On the Stability Analysis of Uncertain Fuzzy Models

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Abstract

This paper deals with the stability and the robust stability of fuzzy models using a class of nonquadratic Lyapunov functions. New sufficient stability conditions are given for continuous-time fuzzy models. The stability conditions are proved to be less conservative than the quadratic case and compared to another classes of nonquadratic Lyapunov functions. The derived conditions are solved using linear matrix inequalities (LMI) and the S-procedure. Examples are given to illustrate the proposed results.

Keywords: Continuous-time fuzzy models, stability, uncertainties, Lyapunov methods, S-procedure, LMI.

1.Introduction

The stability analysis of the Takagi-Sugeno (T-S) models [11] has been actively considered during the last decade. This approach includes the Polytopic Linear Differential Inclusions (PLDI) [1], multiple models approach [8] and in some cases the Linear Parameters Varying (LPV) approach (see e.g. [14] and references therein).

The stability analysis and stabilisation of T-S models is firstly studied using a common quadratic Lyapunov function. However, the class of quadratic Lyapunov function leads to significant conservativeness (see for example [3][4][14]-[19]). To improve these results, stability conditions have been established using a piecewise quadratic Lyapunov function [5][6][15]. Unfortunately, when the supports of the activation functions are infinite, the method does not introduce any relaxation comparing with the quadratic result. Another class of Lyapunov function candidates usually called polyquadratic (or fuzzy quadratic) is also studied for both continuous-time and discrete-time cases (see [2][7][10][12][13][20][21] and references therein).

However, it is interesting to notice the great difference between the results of the continuous domain and the discrete domain. Thus, whereas the stability conditions obtained for the discrete case are global and may be formulated into a LMI form [20][21], the conditions in continuous case are often local and in nonlinear form [2][7][10][12]. In this case, the proposed method needs a priori bound on the state variation. For example, the stability conditions in [7][10][12] are obtained under constraint on state norm or his time-derivative (i.e. time derivative of activation functions).

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This paper deals with the stability analysis of uncertain continuous-time T-S model with infinite support of activation functions. Using another class of nonquadratic Lyapunov function, the aim is to obtain global stability independently of the time derivative of activation functions and easy to solve using classical numerical tools (LMI Toolbox for Matlab for example).

In this paper, the stability of uncertain continuous-time T-S models is considered. New sufficient conditions for global asymptotic stability are obtained using a class of nonquadratic Lyapunov function and the S-procedure. The paper is organized as follows. Section 2 is dedicated to the description of the uncertain continuous-time T-S model. In section 3, sufficient stability conditions for nominal T-S models are given and then extended to robust stability in section 4. To illustrate the effectiveness of the proposed results comparing with others methods, numerical examples are given in section 5.

In the rest of the paper, the following useful notation is used: X^T denotes the transpose of the matrix X, $X > 0$ $(X \geq 0)$ denotes a symmetric positive definite (semi-definite) matrix, I denotes an identity matrix and $I_n = \{1,2,...,n\}$.

2. Preliminary

In this work, we consider the case of uncertain continuous-time T-S model obtained by extending the T–S fuzzy model [11]. An open loop uncertain continuous-time T-S model fuzzy is based on the interpolation between several uncertain LTI (Linear Time Invariant) local models as follows:

Model rule i: If $z_1(t)$ **is** M_{i1} **and…and** $z_q(t)$ **is** M_{iq}

Then $\dot{x}(t) = \left(A_i + \Delta A_i\right) x(t), \quad i \in I_n$

where $x(t) \in \mathbb{R}^p$ is the state vector, $A_i \in \mathbb{R}^{p.p}$ is the i[th] state matrix and the terms $\Delta A_i(t)$ are time-varying matrices representing parametric uncertainties. n is the number of the If-Then rules (i.e. the number of sub-models) and $\left(M_{i1},...,M_{iq}\right)$ are the fuzzy sets. Vector $z(t)=\left(z_1(t),...,z_q(t)\right)^T \in \mathbb{R}^q$ is the vector of the premise variables assumed to be depending on measurable variables.

The final output of the T-S models is interpolated as follows:

$$\dot{x}(t)=\sum_{i=1}^{n}\mu_i(z(t))(A_i+\Delta A_i(t))x(t) \tag{1}$$

where $\mu_i(z(t))=\dfrac{\omega_i(z(t))}{\sum_{i=1}^{n}\omega_i(z(t))}$ and $\omega_i(z(t))=\prod_{j=1}^{q}M_{ij}(z(t))$

The *normalised activation function* $\mu_i(.), i\in I_n$ respect the properties:

$$\sum_{i=1}^{n}\mu_i(z(t))=1,\ \mu_i(z(t))\geq 0 \quad \forall\, i\in I_n \tag{2}$$

The choice of variable $z(t)$ leads to different classes of models. It can depend on the measurable state variables. In this case, system (1) describes a class of nonlinear systems. It can also be an unknown constant value, system (1) then representing a PLDI (Polytopic Linear Differential Inclusion) [1].

