

Observer-Based Adaptive Fuzzy Control of Time-Delay Chaotic Systems

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Abstract

This work presents an observer-based adaptive control approach for time-delay chaotic systems with uncertainty. The chaotic system is initially expressed with the Takagi-Sugeno (T-S) fuzzy model. The design of the adaptive fuzzy observer is introduced based on the assumptions of chaotic system properties. Next, the design principle is applied to stabilization and tracking problems. Based on Lyapunov's stability analysis, the sufficient conditions for synthesizing a stable observer and controller are expressed in the form of adaptive updating laws and linear matrix inequality (LMI). Computer simulation verifies the system, controlled by the proposed method, performs well.

Keywords: T-S fuzzy model, adaptive fuzzy observer.

1. Introduction

A chaotic system is essentially a nonlinear system in which the evolution is sensitive to initial conditions. The specific features of chaos are broadband, noise-like, and contain trajectories that are difficult to predict. For past decades, chaos has drawn much research attention for its potential applications such as secure communication, chemical reactions, biological systems, and information processing [1]. The idea of chaos control was initiated in the works presented by Ott, Grebogi, and Yorke [2]. The control is designed so as to make the chaotic behaviors converge to an unstable equilibrium point or unstable orbit. Since the pioneering works, controlling chaos has been extensively investigated. The relevant issues connected with chaos control and its applications have been recently reviewed in [3]. Most investigations have focused on the chaotic systems without time-delay state. Chaos with time-delay has been rarely investigated. In industry, the phenomenon of time delay is commonly encountered in chemical processes, rolling mill systems, and long pneumatic or hydraulic transmission lines. The existence of time delays is a source of instability and can deteriorate performance of systems. Therefore, the

analysis and design of time-delay chaotic systems are more complicated than that of chaotic systems without time-delay.

Since Mackay and Glass [4] first characterized time-delay chaos, there has been increasing interest in chaotic system with time delay. Wang [5] introduced a modified Chua's circuit that enhanced the complexity of chaotic dynamics through a time-delay feedback. Cherrier [6-7] proposed an observer-based approach for synchronization and secure communication via the modified Chua's circuit, where the problem of synchronization with unknown delay time was solved. Based on the linear matrix inequality method, a delayed feedback controller [8-9] was constructed to stabilize a continuous-time chaotic system on an unstable fixed point. In the literatures mentioned above, the phenomenon of time-delay was mainly due to the time-delay control input. In contrast, the propagation delay of the remote communication may also be a reason of time-delay. Synchronization of such systems was considered in [10-11], where an adaptive robust observer was developed to deal with unknown channel delay and system uncertainties. The same problem was also discussed in [12] and the further exploration was pursued.

This investigation will focus on developing an observer-based control approach for the chaotic system with time-delay state. Motivated by the work of [13], the systems considered herein are represented by the T-S fuzzy model. The first approach is to apply the observation principle to designing an adaptive fuzzy observer for synchronization. The observed system is assumed to contain uncertainty, which is used to represent parameter variations or the parameter differences between the master system (plant) and the slave system (observer). Given the assumptions regarding the properties of the system, sufficient conditions for ensuring the asymptotic stability of the observation error are established by a proper choice of Lyapunov-Krasovskii functional candidate. An attempt towards construction of a control algorithm derived from the observer-based model will be made in the second approach. The stabilization of chaotic time-delay systems is investigated. Moreover, the obtained result is extended to the development of tracking control. The computer simulations on a time-delay Chua's circuit [12] are finally used to illustrate the effectiveness and correctness of the proposed approaches.

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2. Problem Formulation

Consider a time-delay chaotic system, which can be presented by the following T-S fuzzy model with time-delay:

$$\begin{aligned}
 R^l : & \text{If } z_1(t) \text{ is } M_1^l \text{ and } \dots \text{ and } z_j(t) \text{ is } M_j^l \text{ then} \\
 \dot{x}(t) &= (A_{1l} + \Delta A_{1l}(t))x(t) + (A_{2l} + \Delta A_{2l}(t))x(t - \tau), \\
 y(t) &= Cx(t), \quad l = 1, 2, \dots, r,
 \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the output vector, $x(t - \tau)$ is the time-delay state vector, $\tau > 0$ is the constant time-delay, A_{1l} , A_{2l} , and C are system matrices with compatible dimensions, $\Delta A_{1l}(t)$ and $\Delta A_{2l}(t)$ are the corresponding uncertainties that represent parameter variations or parameter differences between the plant (master system) and the observer (slave system), M_k^l is the fuzzy set ($k = 1, 2, \dots, j$), $z(t) = [z_1(t), z_2(t), \dots, z_j(t)]^T$ is the premise variable vector associated with the system states and inputs, and r is the number of fuzzy rules. In accordance with [14-15], many well-known continuous and discrete time chaotic systems can be described by T-S fuzzy models with only one premise variable.

