Takagi-Seguno Fuzzy Systems Based Nonlinear Adaptive Control

Noureddine Goléa, Amar Goléa, and Ibtissem Abdelmalek

Abstract

This paper develops a fuzzy model reference adaptive control of continuous-time nonlinear systems described by Takagi-Seguno (TS) fuzzy model. The tracking performance is assured using a direct TS fuzzy controller. Both full state information and observer-based control schemes are investigated. It is proved, using Lyapunov stability tools, that this adaptive scheme is asymptotically stable and the tracking and observation errors converge to zero. Simulation results illustrate the performance of the proposed approach.

Keywords: Fuzzy control, TS Fuzzy systems, Nonlinear systems, Adaptive Control, Observer, Stability.

1. Introduction

Fuzzy systems have been proved to be universal approximators for nonlinear systems [1-4]. Various algorithms were proposed to train fuzzy systems as nonlinear identifiers, and were useful in the modeling and control of nonlinear plants [1, 5-12]. Since Wang paper [1], various kinds of fuzzy adaptive systems were developed and their stability were achieved using Lyapunov theory [8-12]. More recently, fuzzy model-based control succeeded to exploit the particular structure of TS models. This quasi-linear structure, i.e., the TS fuzzy system being nonlinear superposition of linear local models, has prompted efficient analysis techniques established mainly on the concept of quadratic stability. Based on those features several design issues were addressed in the robust control context and the parallel-distributed (PDC) approach was applied [13-16]. This approach assumes the local controllability of the every model's rule, and the control objective was achieved using the linear matrix inequalities (LMI).

This paper investigates the adaptive model reference approach to control the nonlinear systems represented by Takagi-Sugeno (TS) model [17]. The Controller is implemented using an adaptive TS system, which is not necessary of the same form of the fuzzy model. In contrast to the previous approaches, only global controllability of the fuzzy model is required. Both full state information and observer-based approaches are developed. The stability in both cases is established using Lyapunov tools. It is shown that the tracking and observation errors converge to zero. Also, it is shown that the approach is robust against external disturbance and approximation error. Simulation results, for unstable fuzzy system with some uncontrollable rules, show the capabilities of the proposed approach for both tracking a reference model and estimating the system states.

The rest of the paper is organized as follows. Section 2 poses the control problem. Section 3 develops the full state information approach, section 4 develops the observer-based approach, section 5 presents the simulation results and section 6 concludes the paper.

2. Problem Statement

Consider the TS fuzzy model constituted by a set of if-then rules of the form

$$\mathrm{R}_k\text{: If } z \text{ is } Z_k \text{ Then } \dot{x}_n = a_k x + b_k u, \quad k = 1 \dots m \tag{1}$$

where $a_k \in R^n$, $b_k \in R$ are the k th rule consequence parameters, m is the number of rules, $z \in R^q$ is the fuzzy model input vector, and the fuzzy sets Z_k operate a fuzzy partition of the fuzzy model input space (i.e., the fuzzification operators).

The output of the fuzzy model is inferred as follows

$$\dot{x}_n = \frac{\sum_{k=1}^{m} \mu_k(z)\ (a_k x + b_k u)}{\sum_{k=1}^{m} \mu_k(z)} \tag{2}$$

where $\mu_k(z)$ is the grade of membership of z in Z_k (i.e., the firing strength of the rule k). Then, (2) can be written as

$$\dot{x}_n = \sum_{k=1}^{m} \varphi_k (a_k x + b_k u) \tag{3}$$

where φ_k is the normalized firing strength, given by

$$\varphi_k = \frac{\mu_k}{\sum_{j=1}^{m} \mu_j} \tag{4}$$

Now, consider the continuous-time nonlinear system

$$\dot{x} = Ax + b[f(x) + g(x)u + d] \tag{5}$$

where $f(x)$ and $g(x)$ are smooth unknown functions,

Corresponding Author: Noureddine Goléa is with the Electrical Engineering Institute, Oum El-Bouaghi University, Oum El-Bouaghi, 04000 Algeria.
E-mail: nour_golea@yahoo.fr
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d is a bounded external disturbance, and A, b are defined as

$$A=\begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ & 0_{1\times n} \end{bmatrix}_{n\times n}, \quad b^T=\begin{bmatrix}0 & \dots & 0 & 1\end{bmatrix}_{1\times n}$$

where I_n is $n\times n$ identity matrix.

Based on the universal approximation results [1-4], the nonlinear system (5) can be modeled by the fuzzy model (3) such as

$$\dot{x}=Ax+b\left[\sum_{k=1}^{m}\varphi_k\left(a_k x+b_k u\right)+\eta\right] \tag{6}$$

where η is an uncertainty term due to the approximation error and the external disturbance.

