Observer-Based Direct Adaptive Fuzzy-Neural Control for Anti-lock Braking Systems

Guan-Ming Chen, Wei-Yen Wang, Tsu-Tian Lee, and C. W. Tao

Abstract

In this paper, an observer-based direct adaptive fuzzy-neural controller (ODAFNC) for an anti-lock braking system (ABS) is developed under the constraint that only the system output, i.e., the wheel slip ratio, is measurable. The main control strategy is to force the wheel slip ratio to well track the optimal value, which may vary with the environment. The observer-based output feedback control law and update law for on-line tuning of the weighting factors of the direct adaptive fuzzy-neural controller are derived. By using the strictly-positive-real (SPR) Lyapunov theory, the stability of the closed-loop system can be guaranteed. Simulation results demonstrate the effectiveness of the proposed control scheme for ABS control.

Keywords: anti-lock braking system, slip ratio, fuzzy control, neural networks, nonlinear systems, adaptive control, observer.

1. Introduction

Locking-up during braking has a marked negative effect on the stability of a vehicle, potentially putting driver and passengers at risk. Therefore, ABS has become the most important and necessary safety devices for vehicles [1], [2]. The main goal of most ABS is to attain the optimum negative acceleration rate without sacrificing the stability and steering ability of the vehicle while maintaining the maximal safety to the driver and passengers [3]. Wheel slip ratio, one of the most influential factors on the quality of braking, should be considered to achieve this goal. The main idea of ABS is preventing the wheels from locking by keeping the wheel slip ratio within a certain range when the brake is operated on a slippery road surface. The optimal wheel slip ratio for most road surfaces is between 0.1 and 0.3 [4]. Most ABS control strategies [3] aim at maintaining the wheel slip ratio at a compromise value 0.2. However, in general, optimal slip ratios vary with different road surfaces, such as dry asphalt or icy road. Forcing the wheel slip ratio to track optimal slip ratios, which causes the maximal tire/road friction force, can minimize the vehicle stopping distance. It is worth noting that if the road surface changes during braking, it is impossible to obtain the minimum stopping distance by tracking a constant optimal slip ratio. Therefore, if we want to track variant optimal slip ratio during braking, a stable and robust controller for ABS is needed [32]-[33].

Adaptive control of systems has been an active area of research [5]-[6] for at least a quarter of a century. Recently, there has been a surge of interest in the adaptive control of nonlinear systems. Some adaptive control schemes for nonlinear systems via feedback linearization have been proposed in [7]–[12]. The fundamental idea of feedback linearization is to transform a nonlinear system dynamic into a linear one. Therefore, linear control techniques can be used to acquire the desired performance. Some preliminary results have been presented in [7].

Since neural networks and fuzzy logic are universal approximators [13]-[14], the adaptive control schemes of nonlinear systems that incorporate the techniques of fuzzy logic [15] or neural networks have grown rapidly [16]–[20]. Although both neural networks and fuzzy logic are universal approximators, there are some differences between them. The former possesses characteristics of fault-tolerance, parallelism and learning. The later has characteristics of linguistic information and logic control. However, both of them have the complementary characteristics. The combined algorithms were proposed in [21]–[24]. All these methods assume that the system states are available for measurement, i.e., the nonlinearities approximated by fuzzy logic or neural networks are functions of the system states which are available for measurement.

It has to be pointed out that ABS, which involves high nonlinearities, time-varying parameters and uncertainties, is a highly nonlinear system. Furthermore, in ABS controller design, it is necessary to precisely measure some parameters such as wheel velocity, the wheel acceler-
tion, the brake-line pressure, etc. Utilizing detecting sensors to obtain the measurements is difficult and expensive. Due to the technical difficulties and the economic benefits, we assume that only the vehicle velocity and the wheel velocity are measurable. In other words, only the system output, i.e., the wheel slip ratio, is measurable [33].

Therefore, in this paper, an observer-based direct adaptive fuzzy neural controller (ODAFNC) for the ABS is developed under the constraint that only the system output is available for measurement. An observer-based control law and update law to on-line tune the weighting factors of the adaptive fuzzy-neural controller are derived. Based on the strictly-positive-real (SPR) Lyapunov theory, the stability of the closed-loop system can be verified. Moreover, the overall adaptive control scheme guarantees that all signals involved are bounded and the output of the closed-loop system will asymptotically track the desired optimal slip ratio.

The paper is organized as follows. First, the vehicle model is introduced in Section 2. The problem is formulated in Section 3. A brief description of fuzzy-neural networks is presented in Section 4. The design of the observer-based direct adaptive fuzzy-neural controller is interpreted in Section 5. In Section 6, simulation results are presented to confirm the effectiveness and the applicability of the proposed controller for ABS. Finally, Section 7 concludes the paper.