Based on the center of gravity defuzzification, the output of the fuzzy system is inferred as

$$\begin{aligned}
 \dot{x}(t) &= \sum_{l=1}^r w_l(z) [(A_{1l} + \Delta A_{1l}(t))x(t) \\
 &\quad + (A_{2l} + \Delta A_{2l}(t))x(t - \tau)] / \sum_{i=1}^r w_i(z), \\
 y(t) &= Cx(t),
 \end{aligned} \tag{2}$$

where $w_l(z) = \prod_{i=1}^j M_i^l(z_i)$ and $M_i^l(z_i)$ denotes the grade of membership function M_i^l corresponding to $z_i(t)$. Define

$$\mu_l(z) = w_l(z) / \sum_{l=1}^r w_l(z). \tag{3}$$

Then

$$\begin{aligned}
 \dot{x}(t) &= \sum_{l=1}^r \mu_l(z) [(A_{1l} + \Delta A_{1l}(t))x(t) + (A_{2l} + \Delta A_{2l}(t))x(t - \tau)], \\
 y(t) &= Cx(t).
 \end{aligned} \tag{4}$$

Notably, $\sum_{l=1}^r \mu_l(z) = 1$ and $\mu_l(z) \geq 0$ for $l = 1, 2, \dots, r$.

Motivated by the work of [13], the system model (4) can be viewed as

$$\begin{aligned}
 \dot{x}(t) &= (A_0 + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - \tau), \\
 \text{where } A_0 &= \frac{1}{r} \sum_{l=1}^r A_{1l}, A_d = \frac{1}{r} \sum_{l=1}^r A_{2l},
 \end{aligned}$$

$$\begin{aligned}
 \Delta A(t) &= \sum_{l=1}^r \mu_l(z) (A_{1l} - A_0 + \Delta A_{1l}), \\
 \Delta A_d(t) &= \sum_{l=1}^r \mu_l(z) (A_{2l} - A_d + \Delta A_{2l}).
 \end{aligned} \tag{5}$$

Therefore, the T-S fuzzy system appears as a perturbed linear time-delay system. Suppose that the uncertainties satisfy the matching conditions. That is, there exist uniformly continuous functions $E_A(t)$ and $E_d(t)$ such that $\Delta A(t) = BE_A(t)$ and $\Delta A_d(t) = BE_d(t)$, and $B \in R^{n \times p}$ is a known matrix. Consequently, the terms concerning the uncertainties in (5) can be expressed in the form of

$$\begin{aligned}
 \Delta A(t)x(t) &= B\xi_1(x(t), t), \quad \Delta A_d(t)x(t - \tau) = B\xi_2(x(t - \tau), t), \\
 \text{where } \xi_1 \text{ and } \xi_2 &\text{ denote the lumped uncertainties. Then, we have} \\
 \dot{x}(t) &= A_0x(t) + A_dx(t - \tau) + B\xi_1(x(t), t) + B\xi_2(x(t - \tau), t), \\
 y(t) &= Cx(t).
 \end{aligned} \tag{6}$$

Since the uncertainties ξ_1 and ξ_2 are unknown, we are thus designing adaptive updating laws for the estimation of these uncertainties.

Consider the following Mamdani type fuzzy inference for the estimation of i th component of $\xi_1(x(t), t)$, ξ_{1i} ,

$$\begin{aligned}
 R^j : & \text{If } x_1(t) \text{ is } \tilde{M}_1^j \text{ and } \dots \text{ and } x_n(t) \text{ is } \tilde{M}_n^j \\
 \text{then } \hat{\xi}_{1i} &\text{ is } \tilde{D}_{ij}, \quad j = 1, 2, \dots, q.
 \end{aligned}$$

The output of the inference is given by

$$\begin{aligned}
 \hat{\xi}_{1i}(x | \theta_i) &= \sum_{j=1}^q \theta_{ij} \left(\prod_{h=1}^n \mu_{\tilde{M}_h^j}(x_h) \right) / \sum_{j=1}^q \prod_{h=1}^n \mu_{\tilde{M}_h^j}(x_h) \\
 &= \theta_i^T \omega(x),
 \end{aligned} \tag{7}$$

where $\theta_i = (\theta_{i1}, \theta_{i2}, \dots, \theta_{iq})^T$ is an adjustable parameter vector, θ_{ij} is the center of \tilde{D}_{ij} for $i = 1, 2, \dots, p$, $j = 1, 2, \dots, q$, and $\omega(x)$ is the fuzzy basis function. Accordingly, the estimation of ξ_1 is $\hat{\xi}_1(x | \theta) = \theta^T \omega(x)$, and $\theta \in R^{q \times p}$. Given the fuzzy inference, the optimal parameter matrix is defined as

$$\theta^* = \arg \min_{\theta \in \Omega_\theta} \left\{ \sup_x \left\| \hat{\xi}_1(x | \theta) - \xi_1(x(t), t) \right\| \right\} \tag{8}$$

such that

$$\left\| \hat{\xi}_1(x | \theta^*) - \xi_1(t, x) \right\| \leq \varepsilon_1, \tag{9}$$

where $\Omega_\theta = \left\{ \theta \mid \text{tr}(\theta^T \theta) < M_\theta^2 \right\}$, $\text{tr}(\cdot)$ denotes the trace of a matrix, M_θ is a designed constant, and ε_1 is an unknown upper-bound that can be obtained via an adaptive mechanism. Moreover, $\xi_2(x(t - \tau), t)$ is assumed to be a bounded function. That is, $\left\| \xi_2(x(t - \tau), t) \right\| \leq \varepsilon_2$, and ε_2 can be estimated by an adaptive mechanism as well.