The terms $\Delta A_i(t)$, representing parametric uncertainties, are supposed admissibly norm-bounded, structured and satisfy assumption 1.

Assumption 1 [9]: The considered parameter uncertainties are norm-bounded:

$$\Delta A_i(t)=D_iF_i(t)E_i,\quad F_i(t)^T F_i(t)\leq I \tag{3}$$

where D_i, E_i are known real constant matrices of appropriate dimension, $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and I is the identity matrix.

The following lemmas will be used in the rest of the paper

Lemma 1 (S-*Procedure,* [1]): Let $F_0(x(t)),...,F_n(x(t))$ be quadratic functions of variable $x(t)\in\mathbb{R}^p$.

If there exists scalars $\tau_1\geq 0,..,\tau_n\geq 0$ such that $F_0(x(t))-\sum_{i=1}^{n}\tau_iF_i(x(t))\leq 0$ then $F_0(x(t))\leq 0$ for all $x(t)$ such that $F_i(x(t))\leq 0, \forall\, i\in I_n$.

Lemme 2 [9]: Given constant matrices D and F, unknown constant matrix F of appropriate dimension satisfying constraint $F^TF\leq I$. The following two propositions are equivalent:

i) $DFE+E^TF^TD^T<0$

ii) $\varepsilon DD^T+\varepsilon^{-1}E^TE<0$ for some $\varepsilon>0$. ■

3. Non quadratic stability conditions

Consider the nominal T-S model of (1):

$$\dot{x}(t)=\sum_{i=1}^{n}\mu_i(z(t))A_ix(t) \tag{4}$$

Sometimes, it is possible to prove the stability of the T-S model (4) using a quadratic Lyapunov function $V(x(t))=x^T(t)Px(t), P>0$. Taking account the time-derivative of Lyapunov function $V(x(t))$, T-S model (4) is globally asymptotically stable if there exists a matrix $P>0$ such that

$$A_i^TP+PA_i<0 \qquad \forall i\in I_n \tag{5}$$

Inequalities (5) give a sufficient condition for ensuring the stability of (4). However, it is well known that in a lot of cases, a common positive definite matrix *P* does not exist whereas the T-S model is stable (see example given in section 5).

To overcome this conservatism, nonquadratic Lyapunov functions may be used. Among these functions, we can quote the piecewise quadratic functions [5][15]. This approach allows to reduce the conservatism of the quadratic method by taking into account the partition of the state space induced by activation functions with limited local support of variables (for example trapezoidal or triangular activation functions) [5]. However, in the case of an infinite support (for example Gaussian activation functions), this approach is reduced to the quadratic stability (conditions (5)). Some works also propose another type of nonquadratic Lyapunov function called fuzzy Lyapunov functions of the form $V(x(t))=x(t)^T\sum_{i=1}^{n}\mu_i(x(t))P_ix(t)$ [2][7][10][12][20][21]. In the continuous domain, the obtained results need a priori bound on the state variation which is synonym of conservatism. In order to overcome this restriction and the limitation of the result obtained by piecewise quadratic Lyapunov functions [5] when the activation functions have an infinite or unknown support, the following class of nonquadratic Lyapunov functions is proposed [1]:

$$V(x(t))=\max(V_1(x(t)),...,V_i(x(t)),...,V_n(x(t))) \tag{6}$$

where $V_i(x(t))=x(t)^TP_ix(t),\quad P_i>0, i\in I_n$

In the following, sufficient conditions for global asymptotic stability of T-S model (4) are established by using the S-procedure lemma and a nonquadratic Lyapunov function candidate (6).

Theorem 1: Suppose that there exists matrices $P_i > 0,\ i \in I_n$ and scalars $\tau_{ijk} \geq 0$ such that $\forall (i, j) \in I_n^2$:

$$A_i^T P_j + P_j A_i + \sum_{k=1}^{n} \tau_{ijk} \left(P_j - P_k\right) < 0 \quad (7)$$

Then T-S model (4) is globally asymptotically stable.

Proof: Considering nonquadratic Lyapunov function candidate (6). It follows that

$$V\left(x(t)\right) = V_j\left(x(t)\right) \text{ if } V_j\left(x(t)\right) \geq V_k\left(x(t)\right), \forall\, k \in I_n \quad (8)$$

Consequently

$$\frac{dV\left(x(t)\right)}{dt} = \begin{cases} \bullet\, x(t)^T \sum_{i=1}^{n} \mu_i\left(z(t)\right)\left(A_i^T P_1 + P_1 A_i\right) x(t) \\ \quad when\, \forall x(t) : V_1\left(x(t)\right) \geq V_k\left(x(t)\right), \forall\, k \in I_n \\ \vdots \\ \bullet\, x(t)^T \sum_{i=1}^{n} \mu_i\left(z(t)\right)\left(A_i^T P_n + P_n A_i\right) x(t) \\ \quad when\, \forall x(t) :\ V_n\left(x(t)\right) \geq V_k\left(x(t)\right), \forall\, k \in I_n \end{cases} \quad (9)$$