<table>
<thead>
<tr>
<th>symbol</th>
<th>Quantity</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Wheel slip ratio</td>
<td></td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>Wheel slip ratio of the front wheel</td>
<td></td>
</tr>
<tr>
<td>$a_f$</td>
<td>Distance from center of gravity to front axle</td>
<td>1.2 m</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Effective orifice area of the build valve</td>
<td>0.00316 m$^2$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Effective orifice area of the dump valve</td>
<td>0.00633 m$^2$</td>
</tr>
<tr>
<td>$a_c$</td>
<td>Vehicle linear acceleration</td>
<td>m</td>
</tr>
<tr>
<td>$b_t$</td>
<td>Distance from center of gravity to rear axle</td>
<td>1.2 m</td>
</tr>
<tr>
<td>$C_{r,f}$</td>
<td>Longitudinal tire stiffness of the front wheel</td>
<td>N</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Coefficient of the flow and the time derivative function of hydraulic pressure</td>
<td>10 m$^3$/kgf</td>
</tr>
<tr>
<td>$F_f$</td>
<td>Longitudinal tractive force of the front wheel in half vehicle model</td>
<td>N</td>
</tr>
<tr>
<td>$F_{f,NC}$</td>
<td>Longitudinal tractive force of the rear wheel in half vehicle model</td>
<td>N</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational constant</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of the sprung mass</td>
<td>m</td>
</tr>
<tr>
<td>$I$</td>
<td>Moment of inertia of wheel</td>
<td>3 kg - m$^2$</td>
</tr>
<tr>
<td>$k_g$</td>
<td>Gain between $P_f$ and $T_b$</td>
<td>28</td>
</tr>
<tr>
<td>$M_f$</td>
<td>Vehicle mass</td>
<td>1100 kg</td>
</tr>
<tr>
<td>$N_f$</td>
<td>Normal force of the front wheel in half vehicle model</td>
<td>N</td>
</tr>
</tbody>
</table>

### Table 1. List of notations

<table>
<thead>
<tr>
<th>symbol</th>
<th>Quantity</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_r$</td>
<td>Normal force of the rear wheel in half vehicle model</td>
<td>N</td>
</tr>
<tr>
<td>$P_f$</td>
<td>Hydraulic pressure at the valves from the wheel cylinder</td>
<td>kgf/cm$^2$</td>
</tr>
<tr>
<td>$P_{w,0}$</td>
<td>Constant reservoir pressure</td>
<td>6000 kgf/cm$^2$</td>
</tr>
<tr>
<td>$P_p$</td>
<td>Constant pump pressure</td>
<td>5 kgf/cm$^2$</td>
</tr>
<tr>
<td>PNG</td>
<td>Percent Normalized Gradient</td>
<td>0.833 s/m</td>
</tr>
<tr>
<td>$R$</td>
<td>Wheel radius</td>
<td>M</td>
</tr>
<tr>
<td>$T_b$</td>
<td>Wheel braking torque</td>
<td>N-m</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Tractive force torque</td>
<td>N-m</td>
</tr>
<tr>
<td>$v$</td>
<td>Vehicle linear speed</td>
<td>m/s</td>
</tr>
<tr>
<td>$w$</td>
<td>Wheel speed</td>
<td>1/s</td>
</tr>
<tr>
<td>$W_e$</td>
<td>Wheel weight</td>
<td>kgw</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Friction coefficient</td>
<td></td>
</tr>
<tr>
<td>$\mu_j$</td>
<td>Friction coefficient of the front wheel</td>
<td></td>
</tr>
<tr>
<td>$\mu_k$</td>
<td>Friction coefficient at zero vehicle speed</td>
<td>1</td>
</tr>
</tbody>
</table>

### 2. Vehicle Brake Model

Due to the high nonlinearities, time-varying parameters and uncertainties, such as vibration of suspension systems and wind drag during braking, it is difficult to describe the exact dynamics of a braking vehicle. In this paper, without losing accuracy, a half vehicle model is introduced. In addition, some assumptions to focus the research interest should be stated first: a) the controlled vehicle is assumed to be in straight-line motion on a flat road, b) the motion dynamics arising from rotation of the vehicle about the vertical axis is not considered. Note that lateral tire force and yaw do not exist. Note that in straight line braking. The wheel slip ratio $\lambda$, which is the most influential factor on the quality of braking, is defined as

$$\lambda = 1 - \frac{Rw}{v}.$$  \hspace{1cm} (1)

### A. Half Vehicle Model

Figure 1. shows the free body diagram of the half vehicle brake model. Consider the front wheel first. Assume that the longitudinal tractive force $F_j$ is a nonlinear function of the front wheel slip $\lambda_f$, as shown by the Dugoff tire model [25]

$$F_j = \begin{cases} \frac{C_{r,f} \times \lambda_f}{1 - \lambda_f} & \text{if} \ \frac{C_{r,f} \times \lambda_f}{1 - \lambda_f} < \frac{\mu_j \times N_f}{2} \\ \frac{N_f \times (\mu_j \times N_f) - \mu_j \times N_f}{4 \times C_{r,f} \times \lambda_f} & \text{if} \ \frac{C_{r,f} \times \lambda_f}{1 - \lambda_f} > \frac{\mu_j \times N_f}{2} \end{cases}$$

where

$$C_{r,f} = 10N_f.$$  \hspace{1cm} (3)
and
\[ \mu_f = \mu_0 \times \exp\left[-\frac{PNG \cdot v \cdot \lambda_f}{100}\right]. \quad (4) \]