The reference model is defined by the following stable LTI state equation

$$\dot{x}_m=A_m x_m+b_m r \tag{7}$$

where $x_m\in R^n$ is the state vector, r is a bounded reference input, and A_m, b_m are given by

$$A_m=\begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ & -a_m \end{bmatrix}_{n\times n}, \quad b_m^T=\begin{bmatrix}0 & \dots & 0 & b_{nm}\end{bmatrix}_{1\times n}$$

and $a_m\in R^{1\times n}$, $b_{nm}>0$.

The control problem can be stated as: Design the control input u such that the states of the system (5) follow those of the reference model (7), under the condition that all involved signals in the closed loop remain bounded.

3. Fuzzy Adaptive Approach

A. *TS Fuzzy controller*

The TS fuzzy controller to be designed is a multi-input single-output TS fuzzy system of the form

$$\mathrm{R}_j\text{: If } v \text{ is } V_j \text{ Then } u_f=k_{1j}x+k_{2j}r,\quad j=1\dots q \tag{8}$$

where $k_{1j}\in R^{1\times n}$, $k_{2j}\in R$ are the j th rule consequence parameters, $v\in R^q$ is the fuzzy controller input vector, and the fuzzy sets V_j operate a fuzzy partition of the fuzzy controller input space. The final output of the fuzzy controller is inferred as follows

$$u_f=\frac{\sum_{j=1}^{q}\rho_j\left(v\right)\ \left(k_{1j}x+k_{2j}r\right)}{\sum_{j=1}^{q}\rho_j\left(v\right)} \tag{9}$$

where $\rho_j(v)$ is the grade of membership of v in V_j. In this paper, it assumed that there exists always at least one active rule, i.e. $\sum_{j=1}^{q}\rho_j(v)>0$.

The fuzzy controller output (9) can also be written in following compact form

$$u_f=\sum_{j=1}^{q}\xi_j\left(k_{1j}x+k_{2j}r\right) \tag{10}$$

where

$$\xi_j=\frac{\rho_j}{\sum_{l=1}^{q}\rho_l} \tag{11}$$

Using (6) and (7), the tracking error can be rewritten as

$$\dot{e}=A_m e-b\left[\sum_{k=1}^{m}\varphi_k\left[\left(a_k+a_m\right)x+b_k u\right]-b_{nm}r+\eta\right] \tag{12}$$

To ensure tracking objective and to overcome the uncertainties effects, we use the following control input

$$u=u_f+u_s \tag{13}$$

where u_s is an additional control term to be defined later, and u_f is the fuzzy adaptive controller defined by (10).

Introducing (13) and (10) in (12) yields

$$\begin{aligned}\dot{e}=A_m e-b\Bigg[&\sum_{k=1}^{m}\varphi_k\left(a_k+a_m\right)x+\sum_{k=1}^{m}\varphi_k b_k\sum_{j=1}^{q}\xi_j\left(k_{1j}x+k_{2j}r\right)\\ &-b_{nm}r+\sum_{k=1}^{m}\varphi_k b_k u_s+\eta\Bigg]\end{aligned} \tag{14}$$

which can be arranged as

$$\begin{aligned}\dot{e}=A_m e-b\Bigg[&\sum_{j=1}^{q}\xi_j\left(\sum_{k=1}^{m}\varphi_k\left[a_k+b_k k_{1j}\right]+a_m\right)x\\ &+\sum_{j=1}^{q}\xi_j\left(\sum_{k=1}^{m}\varphi_k b_k k_{2j}-b_{nm}\right)r+\sum_{k=1}^{m}\varphi_k b_k u_s+\eta\Bigg]\end{aligned} \tag{15}$$

At this point, we introduce the following notations

$$\phi_j:=\sum_{k=1}^{m}\varphi_k\left[a_k+b_k k_{1j}\right]+a_m \tag{16}$$

$$\psi_j:=\sum_{k=1}^{m}\varphi_k b_k k_{2j}-b_{nm} \tag{17}$$

Hence, using (16)-(17), (15) can be rewritten as

$$\dot{e}=A_m e-b\left[\sum_{j=1}^{q}\xi_j\phi_j x+\sum_{j=1}^{q}\xi_j\psi_j r+\sum_{k=1}^{m}\varphi_k b_k u_s+\eta\right] \tag{18}$$

B. *Stability analysis*

To establish the stability of the proposed approach, the following assumptions are used.