Therefore when $x(t)^T \left(P_j - P_k\right) x(t) \geq 0, \forall\, k \in I_n$, we obtain

$$\frac{dV\left(x(t)\right)}{dt} = x(t)^T \sum_{i=1}^{n} \mu_i\left(z(t)\right)\left(A_i^T P_j + P_j A_i\right) x(t) \quad (10)$$

Consequently, if we have $A_i^T P_j + P_j A_i < 0$ when $x(t)^T \left(P_j - P_k\right) x(t) \geq 0,\ \forall\, k \in I_n$ we ensure that $\frac{dV\left(x(t)\right)}{dt} < 0 \quad \forall\, x(t) \neq 0$. Using the S-procedure (lemma1) stability conditions (7) are derived.

Remark 1: It should be noted that quadratic conditions (5) are included in conditions derived in (7). So when $P_i = P, \forall\, i \in I_n$ we have $P_j - P_k = 0$ and $V\left(x(t)\right) = \max_{i \in I_n}\left(V_i\left(x(t)\right)\right) = x(t)^T P x(t)$. Then conditions (7) are reduced to the quadratic conditions (5). The same result can be obtained by using nonquadratic Lyapunov function $V\left(x(t)\right) = \min_{i \in I_n}\left(V_i\left(x(t)\right)\right)$ where $V_i\left(x(t)\right)$ is defined in (6), it suffices to choose the scalars τ_{ijk} negatives.

4. Extension to robust stability analysis

In this section we consider uncertain T-S model (1). Using a quadratic Lyapunov function ($V\left(x(t)\right) = x^T(t) P x(t), P > 0$), the global asymptotic stability of (1) can be obtained directly by considering (5) as follows:

$$\left(A_i + \Delta A_i\right)^T P + P\left(A_i + \Delta A_i\right) < 0 \qquad \forall i \in I_n \quad (11)$$

Knowing that $\Delta A_i = D_i F_i(t) E_i$ (assumption 1) and using lemma 2, the inequalities (11) become

$$A_i^T P + P A_i + \varepsilon_{ij} E_i^T E_i + \varepsilon_{ij}^{-1} P D_i D_i^T P < 0 \quad (12)$$

Indeed, if there exist a matrix $P > 0$ and scalars $\varepsilon_{ij} > 0$ such that (12) hold, then the uncertain T-S model (1) is robustly stable. However, the stability conditions (12) are often very conservative (see example given in section 5). In order to reduce this conservatism, the robust stability of such models can be studied directly by using the result of theorem 1. The following result is proposed.

Theorem 2: Suppose that there exists matrices $P_i > 0,\ i \in I_n$ and scalars ε_{ij} and $\tau_{ijk} \geq 0$ such that $\forall (i, j) \in I_n^2$:

$$\begin{pmatrix} A_i^T P_j + P_j A_i + \sum_{k=1}^{n} \tau_{ijk}\left(P_j - P_k\right) + \varepsilon_{ij} E_i^T E_i & P_j D_i \\ D_i^T P_j & -\varepsilon_{ij} I \end{pmatrix} < 0 \quad (13)$$

Then, uncertain T-S model (1) is globally asymptotically stable.

Proof : The stability conditions of the uncertain T-S model (1) are obtained directly from theorem 1, i.e. :

$$\left(A_i + \Delta A_i\right)^T P_j + P_j\left(A_i + \Delta A_i\right) + \sum_{k=1}^{n} \tau_{ijk}\left(P_j - P_k\right) < 0 \quad (14)$$

Taking account assumption 1 (i.e. $\Delta A_i = D_i F_i(t) E_i$), inequalities (14) are equivalent to

$$A_i^T P_j + P_j A_i + E_i^T F_i(t)^T D_i^T P_j + P_j D_i F_i(t) E_i + \sum_{k=1}^{n} \tau_{ijk}\left(P_j - P_k\right) < 0 \quad (15)$$

Using lemma 2, the following propositions are equivalent:

i) $E_i^T F_i(t)^T D_i^T P_j + P_j D_i F_i(t) E_i$, ii) $\varepsilon_{ij} E_i^T E_i + \varepsilon_{ij}^{-1} P_j D_i D_i^T P_j$ for some $\varepsilon_{ij} > 0$

Consequently, inequalities (14) are equivalent to the following inequalities

$$A_i^T P_j + P_j A_i + \sum_{k=1}^{n} \tau_{ijk} \left(P_j - P_k\right) + \varepsilon_{ij} E_i^T E_i + \varepsilon_{ij}^{-1} P_j D_i D_i^T P_j < 0 \quad (16)$$

This is only the Schur complement [1] of the sufficient conditions (13).