PNG (Percent Normalized Gradient) is a constant whose magnitude is relative to the root mean square texture height of the road surface. Note that in (2), if we change the parameters for the front wheel to those for the rear wheel, the resulted equation is exactly the Dugoff tire model for the rear wheel. Owing to the load transfer effect, the normal forces acting on the front and rear tires are unequal and time varying. According to the equilibrium of torque, we have
\[ -F_f - F_r = \frac{M}{2} \times a_v \quad (5) \]
\[ N_f + N_r = \frac{W}{2} \quad (6) \]
\[ N_f (a_f + b_r) - \frac{W}{2} \times a_f = \frac{M}{2} \times a_v \times h \quad (7) \]

From (5)-(7), the normal forces, \( N_f \) and \( N_r \), can be expressed as
\[ N_f = \frac{W}{2} \times a_f + (F_f + F_r) \times h \quad (8) \]
\[ N_r = \frac{W}{2} \times a_f - (F_f + F_r) \times h \quad (9) \]

The acceleration of the vehicle is
\[ a_v = \frac{2}{M_v} \times (-F_f - F_r). \quad (10) \]

B. Hydraulic Brake System Model

The most commonly used brake system on automobiles is the hydraulic brake system shown in Figure 2. [1]. The pressure created by the driver and pump can only be transferred to the wheel cylinder when the build valve is open and the dump valve is closed. On the contrary, if the build valve is closed and the dump valve is open, the pressure in the wheel cylinder decreases because the brake fluid flows back to the low-pressure reservoir. The case that both valves are closed is not allowed. If we regard the flow in and out of the hydraulic circuit as a flow through an orifice [26], the hydraulic brake system dynamics can be obtained as follows [1]:
\[ \frac{dP_i}{dt} = \frac{A_i}{C_f} \left( C_{d1} \sqrt{\frac{2}{\rho} (P_i - P_p)} - A_i \frac{C_{d2}}{C_{f}} \sqrt{\frac{2}{\rho} (P_i - P_{low})} \right) \quad (11) \]

where \( C_{d1} = 1 \) and \( C_{d2} = 0 \) is the case that the build valve is open and the dump valve is closed. On the contrary, \( C_{d1} = 0 \) and \( C_{d2} = 1 \) is the case that the build valve is close and the dump valve is open. On the premise that the initial properties of the fluid and the resistance in the pipes are neglected, the pressure in the wheel cylinder is assumed to be \( P_i \). Brake torque \( T_b \) can be expressed as a linear function of the brake pressure \( P_i \)
\[ T_b = K_i \times P_i. \quad (12) \]

3. Problem Formulation

The discussions in the following can be applied to the case corresponding to the front wheel as well as the rear wheel. For convenience, the subscripts representing the front or the real wheel are omitted.

A. Problem Formulation

Rewrite (1) as
\[ \frac{v(1 - \lambda)}{R} = w. \quad (13) \]

Differentiating both sides of (13) yields
\[ \dot{w} = \frac{1}{R} \left[-v \dot{\lambda} + (1 - \lambda) a_v \right] \quad (14) \]
where \( a_v = \frac{dv}{dt} \). Also, form Figure 1., we obtain
\[
\dot{w} = \frac{T_e - T_k}{I} = \frac{F_R - K_v P_i}{I} .
\] (15)

From (12), (14) and (15), we have
\[
\dot{\lambda} = \frac{1}{v^2} \left[ -F_R y + K_v P_i \right] (1 - \lambda) a_v .
\] (16)

Differentiating (16) yields
\[
\ddot{\lambda} = \Lambda(\lambda, \dot{\lambda}, v, \dot{v}, \ddot{v}, \lambda, \dot{\lambda}) u + d
\] (17)
where \( \lambda = A(\lambda, \dot{\lambda}, v, \dot{v}, \ddot{v}, P_i, \dot{P}_i) \) is a highly nonlinear function. Rewrite (17) as
\[
\ddot{\lambda} = f(\lambda, \dot{\lambda}) + g(\lambda, \dot{\lambda}) u + d
\] (18)
where \( f(\lambda, \dot{\lambda}) \) and \( g(\lambda, \dot{\lambda}) \) are unknown nonlinear functions, \( u = \dot{P}_i \), and \( d = \Lambda(\dot{\lambda}, v, \dot{v}, \ddot{v}, P_i, \dot{P}_i) - f(\lambda, \dot{\lambda}) - g(\lambda, \dot{\lambda}) \). Now we can consider the following dynamics of an anti-lock braking system:
\[
\ddot{\lambda} = f(\lambda, \dot{\lambda}) + g(\lambda, \dot{\lambda}) u + d, \quad y = \lambda
\] (19)
where \( y \in \mathbb{R} \) is the system output. Assume that the solution for (19) exists. In addition, only the system output \( y \) is assumed to be measurable. The control objective is to design an observer-based direct adaptive fuzzy-neural controller such that the system output \( y \) follows a given bounded optimal wheel slip ratio \( y_o \), and all signals involved in the system are bounded.

