hamilton-Jacobi inequality, and LMIs.

1. Introduction

The output feedback H_∞ control problem is to design a controller using output feedback only, which guarantees the L_2 gain less than a prescribed level [1]. On the other hand, the stochastic H_∞ control problems with system models expressed by $\text{It}\hat{o}-\text{type}$ stochastic differential equations have become a popular research topic, and have gained extensive attention [2-5]. Most of the above works are limited to the linear stationary stochastic systems, while [3] and [4] discussed the linear and nonlinear stochastic H_∞ control problems. In [6], the stabilization problem of stochastic fuzzy system with norm bounded uncertainties has been discussed. The state feedback controller, which stabilizes either the nominal system or the uncertain one, is synthesized via solving the LMI problem. However, in [6], the control performance like H_∞ performance was not mentioned and only static state feedback case was studied. Unlike the deterministic case, the Hamilton-Jacobi inequality (HJI) associated with nonlinear stochastic H_∞ control is a second-order (not first-order) nonlinear partial differential inequality due to the effect of the diffusion term, which makes the stochastic H_∞ control problem more complex [5]. In general, it is very difficult to solve the second-order nonlinear Hamilton-Jacobi inequality.

Recently, there have been many applications of fuzzy systems theory in various fields. In most of these applications, the fuzzy systems were thought of as universal approximators for any nonlinear systems. The Takagi and Sugeno (T-S) fuzzy model [7] which has been proved to be a very good representation for a certain class of nonlinear dynamic systems was extensively studied in control systems [8-10]. In this study, using (T-S fuzzy model) fuzzy approach, the output feedback H_∞ fuzzy control design for a class of nonlinear stochastic systems can be given via solving linear matrix inequalities (LMIs) instead of a second-order Hamilton-Jacobi inequality. First, a T-S fuzzy model is proposed to approximate a class of nonlinear stochastic systems. Next, based on the T-S fuzzy model, the output feedback H_∞ fuzzy control design for the nonlinear stochastic systems is characterized in terms of minimizing the attenuation level subject to some linear matrix inequalities (LMIs), which is also called eigenvalue problem (EVP) [11] and can be efficiently solved by the LMI toolbox in Matlab [12]. Simulation example is provided to illustrate the design procedure and expected performance.

For convenience, we adopt the following notations throughout this paper: A' : the trace (transpose) of matrix A. $L^2(R_+, R^l)$: the space of nonanticipative stochastic processes y(t) with respect to filtration F_t satisfying $\|y(t)\|_{L_2}^2 := E\int_0^\infty \|y(t)\|^2 dt < \infty$. $C^2(U)$: class of functions V(x) twice continuously differential with respect to $x \in U$ except possibly at the origin. $\begin{bmatrix} M_{11} & * \\ M_{12}^T & M_{22} \end{bmatrix} \triangleq \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix}$.

2. H_∞ Settings for Nonlinear Stochastic Systems

Consider a class of nonlinear stochastic systems governed by $\text{It}\hat{o}-\text{type}$ stochastic differential

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equations

$$\begin{cases} dx(t) = [f(x(t)) + g(x(t))v(t)]dt + h(x(t))dW(t) \\ \eta(t) = q(x(t)) \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the system state, $y(t) \in R^r$ is the measurement, $\eta(t) \in R^m$ is the controlled output, and $v \in L^2(R_+, R^{n_v})$ stands for the exogenous disturbance signal. $f(x(t)), g(x(t)), h(x(t))$, and $q(x(t))$ are smooth functions with $f(0) = h(0) = q(0) = 0$. $W(\cdot)$ is a standard one-dimensional Wiener process defined on the probability space (Ω, F, P) relative to an increasing family $(F_t)_{t \in R_+}$ of σ-algebras $F_t \subset F$. In (1), the state equation, in engineering terminology, can be written as [13]

$$\dot{x}(t) = f(x(t)) + g(x(t))v(t) + h(x(t))w(t) \quad (2)$$

where w is a stationary white noise.