Notice that the construction of fuzzy estimator relies on the completeness of the state. However, the state con-

sidered herein is assumed to be unavailable for measurement except at the output of the system. This leads to a need for designing an indirect estimator. That is, the state vector x in the premise of the fuzzy rules is replaced by \hat{x} , $\hat{\xi}_1(\hat{x}|\theta)$. Moreover, we assume $\omega(x)$ is continuous and imposed by the Lipschitz condition, $\|\omega(x) - \omega(\hat{x})\| \leq \gamma \|x - \hat{x}\|$, for a known Lipschitz constant γ .

Based on the aforementioned description, the fuzzy observer for time-delay chaotic system (1) is defined as

$$\begin{aligned} \dot{\hat{x}}(t) &= A_0 \hat{x}(t) + A_d \hat{x}(t - \tau) + L(y(t) - \hat{y}(t)) \\ &\quad + B(\hat{\xi}_1(\hat{x}|\theta) + u_1(t) + u_2(t)), \\ \hat{y}(t) &= C\hat{x}(t), \end{aligned} \quad (10)$$

where $L \in R^{n \times m}$ is the constant observer gain, and u_1 and u_2 are the supervisory control that will be given later. Let $e(t) = x(t) - \hat{x}(t)$ be the observation error. The error dynamics is of the form

$$\begin{aligned} \dot{e}(t) &= (A_0 - LC)e(t) + A_d e(t - \tau) + B(\xi_1(x(t), t) \\ &\quad - \hat{\xi}_1(\hat{x}|\theta) - u_1(t)) + B(\xi_2(x(t - \tau), t) - u_2(t)). \end{aligned} \quad (11)$$

For compact expression, in the following derivation the notations of $\xi_1(x(t), t)$ and $\xi_2(x(t - \tau), t)$ are simplified to ξ_1 and ξ_2 , respectively, and a function of t will be succinctly expressed by dropping its argument. The proposed approach is introduced in the subsequent section.

3. Design of Adaptive Observer and Controller

In this section, we will explore the control methods that deal with problems such as synchronization, stabilization, and tracking control of the chaotic time-delay systems. Synchronization is discussed first. In contrast to [10], the synchronization considered herein relies on the design of a fuzzy observer for a chaotic system with time-delay state, excluding the factor of propagation delay. In our derivation, the controlled plant possesses uncertainty so that the designed observer is robust. Since the nonlinear chaotic system is represented by the T-S fuzzy model, it is more general than that adopted in [11-12]. The derived result is based on the fuzzy control theory.

The main results are introduced as follows.

Theorem 1: Consider the time-delay chaotic system described in (1) and the corresponding observer (10). Suppose that there are the positive definite matrices P , S , Q , and feedback gain L , such that $PB = C^T$ and the following conditions hold:

$$(A_0 - LC)^T P + P(A_0 - LC) + S + PA_d S^{-1} A_d^T P < -Q, \quad (12)$$

$$\lambda_{\min}(Q) \geq 2\gamma M_\theta \|C\|, \quad (13)$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of a matrix. Let the supervisory controls be chosen

$$u_1 = -\hat{\varepsilon}_1 B^T P e / \|B^T P e\|, \quad u_2 = -\hat{\varepsilon}_2 B^T P e / \|B^T P e\|, \quad (14)$$

where $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ stand for the estimations of ε_1 and ε_2 , respectively. The asymptotic stability of the system (11) is guaranteed by applying the following adaptive laws

$$\dot{\theta} = 2\eta \omega(\hat{x}) e^T P B, \quad \dot{\hat{\varepsilon}}_1 = 2\eta_1 \|e^T P B\|, \quad \dot{\hat{\varepsilon}}_2 = 2\eta_2 \|e^T P B\|, \quad (15)$$

where η , η_1 , and η_2 denote the positive adaptation constants.

Proof: Define the Lyapunov-Krasovskii functional as

$$\begin{aligned} V(t) &= e^T P e + \frac{1}{2\eta} \text{tr}(\tilde{\theta}^T \tilde{\theta}) + \frac{1}{2\eta_1} \tilde{\varepsilon}_1^2 \\ &\quad + \frac{1}{2\eta_2} \tilde{\varepsilon}_2^2 + \int_{t-\tau}^t e^T(\sigma) S e(\sigma) d\sigma, \end{aligned} \quad (16)$$

where $\tilde{\theta} = \theta - \theta^*$, $\tilde{\varepsilon}_1 = \varepsilon_1 - \hat{\varepsilon}_1$, and $\tilde{\varepsilon}_2 = \varepsilon_2 - \hat{\varepsilon}_2$. The time derivative along the error dynamics (11) is