As it is stated above, conditions (12) are included in conditions (13). So, when $P_i = P, \forall\, i \in I_n$ we have $P_j - P_k = 0$ and stability conditions (13) becomes

$$\begin{pmatrix} A_i^T P + PA_i + \varepsilon_{ij} E_i^T E_i & PD_i \\ D_i^T P & -\varepsilon_{ij} I \end{pmatrix} < 0 \quad (17)$$

It is obvious that conditions (17) are only the Schur complement of (12). This is a proof that the proposed stability conditions (13) are less conservative than conditions (12) derived from the quadratic case.

Remark 2: The derived stability conditions, using the S-procedure lemma and the nonquadratic Lyapunov function (6), lead to $n^2 \frac{(n+1)}{2}$ constraints. These conditions are not jointly convex in $P_i > 0$ and τ_{ijk}. However if we fix scalars τ_{ijk}, we obtain convex problem in $P_i > 0$ and $\varepsilon_{ij} > 0, (i, j) \in I_n^2$.

5. Numerical examples

Consider the T-S model (1) with $n = 3,\ \Delta A_i = E_i F_i D_i,\ F_i^T F_i \le I$ and the following data

$$A_1 = \begin{pmatrix} 0 & 1 \\ -0.1 & -1 \end{pmatrix},\ E_1 = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix},\ D_1 = \begin{pmatrix} 0.0667 & 0 \\ 0 & 0.0667 \end{pmatrix} \quad (18a)$$

$$A_2 = \begin{pmatrix} 0 & 1.2 \\ -1.9 & -1 \end{pmatrix},\ E_2 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix},\ D_2 = \begin{pmatrix} 0.0667 & 0 \\ 0 & 0.0667 \end{pmatrix} \quad (18b)$$

$$A_3 = \begin{pmatrix} 0 & 2 \\ -0.5 & -1.5 \end{pmatrix},\ E_3 = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix},\ D_3 = \begin{pmatrix} 0.0667 & 0 \\ 0 & 0.0667 \end{pmatrix} \quad (18c)$$

The activation functions (figures 1, 2) are as follow

$$\mu_1\left(x_1(t)\right) = \frac{\omega_1\left(x_1(t)\right)}{\omega_1\left(x_1(t)\right) + \omega_2\left(x_1(t)\right) + \omega_3\left(x_1(t)\right)} \quad (18d)$$

$$\mu_2\left(x_1(t)\right) = \frac{\omega_2\left(x_1(t)\right)}{\omega_1\left(x_1(t)\right) + \omega_2\left(x_1(t)\right) + \omega_3\left(x_1(t)\right)} \quad (18e)$$

$$\mu_3\left(x_1(t)\right) = \frac{\omega_3\left(x_1(t)\right)}{\omega_1\left(x_1(t)\right) + \omega_2\left(x_1(t)\right) + \omega_3\left(x_1(t)\right)} \quad (18f)$$

with

$$\omega_1\left(x_1(t)\right) = \exp\left(-\frac{1}{2}\left(\frac{x_1(t)+\text{c}}{d}\right)^2\right)\ \omega_2\left(x_1(t)\right) = \exp\left(-\frac{1}{2}\left(\frac{x_1(t)}{d}\right)^2\right),$$

$$\omega_3\left(x_1(t)\right) = \exp\left(-\frac{1}{2}\left(\frac{x_1(t)-\text{c}}{d}\right)^2\right) \quad (18g)$$

The figures 1 and 2 show the case of activation functions with infinite support and different slope (i.e. switching speed between sub-models related directly to the time-derivative of activation functions). Figure 2 represents the case of activation functions with very large slope in comparison with figure 1.The influence of the activation functions form (i.e. support and switching speed) will be discussed in the following.

A. *Comparison with other results* ($\Delta A_i = 0, i \in I_3$):

Although the three nominal state matrices are stable and the simulation show that stability of example (18) (see figures 5), the quadratic stability conditions (5) fail to prove the stability of this example. This verification can be made easily by solving the dual problem stated in the following lemma 1:

Lemma 3 [1]: If there exists matrices $X_i, \forall\, i \in I_n$ not all zero such that

$$X_i \ge 0 \text{ and } \sum_{i=1}^{n} X_i A_i^T + A_i X_i \ge 0$$

then the inequalities (5) do not admit a solution $P > 0$. ■

Thus, for the given example, the dual problem is

$$X_1 > 0,\ X_2 > 0,\ X_3 > 0$$
$$X_1 A_1^T + A_1 X_1 + X_2 A_2^T + A_2 X_2 + X_3 A_3^T + A_3 X_3 > 0$$

Which is feasible and gives

$$X_1 = \begin{pmatrix} 2041.9 & 639.7 \\ 639.7 & 210.5 \end{pmatrix},\quad X_2 = \begin{pmatrix} 447.4 & -583.7 \\ -583.7 & 768.5 \end{pmatrix},$$
$$X_3 = \begin{pmatrix} 66.5 & 34.5 \\ 34.5 & 27.5 \end{pmatrix}$$

The conditions given in [5] also fail to prove the stability of T-S model (18) because the activation functions (18d-g) are with infinite support; these conditions are reduced to the search of a common global quadratic Lyapunov function in this case.