The following Lemma is a special case of Proposition 1 in [5], which plays an important role in this paper.

Lemma 1: For system (1), if there exists a positive function $V(x) \in C^2(R^n)$ and $V(0) = 0$ solving the following HJI

$$\begin{aligned} &\frac{\partial V'}{\partial x} f + \frac{1}{2}\gamma^{-2}\left(\frac{\partial V'}{\partial x} g\right)\left(g'\frac{\partial V}{\partial x}\right) \\ &+ \frac{1}{2}\|q(x(t))\|^2 + \frac{1}{2} h' \frac{\partial^2 V}{\partial x^2} h < 0, \end{aligned} \quad (3)$$

then (i) The equilibrium point $x \equiv 0$ of the system (1) is globally asymptotically stable in probability in the case of v=0 and (ii) the following inequality

$$\|\eta(t)\|_{L_2}^2 \le 2E[V(x(0))] + \gamma^2 \|v(t)\|_{L_2}^2 \quad (4)$$

$\forall v \in L_F^2(R^+, R^{n_v})$, $v \neq 0$ holds for some $\gamma > 0$ if initial state $x(0) \neq 0$ and

$$\|\eta(t)\|_{L_2}^2 \le \gamma^2 \|v(t)\|_{L_2}^2, \forall v \in L_F^2(R^+, R^{n_v}), v \neq 0 \quad (5)$$

holds for some $\gamma > 0$ if initial state $x(0) = 0$.

Proof. The proof is immediately followed from Proposition 1 and Lemma 1 in [5].

Remark 1: In general, it is difficult to solve the second-order nonlinear Hamilton-Jacobi inequality in (3). In the next section, using fuzzy approach, the H_∞ fuzzy control design for the nonlinear stochastic systems can be given via solving linear matrix inequalities (LMIs) instead of a second-order Hamilton-Jacobi inequality.

3. Output Feedback H_∞ Fuzzy Control Design For Nonlinear Stochastic System

A fuzzy dynamic model has been proposed by Takagi and Sugeno [7] to represent locally linear input/output relations for nonlinear systems. This fuzzy dynamic model is described by fuzzy If-Then rules and will be employed here to deal with the filtering design problem for a class of nonlinear stochastic systems governed by $\text{It}\hat{o} - \text{type}$ stochastic differential equations. The ith rule of the fuzzy model for the nonlinear stochastic systems is proposed as the following form:

Rule i:

If $z_1(t)$ is $F_{i1} \cdots$ and $z_g(t)$ is F_{ig}

Then

$$\begin{aligned} dx(t) &= (A_i x(t) + B_i u(t) + D_{1i} v(t))dt + E_{1i} x(t)dW(t) \\ Y(t) &= (C_i x(t) + D_{2i} v(t)) + E_{2i} x(t)w(t) \end{aligned} \quad (6)$$

for i= 1,…,L where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^{n \times 1}$ denotes the state vector; $Y(t) \in R^{m \times 1}$ is the measurement output; $v \in L^2(R_+, R^{n_v})$ stands for the exogenous disturbance signal; F_{ij} is the fuzzy set; $A_i, B_i, D_{1i}, E_{1i}, C_i, D_{2i}$ and E_{2i} are known constant matrices with appropriate dimension; L is the number of If-Then rules; $z_1(t), \ldots, z_g(t)$ are the premise variables; $W(\cdot)$ is a standard one-dimensional Wiener process defined on the probability space (Ω, F, P) relative to an increasing family $(F_t)_{t \in R_+}$ of σ-algebras $F_t \subset F$; and $w(t) = \dot{W}(t)$ is a stationary white noise.