$$\begin{aligned} \dot{V}(t) &= 2e^T P [(A_0 - LC)e + A_d e(t - \tau) + B(\xi_1 - \hat{\xi}_1(\hat{x}|\theta) \\ &\quad - u_1(t)) + B(\xi_2 - u_2)] - \frac{1}{\eta} \text{tr}(\tilde{\theta}^T \dot{\tilde{\theta}}) - \frac{1}{\eta_1} \tilde{\varepsilon}_1 \dot{\tilde{\varepsilon}}_1 - \frac{1}{\eta_2} \tilde{\varepsilon}_2 \dot{\tilde{\varepsilon}}_2 \\ &\quad + e^T S e - e^T(t - \tau) S e(t - \tau) \\ &= e^T [(A_0 - LC)^T P + P(A_0 - LC)] e + 2e^T P A_d e(t - \tau) \\ &\quad + e^T S e - e^T(t - \tau) S e(t - \tau) + 2e^T P B [\xi_1 - \hat{\xi}_1(\hat{x}|\theta) \\ &\quad + \hat{\xi}_1(\hat{x}|\theta^*) - \hat{\xi}_1(\hat{x}|\theta^*) + \hat{\xi}_1(x|\theta^*) - \hat{\xi}_1(x|\theta^*) - u_1] \\ &\quad + 2e^T P B (\xi_2 - u_2) - \frac{1}{\eta} \text{tr}(\tilde{\theta}^T \dot{\tilde{\theta}}) - \frac{1}{\eta_1} \tilde{\varepsilon}_1 \dot{\tilde{\varepsilon}}_1 - \frac{1}{\eta_2} \tilde{\varepsilon}_2 \dot{\tilde{\varepsilon}}_2 \end{aligned} \quad (17)$$

Notice that

$$2e^T P A_d e(t - \tau) \leq e^T P A_d S^{-1} A_d^T P e + e^T(t - \tau) S e(t - \tau), \quad (18)$$

and

$$\begin{aligned} &\xi_1 - \hat{\xi}_1(\hat{x}|\theta) + \hat{\xi}_1(\hat{x}|\theta^*) - \hat{\xi}_1(\hat{x}|\theta^*) + \hat{\xi}_1(x|\theta^*) - \hat{\xi}_1(x|\theta^*) \\ &= (\xi_1 - \hat{\xi}_1(x|\theta^*)) + (\hat{\xi}_1(\hat{x}|\theta^*) - \hat{\xi}_1(\hat{x}|\theta)) + (\hat{\xi}_1(x|\theta^*) - \hat{\xi}_1(\hat{x}|\theta^*)) \\ &= (\xi_1 - \hat{\xi}_1(x|\theta^*)) + \tilde{\theta}^T \omega(\hat{x}) + \theta^* (\omega(x) - \omega(\hat{x})). \end{aligned} \quad (19)$$

Considering these relations and substituting (12) into (17), we have

$$\begin{aligned} \dot{V}(t) &\leq e^T [(A_0 - LC)^T P + P(A_0 - LC) + S + PA_d S^{-1} A_d^T P] e \\ &\quad + 2e^T P B (\xi_1 - \hat{\xi}_1(x|\theta^*)) + 2e^T P B (\tilde{\theta}^T \omega(\hat{x})) \\ &\quad + \theta^* (\omega(x) - \omega(\hat{x})) - 2e^T P B u_1 + 2e^T P B (\xi_2 - u_2) \\ &\quad - \frac{1}{\eta} \text{tr}(\tilde{\theta}^T \dot{\tilde{\theta}}) - \frac{1}{\eta_1} \tilde{\varepsilon}_1 \dot{\tilde{\varepsilon}}_1 - \frac{1}{\eta_2} \tilde{\varepsilon}_2 \dot{\tilde{\varepsilon}}_2 \\ &\leq -e^T Q e + 2 \|e^T P B\| \varepsilon_1 + \frac{1}{\eta} \text{tr}[\tilde{\theta}^T (2\eta \omega(\hat{x}) e^T P B - \dot{\tilde{\theta}})] \end{aligned}$$

$$\begin{aligned}
 &+ 2\gamma M_\theta \|C\| \|e\|^2 - 2e^T P B u_1 + 2e^T P B (\xi_2 - u_2) \\
 &- \frac{1}{\eta_1} \tilde{\varepsilon}_1 \dot{\hat{\varepsilon}}_1 - \frac{1}{\eta_2} \tilde{\varepsilon}_2 \dot{\hat{\varepsilon}}_2. \tag{20}
 \end{aligned}$$

Applying the supervisory controls (14) leads to

$$\begin{aligned}
 \dot{V}(t) &\leq -(\lambda_{\min}(Q) - 2\gamma M_\theta \|C\|) \|e\|^2 \\
 &+ \frac{1}{\eta_1} (2\eta_1 \|e^T P B\| - \dot{\hat{\varepsilon}}_1) + \frac{1}{\eta_2} (2\eta_2 \|e^T P B\| - \dot{\hat{\varepsilon}}_2). \tag{21}
 \end{aligned}$$

By applying (12) and (15), we have

$$\dot{V}(t) \leq -\beta e^T e, \quad \beta > 0. \tag{22}$$

That refers to $V \in L_\infty$, and all of the parameters, $e, \tilde{\theta}, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2, u_1$, and u_2 , are bounded. Integrating (22) from 0 to ∞ results in

$$\beta \int_0^\infty e^T e dt \leq V(0) - V(\infty) < \infty, \tag{23}$$

which implies $e \in L_2$. Based on the Lipschitz condition of $\omega(x)$, it is clearly that $\dot{e} \in L_\infty$. By using the Barbalat's lemma [16], one may conclude $e \rightarrow 0$ as $t \rightarrow \infty$. The proof is completed.