The derived stability conditions using polyquadratic Lyapunov functions ($V(x(t)) = x(t)^T \sum_{i=1}^{n} \mu_i(x(t)) P_i x(t)$) [7][10][12] also fail to prove the global stability of the T-S model (18), because the stability conditions depend on the activation functions dynamics. The obtained stability conditions are derived under bound of state time-derivative and are local only. Moreover, the method is very conservative in the case of activation functions with

large time-derivative (see [7][10][12] for examples). Thus, whereas the proposed example is globally asymptotically stable (see figure 5)-as we will prove it sub-section 5.2- these methods depend on switching speed between sub-models (γ) and need a priori bound on state variation which is synonym of conservatism.
For example using the stability conditions (theorem 1) of [12], the T-S model (18) is asymptotically stable if the constraint on the state variation: $\|\dot{x}_1(t)\| \le \frac{0.0219}{\gamma}$ is satisfied where $\gamma = \max_{i \in I_3} \left\| \frac{\partial \mu_i(x_1(t))}{\partial x_1(t)} \right\|$ is the upper bound of the derivative of the activations functions (see figure 3 and figure 4). Indeed, the stability is local only and depends on switching speed between sub-models ($\|\dot{x}_1(t)\| \le 0.15$ for activation functions (18d-g) with $c = 9,\ d = 4$ and $\|\dot{x}_1(t)\| \le 0.0118$ for activation functions (18d-g) with $c = 9,\ d = 1$). It is obvious that when the switching speed between sub-models is very large, even a local stability is impossible to find.

In the following sub-sections, by using the derived stability conditions of theorem 1 and theorem 2, we prove that the given example is globally asymptotically stable independently of the activation functions form.

B. *Stability analysis ($\Delta A_i = 0, i \in I_3$):*

Now by considering stability conditions of theorem 1, we obtain nine constraints in $P_1 > 0, P_2 > 0$, $P_3 > 0$ and $\tau_{ijk} > 0, (i, j, k) \in I_n^3$. By an iterative method fixing the positive scalars $\tau_{ijk}, (i, j, k) \in I_n^3$, stability conditions (7) lead to an LMI problem in $P_1 > 0, P_2 > 0$, $P_3 > 0$. Solving this LMI problem, we get:

$$P_1 = \begin{pmatrix} 149.4433 & 84.1699 \\ 84.1699 & 162.2407 \end{pmatrix},\quad P_2 = \begin{pmatrix} 161.1414 & 99.1047 \\ 99.1047 & 174.9105 \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 151.4240 & 57.5350 \\ 57.5350 & 179.8239 \end{pmatrix} \qquad (19)$$

$$\tau_{112} = \tau_{113} = 0,\ \tau_{212} = \tau_{213} = 0.5,\ \tau_{312} = \tau_{313} = 0 \qquad (20a)$$

$$\tau_{121} = \tau_{123} = 0,\ \tau_{221} = \tau_{223} = 1,\ \tau_{321} = \tau_{323} = 0 \qquad (20b)$$

$$\tau_{131} = \tau_{132} = 0.5,\ \tau_{231} = \tau_{232} = 0,\ \tau_{331} = \tau_{332} = 0 \qquad (20c)$$

which allows affirming the global asymptotic stability of the nominal T-S model of (18).

C. *Robust stability analysis*

By an iterative method fixing the positive scalars $\tau_{ijk}, (i, j, k) \in I_n^3$, stability conditions of theorem 2 lead to nine LMI in $P_1 > 0, P_2 > 0$, $P_3 > 0$ and $\varepsilon_{ij}\ (i, j) \in I_n^2$. Solving this LMI problem, we get:

$$P_1 = \begin{pmatrix} 39.57 & 26.39 \\ 26.39 & 47.11 \end{pmatrix},\quad P_2 = \begin{pmatrix} 35.99 & 10.51 \\ 10.51 & 40.90 \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 37.87 & 10.53 \\ 10.53 & 42.87 \end{pmatrix} \qquad (21)$$

$$\tau_{112} = \tau_{113} = 0,\ \tau_{212} = \tau_{213} = 1,\ \tau_{312} = \tau_{313} = 0 \qquad (22a)$$

$$\tau_{121} = \tau_{123} = 0.5,\ \tau_{221} = \tau_{223} = 0,\ \tau_{321} = \tau_{323} = 1 \qquad (22b)$$

$$\tau_{131} = \tau_{132} = 1,\ \tau_{231} = \tau_{232} = 0,\ \tau_{331} = \tau_{332} = 1 \qquad (22c)$$

$$\varepsilon_{11} = 0.7912,\ \varepsilon_{12} = 0.7597,\ \varepsilon_{13} = 1.1174 \qquad (23a)$$

$$\varepsilon_{21} = 0.2281,\ \varepsilon_{22} = 0.2909,\ \varepsilon_{23} = 0.3137 \qquad (23b)$$

$$\varepsilon_{31} = 0.2810,\ \varepsilon_{32} = 0.9569,\ \varepsilon_{33} = 0.4369 \qquad (23c)$$

Which shows the global asymptotic stability of the uncertain T-S models (18). It is important to recall that the quadratic stability conditions (12) fail to prove the robust stability of (18).