Assumption 1: The fuzzy model (3) is globally controllable, i.e., $\sum_{k=1}^{m}\varphi_k b_k>0\forall x$.

Assumption 2: $\mu_k(z)\in C^1$, i.e. φ_k has a bounded first order derivative.

Assumption 3: The uncertainty term is bounded by $|\eta|<\eta_0$, where η_0 is known constant.

Consider the following Lyapunov function

$$V=\frac{1}{2}e^T Pe+\frac{1}{2\gamma_1}\sum_{j=1}^{q}\phi_j\phi_j^T+\frac{1}{2\gamma_2}\sum_{j=1}^{q}\psi_j^2 \tag{19}$$

where γ_1, $\gamma_2>0$ are design parameters, and $P=P^T>0$ is the solution, for a given $Q=Q^T>0$, of the Lyapunov equation

$$A_m^T P+PA_m=-Q \tag{20}$$

The differentiation of (19) along the trajectory of (18) yields

$$\dot{V} = -\frac{1}{2}e^T Qe - e^T Pb\left[\sum_{j=1}^{q}\xi_j\phi_j x + \sum_{j=1}^{q}\xi_j\psi_j r + \sum_{k=1}^{m}\varphi_k b_k u_s + \eta\right] + \frac{1}{\gamma_1}\sum_{j=1}^{q}\phi_j \dot{\phi}_j^T + \frac{1}{\gamma_2}\sum_{j=1}^{q}\psi_j \dot{\psi}_j \quad (21)$$

which can be arranged as

$$\dot{V} = -\frac{1}{2}e^T Qe - e^T Pb\left[\sum_{k=1}^{m}\varphi_k b_k u_s + \eta\right] + \frac{1}{\gamma_1}\sum_{j=1}^{q}\phi_j\left(\dot{\phi}_j^T - \gamma_1\xi_j x^T b^T Pe\right) + \frac{1}{\gamma_2}\sum_{j=1}^{q}\psi_j\left(\dot{\psi}_j - \gamma_2\xi_j r b^T Pe\right) \quad (22)$$

The differentiation of (16) and (17) with respect to time yields

$$\dot{\phi}_j = \sum_{k=1}^{m}\dot{\varphi}_k\left[a_k + b_k k_{1j}\right] + \sum_{k=1}^{m}\varphi_k b_k \dot{k}_{1j} \quad (23)$$

$$\dot{\psi}_j = k_{2j}\sum_{k=1}^{m}\dot{\varphi}_k b_k + \dot{k}_{2j}\sum_{k=1}^{m}\varphi_k b_k \quad (24)$$

For the third and fourth terms in (22) to be null, we chose

$$\dot{k}_{1j} = \frac{1}{\sum_{k=1}^{m}\varphi_k b_k}\left[\gamma_1\xi_j e^T Pbx - \sum_{k=1}^{m}\dot{\varphi}_k\left[a_k + b_k k_{1j}\right]\right] \quad (25)$$

$$\dot{k}_{2j} = \frac{1}{\sum_{k=1}^{m}\varphi_k b_k}\left[\gamma_2\xi_j e^T Pbr - k_{2j}\sum_{k=1}^{m}\dot{\varphi}_k b_k\right] \quad (26)$$

Then, introducing (25)-(26) in (22) yields

$$\dot{V} = -\frac{1}{2}e^T Qe - e^T Pb\left[\sum_{k=1}^{m}\varphi_k b_k u_s + \eta\right] \quad (27)$$

Now, choosing the switching term as

$$u_s = \frac{\eta_0}{\sum_{k=1}^{m}\varphi_k b_k}\operatorname{sgn}\left(e^T Pb\right) \quad (28)$$

and substituting in (27) gives

$$\dot{V} \le -\frac{1}{2}e^T Qe \quad (29)$$

The stability results for this approach are summarized by the following theorem.

Theorem 1: The control system composed by the nonlinear system (5), the reference model (7), the fuzzy model (6), the fuzzy controller (10) and the update laws (25)-(26), is asymptotically stable and the tracking error converges to zero.

Proof: from (29) $\dot{V}$ is always negative in the e space if $e \neq 0$, then e, ϕ_j and $\psi_j \in L_\infty$, therefore $V \in L_\infty$. Since all variables in the right-hand side of (18) are bounded, $\dot{e}$ is bounded, i.e., $\dot{e} \in L_\infty$. Integrating both sides of (29) yields

$$\int_0^\infty |e|^2\, dt \le \frac{2}{\lambda_{\min}(Q)}V(0) \quad (30)$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of Q. Since the right side of (30) is bounded, $e \in L_2$. Using Barbalat's lemma [18], we have that the error converges asymptotically to zero, i.e., $\lim_{t\to\infty} e = 0$.