B. Convert the Tracking Problem to Regulation Problem

Rewritten (19) as
\[
\dot{x} = A \lambda + B(f(\lambda) + g(\lambda) u + d)
\]
\[
y = C^T \dot{x}
\] (20)
where
\[
A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix},
\]
and \( \lambda = [\lambda \dot{\lambda}]^T \in \mathbb{R}^2 \) is the state vector. Define the output tracking error \( e = y_m - y \) and the tracking error vector \( \epsilon = [e \dot{\epsilon}]^T \in \mathbb{R}^2 \). According to the certainty equivalence approach, the optimal control law is
\[
u^* = \frac{1}{g} [-f + \dot{y}_m + \epsilon \cdot \hat{\dot{e}}],
\] (21)
where \( \hat{e} \) denotes the estimate of \( e \), and \( K_v = [k_v^1 \ k_v^2]^T \) is the feedback gain vector, chosen such that the characteristic polynomial of \( A - BK_v^T \) is Hurwitz because \( (A, B) \) is controllable. Since only the system output \( y \) is measurable and \( f \) is unknown, the optimal control law (21) cannot be implemented. Thus, suppose the control input \( u \) is
\[
u = u_F + u_w \]
(22)
where \( u_F \) is designed to approximate the optimal control law (21), and \( u_w \) is employed to compensate the external disturbance and the modeling error. From (20)-(22), we have
\[
\dot{\hat{e}} = (A - K_v C^T) \hat{e} + B [g u^* - gu_F - gy - d] \]
(23)
Equation (23) indicates that we have converted the tracking problem into the regulation problem. That is, designing a state observer to estimate the state vector \( \epsilon \) and thus \( e_i \) can be regulated to zero.

C. State Observer Design

Considering the following observer that estimates the state vector \( \epsilon \) in (23)
\[
\dot{\epsilon} = A \epsilon - B K_v \hat{e} + B (gy - gu_w) + K_v (e_i - \hat{e}_i)
\]
(24)
where \( K_v = [k_v^1 \ k_v^2]^T \) is the observer gain vector, chosen such that the characteristic polynomial of \( A - K_v C^T \) is strictly Hurwitz because \( (C, A) \) is observable. The control term \( v \) is employed to compensate the external disturbance \( d \) and the modeling error. Define the observation errors \( \tilde{e} = \epsilon - \hat{e} \) and \( \tilde{e}_i = e_i - \hat{e}_i \). Subtracting (24) from (23), we have
\[
\dot{\tilde{e}} = (A - K_v C^T) \tilde{e} + B [g u^* - gu_F - g y - d]
\]
(25)
Furthermore, the output error dynamics of (25) can be given as
\[
\tilde{e}_i = H(s) [g u^* - bu_F - by - d]
\] (26)
where \( s \) is the Laplace variable, and \( H(s) = C^T (sI - (A - K_v C^T))^{-1} B \) is the transfer function of (25).

4. Description of Fuzzy-Neural Network

The basic configuration of fuzzy logic systems con-
consists of some fuzzy IF-THEN rules and a fuzzy inference engine. In this paper, the fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector \( e \in \mathbb{R}^2 \) to an output \( u_f \in \mathbb{R} \). The \( i \)th fuzzy IF-THEN rules is written as

\[
R^{(i)}: \text{If } e_1 \text{ is } A_{1}^{(i)} \text{ and } e_2 \text{ is } A_{2}^{(i)} \text{ then } u_f \text{ is } B^{(i)}
\]

where \( A_{1}^{(i)}, A_{2}^{(i)} \) and \( B^{(i)} \) are fuzzy sets [15], [23]. By using produce inference, center-average and singleton fuzzifier, the output of the fuzzy-neural network can be expressed as

\[
u_f = \frac{\sum_{j=1}^{h} \prod_{i=1}^{n} \mu_{e_j}^{(i)}(e_j)}{\sum_{j=1}^{h} \prod_{i=1}^{n} \mu_{e_j}^{(i)}(e_j)} \rightleftharpoons \theta^T \varphi(e) \tag{28}
\]

where

- \( \mu_{e_j}^{(i)}(e_j), j = 1, 2 \)
- the membership function value of the fuzzy variable \( e_j \)
- \( h \)
- the number of the IF-THEN rules
- \( \overline{u}^{i} \)
- the point where \( \mu_{\theta}^{(i)}(\overline{u}^{i}) = 1 \)
- \( \theta = [\overline{u}^{1}, \overline{u}^{2}, \ldots, \overline{u}^{h}]^T \)
- adjustable parameter vector
- \( \varphi = [\varphi^{1}, \varphi^{2}, \ldots, \varphi^{h}]^T \)
- fuzzy basis vector

where \( \varphi^{i} \) is defined as

\[
\varphi^{i}(e) = \frac{\prod_{i=1}^{n} \mu_{e_j}^{(i)}(e_j)}{\sum_{j=1}^{h} \prod_{i=1}^{n} \mu_{e_j}^{(i)}(e_j)}.
\tag{29}
\]

When the inputs are given into the fuzzy-neural network shown in Figure 3., the truth value \( \varphi^{i} \) (layer III) of the antecedent part of the \( i \)th implication is calculated by (29). Among the commonly used defuzzification strategies, the outputs (layer IV) of the fuzzy-neural system are expressed as (28). The fuzzy logic approximator based on neural networks can be established [22], [24].