Remark 3: It is important to note that the proposed method (theorems 1 and 2) do not depend on time-derivative of activation functions (i.e. switching speed between models) in comparison with the polyquadratic approach ($V(x(t)) = x(t)^T \sum_{i=1}^{n} \mu_i(x(t)) P_i x(t)$). To show concretely this fact, the example (18) is simulated with activation functions of different switching speed between sub-models (figures 1 & 2). It is checked that when the proposed stability conditions hold, the global stability of the system is guaranteed independently of the activation functions form.
Indeed, the proposed method guarantees a global asymptotic stability independently of the activation functions dynamics. This contribution is particularly more interesting when the switching speed between models is very large (i.e. the case of activation functions $\mu_i(.), i \in I_n$ with large slope, example of figure 2). The given stability conditions are also shown independent of the support of the functions of activation (figures 1, 2 are with infinite supports).

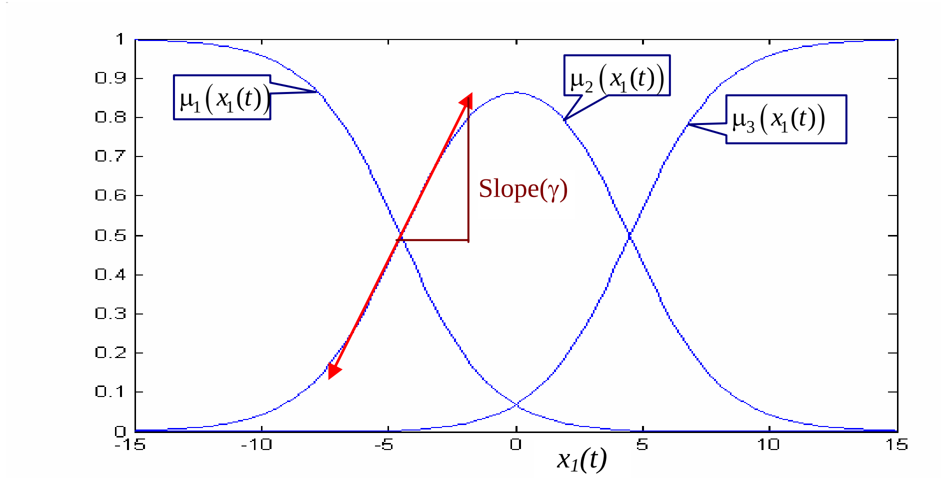

Figure 1. Activation functions (18d-g) with $c = 9$, $d = 4$

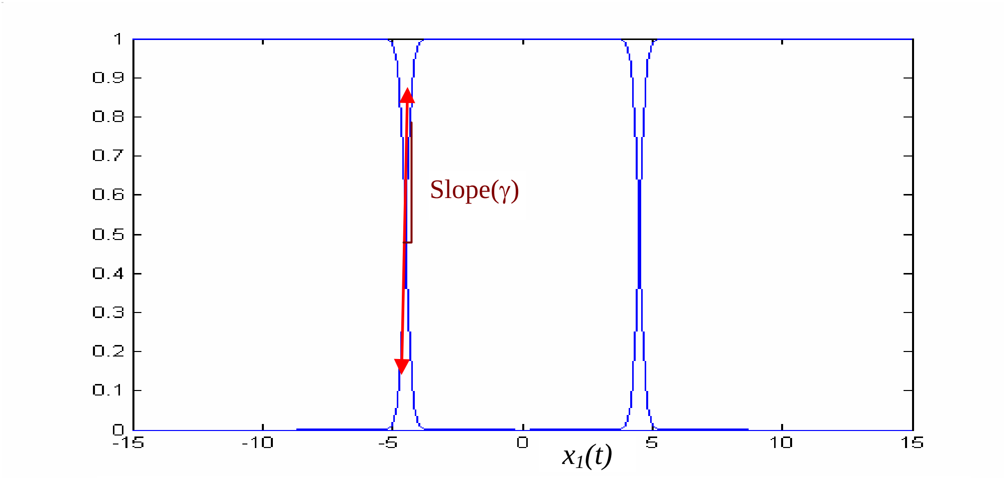

Figure 2. Activation functions (18d-g) with $c = 9$, $d = 1$