Notably, the assumption $PB = C^T$ is the basis of the adaptation via the output feedback. However, this condition is equivalent to

$$(PB - C^T)^T (PB - C^T) = 0, \tag{24}$$

and the equation can be converted into the following linear matrix inequality [2]:

$$\begin{aligned}
 &\underset{X}{\text{minimize}} \quad \delta \\
 &\text{subject to } X > 0, \quad \delta > 0 \tag{25}
 \end{aligned}$$

$$\begin{bmatrix} \delta I & (PBX - C^T X)^T \\ (PBX - C^T X) & I \end{bmatrix} > 0.$$

If the solution is feasible and the elements of δX^{-2} are near zero, then the relation (24) holds. Therefore, the asymptotic stability of error dynamics can be guaranteed by satisfying (12) and (25). This is a generalized eigenvalue minimization problem and can be efficiently solved by LMI optimization.

With the aforementioned result, the designing work is then extended to stabilization of a time-delayed chaotic system. Consider the chaotic system with forced input,

$$\begin{aligned}
 \dot{x}(t) &= \sum_{l=1}^r \mu_l(z) [(A_{1l} + \Delta A_{1l}(t))x(t) + (A_{2l} + \Delta A_{2l}(t))x(t - \tau)] \\
 &+ Bu(t), \quad y(t) = Cx(t), \tag{26}
 \end{aligned}$$

or equivalently,

$$\begin{aligned}
 \dot{x}(t) &= A_0 x(t) + A_d x(t - \tau)(t) \\
 &+ B[\xi_1(x(t), t) + \xi_2(x(t - \tau), t) + u(t)], \quad y(t) = Cx \tag{27}
 \end{aligned}$$

The corresponding observer is defined as

$$\begin{aligned}
 \dot{\hat{x}}(t) &= A_0 \hat{x}(t) + A_d \hat{x}(t - \tau) - BK \hat{x}(t) + L(y(t) - \hat{y}(t)), \\
 \hat{y}(t) &= C \hat{x}, \tag{28}
 \end{aligned}$$

where K is the feedback control gain matrix. The design of stabilization and model tracking are discussed as follows.

Theorem 2: Let the controller in (27) be chosen as

$$u(t) = -K \hat{x}(t) - L^T M \hat{x}(t) - \hat{\xi}_1(\hat{x} | \theta) - u_1 - u_2, \tag{29}$$

where M is a positive definite matrix. Suppose that the definitions of $\hat{\xi}_1(\hat{x} | \theta)$, u_1 , and u_2 and the stability conditions stated in Theorem 1 hold. If there exist the positive definite matrices M , W , and N such that the following conditions are satisfied

$$\begin{aligned}
 (A_0 - BK)^T M + M(A_0 - BK) + W \\
 + MA_d W^{-1} A_d^T M < -N, \tag{30}
 \end{aligned}$$

then the closed-loop system is asymptotically stable.

Proof: The error dynamics related to observation error $e(t) = x(t) - \hat{x}(t)$ is presented by

$$\begin{aligned}
 \dot{e}(t) &= (A_0 - LC)e(t) + A_d e(t - \tau) - BL^T M \hat{x}(t) \\
 &+ B(\xi_1 - \hat{\xi}_1(\hat{x} | \theta) - u_1) + B(\xi_2 - u_2). \tag{31}
 \end{aligned}$$

Construct a Lyapunov-Krasovskii functional candidate of the form

$$\begin{aligned}
 V(t) &= \hat{x}^T M \hat{x} + e^T P e + \frac{1}{2\eta} \text{tr}(\tilde{\theta}^T \tilde{\theta}) + \frac{1}{2\eta_1} \tilde{\varepsilon}_1^2 \\
 &+ \frac{1}{2\eta_2} \tilde{\varepsilon}_2^2 + \int_{t-\tau}^t e^T(\sigma) S e(\sigma) d\sigma + \int_{t-\tau}^t \hat{x}^T(\sigma) W \hat{x}(\sigma) d\sigma. \tag{32}
 \end{aligned}$$

The derivative of (32) along the trajectories of (28) and (31) is given by

$$\begin{aligned}
 \dot{V}(t) &= 2\hat{x}^T M \dot{\hat{x}} + 2e^T P \dot{e} - \frac{1}{\eta} \text{tr}(\tilde{\theta}^T \dot{\tilde{\theta}}) - \frac{1}{\eta_1} \tilde{\varepsilon}_1 \dot{\hat{\varepsilon}}_1 - \frac{1}{\eta_2} \tilde{\varepsilon}_2 \dot{\hat{\varepsilon}}_2 \\
 &+ e^T S e - e^T(t - \tau) S e(t - \tau) + \hat{x}^T W \dot{\hat{x}} - \hat{x}^T(t - \tau) W \hat{x}(t - \tau). \tag{33}
 \end{aligned}$$

Consider the following relations

$$\begin{aligned}
 2\hat{x}^T M \dot{\hat{x}} &\leq \hat{x}^T [(A_0 - BK)^T M + M(A_0 - BK) + MA_d W^{-1} A_d^T M] \hat{x} \\
 &+ \hat{x}^T(t - \tau) W \hat{x}(t - \tau) + 2\hat{x}^T M L C e, \tag{34}
 \end{aligned}$$

$$\begin{aligned}
 2e^T P \dot{e} &\leq e^T [(A_0 - LC)^T P + P(A_0 - LC) + PA_d S^{-1} A_d^T P] e \\
 &+ e^T(t - \tau) S e(t - \tau) - 2e^T P B L M \hat{x} + 2e^T P B (\xi_1 - \hat{\xi}_1(\hat{x} | \theta) - u_1) \\
 &+ 2e^T P B (\xi_2 - u_2). \tag{35}
 \end{aligned}$$