To obtain a tractable mathematical interpretation of Y(t) in (6), we introduce

$$y(t) = \int_0^t Y(s)ds \quad (7)$$

and thereby we obtain the stochastic integral representation

Rule i:

If $z_1(t)$ is $F_{i1} \cdots$ and $z_g(t)$ is F_{ig}

Then

$$\begin{aligned} dx(t) &= (A_i x(t) + B_i u(t) + D_{1i} v(t))dt + E_{1i} x(t)dW(t) \\ dy(t) &= (C_i x(t) + D_{2i} v(t))dt + E_{2i} x(t)dW(t) \end{aligned} \quad (8)$$

Remark 2 : The fuzzy model in (8), in engineering terminology, can be expressed as follows:

Rule i:

If $z_1(t)$ is $F_{i1} \cdots$ and $z_g(t)$ is F_{ig}

Then

$$\dot{x}(t) = (A_i x(t) + B_i u(t) + D_{1i} v(t)) + E_{1i} x(t) w(t)$$
$$Y(t) = (C_i x(t) + D_{2i} v(t)) + E_{2i} x(t) w(t) \quad (9)$$

The final output of the fuzzy system is inferred as follows:

$$dx(t) = \sum_{i=1}^{L} h_i(z(t))[(A_i x(t) + B_i u(t) + D_{1i} v(t))dt + E_{1i} x(t) dW(t)] \quad (10)$$

and

$$dy(t) = \sum_{i=1}^{L} h_i(z(t))[(C_i x(t) + D_{2i} v(t))dt + E_{2i} x(t) dW(t)] \quad (11)$$

where

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{L} \mu_i(z(t))} \quad (12)$$

$$\mu_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j(t)), z(t) = [z_1(t), .., z_g(t)]$$

and $F_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in F_{ij}.

It is assumed that $\mu_i(z(t)) \geq 0$ and $\sum_{i=1}^{L} \mu_i(z(t)) > 0$.

Therefore, we get

$$h_i(z(t)) \geq 0, \text{ for } i = 1,2,\ldots L \quad (13)$$

and

$$\sum_{i=1}^{L} h_i(z(t)) = 1 \quad (14)$$

Based on the fuzzy model (8), the following fuzzy estimator is proposed to deal with the state estimation for the nonlinear stochastic systems

Observer Rule i:

If $z_1(t)$ is $F_{j1} \cdots$ and $z_g(t)$ is F_{jg}

Then

$$d\hat{x}(t) = A_i \hat{x}(t)dt + B_i u(t)dt + L_i[dy(t) - d\hat{y}(t)] \quad (15)$$

where L_i is the fuzzy observer gain for the ith observer rule and $d\hat{y}(t) = \sum_{i=1}^{L} h_i(z(t)) C_i \hat{x}(t)dt$.

The overall fuzzy estimator is written as

$$d\hat{x}(t) = \sum_{i=1}^{L} h_i(z(t))\{A_i \hat{x}(t)dt + B_i u(t)dt + L_i[dy(t) - d\hat{y}(t)]\} \quad (16)$$

Remark 3: In practice, the fuzzy estimator is implemented as follows

$$\dot{\hat{x}}(t) = \sum_{i=1}^{L} h_i(z(t))\{A_i \hat{x}(t)dt + B_i u(t) + L_i[Y(t) - \hat{Y}(t)]\} \quad (17)$$

Hence, the fuzzy observer-based fuzzy controller is proposed as

$$u(t) = \sum_{j=1}^{L} h_j(z(t)) K_j \hat{x}(t) \quad (18)$$

where K_j is the control gain for the jth controller rule.

Let us denote the estimation errors as

$$de(t) = dx(t) - d\hat{x}(t) \quad (19)$$

By substituting (10) and (16) into (19), we get

$$de(t) = \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t)) h_j(z(t))\{[(A_i - L_i C_j)e(t) + (D_{1i} - L_i D_{2j})v(t)]dt + (E_{1i} - L_i E_{2j})x(t)dW(t)\} \quad (20)$$

After manipulation, the augmented system can be expressed as the following form:

$$d\tilde{x}(t) = \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))[(A_{ij}\tilde{x}(t) + D_{ij} v(t))dt + E_{ij}\tilde{x}(t)dW(t)] \quad (21)$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, D_{ij} \begin{bmatrix} D_{1i} \\ D_{1i} - L_i D_{2j} \end{bmatrix}$$
$$A_{ij} = \begin{bmatrix} A_i + B_i K_j & -B_i K_j \\ 0 & A_i - L_i C_j \end{bmatrix}$$
$$E_{ij} = \begin{bmatrix} E_{1i} & 0 \\ E_{1i} - L_i E_{2j} & 0 \end{bmatrix} \quad (22)$$