4. Observer-Based Control

A. *Observer Design*

The above control design was based on full state information. Since, this condition is difficult even impossible in many practical situations; the problem will be solved using only the measured output as available information.

To estimate the system state vector, we define the following observer

$$\dot{\hat{x}} = Ax + b\sum_{k=1}^{m}\varphi_k\left[a_k\hat{x} + b_k u + b_k u_s\right] + h\left(y - y\right) \quad (31)$$

$$\hat{y} = c^T x \quad (32)$$

where $c^T = [1 \;\; 0 \;\; 0 \;\; 0] \in R^{1\times n}$, $\hat{x}$ and $\hat{y}$ are the estimated states and output, and $h \in R^{n\times 1}$ is selected such as $\left[A - hc^T\right]$ is a Hurwitz matrix.

Let's define the observation error $\tilde{x} = x - \hat{x}$, then, substracting (32) from (6) gives

$$\dot{\tilde{x}} = \left[A - hc^T\right]\tilde{x} + b\left[\sum_{k=1}^{m}\varphi_k a_k\tilde{x} - \sum_{k=1}^{m}\varphi_k b_k u_s + \eta\right] \quad (33)$$

$$\tilde{y} = c^T\tilde{x} \quad (34)$$

The input-output transfer function of (33)-(34) is given by

$$\tilde{y} = c^T\left[sI_n - A_o\right]^{-1} b\left[\sum_{k=1}^{m}\varphi_k a_k\tilde{x} - \sum_{k=1}^{m}\varphi_k b_k u_s + \eta\right] \quad (35)$$

where s is the Laplace operator and $A_o = \left[A - hc^T\right]$.

Let's define the Hurwitz polynomial $l(s) = s^{n-1} + \alpha_1 s^{n-2} + \ldots + \alpha_{n-2}s + \alpha_{n-1}$ such that $l^{-1}(s)$ is proper stable transfer function. Then multiplying and devising (35) by $l(s)$ yields

$$\tilde{y} = G(s)\left[\sum_{k=1}^{m}\varphi_k^f a_k\tilde{x} - \sum_{k=1}^{m}\varphi_k^f b_k u_s + \eta^f\right] \quad (36)$$

where

$$G(s) = l(s)c^T\left[sI_n - A_o\right]^{-1} b \quad (37)$$

$$\varphi_k^f = l^{-1}(s)\varphi_k \quad (38)$$

$$\eta^f = l^{-1}(s)\eta \quad (39)$$

Hence, (36) can be rewritten in the following state space form

$$\dot{\tilde{x}} = A_o\tilde{x} + b_o\left[\sum_{k=1}^{m}\varphi_k^f a_k\tilde{x} - \sum_{k=1}^{m}\varphi_k^f b_k u_s + \eta^f\right] \quad (40)$$

$$\tilde{y} = c^T\tilde{x} \quad (41)$$

where $b_o^T = [1 \;\; \alpha_1 \;\; \ldots \;\; \alpha_{n-1}] \in R^{1\times n}$.

Using (7) and (31), the estimated tracking error is given by

$$\dot{\hat{e}} = A_m e - hc^T \tilde{x} - b\left[\sum_{k=1}^{m}\varphi_k\left(a_k \hat{x} + b_k u\right) + a_m \hat{x} - b_{nm} r + \sum_{k=1}^{m}\varphi_k b_k u_s\right] \quad (42)$$

Now, we define the control input as in (13), with

$$u_f = \sum_{j=1}^{q}\xi_j\left(k_{1j}\hat{x} + k_{2j} r\right) \quad (43)$$

Then, using the same notation (16)-(17), (42) can be arranged as

$$\dot{\hat{e}} = A_m e - b\left[\sum_{j=1}^{q}\xi_j \phi_j \hat{x} + \sum_{j=1}^{q}\xi_j \psi_j r\right] - hc^T \tilde{x} \quad (44)$$

B. *Stability Analysis*

To establish the stability of the observer-based approach, the following lemma is required

Lemma 1 [18]: If the transfer function $G(s) = c^T\left(sI_n - A_o\right)^{-1} b_o$ is strictly positive real (SPR), then there exists two symmetric positive definite matrices P_o and Q_o such that the following equations are verified

$$P_o A_o + A_o^T P_o = -Q_o \qquad b_o^T P_o = c^T \quad (45)$$

Since A_o is Hurwitz, and the pairs $(A_o,\ b)$, $(A_o,\ c)$ are controllable and observable respectively, $l(s)$ can be always chosen such that the SPR condition is verified.