5. Observer-Based Direct Adaptive Fuzzy-Neural Controller (ODAFNC)

In this section, our task is to use the fuzzy neural network to approximate the ideal optimal control (21), and develop an adaptive control law to adjust the parameters of fuzzy neural networks for the purpose of regulating the observation error \( \tilde{e}_i \) to zero.

First, replace \( u_f \) in (22) by the output of the fuzzy-neural network, \( \theta^T \varphi(e) \), in (28), i.e.,

\[
u_f (\hat{e} | \theta) = \theta^T \varphi(e).
\tag{30}
\]

In order to derive the direct adaptive update law, the following assumptions and lemmas should be stated first.

**Assumption 1** [10]: Let \( e \) and \( \hat{e} \) belong to the compact sets

\[
U_e = \left\{ e \in \mathbb{R}^2 : \|e\| \leq m_e < \infty \right\}
\]

and

\[
U_{\hat{e}} = \left\{ \hat{e} \in \mathbb{R}^2 : \|\hat{e}\| \leq m_{\hat{e}} < \infty \right\},
\]

respectively, where \( \hat{e} \) denotes the estimate of \( e \) and \( m_e \) and \( m_{\hat{e}} \) are designed parameters. It is known a priori that the optimal parameter vector \( \theta^* = \arg \min_{\theta} \sup_{k \in [0, T]} \|e_k - u(\hat{e} | \theta)\| \) lies in some convex region \( M_0 = \left\{ \theta \in \mathbb{R}^2 : \|\theta\| \leq m_0 \right\} \), where the radius \( m_0 \) is constant.

According to Assumption 1 and (30), (25) can be rewritten as

\[
\tilde{e} = (A - K_s C^T) \hat{e} + B[g_u (\hat{e} | \theta^*) - g_u (\hat{e} | \theta) - g_v + w - d] \]

\[
\tilde{e}_i = C^T \hat{e}
\tag{31}
\]

where \( [u^* - u_f (\hat{e} | \theta^*)] \) is the approximation error and \( w = g[u^* - u_f (\hat{e} | \theta^*)] \). Rewrite (31) as
where $\hat{e} = (A - K_c C^T) \tilde{e} + B [g \bar{\theta}^T \phi(\hat{e}) - gw + w - d]$ (32)
\[
\tilde{e}_i = C^T \hat{e}
\]
where $\bar{\theta} = \theta^* - \theta$. Since only the output $\tilde{e}_i$ in (32) is assumed to be measurable, we use the strictly-positive-real (SPR) Lyapunov design approach to analyze the stability of (32) and derive the direct adaptive update law for $\tilde{e}_i$. The output dynamics of (32) can be rewritten as
\[
\tilde{e}_i = H(s)[g \bar{\theta}^T \phi(\hat{e}) - gw + w - d]
\]
where $H(s) = C^T(s I - (A - K_c C^T))^{-1} B$ is a known stable transfer function. In order to employ the SPR-Lyapunov design approach, (33) can be written as
\[
\tilde{e}_i = H(s) L(s) [\theta^T \phi(\hat{e}) - v_f + w_f]
\]
where $\phi(\hat{e}) = L^{-1}(s) [\theta^T \phi(\hat{e})]$, $v_f = L^{-1}(s) g \bar{\theta}^T \phi(\hat{e})$, and $w_f = L^{-1}(s) [w - d + g \bar{\theta}^T \phi(\hat{e})] - \bar{\theta}^T \phi(\hat{e})$. $L(s)$ is chosen so that $L^{-1}(s)$ is a proper stable transfer function and $H(s) L(s)$ is a proper SPR transfer function. Suppose that $L(s) = s + b_i$, where $b_i$ is a constant, such that $H(s) L(s)$ is a proper SPR transfer function. Then the state-space realization of (34) can be written as
\[
\hat{e} = A_c \hat{e} + B_c [\bar{\theta}^T \phi(\hat{e}) - v_f + w_f]
\]
where $A_c = (A - K_c C^T) \in \mathbb{R}^{n_2}$, $B_c = [0 \ b_i] \in \mathbb{R}^n$ and $C_c = [1 \ 0] \in \mathbb{R}^2$. In the following, some assumptions and lemmas are stated for the purpose of stability analysis of the whole control scheme.