The nonlinear stochastic H_∞ fuzzy control problem can be stated as follows: Find the controller gain K_j and the observer gain L_i (for i,j…=1,…L) such that

(i) The equilibrium point $\tilde{x}(t) \equiv 0$ of the augmented system (21) is globally asymptotically stable in probability in the case of $v(t) = 0$.

(ii) For a given disturbance attenuation level $\gamma > 0$, the following relation (H_∞ performance) holds.

$$\|\tilde{x}(t)\|_{L_2}^2 \leq 2E[V(\tilde{x}(0))] + \gamma^2 \|v(t)\|_{L_2}^2 \quad (23)$$

$\forall v(t) \in L_F^2(R^+, R^{n_v})$, $v(t) \neq 0$, where $V(\tilde{x}) \in C^2(R^n)$ is a positive function.

Then, we obtain the following result.

Theorem 2 : For the augmented system in (21), if there exists a symmetric positive definite matrix P solving the following inequalities

$$\begin{bmatrix} P(\frac{A_{ij}+A_{ji}}{2})+(\frac{A'_{ij}+A'_{ji}}{2})P+I & * & * \\ P(\frac{E_{ij}+E_{ji}}{2}) & -P & * \\ (\frac{D_{ij}+D_{ji}}{2})'P & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (24)$$

for all $i \le j$ (i,j = 1,…,L), then (i) $\tilde{x}(t) \equiv 0$ of the augmented system (21) is globally asymptotically stable in probability in the case of v = 0 and (ii) the following H_∞ performance

$$\|\tilde{x}(t)\|_{L_2}^2 \le E[\tilde{x}'(0)P\tilde{x}(0)] + \gamma^2 \|v(t)\|_{L_2}^2 \quad (25)$$

$\forall v(t) \in L_F^2(R^+, R^{n_v})$, $v(t) \ne 0$ holds for some $\gamma > 0$.

Proof. By Lemma 1, if there exists a positive function $V(\tilde{x}) \in C^2(R^n)$ and V(0) = 0 such that

$$\begin{cases} \Big\{\frac{\partial V'}{\partial \tilde{x}}(\sum_{i=1}^{L} h_i(z(t))\sum_{j=1}^{L} h_j(z(t)A_{ij}\tilde{x}(t)) \\ +\frac{1}{2}\gamma^{-2}(\frac{\partial V'}{\partial \tilde{x}}(\sum_{i=1}^{L} h_i(z(t))\sum_{j=1}^{L} h_j(z(t)D_{ij}) \\ \times((\sum_{m=1}^{L} h_m(z(t))\sum_{n=1}^{L} h_n(z(t)D_{mn})'\frac{\partial V}{\partial \tilde{x}}) \\ +\frac{1}{2}(\tilde{x}(t))'(\tilde{x}(t)) \\ +\frac{1}{2}(\sum_{i=1}^{L} h_i(z(t))\sum_{j=1}^{L} h_j(z(t)E_{ij}\tilde{x}(t))'\frac{\partial^2 V}{\partial \tilde{x}^2} \\ \times(\sum_{m=1}^{L} h_m(z(t))\sum_{n=1}^{L} h_n(z(t)E_{mn}\tilde{x}(t))\} < 0 \end{cases} \quad (26)$$

then (i) and (ii) hold. Let

$$V(\tilde{x}) = \frac{1}{2}\tilde{x}'P\tilde{x} \quad (27)$$

By (27), (26) holds, if

$$\begin{aligned} &P(\frac{A_{ij}+A_{ji}}{2})+(\frac{A'_{ij}+A'_{ji}}{2})P+\gamma^{-2}[P(\frac{D_{ij}+D_{ji}}{2})] \\ &\times[(\frac{D_{ij}+D_{ji}}{2})'P]+I+(\frac{E_{ij}+E_{ji}}{2})'P(\frac{E_{ij}+E_{ji}}{2}) < 0 \end{aligned} \quad (28)$$

for all $i \le j$ $(i, j = 1, \ldots L)$.