To establish the stability, let's define the Lyapunov function as

$$V = V_1 + V_2 \quad (46)$$

with

$$V_1 = \frac{1}{2}\tilde{x}^T P_o \tilde{x} \quad (47)$$

and

$$V_2 = \frac{1}{2}\hat{e}^T P e + \frac{1}{2\gamma_1}\sum_{j=1}^{q}\phi_j \phi_j^T + \frac{1}{2\gamma_2}\sum_{j=1}^{q}\psi_j^2 \quad (48)$$

where P and P_o are solutions of (20) and (45) respectively.

The differentiation of (47) along (40) yields

$$\dot{V}_1 = -\frac{1}{2}\tilde{x}^T Q_o \tilde{x} + \tilde{x}^T P_o b_o\left[\sum_{k=1}^{m}\varphi_k^f a_k \tilde{x} - \sum_{k=1}^{m}\varphi_k^f u_s + \eta^f\right] \quad (49)$$

Using the fact that $\tilde{x}^T P_o b_o = \tilde{y}$, the switching term is chosen as

$$u_s = \frac{\eta_0}{\sum_{k=1}^{m}\varphi_k^f}\operatorname{sgn}\left(\tilde{y}\right) \quad (50)$$

Then, substituting (50) in (49) gives

$$\dot{V}_1 \le -\frac{1}{2}\tilde{x}^T Q_o \tilde{x} + \tilde{x}^T P_o b_o \sum_{k=1}^{m}\varphi_k^f a_k \tilde{x} \quad (51)$$

Using (45), and noting $a := \sum_{k=1}^{m}\varphi_k^f a_k$ yields

$$\dot{V}_1 \le -\frac{1}{2}\tilde{x}^T Q_o \tilde{x} + \tilde{x}^T ca\tilde{x} \quad (52)$$

Applying norm on (52) gives

$$\dot{V}_1 \le -\frac{1}{2}\lambda_{\min}\left(Q_o\right)\left|\tilde{x}\right|^2 + \overline{a}\left|\tilde{x}\right|^2 \quad (53)$$

where we have used the fact that $|c| = 1$, $\sum_{k=1}^{m}\left|\varphi_k^f\right| \le 1$, and $\overline{a} = \sum_{k=1}^{m}|a_k|$. Hence, if the matrix Q_o is chosen such that

$$\lambda_{\min}\left(Q_o\right) - 2\overline{a} \ge \kappa > 0 \quad (54)$$

for some positive constant κ, it follows that

$$\dot{V}_1 \le -\frac{\kappa}{2}\left|\tilde{x}\right|^2 \quad (55)$$

The differentiation of (48) along the trajectory of (44) yields

$$\dot{V}_2 = -\frac{1}{2}\hat{e}^T Q e - \hat{e}^T Phc^T \tilde{x} + \frac{1}{\gamma_1}\sum_{j=1}^{q}\phi_j\left(\dot{\phi}_j^T - \gamma_1 \xi_j \hat{x}^T b^T Pe\right) + \frac{1}{\gamma_2}\sum_{j=1}^{q}\psi_j\left(\dot{\psi}_j - \gamma_2 \xi_j r b^T P\hat{e}\right) \quad (56)$$

Then, using the update laws

$$\dot{k}_{1j} = \frac{1}{\sum_{k=1}^{m}\varphi_k b_k}\left[\gamma_1 \xi_j \hat{e}^T Pbx - \sum_{k=1}^{m}\dot{\varphi}_k\left[a_k + b_k k_{1j}\right]\right] \quad (57)$$

$$\dot{k}_{2j} = \frac{1}{\sum_{k=1}^{m}\varphi_k b_k}\left[\gamma_2 \xi_j \hat{e}^T Pbr - k_{2j}\sum_{k=1}^{m}\dot{\varphi}_k b_k\right] \quad (58)$$

in (56) yields

$$\dot{V}_2 = -\frac{1}{2}\hat{e}^T Q e - \hat{e}^T Phc^T \tilde{x} \quad (59)$$

Again, applying norm to (59) gives

$$\dot{V}_2 \le -\frac{1}{2}\lambda_{\min}\left(Q\right)\ \left|\hat{e}\right|^2 + \kappa_2\left|e\right|\left|\tilde{x}\right| \quad (60)$$

where $\kappa_2 = \lambda_{\max}(P)\left|h\right|$.

The stability for the observer-based approach is summarized by the following theorem.