**Lemma 1** [15], [28]: Suppose that the adaptive laws are chosen as
\[
\dot{\theta} = \begin{cases} 
\gamma \tilde{e}_i \bar{\theta} \phi(\hat{e}), & \text{if } \|\theta\| < m_\theta \text{ or } \|\theta\| = m_\theta \text{ and } \bar{\theta}^T \phi(\hat{e}) \geq 0 \\
\text{Pr}(\gamma \tilde{e}_i \bar{\theta} \phi(\hat{e})), & \text{if } \|\theta\| = m_\theta \text{ and } \bar{\theta}^T \phi(\hat{e}) < 0 
\end{cases}
\]
where the projection operator is given as [15]
\[
\text{Pr}(\gamma \tilde{e}_i \bar{\theta} \phi(\hat{e})) = \gamma \tilde{e}_i \bar{\theta} \phi(\hat{e}) - \frac{\gamma \bar{\theta}^T \phi(\hat{e})}{\|\theta\|} \theta.
\]
Then $\|\theta\| \leq m_\theta$ and $\|\theta\| \leq 2m_\theta$.

Assumption 2: The unknown function $g(\lambda)$ is bounded by
\[
\beta_1 \leq \|g(\lambda)\| \leq \beta_2
\]
where $\beta_1$ and $\beta_2$ are positive constants.

Assumption 3: $w_f$ is assumed to satisfy
\[
|w_f| \leq \varepsilon
\]
where $\varepsilon$ is a positive constant.

**Remark 1:** Assumption 3 is reasonable due to the universal approximation theorem.

On the basis of the above discussions, the following theorems can be obtained.

**Theorem 1:** Consider the system (35) that satisfies Assumptions 1-3. Let $\theta$ be adjusted by the update law (36), and let $v$ be given as
\[
v = \begin{cases}
\rho \text{ if } \tilde{e}_i \geq 0 \\
-\rho \text{ if } \tilde{e}_i < 0
\end{cases}
\]
where $\rho \geq \frac{\varepsilon}{\beta_1}$. Then $\tilde{e}_i(t)$ converges to zero as $t \to \infty$.

**Proof:** Given in Appendix.

**Theorem 2:** Consider the nonlinear ABS dynamic system (20) that satisfies Assumptions 1-3. Suppose that the control law is
\[
u = u_f (\hat{e} | \theta) + u_v
\]
with the state observer (24), the adaptive law (36), and $u_v = v$, given as (40). Then all signals in the closed-loop system are bounded, and $e_1(t)$ converges to zero as $t \to \infty$.

**Proof:** Given in Appendix.

According to the aforementioned discussion, the design algorithm of the ODAFNC is described as following.

[Step 1] Select the feedback and observer gain vectors $K_c$, $K_v$ such that the matrices $A - BK_c^T$ and $A - K_c C^T$ are Hurwitz matrices, respectively.

[Step 2] Choose an appropriate value $\rho$ in (40) and $\varepsilon$ in (36). In order to eliminate the control chattering, (42) can be modified as
\[
\begin{cases}
\rho \text{ if } \tilde{e}_i \geq 0 \text{ and } |\tilde{e}_i| > \alpha,
\varepsilon = \rho \text{ if } \tilde{e}_i < 0 \text{ and } |\tilde{e}_i| > \alpha
\end{cases}
\]
where $\alpha$ is a positive constant.

[Step 3] Solve the state observer in (24), where $v$ in (40) or (42).

[Step 4] Construct fuzzy sets for $\hat{e}(t)$, and then compute the fuzzy basis vector $\phi$ by (29).

[Step 5] Calculate the control law (41), and the update law (36).

**Remark 2:** The initial value $\theta(0)$ should be determined before solving the adaptive laws in (36). The value of $\gamma$
in (36) is obtained by an trial and error procedure according to $\theta(0)$. In order to compute the controller in (41), we need to determine $\rho$. Under Assumptions 2-3, $\rho$ is also chosen by a trial and error procedure without using any adaptive tuning procedure. Larger $\rho$ results in larger control input according to (42). From (40), we see that the absolute value of the control term $u$ is the value of $\rho$. The control term $v$ is employed to compensate for external disturbance and modeling error.

To summarize, Figure 4. shows the overall scheme of the proposed observer-based direct adaptive fuzzy-neural controller for an ABS.

6. Simulations

In this section, the effectiveness of the proposed ODAFNC for anti-lock braking system are examined on half vehicle models. Figure 5. shows longitudinal tractive forces ($F_x$) with respect to wheel slip ratio. We can observe clearly that $F_x$ increases rapidly near a zero wheel slip ratio. However, after reaching a peak value, $F_x$ decreases gradually with the increasing slip ratio. The optimal slip ratios corresponding to different road conditions are determined by choosing the slip ratios corresponding to the peak tractive forces.