By the Schur complements [11], (28) is equivalent to

$$\begin{bmatrix} P(\frac{A_{ij}+A_{ji}}{2})+(\frac{A'_{ij}+A'_{ji}}{2})P+I & * & * \\ P(\frac{E_{ij}+E_{ji}}{2}) & -P & * \\ (\frac{D_{ij}+D_{ji}}{2})'P & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (29)$$

Therefore, the proof is complete.

For the convenience of design, let

$$P = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix} \quad (30)$$

then (24) can be rewritten as the following LMIs

$$\begin{bmatrix} \{\frac{(P_{11}A_i+P_{11}B_iK_j)+(P_{11}A_j+P_{11}B_jK_i)}{2} + \frac{(A'_iP_{11}+K'_jB'_iP_{11})+(A'_jP_{11}+K'_iB'_jP_{11})}{2}+I\} & * & * & * & * \\ \frac{-K'_jB'_iP_{11}-K'_iB'_jP_{11}}{2} & \{\frac{(P_{22}A_i-Y_iC_j)+(P_{22}A_j-Y_jC_i)}{2} + \frac{(A'_iP_{22}-C'_jY'_i)+(A'_jP_{22}-C'_iY'_j)}{2} & * & * & * \\ P_{11}E_{1i} & 0 & -P_{11} & * & * \\ \frac{(P_{22}E_{1i}-Y_iE_{2j})+(P_{22}E_{1j}-Y_jE_{2i})}{2} & 0 & 0 & -P_{22} & * \\ D'_{1i}P_{11} & \frac{(D'_{1i}P_{22}-D'_{2j}Y'_i)+(D'_{1j}P_{22}-D'_{2i}Y'_j)}{2} & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (31)$$

where $Y_i = P_{22}L_i$.

Note that (31) implies that

$$\frac{(P_{11}A_i+P_{11}B_iK_j)+(P_{11}A_j+P_{11}B_jK_i)}{2} + \frac{(A'_iP_{11}+K'_jB'_iP_{11})+(A'_jP_{11}+K'_iB'_jP_{11})}{2}+I<0 \quad (32)$$

And (32) is equivalent to

$$\begin{bmatrix} \{\frac{(A_iQ_{11}+B_iZ_j)+(A_jQ_{11}+B_jZ_i)}{2} + \frac{(Q_{11}A'_i+Z'_jB'_i)+(Q_{11}A'_j+Z'_iB'_j)}{2}\} & * \\ Q_{11} & -I \end{bmatrix} < 0 \quad (33)$$

where $Q_{11} = P_{11}^{-1}$ and $Z_j = K_jQ_{11}$.

Design Procedure : Based on the analysis above, the design procedure for the H_∞ fuzzy control design is summarized as follows:

Step 1 : Construct the fuzzy model rules (9).
Step 2 : Solve the LMIP in (33) first to obtain Q_{11} and Z_j and (thus $K_j = Z_j Q_{11}^{-1}$ and $P_{11} = Q_{11}^{-1}$).
Step 3 : Substitute P_{11} and K_j into (31) and then solve the EVP by minimizing the attenuation level γ^2 as the following minimization problem to obtain P_{22} and Y_i (thus $L_i = P_{22}^{-1} Y_i$).

$$\min_{P_{22}} \quad \gamma^2 \qquad (34)$$
$$\text{subject to} \quad P_{22} = P_{22}^T > 0 \quad \text{and} \quad (31)$$

Step 4 : Construct the fuzzy estimator in (17).
Step 5 : Construct the fuzzy controller in (18).