Theorem 2: the feedback system composed of the nonlinear system (5), the reference model (7), the control input (13), the fuzzy model-based observer (31), and the update laws (57)-(58), is asymptotically stable and the observation and tracking errors converge to zero.

Proof: Under condition (54), $\dot{V}_1 \le 0$, which implies that $\tilde{x} \in L_\infty$. Since, the right side of (40) is bounded, then $\dot{\tilde{x}} \in L_\infty$, therefore $\tilde{x}$ is uniformly continuous. Integrating both sides of (55) yields

$$\int_0^\infty \left|\tilde{x}\right|^2 \le \frac{2}{\kappa}V_1(0) \quad (61)$$

which implies that $\tilde{x} \in L_2$, by Barbalat's lemma [18] it follows that $\lim_{t\to\infty}\tilde{x} = 0$. Hence, applying this result to

(60) yields that $\dot{V}_2 \le 0$ which implies V_2, $\hat{e}$, k_{1j} and $k_{2j} \in L_\infty$. Since all terms in the right side of (44) are bounded $\dot{\hat{e}} \in L_\infty$, with implies that $\hat{e}$ is uniformly continuous. Integrating both sides of (60) yields

$$\int_0^\infty |\hat{e}|^2 \le \frac{2}{\lambda_{\min}(Q)} V_2(0) \quad (62)$$

with implies that $\hat{e} \in L_2$. Then, using Barbalat's lemma [18] yields that $\lim_{t\to\infty} \hat{e} = 0$. Hence, the convergence of the tracking error $e = \tilde{x} - \hat{e}$ follows.

5. Simulation

The described approach is tested on the following nine rules fuzzy model

$$\begin{aligned} &\mathrm{R}_k: \text{If } x_1 \text{ is } Z_{1i} \text{ and } x_2 \text{ is } Z_{2j} \\ &\text{Then } \dot{x}_2 = a_{1k}x_1 + a_{2k}x_2 + b_k u, \quad k = 1..9 \end{aligned} \quad (63)$$

where Z_{1i} and Z_{2j} i, $j = 1..3$, are fuzzy sets defined by the membership functions on fig. 1(a). The disturbance term is $\eta = 0.5\sin(\pi t)$. The consequence parameters of the nine local linear models are given in Table I. It can be seen that, the rules 2, 5 and 8 are uncontrollable. The global fuzzy system (63) as unstable as it is shown by its autonomous response depicted in fig. 2.

The fuzzy model (63) is requested to track the reference model given by $a_m = [1 \quad 2]$ and $b_{nm} = 1$.

To achieve the control task, we use the following three rules fuzzy controller

$$\mathrm{R}_j: \text{If } x_1 \text{ is } V_j \text{ Then } u_f = k_{1j}\hat{x} + k_{2j}r, \quad j = 1..3 \quad (64)$$

where the fuzzy sets V_j are defined by the membership functions in fig. 1(b). To show the performance of the adaptive fuzzy controller alone, the switching term is not used. The state observer is defined as in (31), with $h^T = [10 \quad 25]$ and $l(s) = s + 4$. The fuzzy controller parameters are updated using (57)-(58) with $\gamma_1 = \gamma_2 = 4$.

For comparative purpose, the combined PDC-LMI approach [14-15] is investigated, using the following nine rules fuzzy controller

$$\begin{aligned} &\mathrm{R}_l: \text{If } x_1 \text{ is } Z_{1i} \text{ and } x_2 \text{ is } Z_{2j} \\ &\text{Then } u = -K_l x, \quad l = 1..9 \end{aligned} \quad (65)$$

To determine the control gains K_l and check the closed loop stability the following LMIs are to be verified:

Find $X > 0$ and $M_i, i = 1..9$ satisfying

$$-XA_i^T - A_iX + M_i^T b_i^T + b_i M_i > 0, i = 1..9$$

$$-XA_i^T - A_iX - XA_j^T - A_jX + M_j^T b_i^T + b_j M_i \ge 0$$

$$i < j, j = 1..9$$

where $X = P^{-1}$ and $M_i = K_i X^{-1}$.

Unfortunately, using the Matlab LMI toolbox the above LMIs were found unfeasible even when the uncontrollable rules are removed, which means that the fuzzy system (63) is not stabilisable using the PDC compensation approach.

The first test concerns the full state information approach. Figure 3.a shows the performance for the nominal case, i.e. when the fuzzy model represents perfectly the controlled system; figure 3.b shows the effect of the external disturbance, figure 3.c illustrates the effect of the parametric uncertainty, where deviations of $a_{ij} - 0.5a_{ij}$ and $b_i - 0.5b_i$ is introduced, and figure 3.d shows the combined effect of the disturbance and uncertainty. As can be seen from those results, the control performance is little affected by the parametric uncertainty effect. The external disturbance (with a magnitude 25% of the reference value) affects more the x_2 state, but the state x_1 achieves good regulation performance, which means that the fuzzy adaptive controller rejects partially the disturbance effect.