To carry out the simulation successfully, the optimal wheel slip ratio function during braking, which is assumed to be obtained from an optimizer [29] or an observer [30], should be defined first. The road conditions are altered in the following order: dry surface for 2.1 seconds, wet surface for 1.15 seconds, snowy surface for 1.45 seconds, and icy surface for the rest of the braking, with corresponding optimal slip ratios of 0.28, 0.25, 0.13, and 0.1, respectively. The optimal slip ratio curve, which shapes like stairs, is shown in Figure 6. It can be expressed as

$$\dot{y}_m = -\frac{2c_1}{c_2} y_m (y_m - c_1)$$

where $c_1$, $c_2$, and $c_3$ are designed constants. Figure 7 (a) shows the curve of the vehicle speed reduced from 33.33 m/sec (about 120 km/hr) to 0 m/sec at 6.17 seconds. The curve of wheel speed during braking is shown in Figure 7 (b). Figure 7 (c) shows the control input. Note that those peaks of control input are reasonable, because they occur at the times when the road surface changes suddenly. In Fig 7 (d), it can be observed that after periods of transient response, the wheel slips $\lambda$ can still approach to the optimal values of different road surfaces. This fact proves the robustness of the ODAFNC under parameter variations. Figure 7 (e) shows the stopping distances in to two cases: the case of tracking a fixed wheel slip ratio 0.2 and the case of tracking the optimal wheel slip ratio variant with different road surfaces. The former stops at 7.88 seconds with a stopping distance 88.88 m, while the latter stops at 6.16 seconds with a stopping distance 75.33 m. It is clear that in the latter case, both braking time and distance are shorter. In other words, the correctness and effectiveness of the control strategy is verified. It is worth noting that the tracking trajectory of the slip ratio can converge to the optimal values in the presence of suddenly variation of road surfaces.

![Diagram](image-url)
G.-M. Chen, et al.: Observer-Based Direct Adaptive Fuzzy-Neural Control for Anti-lock Braking Systems

Figure 6. Optimal slip ratio curve

Figure 7 Simulation results of the ABS with an ODAFNC
6. Appendix

A. Problem Formulation

Consider the Lyapunov-like function candidate
\[ V = \frac{1}{2} \tilde{e}^T P \tilde{e} + \frac{1}{2} \gamma \tilde{\theta}^T \tilde{\theta} \]  
(A.1)
where \( P = P^T > 0 \). Differentiating (A.1) with respect to time and inserting (35) yield
\[ \dot{V} = \frac{1}{2} \tilde{e}^T (A^T P + PA) \dot{\tilde{e}} + \tilde{e}^T PB \left[ \tilde{\theta}^T \phi - v_f + w_f \right] + \frac{1}{2} \gamma \tilde{\theta}^T \tilde{\theta} . \]  
(A.2)

Because \( H(s)L(s) \) is SPR, there exists \( P = P^T > 0 \) such that
\[ A^T P + PA = -Q \]  
(A.3)
\[ PB = C_e \]  
where \( Q = Q^T > 0 \). By using (A.3), (A.2) becomes
\[ \dot{V} = -\frac{1}{2} \tilde{e}^T Q \tilde{e} + \tilde{e}^T \left[ \tilde{\theta}^T \phi - v_f + w_f \right] + \frac{1}{2} \tilde{\theta}^T \tilde{\theta} . \]  
(A.4)

By using Assumptions 2-3, (40) and the fact \( \lambda_{\min}(Q) \| \tilde{e} \|^2 \geq \lambda_{\max}(Q) | \tilde{e} |^2 \), where \( \lambda_{\max}(Q) \) denotes the smallest eigenvalue of \( Q \) and \( \lambda_{\min}(Q) > 0 \), we have
\[ \dot{V} \leq -\frac{1}{2} \lambda_{\min}(Q) \| \tilde{e} \|^2 + \tilde{e}^T \tilde{\theta} R \tilde{\theta} + \frac{1}{2} \tilde{\theta}^T \tilde{\theta} . \]  
(A.5)

Inserting (36) in (A.5) and after some manipulation yield
\[ \dot{V} \leq -\frac{1}{2} \lambda_{\min}(Q) \| \tilde{e} \|^2 . \]  
(A.6)

(A.1) and (A.6) only guarantee that \( \tilde{e} \in L_{\infty} \) and \( \tilde{e}(t) \in L_{\infty} \), but do not guarantee the convergence. Because all variables in the right-hand side of (35) are bounded, \( \dot{\tilde{e}}(t) \) is bounded, i.e., \( \tilde{e}(t) \in L_{\infty} \). Integrating both side of (A.6) and after some manipulation yields
\[ \int_0^t \| \tilde{e}(t) \|^2 dt \leq \frac{V(0) - V(\infty)}{2 \lambda_{\min}(Q)} . \]  
(A.7)

Since the right side of (A.7) is bounded, so \( \tilde{e}(t) \in L_2 \). Using Barbalat’s lemma [31], we have \( \lim_{t \to \infty} \| \tilde{e}(t) \| = 0 \). This completes the proof.

B. Proof of theorem 2

First, from Theorem 1, we have \( \lim_{t \to \infty} \| \tilde{e}(t) \| = 0 \). Using (24) and the fact \( u_{av} = v \), we obtain
\[ \tilde{e} = (A - BK_e) \tilde{e} + K_o C_e \tilde{e} . \]  
(A.8)

Similarly, because \( A - BK_e \) is a Hurwitz matrix and \( \tilde{e}(t) \) is bounded, \( \tilde{e}(t) \) is bounded. From \( \tilde{e} = e - \dot{\tilde{e}} \), it follows that \( e, \tilde{e} \in L_{\infty} \) and \( e(t) \to 0 \) as \( t \to \infty \).