4. Simulation Example

To illustrate the proposed fuzzy control approach, a control problem of balancing an inverted pendulum on a cart is considered in this study. For this example, the state equations of the inverted pendulum are given by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{[(M+m)(J+ml^2) - (ml\cos x_1)^2]} \times [(M+m)mgl\sin x_1 - (mlx_2)^2 \sin x_1 \cos x_1 - ml\cos x_1 u + v \text{-} 10^{-3}(M+m)x_2(7+w)]$$
$$Y = x_1 + 0.02v + 10^{-4} x_1 w \qquad (35)$$

where x_1 denotes the angle (rad) of the pendulum from the vertical, x_2 is the angular velocity (rad/s), $g = 9.8 m/s^2$ is the gravity constant, m is the mass (kg) of the pendulum, M is the mass (kg) of the cart, w is the uncertain friction factor (N/rad/s) of the pendulum, which is assumed to be a stationary white noise, l is the length (m) from the center of mass of the pendulum to the shaft axis, J is the moment of inertia (kgm^2) of the pendulum, u is the force (N) applied to the cart, and v is external disturbance. The parameters in this example are assumed to be m=1.0, M=10, l=0.5, and J=0.005, and $v = 2e^{-t}\sin 5t$.

To minimize the design effort and complexity, we try to use as few rules as possible. Hence, we approximate the system by the following four-rule fuzzy model.

Rule 1 : IF x_1 is about 0
THEN $\dot{x} = A_1 x + B_1 u + D_{11} v + E_{11} xw$ and
$Y = C_1 x + D_{21} v + E_{21} xw$
Rule 2 : IF x_1 is about $\pm\pi/9$
THEN $\dot{x} = A_2 x + B_2 u + D_{12} v + E_{12} xw$ and
$Y = C_2 x + D_{22} v + E_{22} xw$
Rule 3 : IF x_1 is about $\pm 2\pi/9$
THEN $\dot{x} = A_3 x + B_3 u + D_{13} v + E_{13} xw$ and
$Y = C_3 x + D_{23} v + E_{23} xw$
Rule 4 : IF x_1 is about $\pm\pi/3$
THEN $\dot{x} = A_4 x + B_4 u + D_{14} v + E_{14} xw$ and
$Y = C_4 x + D_{24} v + E_{24} xw$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 21.0959 & -0.0301 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ -0.1957 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ 20.4362 & -0.0298 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -0.1818 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & 1 \\ 18.6688 & -0.0290 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ -0.1441 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 0 & 1 \\ 16.2534 & -0.0281 \end{bmatrix}, B_4 = \begin{bmatrix} 0 \\ -0.0912 \end{bmatrix},$$
$$D_{11} = \begin{bmatrix} 0 \\ 0.3914 \end{bmatrix}, E_{11} = \begin{bmatrix} 0 & 0 \\ 0 & -0.0043 \end{bmatrix},$$
$$D_{12} = \begin{bmatrix} 0 \\ 0.3870 \end{bmatrix}, E_{12} = \begin{bmatrix} 0 & 0 \\ 0 & -0.0043 \end{bmatrix},$$
$$D_{13} = \begin{bmatrix} 0 \\ 0.3762 \end{bmatrix}, E_{13} = \begin{bmatrix} 0 & 0 \\ 0 & -0.0041 \end{bmatrix},$$
$$D_{14} = \begin{bmatrix} 0 \\ 0.3646 \end{bmatrix}, E_{14} = \begin{bmatrix} 0 & 0 \\ 0 & -0.0040 \end{bmatrix},$$
$$C_i = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{2i} = 0.02, E_{2i} = \begin{bmatrix} 10^{-4} & 0 \end{bmatrix}$$

for i = 1,…,4. Triangle type membership functions are used for Rule 1-Rule 4. The LMIP in (33) and the EVP in (34) are solved using the LMI optimization toolbox in Matlab. In this case, $\gamma_{\min}^2 = (0.5016)^2$ and

$$Q_{11} = \begin{bmatrix} 0.2662 & -0.3870 \\ -0.3870 & 0.7290 \end{bmatrix}$$
$$P_{22} = \begin{bmatrix} 124.2369 & -132.3597 \\ -132.3597 & 142.1074 \end{bmatrix}$$