The second test concerns the observer-based approach. As in the first case, figure 3.a, b, c and d show the performance for the nominal case, the external disturbance effect, the parametric uncertainty, and the combined uncertainties effect. In those results one can remark the following: First, after a brief transient stage the estimated states follows the real one except for a small steady state error due to the external disturbance effect. Second, the tracking performance is not affected by the parametric uncertainty, which indicates that the adaptive control was effective in compensating for this kind of uncertainty. Third, the external disturbance affects more the control performance due to its negative effect on the observer performance.

Finally, the norms of the fuzzy parameters controller (note that $\mathrm{norm}(k_i) = \sqrt{\sum_{j=1}^{q} k_{ij}^2}, i = 1,2$) evolutions indicate that after a period of fast variations, due to the large initial tracking error and rapid variation of the membership functions, converge to practically constant values.

Table I. Fuzzy system Parameters

Fuzzy system rules	a_{i1}	a_{i2}	b_i
1	2	1	2
2	2	0	0
3	0.2	0.8	2.5
4	3	-1	1
5	0	2	0
6	0.7	0.2	5.3
7	-2	-3	4.4
8	0.1	1	0
9	-2	-4	6.8

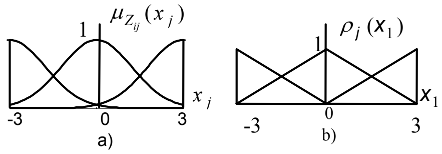


Figure 1. Membership functions: a) model, b) Controller.

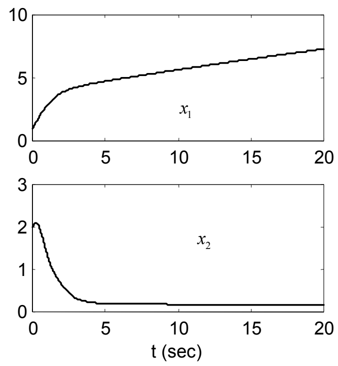


Figure 2. Fuzzy system autonomous response.