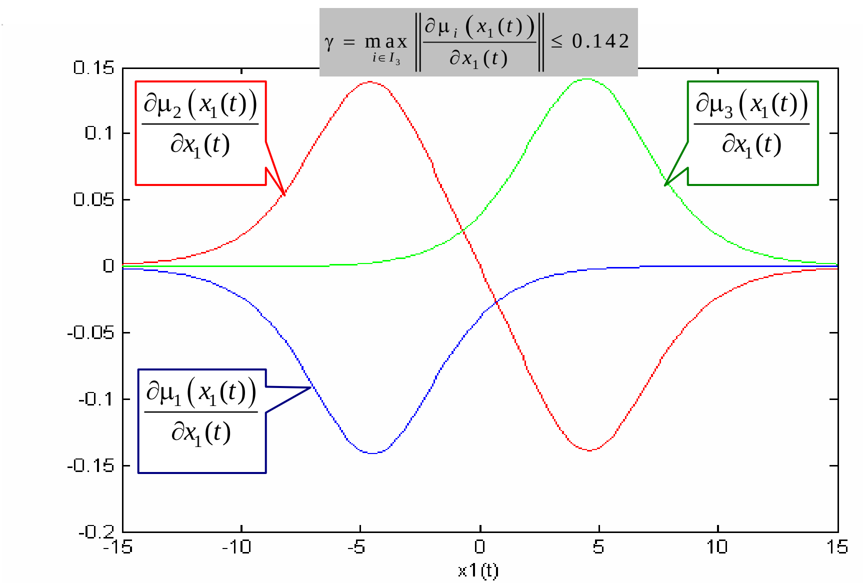

Figure 3. Derivative of activation functions (18d-g) $\frac{\partial \mu_i(x_1(t))}{\partial x_1(t)}$ with $c = 9$, $d = 4$

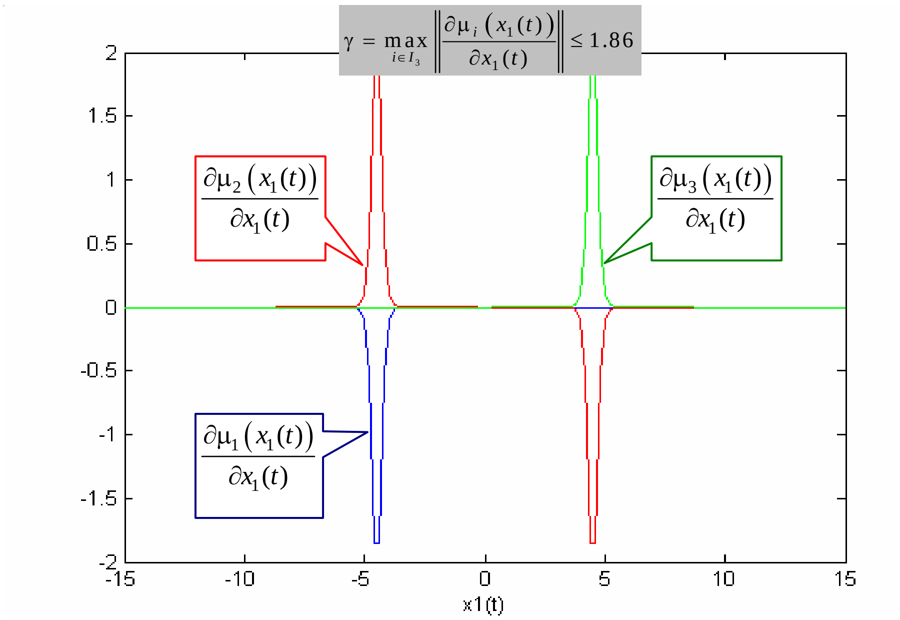

Figure 4. Derivative of activation functions (18d-g) $\frac{\partial \mu_i(x_1(t))}{\partial x_1(t)}$ with $c = 9$, $d = 1$

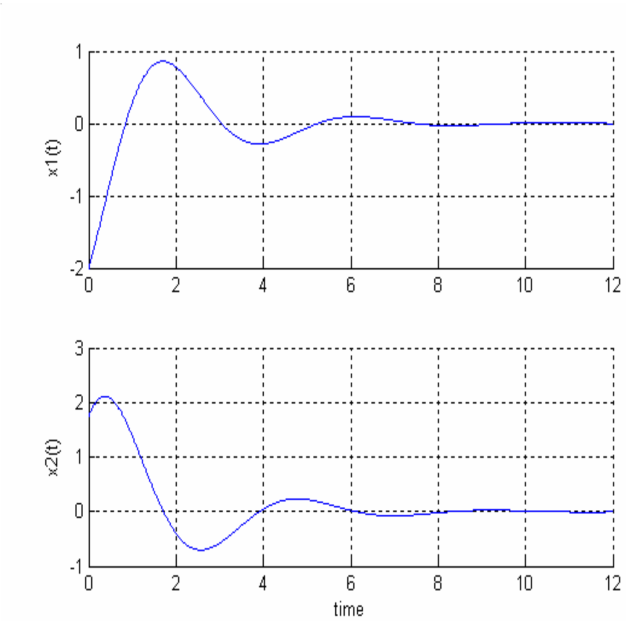

Figure 5. Example of simulation of the nominal T-S model (18) with activation functions (18) and initial condition $x(0)^T = (-2, 1.75)$

6. Conclusion

In this paper, the stability of uncertain continuous-time T-S fuzzy model is considered. Using the S-procedure and a class of nonquadratic Lyapunov function, sufficient conditions for global asymptotic stability are derived. The derived stability conditions allow to reduce the conservatism of the results obtained by (i) the quadratic method, (ii) the nonqudartic method [5] with unknown activations functions $\mu_i(.), i \in I_n$ or known with infinite support, (iii) and the polyquadratic methods, particularly in the case of very large speed between sub-models. A numerical example is given to illustrate the effectiveness of the proposed method in comparison with some others existing methods.