The observe gains are found to be

$$L_1 = \begin{bmatrix} 894.6338 \\ 853.2804 \end{bmatrix}, L_2 = \begin{bmatrix} 879.3905 \\ 838.9958 \end{bmatrix}$$
$$L_3 = \begin{bmatrix} 880.0687 \\ 839.1571 \end{bmatrix}, L_4 = \begin{bmatrix} 1007.106 \\ 957.7221 \end{bmatrix}$$

The control gains are found to be

$$K_1 = [213.6 \quad 70.2], K_2 = [228.9 \quad 76.4]$$
$$K_3 = [276.5 \quad 96.5], K_4 = [355.8 \quad 132.2]$$

The trajectories of x_1, $\hat{x}_1$, and external disturbance v are shown in Figure 1 and the trajectories of x_2 and $\hat{x}_2$ are shown in Figure 2. From the simulation results, the proposed fuzzy control clearly results in desired H_∞ control performance.

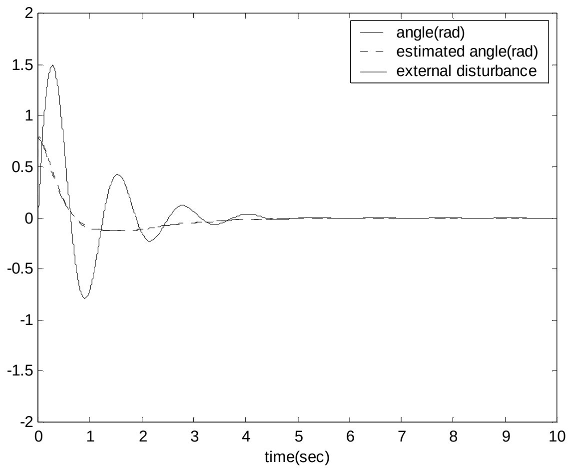

Figure 1. The trajectories of x_1 and $\hat{x}_1$, and external disturbance v.

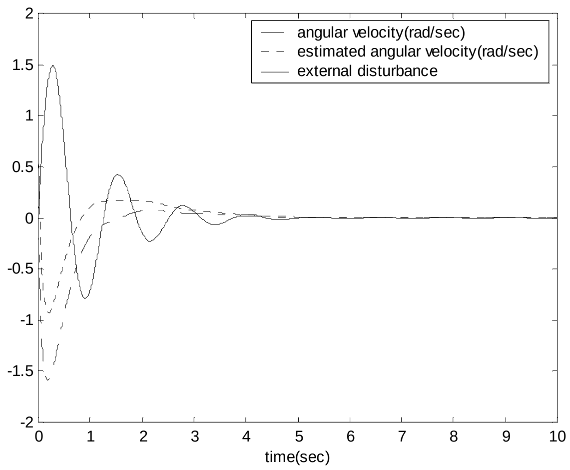

Figure 2. The trajectories of x_2 and $\hat{x}_2$.

5. Conclusions

In this paper, based on a T-S fuzzy model, the output feedback H_∞ fuzzy control problems for a class of nonlinear stochastic systems governed by $\text{Itô}-\text{type}$ stochastic differential equations are studied. Using fuzzy approach, the output feedback H_∞ fuzzy control design for the nonlinear stochastic systems can be given via solving linear matrix inequalities (LMIs) instead of a second-order Hamilton-Jacobi inequality.

This study extends the output feedback H_∞ control design from linear stochastic systems to a class of nonlinear stochastic systems using fuzzy techniques. LMI-based design procedure for the output feedback H_∞ fuzzy control problems for the nonlinear stochastic systems is developed systematically. The proposed design procedure is very simple and can be performed efficiently using the LMI optimization toolbox in Matlab. Simulation example is provided to illustrate the design procedure and expected performance. Therefore, the proposed method is very suitable for practical applications in the nonlinear stochastic systems.

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