Substituting (34) and (35) into (33) and using the result derived from the previous theorem, we have

$$\begin{aligned}
 \dot{V}(t) &\leq -\beta e^T e + \hat{x}^T [(A_0 - BK)^T M + M(A_0 - BK) \\
 &+ W + MA_d W^{-1} A_d^T M] \hat{x}. \tag{36}
 \end{aligned}$$

By the stability condition (30), it can be verified clearly that

$$\dot{V}(t) \leq -\beta e^T e - \lambda_{\min}(N) \|\hat{x}\|^2 \leq -\alpha \hat{x}^T \hat{x}, \quad \alpha > 0. \tag{37}$$

Based on the Barbalat's lemma, we may conclude that both e and \hat{x} will eventually approach zero.

Inspired by the work of observer-based fuzzy control, we propose the following algorithm for designing a tracking controller. Consider the reference model as follows:

$$\begin{aligned} \dot{x}_m(t) &= A_{1m}x_m(t) + A_{2m}x_m(t - \tau) + B_m r(t), \\ y_m(t) &= Cx_m(t), \end{aligned} \quad (38)$$

where $A_{1m} = A_0 - BK$, $A_{2m} = A_d$, $B_m = BK_m$, K_m is a known real matrix, and $r(t)$ is a reference input. The corresponding observer is defined as

$$\begin{aligned} \dot{\hat{x}}(t) &= (A_0 - BK)\hat{x}(t) + A_d\hat{x}(t - \tau) + L(y(t) - \hat{y}(t)) + BK_m r(t), \\ \hat{y}(t) &= C\hat{x}(t). \end{aligned} \quad (39)$$

Let us denote the error vectors as $e(t) = x(t) - \hat{x}(t)$ and $\bar{x}(t) = x_m(t) - \hat{x}(t)$. Choose the following control law

$$\begin{aligned} u(t) &= -K\hat{x}(t) + L^T M\bar{x}(t) + K_m r(t) \\ &\quad - \hat{\xi}_1(\hat{x} | \theta) - u_1(t) - u_2(t). \end{aligned} \quad (40)$$

The error dynamics are thus written as

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^r \mu_i [(A_{1i} - L_i C)e(t) + A_{2i}e(t - \tau) + BL_i^T M\bar{x}(t)] \\ &\quad + B(\xi_1 - \hat{\xi}_1(\hat{x} | \theta) - u_1(t)) + B(\xi_2 - u_2(t)), \end{aligned} \quad (41)$$

$$\dot{\bar{x}}(t) = \sum_{i=1}^r \mu_i (\bar{A}_{1i}\bar{x}(t) + \bar{A}_{2i}\bar{x}(t - \tau) - L_i C e(t)). \quad (42)$$

Theorem 3: Suppose all the hypotheses of Theorem 1 and Theorem 2 are satisfied. The asymptotic stabilities of the closed-loop systems (41) and (42) are guaranteed by the adaptive controller (40) with the updating laws (15).

Proof: The proof of this theorem is in the same spirit as that derived in the previous theorems. One may choose the following Lyapunov-Krasovskii functional candidate

$$\begin{aligned} V(t) &= \bar{x}^T M\bar{x} + e^T P e + \frac{1}{2\eta} \text{tr}(\tilde{\theta}^T \tilde{\theta}) + \frac{1}{2\eta_1} \tilde{\xi}_1^2 + \frac{1}{2\eta_2} \tilde{\xi}_2^2 \\ &\quad + \int_{t-\tau}^t e^T(\sigma) S e(\sigma) d\sigma + \int_{t-\tau}^t \bar{x}^T(\sigma) W \bar{x}(\sigma) d\sigma \end{aligned} \quad (43)$$

and proceed the aforementioned procedure to obtain the result.

4. Numerical Example

In this section, we shall consolidate the design principle with numerical example, so as to verify the effectiveness of the proposed algorithm.

Consider a time-delay Chua's circuit described by

$$\begin{aligned} \dot{x}_1 &= \sigma_1(-x_1 + x_2 - f(x_1)) - cx_1(t - \tau) + d(t) \\ \dot{x}_2 &= x_1 - x_2 + x_3 + cx_2(t - \tau) \\ \dot{x}_3 &= -\sigma_2 x_2 + c(2x_1(t - \tau) + x_3(t - \tau)) \\ d(t) &= (0.8 \sin 2t)x_1 + (1 + 0.3 \sin 2t)x_2 + (-1 + 0.5 \cos t)x_3 \\ &\quad + (1 + \sin 3t)x_1(t - \tau) + 0.4x_2(t - \tau) + (0.1 \cos 3t)x_3(t - \tau), \end{aligned}$$

where $f(x_1) = bx_1 + 0.5(a - b)(|x_1 + 1| - |x_1 - 1|)$, $\sigma_1 = 10$, $\sigma_2 = 15$, $a = -1.28$, $b = -0.69$, and $c = 0.1$.