7. Acknowledgement

Thanks to Prof. Lionel Guyot, UPJV, for reviewing the English version of this paper.

8. References

[1] S. Boyd et al. *Linear matrix inequalities in systems and control theory*, Philadelphia, PA: SIAM, 1994.
[2] T. A. Johansson, "Computation of Lyapunov functions for smooth nonlinear systems using convex optimisation", *Automatica*, 36(11), pp. 1617-1626, 2000.
[3] S. K. Hong, R. Langari, "An LMI-based H∞ fuzzy control system design with T-S framework", *Information science*, no. 123, pp. 163-179, 2000.
[4] M. Akar, and U. Ozguner, "Decentralized techniques for the analysis and control of T-S fuzzy systems", *IEEE Trans. on Fuzzy Systems*, vol. 8, no. 6, pp. 691-704, 2000.
[5] M. Johansson, A. Rantzer, K. Arzén, "Piecewise quadratic stability of fuzzy systems", *IEEE Trans. on Fuzzy Systems*, 7(6), pp. 713-722, 1999.
[6] S. Petterson, B. Lennartson, "An LMI approach for stability analysis for nonlinear systems". In *Proc. of*

the European Control Conference, Bruxelles, Belgium, pp. 596-661, 1997.

[7] A. Jadbabaie, "A reduction in conservatism in stability and L2 gain analysis of T-S fuzzy systems via linear matrix inequalities", 14th triennial World IFAC Congress, Beijing, China. pp. 285-289, 1999.

[8] R. Murray-Smith, T. Johansen. *A Multiple model approaches to modelling and control*, Taylor & Francis, 1997.

[9] K. Zhou and J. C. Doyle. *Essentials of robust control*. Prentice-Hall, 1998.

[10] K. Tanaka, T. Hori, H.O. Wang, "A multiple Lyapunov function approach to stabilization of fuzzy control systems", *IEEE Transactions on Fuzzy Systems*, Vol. 11, no. 4, pp. 582 – 589, 2003.

[11] M. Takagi and M. Sugeno, "Fuzzy identification of systems and its application to moddeling and control", *IEEE Trans. on Systems Man and Cybernetics-part C*, 15 (1), pp. 116-132, 1985.

[12] M. Chadli, D. Maquin, J. Ragot, "Stability analysis and design for continuous-time Takagi-Sugeno control systems", *International Journal of Fuzzy Systems*, vol. 7, no. 3, 2005.

[13] M. Chadli, D. Maquin, J. Ragot, "Nonquadratic stability analysis of Takagi-Sugeno models", *41th IEEE CDC*, Las Vegas, Nevada, pp. 2143-2148, 2002.

[14] K. S. Narendra, J. Balakrishnan, "A common Lyapunov function for stable LTI systems with commuting A-matrices" *IEEE Trans. on Automatic Control*, vol. 39, no. 12, pp. 2469-2471, 1994.

[15] S. C Cao, N. W. Rees, G. Feng. "Hinf control of uncertain fuzzy continuous-time systems", *Fuzzy sets and systems*, 115, pp. 171-190, 2000.

[16] K. Tanaka, T. Ikeda, H. O. Wang, "Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based design", *IEEE Trans. on Fuzzy Systems*, vol. 6, no. 2, pp. 250-256, 1998.

[17] J. Park, J. Kim, D. Park "LMI-based design of stabilitizing fuzzy controllers for nonlinear systems described by T-S fuzzy model", *Fuzzy Set and Systems*, no. 122, pp 73-82, 2001.

[18] E. Kim, H. Lee. "New approaches to relaxed quadratic stability condition of fuzzy control systems", *IEEE trans. on Fuzzy Sets*, vol. 8, no. 5, pp. 523-534, 2000.

[19] L. Xiaodiong, Z. Qingling, "New approach to Hinf controller designs based on observers for T-S fuzzy systems via LMI", *Automatica*, 39, pp. 1571-1582, 2003.

[20] T. M. Guerra and L. Vermeiren, "LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi-Sugeno's form", *Automatica*, vol.40, no. 5, pp. 823-829, 2004.

[21] B. C. Ding, H. Sun and P. Yang, "Further studies on LMI-based relaxed stabilization conditions for nonlinear systems in Takagi-Sugeno's form", *Automatica*, vol.42, no. 3, pp. 503-508, 2006.