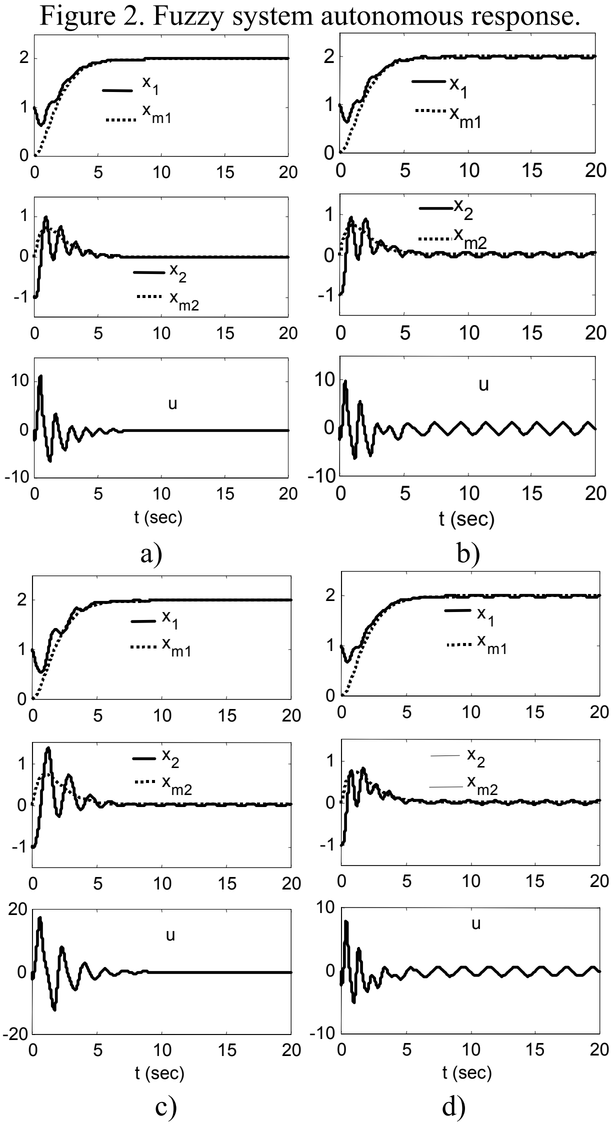
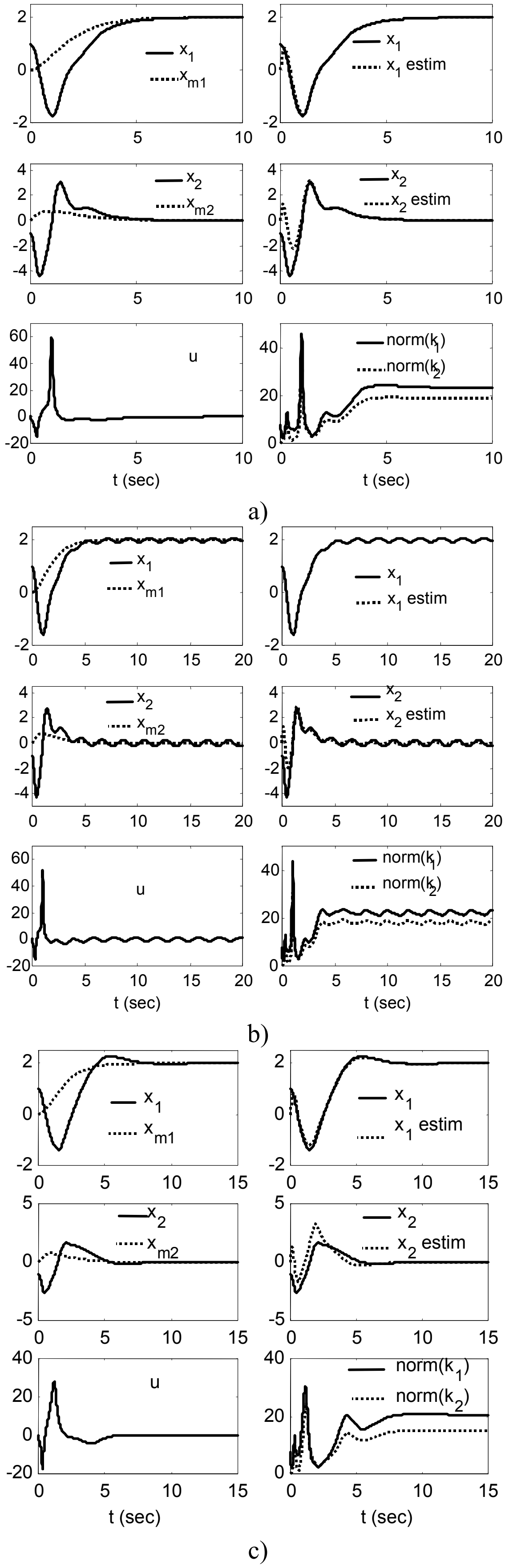
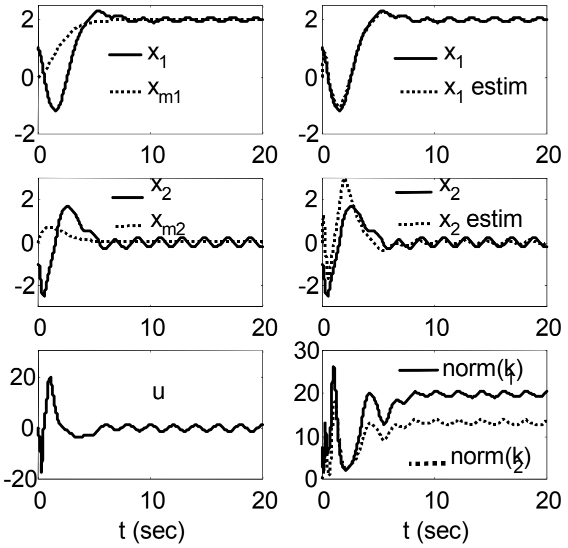


Figure 3. State feedback performance: a) nominal fuzzy model, b) disturbance effect, c) uncertainty effect, d) disturbance + uncertainty effect.





d)

Figure 4. Observer-based performance: a) nominal fuzzy model, b) disturbance effect, c) uncertainty effect, d) disturbance + uncertainty effects.

6. Conclusion

This paper has presented fuzzy model-based observer adaptive control of nonlinear systems modeled by fuzzy systems. Unlike PDC approach, local rules controllability is not necessary and the fuzzy controller is not required to share the same fuzzy sets with the fuzzy model. The stability and robustness of the proposed scheme are proved using Lyapunov tools. Simulation results have demonstrated the efficiency of the proposed approach, with small adaptive fuzzy controller, for unstable fuzzy system under only global controllability condition.

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