The T-S fuzzy model for the system is given by [15]

$$\begin{aligned} R^l : &\text{ If } x_1(t) \text{ is } M_l \\ &\text{ then } \dot{x}(t) = (A_{1l} + \Delta A_{1l})x(t) + (A_{2l} + \Delta A_{2l})x(t - \tau), \\ &\quad y(t) = x_1(t), \quad \text{for } l = 1, 2, \end{aligned}$$

where

$$A_{11} = \begin{bmatrix} (\alpha - 1)\sigma_1 & \sigma_1 & 0 \\ 1 & -1 & 1 \\ 0 & -\sigma_2 & 0 \end{bmatrix}, A_{12} = \begin{bmatrix} -(\alpha + 1)\sigma_1 & \sigma_1 & 0 \\ 1 & -1 & 1 \\ 0 & -\sigma_2 & 0 \end{bmatrix},$$

$$A_{21} = A_{22} = \begin{bmatrix} -c & 0 & 0 \\ 0 & c & 0 \\ 2c & 0 & c \end{bmatrix},$$

$$\Delta A_{11} = \Delta A_{12} = \begin{bmatrix} 0.8 \sin(2t) & 1 + 0.3 \sin(2t) & -1 + 0.5 \cos(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Delta A_{21} = \Delta A_{22} = \begin{bmatrix} 1 + \sin(3t) & 0.4 & 0.1 \cos(3t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The related fuzzy sets are defined as $M_1(x_1) = (1 - (g(x_1)/\alpha))/2$ and $M_2(x_1) = 1 - M_1(x_1)$ with $\alpha = \sup_{x \in \Omega} |g(x_1)| = 1.28$ and

$$g(x_1) = \begin{cases} f(x_1)/x_1, & x_1 \neq 0 \\ b, & x_1 = 0. \end{cases}$$

Then, the perturbed linear model is represented by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -\sigma_1 & \sigma_1 & 0 \\ 1 & -1 & 1 \\ 0 & -\sigma_2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} -c & 0 & 0 \\ 0 & c & 0 \\ 2c & 0 & c \end{bmatrix} x(t - \tau) + \begin{bmatrix} d(t) \\ 0 \\ 0 \end{bmatrix} \\ &= A_0 x(t) + A_d x(t - \tau) + B(\xi_1(x(t), t) + \xi_2(x(t - \tau), t)), \\ y(t) &= Cx(t). \end{aligned}$$

Accordingly, the system matrices B and C are determined to be $[1 \ 0 \ 0]^T$ and $[1 \ 0 \ 0]$, respectively. Figure 1 shows the phase portrait of the time-delay Chua's circuit without uncertainties.

The linear matrix inequalities (12) and (25) are solved to obtain the following matrices

$$L = \begin{bmatrix} 515.6043 \\ 0.9942 \\ 0.0626 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1702.2 & -8.41 \\ 0 & -8.41 & 118.4 \end{bmatrix},$$

$$Q = \begin{bmatrix} 352.8478 & 0 & 0 \\ 0 & 247.9827 & 1.9104 \\ 0 & 1.9104 & 46.8796 \end{bmatrix},$$

$$S = \begin{bmatrix} 351.9014 & -6.0883 & -2.0326 \\ -6.0883 & 310.2571 & -0.3932 \\ -2.0326 & -0.3932 & 74.765 \end{bmatrix}$$

The Lipschitz constant γ has a value of 0.7756, which is determined by calculating norm value of the Jacobian matrix of $\omega(x)$. The constants related to the adaptive

laws are chosen as: $M_\theta = 20$, $\eta = 10$, and $\eta_1 = \eta_2 = 0.2$. Moreover, we employ five fuzzy sets, which are spreading equally over the operating region, for the estimation of ξ_1 . The initial value of θ is given by $[-8 \ -2 \ 0 \ 2 \ 8]^T$, and the time-delay constant τ is set to 1. With these chosen parameters, the condition (13) is satisfied. Given the initial states $x(0) = [1 \ -0.3 \ 0.2]^T$ and $\hat{x}(0) = [-1 \ -1 \ -1]^T$, the performance of synchronization are shown in Fig. 2. The phase portrait of time-delay Chua's circuit with uncertainty is plotted in Fig. 3. It is different from a double-scroll attractor as sketched in Fig.1; however, the dynamics is bounded in a region.

For the stabilization and tracking control, the following matrices are determined to satisfy (30):

$$K = [38.5339 \ 167.4579 \ -13.8471],$$

$$M = \begin{bmatrix} 1.0141 & 3.0848 & -0.3483 \\ 3.0848 & 10.8492 & -1.2022 \\ -0.3483 & -1.2022 & 0.2562 \end{bmatrix},$$

$$N = \begin{bmatrix} 28.9768 & 92.2347 & -10.2269 \\ 92.2347 & 294.9688 & -32.6264 \\ -10.2269 & -32.6264 & 3.7179 \end{bmatrix},$$

$$W = \begin{bmatrix} 34.4324 & 112.1828 & -12.4584 \\ 112.1828 & 367.5157 & -40.7118 \\ -12.4584 & -40.7118 & 4.668 \end{bmatrix}.$$

The result of stabilization control is displayed in Fig. 4. In the tracking control, we assume that $\bar{B}_i = B, i = 1,2$, and the reference input $r(t)$ is $20\cos(\pi/4)$. Given the initial state $x_m(0) = [0 \ 0 \ 0]^T$, Fig. 5 gives the response curves of the states. From the simulations, we have shown that the system controlled by the proposed methods can perform well. The effectiveness and correctness of presented method are demonstrated.