From \( e, \dot{e}, y_m \in L_{\infty} \), it follows that \( x, \dot{x}, e, \tilde{e} \in L_{\infty} \). The boundedness of \( y(t) \) follows that of \( e(t) \) and \( y_m(t) \).

This completes the proof.

Acknowledgement

This work was financially supported by the National Science Council of Taiwan, R.O.C., under Grants NSC 91-2213-E-030-002 and NSC 93-2218-E-030-001.

7. References


Guan-Ming Chen was born in Kaohsiung, Taiwan, R.O.C., in 1975. He received B.S. degree in electrical and control engineering from Chiao Tung University, Hsinchu, Taiwan in 1999 and M. S. degree in electronic engineering from Fu-Jen Catholic University, Taipei, Taiwan in 2003. Since September 2003, he has been pursuing the Ph.D. degree in the Department of Electrical and Control Engineering at Chiao Tung University, Taiwan. His research interests include neural network, fuzzy logic system, adaptive control, and intelligent control.

Wei-Yen Wang received the M.S. and Ph.D. degrees in electrical engineering from National Taiwan University of Science and Technology, Taipei, Taiwan, in 1990 and 1994, respectively. From 1990 to 2006, he served concurrently as a patent screening member of the National Intellectual Property Office, Ministry of Economic Affairs, Taiwan. In 1994, he was appointed as Associate Professor in the Department of Electronic Engineering, St. John’s and St. Mary’s Institute of Technology, Taiwan. From 1998 to 2000, he worked in the Department of Business Mathematics, Soochow University, Taiwan. Currently, he is a Profes-
Dr. Wang is an IEEE Senior Member, an Associate Editor of the IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, an Associate Editor of the International Journal of Fuzzy Systems, and a member of Editorial Board of International Journal of Soft Computing.

Tsu-Tian Lee was born in Taipei, Taiwan, R.O.C., in 1949. He received the B.S. degree in control engineering from the National Chiao Tung University (NCTU), Hsinchu, Taiwan, in 1970, and the M.S. and Ph.D. degrees in electrical engineering from the University of Oklahoma, Norman, OK in 1972 and 1975, respectively.

In 1975, he was appointed Associate Professor and in 1978 Professor and Chairman of the Department of Control Engineering at NCTU. In 1981, he became Professor and Director of the Institute of Control Engineering, NCTU. In 1986, he was a Visiting Professor and in 1987, a Full Professor of Electrical Engineering at University of Kentucky, Lexington. In 1990, he was a Professor and Chairman of the Department of Electrical Engineering, National Taiwan University of Science and Technology (NTUST). In 1998, he became the Professor and Dean of the Office of Research and Development, NTUST. In 2000, he was with the Department of Electrical and Control Engineering, NCTU, where he served as a Chair Professor. Since 2004, he has been with National Taiwan University of Technology (NTUT), where he is now the President of NTUT. He has published more than 200 refereed journal and conference papers in the areas of automatic control, robotics, fuzzy systems, and neural networks. His current research involves motion planning, fuzzy and neural control, optimal control theory and application, and walking machines.

Prof. Lee received the Distinguished Research Award from National Science Council, R.O.C., in 1991-1992, 1993-1994, 1995-1996, and 1997-1998, respectively, the TECO Sciences and Technology Award from TECO Foundation in 2003, the Academic Achievement Award in Engineering and Applied Science from the Ministry of Education, Republic of China, in 1998, and the National Endow Chair from Ministry of Education, Republic of China, in 2003. He was elected to the grade of IEEE Fellow in 1997 and IEE Fellow in 2000, respectively. He became a Fellow of New York Academy of Sciences (NYAS) in 2002. His professional activities include serving on the Advisory Board of Division of Engineering and Applied Science, National Science Council, serving as the Program Director, Automatic Control Research Program, National Science Council, and serving as an Advisor of Ministry of Education, Taiwan, and numerous consulting positions.

Prof. Lee has been actively involved in many IEEE activities. He has served as Member of Technical Program Committee and Member of Advisory Committee for many IEEE sponsored international conferences. He is now the Vice President for Conference and Meeting of the IEEE Systems, Man and Cybernetics Society.

C.W. Tao received the B.S. degree in electrical engineering from National Tsing Hua University, Hsinchu, Taiwan, R.O.C., in 1984, and the M.S. and Ph.D. degrees in electrical engineering from New Mexico State University, Las Cruces, in 1989 and 1992, respectively.

He is currently a Professor with the Department of Electrical Engineering, National I-Lan University, I-Lan, Taiwan, R.O.C. He is an Associate Editor of the IEEE Transactions on Systems, Man, and Cybernetics. His research interests are on the fuzzy neural systems including fuzzy control systems and fuzzy neural image processing.

Dr. Tao is an IEEE Senior Member.