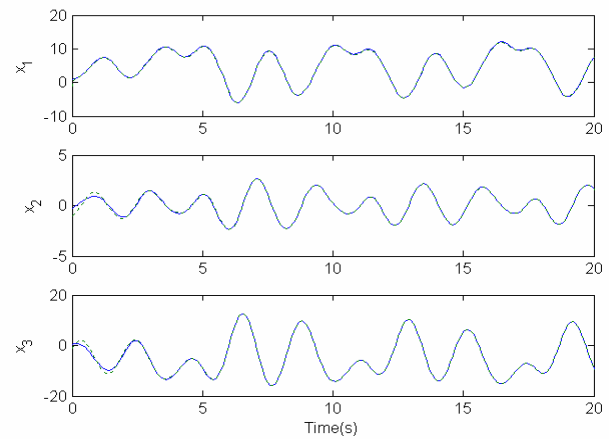


Figure 2. The performance of synchronization (solid line: $x(t)$, dashed line: $\hat{x}(t)$).

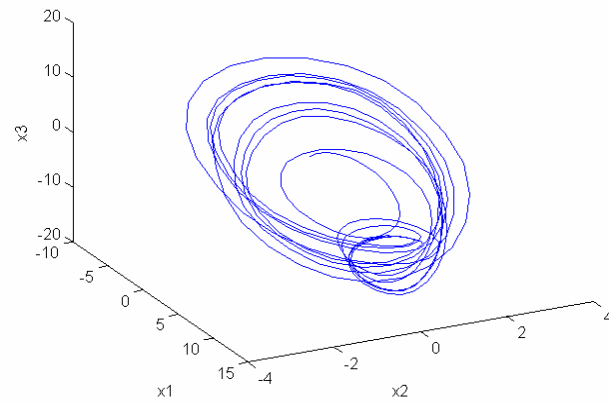


Figure 3. The phase portrait of the time-delay Chua's circuit with uncertainties.

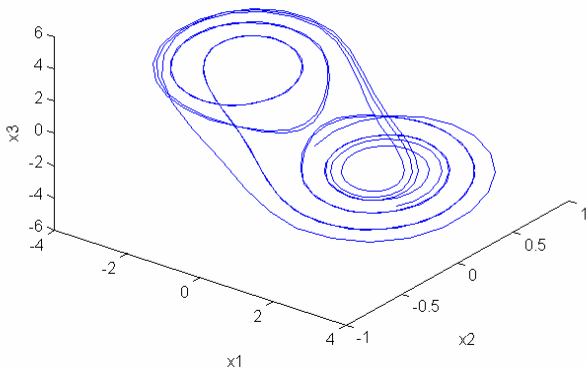


Figure 1. The phase portrait of the time-delay Chua's circuit without uncertainties.

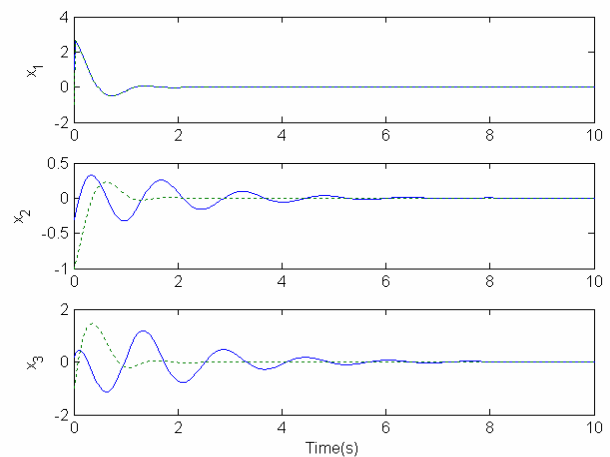


Figure 4. The performance of stabilization (solid line: $x(t)$, dashed line: $\hat{x}(t)$).

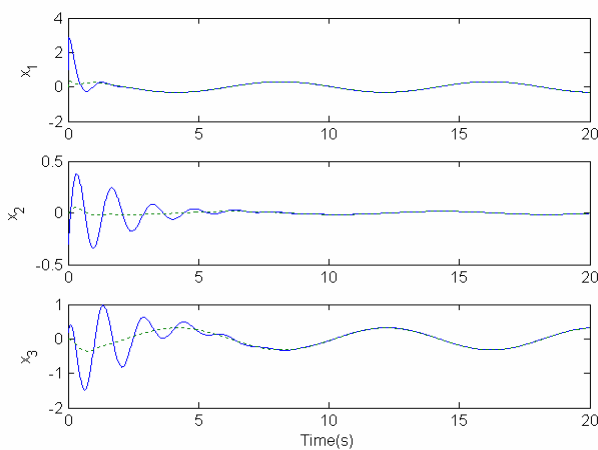


Figure 5. The performance of tracking control (solid line: $x(t)$, dashed line: $x_m(t)$).

5. Conclusions

This investigation introduces an observer-based adaptive fuzzy control approach for a time-delay chaotic system. Because the effect of parameter uncertainty has been considered, a robust design is discerned. Specifically, the uncertainty can be indirectly estimated by Mamdani type fuzzy inference. With the adaptive updating laws, the asymptotic stability of the closed-loop system is guaranteed by the proposed control scheme. Unlike earlier studies that are based on the explicit nonlinear model, the chaotic system considered herein is represented by the T-S fuzzy model. The effectiveness of proposed approach is illustrated by computer simulation on a time-delay Chua's circuit.

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